CERN **A**ccelerator **S**chool

"Superconductivity for Accelerators"

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Heat transfer and cooling techniques at low temperature

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Outline

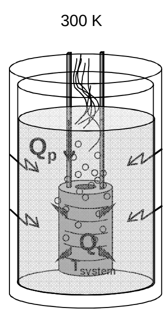
- Heat transfer at low temperature (Lecture 1)
  - Conduction
  - Radiation
  - Convection
- Cooling techniques at low temperature (Lecture 2)
  - Different classifications of system with respect to cooling
  - Different methods of cooling
  - Some examples

Heat transfer at low temperature (Lecture 1)

- Content
  - Review of different fundamental modes of heat transfer
  - Specificity to the low temperature domain
  - Some practical cases
  - Useful data and references
- Not covered by this lecture
  - Thermodynamics
  - Properties of materials
  - Superfluid helium heat transfer
  - Production of cryogens
- Present until the end of the school, do not hesitate to ask.

Cooling to low temperature (1/2)

- Primary goal : maintain a system at a temperature T≪ room temperature
  - Thermal stability in steady-state regime → T_{system} ≈ constant
  - Protecting your system against transient events →$T_{system}<T_{max}$
- System defined by thermophysical properties
  - Density (kg/m³), Heat capacity (J/kg.K), Thermal conductivity (W/m.K)…
- System subjected to permanent heat input (heat losses), $\mathbf{Q_p}$
  - Thermal radiation (room temperature to T_{system})
  - Conduction through supports, current leads, …
  - Internal dissipation (Joule effect, AC losses, beam losses…)
- System subjected to transient heat perturbation, $\mathbf{Q_t}$
  - Quench of a superconducting cavity or magnet
- Cooling power provided, $\mathbf{Q_R}$
- In the system design at low temperature conditions
  - Minimize heat input : Minimization of the heat transfer at constant ΔT
  - Maximize heat extraction : Minimization of ΔT at a constant heat transfer



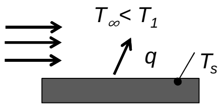
Cooling to low temperature (2/2)

- Three modes of heat transfer
  - Conduction: heat transferred in solid or fluid at rest

$$q = -k(T)\vec{\nabla}T$$

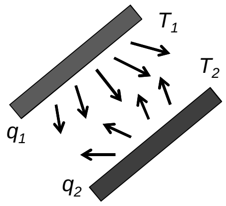
  - Convection: heat transferred by movement of fluid

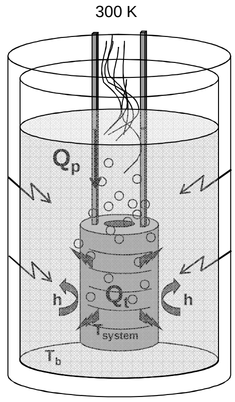
$$q = hA(T_s - T_\infty)$$



  - Radiation : Heat transferred by electromagnetic wave

$$q = \varepsilon.\sigma.(T_2^4 - T_1^4)$$





Outline| Conduction

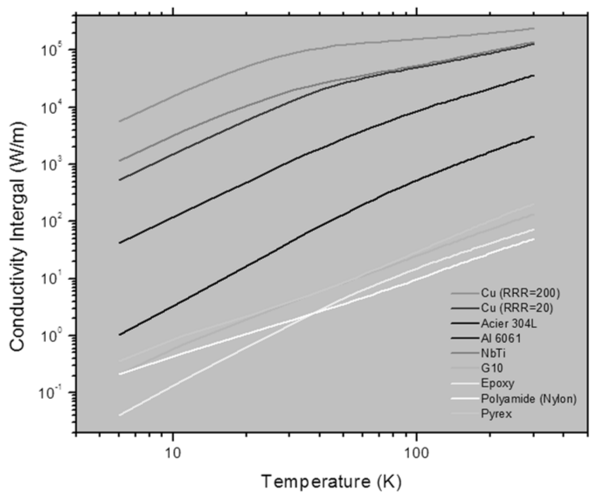
- Heat transfer at low temperature (Lecture 1)
  - Conduction
    - Fourrier's law
    - Thermal conductivity integral – case of a support
    - Thermal resistance
    - Thermal contact resistance
    - Transient heat conduction
    - Conduction in liquid
    - Conduction in gas
  - Radiation
  - Convection

Conduction | Fourier's Law

- Heat transfer without mass transfer in solid, liquid or fluid at rest
- For steady-state regime : Fourier's law $q = -k(T)\vec{\nabla} T$
  - Heat is flowing from the hot to the cold source.
- In 1D with constant geometry: $q = -k(T)\dfrac{dT}{dx} \Rightarrow \dfrac{Q}{A} = \dfrac{1}{L}\displaystyle\int_{T_{cold}}^{T_{hot}} k(T)\, dT$

- In 1D with non constant geometry: $q = -k(T)\dfrac{dT}{dx} \Rightarrow Q\displaystyle\int_{0}^{L} \dfrac{dx}{A} = \int_{T_{cold}}^{T_{hot}} k(T)\, dT$
- $\int k(T)\, dT$ is the integral conductivity. Very important since the thermal conductivity varies between room temperature and low temperature

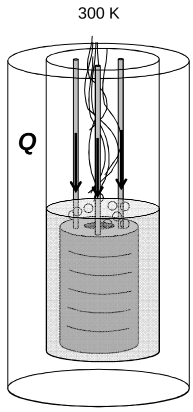
Conduction | Thermal conductivity integral



Cryocomp. Eckels Engineering. 3.06. Cryodata Inc. Florence SC, USA 29501

Conduction | Case of a support (1/2)

- Use of conductivity integral
  - Heat leak, temperature profile



- Heat input on the liquid helium bath cooling a magnet?
- If the magnet is suspended by three rods of 304 stainless steel from the 300 K top flange
  - Rods : Ø=10 mm and *L*=1 m

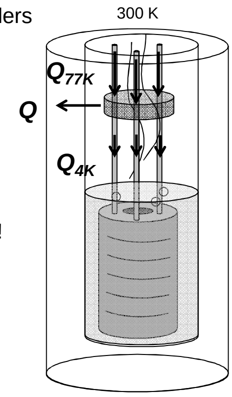
$$Q_{4K} = \frac{A}{L}\int_{4.2}^{300} k_{SS}(T)dT = \frac{2.36\,10^{-4}}{1}3.07\,10^{3} = 0.7\ \text{W}$$

  - It corresponds to a consumption of 1 l/h of liquid helium

- If the rods are made of
  - Copper (RRR=20) with a conductivity integral of 1.26 10^5 W/m, then $Q_{4K}\approx$20 W
  - G10 (Epoxy fiberglass tape) with an integral of 167 W/m, then $Q_{4K}\approx$26 mW

Conduction | Case of a support (2/2)

- To reduce the heat load on the helium bath
  - Heat interception with another cold source at an intermediate constant temperature (thermalization)
  - Boiling nitrogen or temperature regulated cold stage of cryocoolers



- If the interception is made with boiling nitrogen @ 77 K at 1/3 of the length from the top
  - $Q_{4K} = \frac{A}{L}\int_{4.2}^{77} k(T)dT = \frac{2.36\,10^{-4}}{0.75}325 = 0.1\ \text{W}$

  which corresponds to a consumption of liquid helium divided by 7!

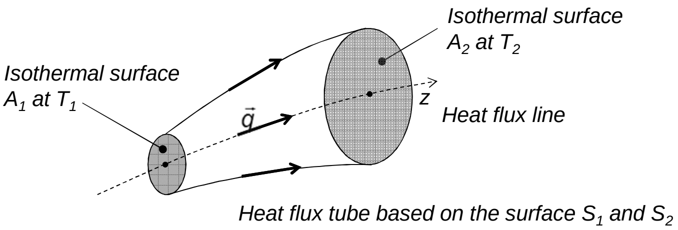
  - $Q_{77K} = \frac{A}{L}\int_{77}^{300} k(T)dT = \frac{2.36\,10^{-4}}{0.25}2.75\,10^{3} = 2.6\ \text{W}$

  which corresponds to a consumption of liquid nitrogen of 0.06 l/h

- Optimization depends on many parameters such as the thermalization temperature, the properties of the materials, the geometry…

Conduction | Thermal resistance (1/2)

- In the case of steady-state and without internal dissipation, a thermal resistance can be defined:



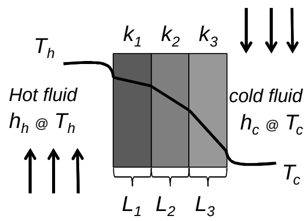
$$Q\int_{Z_1}^{Z_2}\frac{dz}{A(z)} = -\int_{T_1}^{T_2}k(T)\ dT = \bar{k}(T_1 - T_2) \Rightarrow R_{th} = \frac{T_1 - T_2}{Q} = \frac{1}{\bar{k}}\int_{Z_1}^{Z_2}\frac{dz}{A(z)} \quad \left[\frac{K}{W}\right]$$

- For a slab with constant section $R_{th} = \frac{L}{\bar{k}A}$, a cylinder $R_{th} = \frac{\ln(R_2/R_1)}{\bar{k}2\pi L}$
- For a convective boundary $R_{th} = \frac{1}{hA}$

Conduction | Thermal resistance (2/2)

- Case of a composite wall, R_{th} are in series so $R_{total} = \sum R_i$
- Case of a composite wall with heat transfer coefficients at boundaries

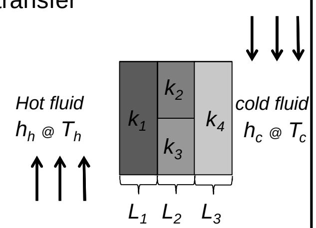
$$Q = \frac{T_h - T_c}{\sum_i R_i} = A\frac{T_h - T_c}{(1/h_h) + (L_1/\bar{k}_1) + (L_2/\bar{k}_2) + (L_3/\bar{k}_3) + (1/h_c)}$$



- In the case of parallel components
  - R_{th} are in parallel so $1/R_{total} = \sum 1/R_i$
- Case of a composite (series/parallel) wall with heat transfer coefficients at boundaries

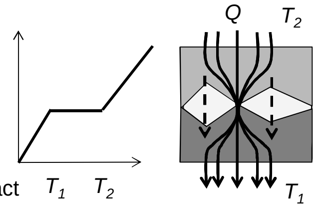
$$Q = A\frac{T_h - T_c}{(1/h_h) + (L_1/\bar{k}_1) + (2L_2/(\bar{k}_2 + \bar{k}_3)) + (L_4/\bar{k}_4) + (1/h_c)}$$

$$L_2 = L_3 \text{ and } A_2 = A_3 = \frac{A}{2}$$



Conduction | Contact resistance (1/2)

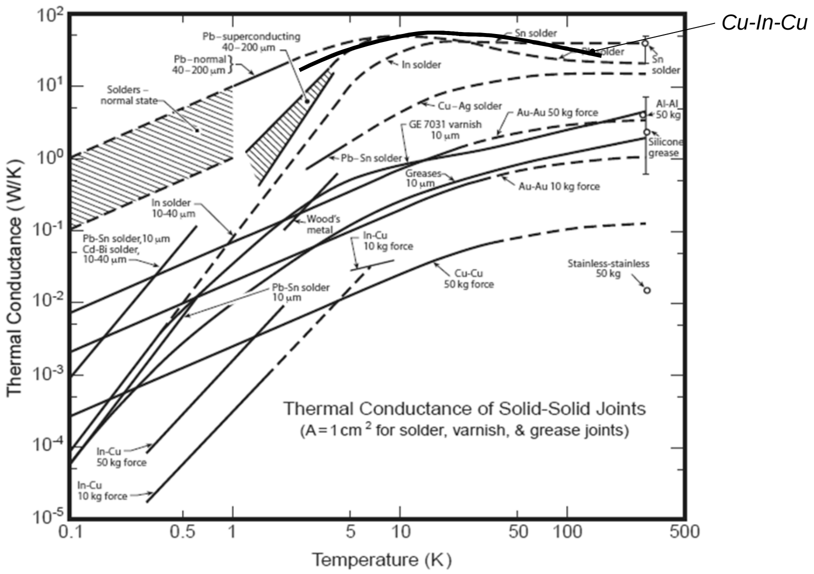
- Imperfect contact characterized by a temperature drop resulting from
  - Local contact creating constriction of the flux lines
  - Phonon scattering at the solid-solid contact (Kapitza resistance)
  - Heat transfer via interstitial elements
- Overall thermal resistance is defined $R_c = \frac{T_2 - T_1}{Q}$
- R_c depends on surface condition, nature of the materials, temperature, interstitial materials, compression force…
  - Proportional to force, not to pressure (number of contact points increases with force)
  - Reduces with increasing force
  - Increases by several orders of magnitude from 200 to 20 K



⚠ At low temperature R_c can be the largest thermal resistance

- Can be reduced by strong tightening or Inserting conductive and malleable fillers (charged grease, indium or coatings)
- Modeling is very difficult, the use of experimental data is recommended

Conduction | Contact resistance (2/2)



Ekin JW. Experimental Techniques for Low Temperature Measurements. Oxford: Oxford University Press; 2006.

Conduction | Transient – Time constant

- Energy conservation equation

$$\rho C \frac{\partial T}{\partial t} = \nabla.\left(-k(T)\vec{\nabla} T\right) + Q \quad \left[\frac{W}{m^3}\right]$$

Change of energy — Heat conduction — Volume heat generation

- In 1D with constant coefficient, one can identify thermal diffusivity:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C}\frac{\partial^2 T}{\partial x^2} + Q^* \Rightarrow D = \frac{k}{\rho C} \quad \left[\frac{m^2}{s}\right]$$

- And a time constant $\tau \approx \frac{4}{\pi^2}\frac{L^2}{D}$ ($T=0.63T_{final}$) for $T=0.95T_{final}$ then $t=3\tau$

Thermal diffusivity in cm^2/s

	300 K	77 K	4 K
Cu OFHC (RRR=150)	1.2	3.2	11700
Pur Al (RRR=800)	1	4.7	42000
Commercial Al (6061)	0.7	1.3	1200
SS 304 L	0.04	0.05	0.15

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Conduction in liquid

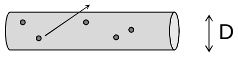
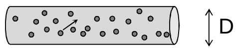
- As at room temperature, liquids are bad thermal conductor at low temperature
- Conductivity decreases with temperature
- Conduction in liquid is negligible compared to convection or phase change phenomena
- Except for superfluid helium, where the k_{eq}~1000 higher than high purity copper

Thermal conductivity of some cryogens at atmospheric pressure (W/m.K)

O_2 (T=90 K)	N_2 (T=77 K)	H_2 (T=20 K)	He (T=4.2 K)
0.152	0.14	0.072	0.019

R.F. Barron, Cryogenic Heat transfer, Taylor&Francis, 1999
V. C. Johnson, *A Compendium of the Properties of Materials at Low Temperatures*, Wadd Tech. Rep. 60-56, 1960

Conduction in gas (1/3)

- Two regimes depending on the ratio of the mean free path of the molecule (ℓ) and the distance between the two surfaces (D) involved in the heat transfer
  - $\ell \gg D$ **Free molecular regime** 
  - $\ell \ll D$ **Hydrodynamic regime** 
- The mean free path for ideal gas $\lambda = \dfrac{RT}{\sqrt{2}\pi d^2 N_A p}$ $\begin{cases} R = \text{universal gas constant} = 8.31\ \text{J / mol.K} \\ N_A = \text{Avogadro's number} = 6.02\ 10^{23}\ /\text{mol} \\ d = \text{Molecule diameter} \end{cases}$
- At constant temperature for a material,
  - The **free molecular regime** is obtained for the low residual pressure
  - Heat transfer depends on the residual gas pressure and independent of D

  - The **hydrodynamic regime** is obtained for high residual gas pressure
  - Heat transfer is independent of pressure and described by a Fourier law

Conduction in gas (2/3)

- Free molecular regime : **Kennard's law**

$$Q = A\alpha\left(\frac{\gamma+1}{\gamma-1}\right)\sqrt{\frac{R}{8\pi M}}\frac{\Delta T}{\sqrt{T}}p \quad \text{with} \quad \gamma = \frac{C_p}{C_v}$$

- α is the accommodation coefficient which relates the degree of thermal equilibrium between the gas and the wall
- Prediction for helium $\alpha \leq 0.5$, argon $\alpha \sim 0.78$ and nitrogen $\alpha \sim 0.78$

- Hydrodynamic regime : **Kinetic theory** $q = -k\vec{\nabla}T$

$$k = \frac{1}{3}\ell v C \qquad \{v : \text{mean velocity of the molecules}$$

Thermal conductivity k [mWm^{-1} K^{-1}] @ 1 atm

T [K]	^4He	H_2	N_2
300	150.7	176.9	25.8
75*	62.4	51.6	7.23
20	25.9	15.7	
5	9.7		

*T=77.36 for Nitrogen

Cryogenic Heat transfer, R.F. Barron, Taylor&Francis, 1999

V. C. Johnson, *A Compendium of the Properties of Materials at Low Temperatures*, Wadd Tech. Rep. 60-56, 1960)

Conduction in gas (3/3)

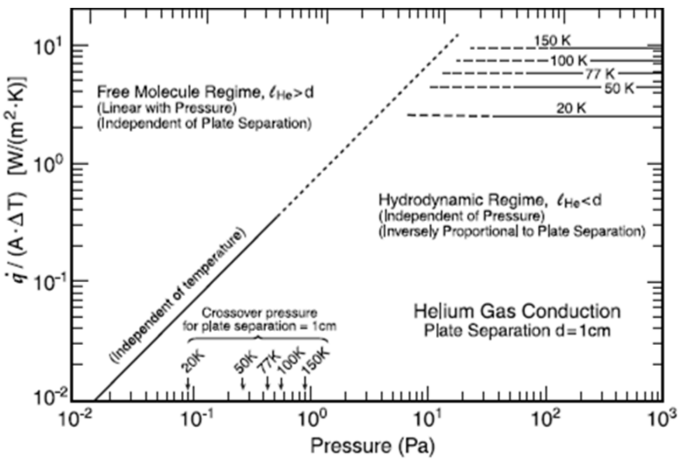


FIG. 2.2 Heat conduction through helium gas between two parallel copper plates spaced 1 cm apart as a function of pressure at various gas temperatures. The heat conduction shows a linear dependence on gas pressure below about 1 Pa and no dependence at higher pressure. The crossover pressure is inversely proportional to the plate separation *d* (determined by setting *l* = *d* in Eq. 2.5).

From *Experimental Techniques in Low Temperature Measurements*, Jack W. Ekin, Oxford Univ. Press (2006, 2007)

Outline | Radiation

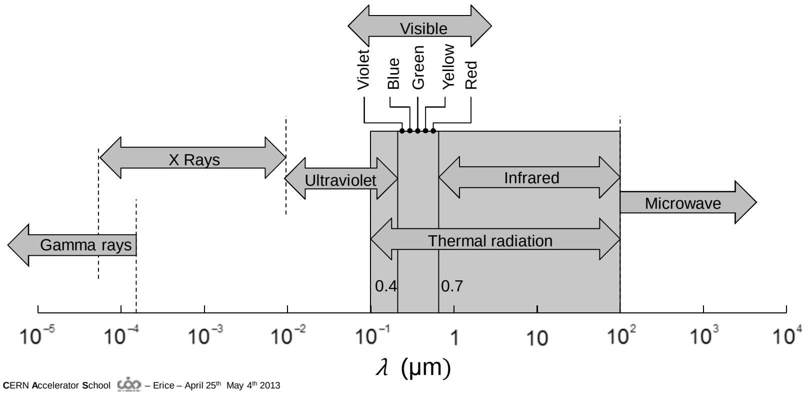
- Heat transfer at low temperature (Lecture 1)
  - Conduction
  - Radiation
    - Introduction
    - Blackbody radiation
    - Surface emission
    - Emissivity
    - Radiation exchange between two surfaces
    - Shielding
    - Multi-layer insulation
  - Convection

Radiation | Introduction (1/2)

- Heat transfer by electromagnetic waves
- Radiated energy propagates through a medium with wave length λ

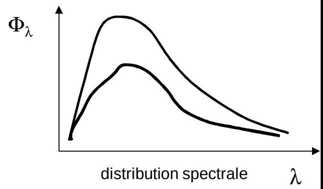
$$\lambda = \frac{c}{\nu} \quad \begin{cases} c = \text{speed of light in the medium (m/s)} \\ \nu = \text{frequency } (s^{-1}) \end{cases}$$

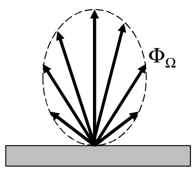
- Wave length associated with thermal radiation: 0.1 μm to 100 μm



Radiation | Introduction (2/2)

- Heat transfer depends on the wave length, λ
  - Emitted radiation consists of a continuous non uniform distribution of monochromatic components
  - Spectral distribution and the magnitude depend on the nature and the temperature of the emitting surface





Directional distribution

- Heat transfer depends on the direction
  - Directional distribution of the emitted radiation

- To characterize radiation heat transfer both spectral and directional dependence (as a function of temperature and surface) must to be known

Radiation | Blackbody radiation

- A perfect emitter and an absorber
  - Absorbs all incident radiation regardless of the wave length and direction
  - At a prescribed temperature and wave length, emission is maximum
  - Diffuse emitter (no directional dependence)
- Emissive power (Emittance) : Planck distribution

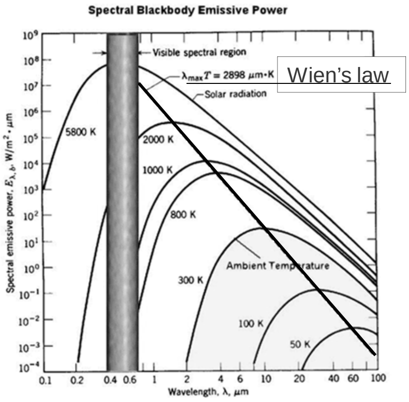
$$E_\lambda^0 = \frac{C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)}$$

$$\begin{cases} C_1 = 2\pi h C_0^{\,2} = 3.74210^{-16} \ \text{Wm}^4/\text{m}^2 \\ C_2 = hC_0 / k = 1.438810^{-16} \ \text{Wm}^4/\text{m}^2 \end{cases}$$

- Stefan-Boltzmann Law

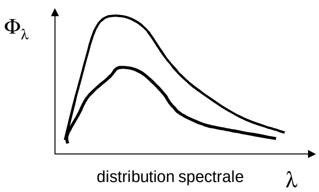
$$E^0 = \int_0^\infty \frac{C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)} d\lambda = \sigma T^4$$

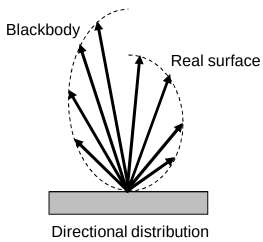
with $\sigma = 5.67\ 10^{-8}\ \text{W/m}^2\text{K}^4$



Radiation | Surface emission

- From a perfect emitter to a real surface
  - Emissivity is the ratio of the real surface to the blackbody radiation intensity
  - A spectral, monochromatic directional emissivity can be defined as ε(λ, θ, φ, T)
  - A spectral emissivity as ε(λ, T)
  - A total emissivity as ε(T)





- Special case (approximation)
  - Grey body : ε(θ, φ, T) independent of λ
  - Diffuse body : ε(λ, T) independent of direction
- Real emissivity depends on the direction and wavelength

Radiation | Emissivity (1/2)

- Emissivity decreases with temperature
- Emissivity increases with oxidation, impurities, dirt

- To achieve the lowest emissivity value
  - Highly polished surface
  - High conductivity surfaces (gold, silver copper or aluminum)

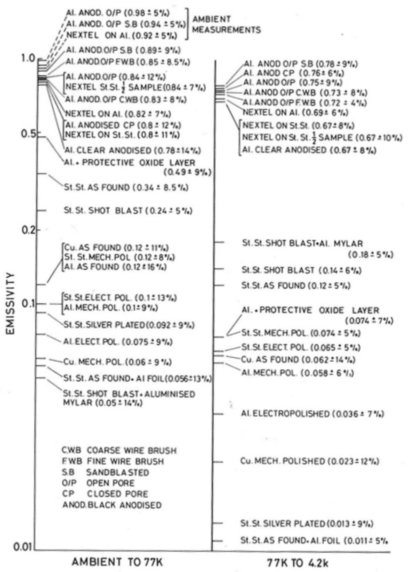
- Many data can be found in the literature

Total emissivity of various metal

	300 K	78 K	4,2 K
3M Black paint (80 µm) on copper surface	0,94	0,91	0,89
Polished A**luminum** (33 µm in rough.)	0,05	0,23	0,018
Polished **Copper** (41 µm in rough.)	0,10	0,07	0,05
304 Polished **Stainless steel** (27 µm in rough.)	0,17	0,13	0,08

K H Hawks & W Cottingham: Total Normal Emittances of Some Real Surfaces at Cryogenic Temperatures, Advances In Cryogenic Engineering, Vol 16, 1970, pp 467-474.

Radiation | Emissivity (2/2)



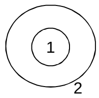
Obert W. Emissivity measurements of metallic surfaces used in cryogenic applications, Adv. Cryo. Eng. 27, Plenum Press 1982 p. 293-300

Radiation | Radiation exchange between two surfaces

- Fraction of the radiation leaving surface i and intercepting surface j

View factor F_{ij}

$F_{11}=0$; $F_{12}=1$



$F_{22}=1-A_1/A_2$; $F_{21}=A_1/A_2$ $\quad q_{12}=\varepsilon_1\sigma A_1(T_1^4-T_2^4)$

–Reciprocity relation $A_iF_{ij}=A_jF_{ji}$

- Heat exchange between diffuse grey two-surface enclosure

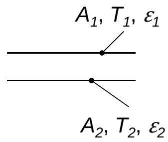
$$q_{12}=\frac{\sigma(T_1^4-T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1A_1}+\frac{1}{A_1F_{12}}+\frac{1-\varepsilon_2}{\varepsilon_2A_2}}$$

Large parallel plates

$A=A_1=A_2$

$F_{12}=1$

$$q_{12}=\frac{\sigma A_1(T_1^4-T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1}$$

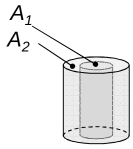


Long concentric cylinders

$\frac{A_1}{A_2}=\frac{r_1}{r_2}$

$F_{12}=1$

$$q_{12}=\frac{\sigma A_1(T_1^4-T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)}$$



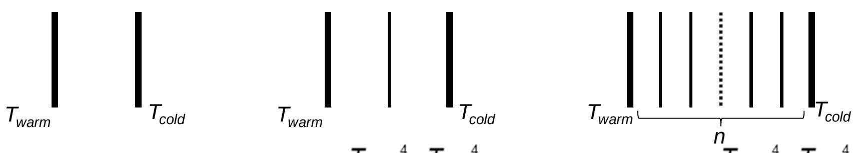
Radiation | Shielding at low temperature

- **Blackbody** heat transfer from room temperature
  - From 300 K to 77 K : q=457 W/m²
  - From 300 K to 4.2 K : q=459 W/m²

$$q=\sigma(T_{warm}^4-T_{cold}^4)$$

- **Blackbody** heat transfer from Nitrogen temperature
  - From 77 K to 4.2 K : q=2 W/m² (q ~200 times lower than 300 K)
- To reduce heat load at low temperature : intermediate surface at intermediate temperature

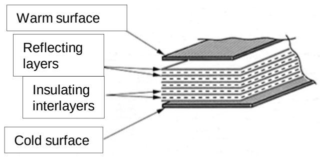
$$q=\frac{\varepsilon\sigma}{2-\varepsilon}(T_{warm}^4-T_{cold}^4) \qquad q=\frac{1}{2}\frac{\sigma\varepsilon}{2-\varepsilon}(T_{warm}^4-T_{cold}^4) \qquad q=\frac{1}{n+1}\frac{\sigma\varepsilon}{2-\varepsilon}(T_{warm}^4-T_{cold}^4)$$



$$T^4=\frac{T_{warm}^4-T_{cold}^4}{2} \qquad T_i^4=T_{cold}^4+\frac{T_{warm}^4-T_{cold}^4}{i+1}$$

Radiation | Multi-layer insulation (1/2)

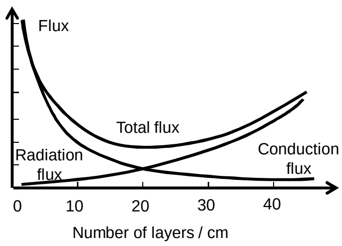
- **MLI** or **Superinsulation**
  - Reflecting layers to reduce heat transfer by radiation
  - Insulating interlayer to reduce heat transfer between reflecting layers
  - High vacuum to reduce convection and residual gas conduction
- **MLI** materials
  - Reflecting layers: mostly aluminum metallized Mylar films (both sides)
    - Thermal conductivity anisotropy
  - Insulating interlayer : mostly net of polyester or fiber glass, paper silk



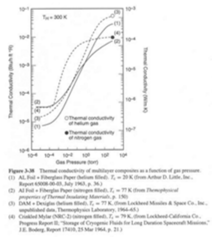
- Heat transfer parallel to the layers is several order of magnitude higher than normal to layers due to the pure aluminum
- Bad vacuum than residual conduction becomes important
- Low temperature boundary (77 K to 4 K)
  - Radiation negligible, heat transfer dominated by conduction
- High temperature boundary (300 K to 80 K)
  - Heat transfer dominated by radiation : MLI efficient

Radiation | Multi-layer insulation (2/2)

- Typical value of heat transfer for 20 layers
  - 1 to 3 W/m² from 300 K to 80 K (5 W/m² if compressed)
  - Lower than 100 mW/m² between 80 K and 4 K
- To optimize the use of MLI
  - Number of layers/cm max : 20-30
  - Isothermal contact points



  - No gaps to have uniform heat transfer
  - No mechanical stress → contact point increases the conduction
  - Perforated MLI to have low residual pressure

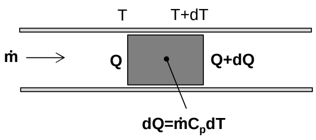


Outline | Convection

- Heat transfer at low temperature (Lecture 1)
  - Conduction
  - Radiation
  - Convection
    - Introduction to single phase convection
    - Natural convection
    - Forced convection
    - Introduction to boiling
    - Boiling heat transfer
    - Two-phase convection

Convection | Introduction to single phase flow (1/4)

- Heat is transferred in the fluid by the movement of matter
  - Quantity of energy is advected within the fluid



- The movement of matter can be created externally by a pump or a pressurization system: **forced convection**
- Equations for convection in the Boussinesq approximation (Steady-State)

Continuity $\nabla.\mathbf{v} = 0$ → $\nabla.\mathbf{v}^* = 0$

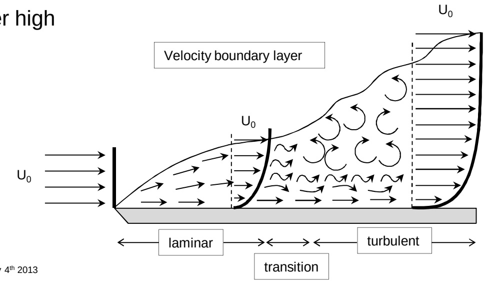
Navier-Stokes $\rho\mathbf{v}.\nabla\mathbf{v} = -\nabla p + \mu\nabla^2\mathbf{v} + \mathbf{f}$ —Dimensionless→ $\mathbf{v}^*.\nabla\mathbf{v}^* = -\nabla p^* + \mathrm{Re}^{-1}\nabla^2\mathbf{v}^*$

Energy $\rho C\mathbf{v}.\nabla T = k\nabla^2 T + Q$ → $\mathbf{v}^*.\nabla T^* = \mathrm{Re}^{-1}\mathrm{Pr}^{-1}\nabla^2 T^*$

- Reynolds number $\mathrm{Re} = \frac{\rho LU}{\mu} = \frac{\text{inertia forces}}{\text{viscous forces}}$ Nature of the flow
- Prandtl number $\mathrm{Pr} = \frac{\mu C}{k} = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}}$ Thermophysical properties of the fluid

Convection | Introduction to single phase flow (2/4)

- Laminar and turbulent regimes
  - Essential to know in which regime the flow is since the surface **heat transfer** and **friction** depend strongly on it
- Laminar regime for Re<2300
  - Viscous forces dominate, flow motion ordered (streamline)
  - Surface heat transfer low
  - Surface friction low
- Turbulent regime for Re>5 10^5
  - Inertia forces dominate, flow motion highly irregular (velocity fluctuation)
  - Surface heat transfer high
  - Surface friction high



Convection | Introduction to single phase flow (3/4)

- The fluid movement can be created internally by a decrease or increase of the fluid density or the buoyancy effect: **natural convection**
- Equations for convection in the Boussinesq approximation (Steady-State)

		Dimensionless →
Continuity	$\nabla.\mathbf{v} = 0$	$\nabla.\mathbf{v}^* = 0$
Navier-Stokes	$\rho\mathbf{v}.\nabla\mathbf{v} = -\nabla p + \mu\nabla^2\mathbf{v} + \beta\Delta T\mathbf{g}$	$\mathbf{v}^*.\nabla\mathbf{v}^* = -\nabla p^* + \mathrm{Re}^{-1}\nabla^2\mathbf{v}^* + \mathrm{Gr}\,\mathrm{Re}^{-2}\,T^*$
Energy	$\rho C\mathbf{v}.\nabla T = k\nabla^2 T$	$\mathbf{v}^*.\nabla T^* = \mathrm{Re}^{-1}\mathrm{Pr}^{-1}\nabla^2 T^*$

–Grashof number $\mathrm{Gr} = \dfrac{g\beta\Delta TL^2}{\mu^2} = \dfrac{\text{buoyancy forces}}{\text{viscous forces}}$

–When $\mathrm{GrRe}^{-2} \gg 1$, then forced convection negligible

–If $\mathrm{GrRe}^{-2} \approx 1$, then mixed convection

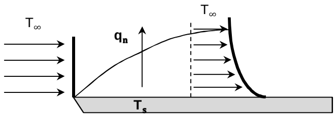
–Gr has the same role for natural convection as Re for forced convection

–Turbulence has a strong effect as in forced convection and reached for

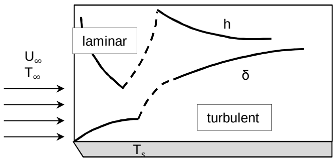
$$\mathrm{Gr.Pr} \geq 10^5$$

Convection | Introduction to single phase flow (4/4)

- Heat is transferred to solid elements
  - Quantity of energy transfer in or out of the fluid to the solid
  - Newton's law $q=h(T_s-T_\infty)$
- At the boundary, the local heat flux is $q_n=-k.\nabla T_n$



- Dimensionless, it is the Nusselt number $Nu=\frac{hL}{k}=\left.\frac{\partial T^*}{\partial n^*}\right|_0$
- The Nusselt number is to the thermal boundary as the friction coefficient is to the velocity boundary
- Nu=f(Re, Pr, L) for forced convection and Nu=f(Gr, Pr, L) for natural convection
- Nu different for turbulent or laminar (different correlation)



Convection | Natural convection heat transfer

- Heat flux is computed with correlation Nu=(**Gr**, Pr, L)
  - Thermophysical properties are established at average temperature $T=(T_w+T_\infty)/2$
    - T_w = solid temperature; T_∞ temperature of the fluid
- The simplest correlations are $Nu=c(Gr.Pr)^n$
  - For laminar regime n=1/4
  - Turbulent regime =1/3
- Few data exit for cryogenic fluid since two-phase phenomena take over
- Results not very different from classic fluids

	c	n
Supercritical helium Vertical orientation Turbulent	0.615	0,258
Liquid Nitrogen different orientations Turbulent	0,14	1/3
Liquid Hydrogen Different configurations turbulent	0,096	0,352

Hilal MA, Boom RW. An experimental investigation of free convection heat transfer in supercritical helium. Int. J Mass Trans. 1980; **23** 697-705.

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Convection | Forced convection heat transfer

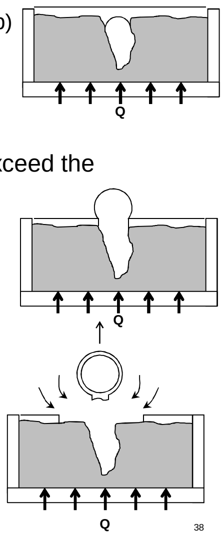
- Heat flux is computed with correlation Nu=(**Re**, Pr, L)
  - Thermophysical properties are established at average temperature $T=(T_s+T_\infty)/2$
- Correlation used for non cryogenic fluid works at low temperature
- Turbulent flow in pipes : The Dittus-Boetler correlation $Nu=0.023Re^{0.8}Pr^{0.4}$
  - Hydrogen : $Nu=0.023Re^{0.8}Pr^{0.4}$ — Tatsumoto H, et al. Forced Convection Heat Transfer of Liquid Hydrogen Through a 200-mm Long Heated Tube. Physics Procedia. 2012;36(0):1360-5.
  - Supercritical helium : $Nu=0.022Re^{0.8}Pr^{0.4}$ — Giarratano PJ, et al. Forced convection heat transfer to subcritical helium I. Adv. Cryo. Eng. **19**, Plenum Press; 1974. p. 404-16.
  - Nitrogen : $Nu=0.027Re^{0.8}Pr^{0.14/3}\ (\mu_f/\mu_w)^{0,14}$ — Ohira K, et al. Pressure-drop reduction and heat-transfer deterioration of slush nitrogen in horizontal pipe flow. Cryogenics. 2011;51(10):563-74
- Laminar flow in pipes : very rare, excepted in porous media

Convection | Introduction to boiling

- Heat is transferred between a surface and the fluid by the conjunction of phase change and the vapor bubble movement in the vicinity of the surface
  - At the heated surface the fluid must be superheated Tf>Tsat(p)
  - Imperfections in the surface where bubbles can form
- For bubbles to be stabilized, the pressure inside must exceed the saturation pressure to overcome the surface tension, *σ*

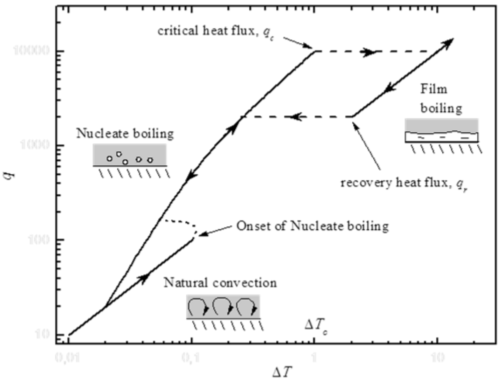
$$p_v - p_{sat} = \frac{2}{r}\sigma$$

- Heat transfer combines natural convection in the liquid, latent heat due to the bubble formation and the bubble hydrodynamics
  - Depends on the bubble growth rate, detachment frequency, number of nucleation sites, surface conditions…



Convection | Pool boiling – boiling curve

- Horizontal flat surface with liquid near saturation



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Shirai Y, et al. Boiling heat transfer from a horizontal flat plate in a pool of liquid hydrogen. Cryogenics. 2010;**50(6–7)**:410-6.

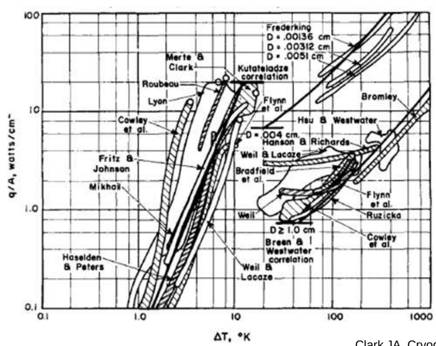
	ΔT_c (K)	q_c (kW/m²)	q_r (kW/m²)
Helium	1	10	
Nitrogen	10	100	
Hydrogen	5	100	10

Convection | Pool boiling heat transfer (1/2)

- Heat transfer in nucleate boiling : Kutateladze correlation q=*f*(p).ΔT$^{2.5}$

$$\frac{h}{k_l}\left(\frac{\sigma}{g\rho_l}\right)^{1/2} = 3.25\ 10^{-4}\left[\frac{qC_{pl}\rho_l}{h_{lv}\rho_v k_l}\left(\frac{\sigma}{g\rho_l}\right)^{1/2}\right]^{0.6}\left[g\left(\frac{\rho_l}{\mu_l}\right)^2\left(\frac{\sigma}{g\rho_l}\right)^{3/2}\right]^{0.125}\left(\frac{p}{\left(\sigma g\rho_l\right)^{1/2}}\right)^{3/2}$$

- Depends on the orientation, fluid, pressure, surface state, …
- Works for most cryogenic fluids within one order of magnitude



Pool boiling for Nitrogen at 1 bar

Clark JA. Cryogenic heat transfer. In: Advances in Heat Transfer. New York: Academic Press; 1969. p. 325-517.

Convection | Pool boiling heat transfer (2/2)

- Critical heat flux : Correlation of Kutateladze

$$q_c = 0.16\ h_{lv}\ \rho_v^{1/2}\left[\sigma g\left(\rho_l - \rho_v\right)\right]^{1/4}$$

  - Works for helium, nitrogen, oxygen and hydrogen

Shirai Y, et al. Boiling heat transfer from a horizontal flat plate in a pool of liquid hydrogen. Cryogenics. 2010;50(6–7):410-6.
Lyon DN. Boiling heat transfer and peak nucleate boiling fluxes in saturated liquid helium between the I and critical temperatures. 10, 1964. p. 371-9.

  - Not valid when the fluid is sub-cooled *i.e.* pressure above the heated surface is higher than saturated pressure

$$\frac{q_{c,sub}}{q_{c,sat}} = 0.2\left[1 + 0.15\left(\frac{\rho_l}{\rho_v}\right)^{3/4}\frac{C_p \Delta T_{sub}}{h_{lv}}\sigma\right]$$

Kirichenko YA, et al. Heat transfer in subcooled liquid cryogens. Cryogenics. 1983;23(4):209-11.

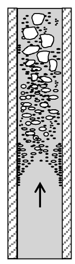
- Film boiling : An order of magnitude lower heat transfer coefficient

Convection | Two-phase flow heat transfer

- Two-phase forced flow heat transfer
  - Modeling must take into account the boiling heat transfer depending on the surface heat transfer, and the forced convection depending on the vapor quality ($x=\dot{m}_v/\dot{m}_t$) and the mass flow rate ($\dot{m}_t$)
  - Boiling tends to be dominant for low quality and high heat flux
  - Forced convection tends to be dominant for large vapor quality and mass flow rate

  - Several general correlations and specific to cryogenic fluid exit
  - Better to try more than one to evaluate the heat transfer rate

  - Superposition method (Chen) $h_{TP} = h_{nb} + h_l$
  - Intensification model (Shah) $h_{TP} = E.h_l$
  - Asymptotic model (Liu et Winterton n=2) $h_{TP} = \left[\left(F_{nb}h_{nb}\right)^n + \left(F_l h_l\right)^n\right]^{1/n}$



- The Steiner-Taborek (n=3) correlation is considered as the most accurate including the cryogenic fluids (helium, hydrogen, nitrogen, oxygen, …)

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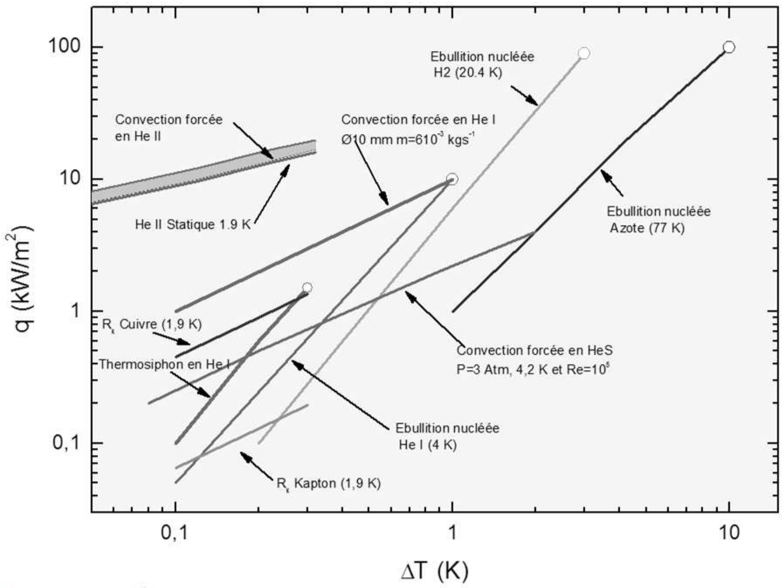
Cooling modes | Comparison



Lecture 1 | References & Acknowledgement

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  - Proceedings of the International Cryogenic Engineering Conference (Europe/Asia)
- **Data bases**
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  - Hepak, Gaspak, MetalPak, Cryodata inc.
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