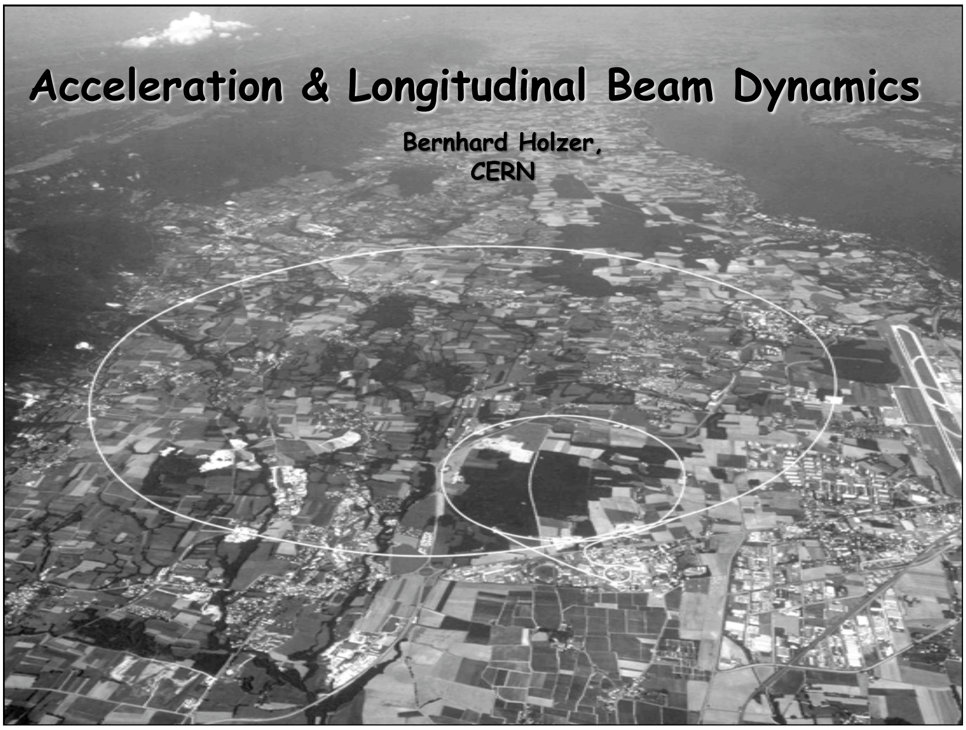

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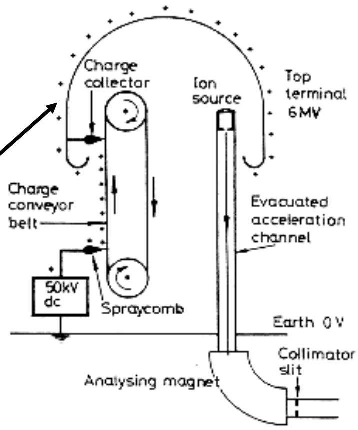
1.) Electrostatic Machines:
(Tandem –) van de Graaff Accelerator (1930 ...)

creating high voltages by mechanical transport of charges



* Terminal Potential: U ≈ 12 ...28 MV
  using high pressure gas to suppress discharge (SF_6)

Problems: * Particle energy limited by high voltage discharges
* high voltage can only be applied once per particle ...
... or twice ?

Energy Gain

... we have to start again from the basics

Lorentz force $\vec{F} = q*(\vec{E} + \vec{v} \times \vec{B})$ — in long. direction the B-field creates no force ($v \parallel B$)

$$\vec{F} = \frac{d\vec{p}}{dt} = e\vec{E}$$

acc. force is given by the electr. Field

In relativistic dynamics, energy and momentum satisfy the relation:

$$E^2 = E_0^2 + p^2c^2 \qquad (E = E_0 + W)$$

Hence:

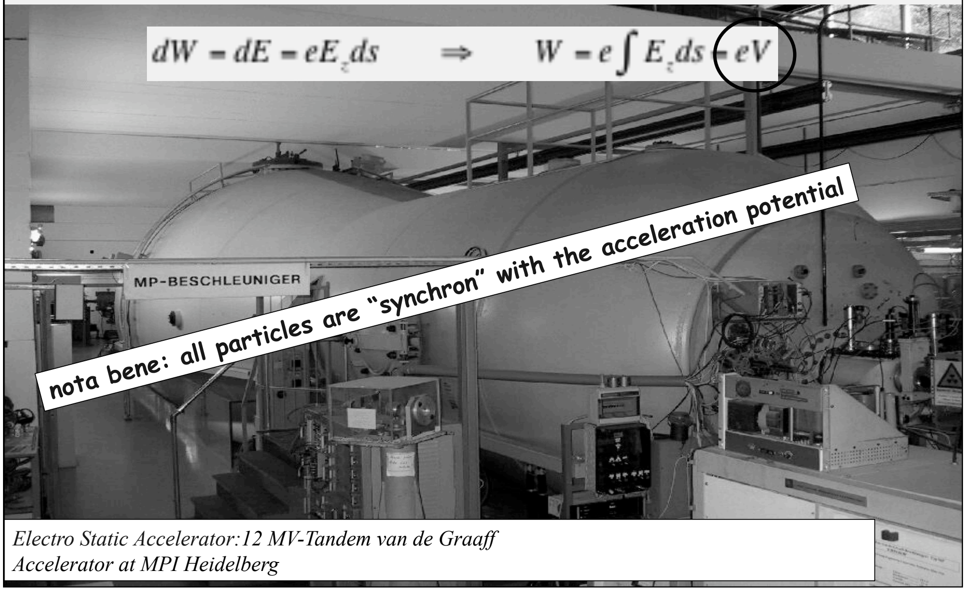
$$dE = \int F ds = v dp$$

and the kinetic energy gained from the field along the z path is:

$$dW = dE = eE_z ds \quad \Rightarrow \quad W = e\int E_z ds = eV$$

The „Tandem principle": Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

$$dW = dE = eE_z ds \quad \Rightarrow \quad W = e\int E_z ds = eV$$

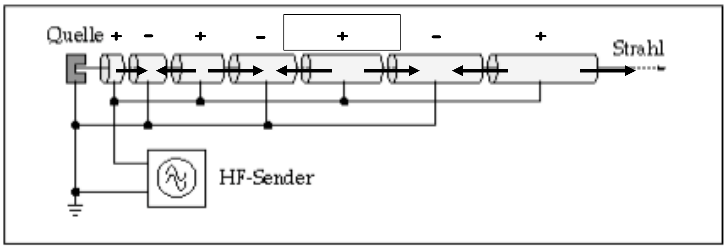


Electro Static Accelerator:12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

2.) The first RF-Accelerator: „Linac"

1928, Wideroe: how can the acceleration voltage be applied several times to the particle beam

schematic Layout:



Energy gained after n acceleration gaps

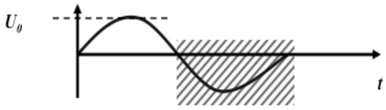
$$E_n = n * q * U_0 * \sin\psi_s$$

n number of gaps between the drift tubes
q charge of the particle
$\mathbf{U_0}$ Peak voltage of the RF System
$\boldsymbol{\Psi_S}$ *synchronous phase of the particle*

*** the problem of synchronisation ... between the particles and the rf voltage**
*** „voltage has to be flipped" to get the right sign in the second gap**
→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



Time span of the negative half wave: $\tau_{RF}/2$

Length of the Drift Tube: $l_i = v_i * \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_i = \frac{1}{2} m v^2$

$\rightarrow v_i = \sqrt{2E_i/m}$

$l_i = \frac{1}{v_{rf}} * \sqrt{\frac{i * q * U_{0*\sin\psi_s}}{2m}}$

valid for non relativistic particles ...

Alvarez-Structure: 1946, surround the whole structure by a rf vessel

Energy: ≈ 20 MeV per Nucleon β ≈ 0.04 … 0.6, Particles: Protons/Ions

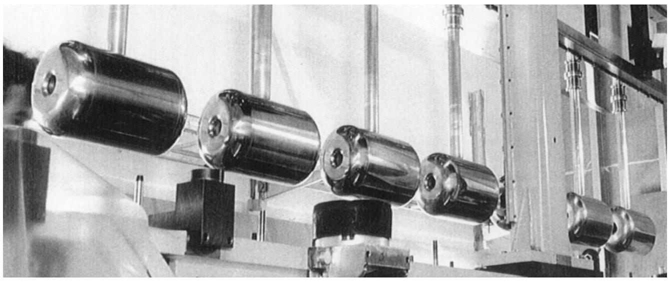
Example: DESY *Accelerating structure of the Proton Linac*

$E_{total} = 988\ MeV$

$m_0c^2 = 938\ MeV$

$p = 310\ MeV\ /\ c$

$E_{kin} = 50\ MeV$



Beam energies

1.) reminder of some relativistic formula

rest energy $E_0 = m_0c^2$

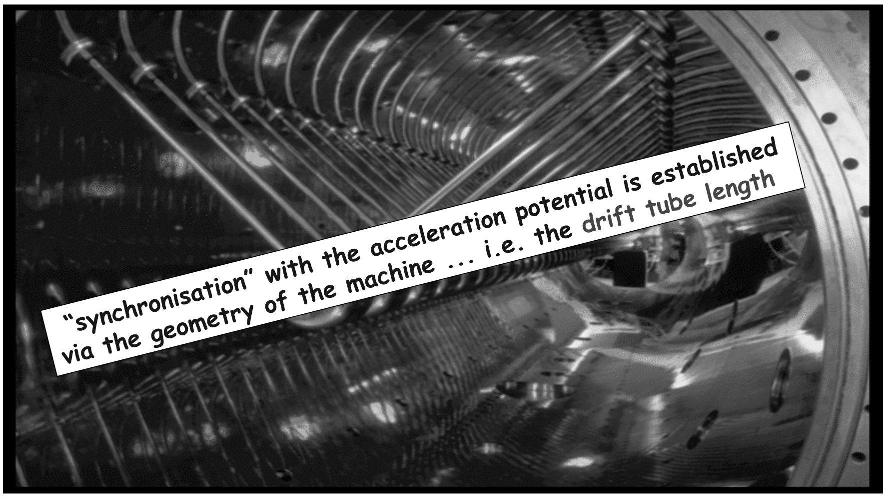
total energy $E = \gamma * E_0 = \gamma * m_0c^2$

momentum $E^2 = c^2p^2 + m_0^2c^4$

kinetic energy $E_{kin} = E_{total} - m_0c^2$

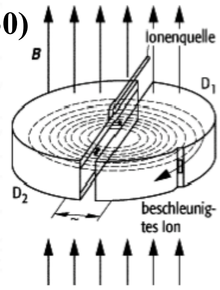
GSI: Unilac, typical Energie ≈ 20 MeV per Nukleon, $\beta \approx 0.04 \ldots 0.6$, Protons/Ions, ν = 110 MHz

Energy Gain per „Gap“: $W = q\, U_0 \sin \omega_{RF} t$



Application: until today THE standard proton / ion pre-accelerator
CERN Linac 4 is being built at the moment

3.) The Cyclotron: (Livingston / Lawrence ~1930)



Idea: B = const, RF = const
Synchronisation particle / RF via orbit

Lorentzforce

$$\vec{F} = q * (\vec{v} \times \vec{B}) = q * v * B$$

circular orbit

$$q * v * B = \frac{m * v^2}{R} \quad \rightarrow \quad B * R = p/q$$

increasing radius for increasing momentum
➔ Spiral Trajectory

revolution frequency

$$\omega_z = \frac{v}{R} = \frac{q}{m} * B_z$$

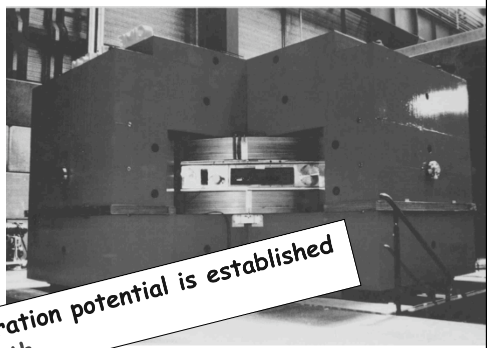
the cyclotron (rf-) frequency is independent of the momentum

rf-frequency = h* revolution frequency, h = "harmonic number"

Cyclotron:

exact equation for revolution frequency:

$$\omega_z = \frac{v}{R} = \frac{q}{\gamma * m} * B_z$$



Cyclotron SPIRAL at GANIL

1.) if v << c ⇒ $\gamma \cong 1$

2.) γ increases with the energy
⇒ no exact synchroni...

"synchronisation" with the acceleration potential is established via the spiraling orbit length

Sy... ...on

B = constant
$\gamma\ \omega_{RF}$ = constant
ω_{RF} decreases with time

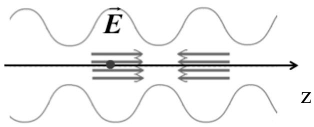
$$\omega_s(t) = \omega_{rf}(t) = \frac{q}{\gamma(t) * m_0} * B$$

keep the synchronisation condition by varying the rf frequency

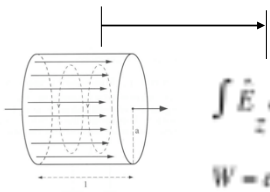
4.) RF Cavities, Acceleration and Energy Gain

$$dW = dE = eE_z ds \quad \Rightarrow \quad W = e\int E_z ds = eV$$

RF acceleration: $V \neq const$



In this case the electric field is oscillating. So it is for the potential. The energy gain will depend on the RF phase experienced by the particle.



$$\int \hat{E}_z\, dz = \hat{V}$$

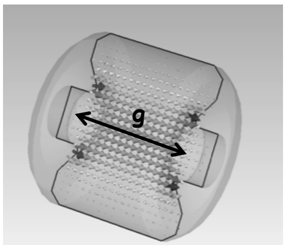
$$E_z = \hat{E}_z \cos \omega_{RF} t \quad = \hat{E}_z \cos \Phi(t)$$

$$W = e\hat{V}\cos\Phi$$

Z

Neglecting the transit time in the gap.

Energy Gain in RF structures: Transit Time Factor



Oscillating field at frequency ω (amplitude is assumed to be constant all along the gap)

$$E_z = E_0 \cos\omega t = \frac{V}{g}\cos\omega t$$

Consider a particle passing through the middle of the gap at time t=0 : $z = vt$

The total energy gain is: $$\Delta W = \frac{eV}{g}\int_{-g/2}^{g/2}\cos\omega\frac{z}{v}dz$$

$$\Delta W = eV\frac{\sin\theta/2}{\theta/2} = eVT$$

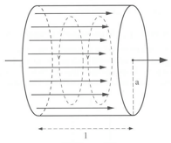
$T = \frac{\sin\theta/2}{\theta/2}$ transit time factor (0 < T < 1)

$\theta = \frac{\omega g}{v}$ *transit angle*

ideal case: $T = \frac{\sin\theta/2}{\theta/2} \to 1 \quad \Leftrightarrow \quad \theta/2 \to 0$

el. static accelertors $\omega \to 0$

minimise acc. gap $g \to 0$



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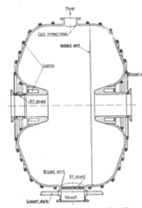
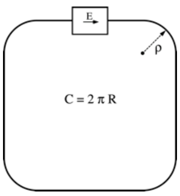


5.) The Synchrotron (Mac Millan, Veksler, 1945)



The synchrotron: Ring Accelerator of const. R where the increase in momentum (i.e. B-field) is automatically synchronised with the correct synchronous phase of the particle in the ... ies

"synchronisation" as basic principle of the machine

$e\dot{V}$ ... → Energy gain per turn

$\Phi = \Phi_s = cte$ → Synchronous particle

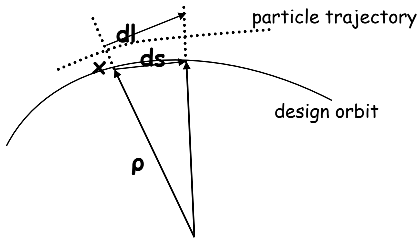
$\omega_{RF} = h\omega_r$ → RF synchronism

$\rho = cte \quad R = cte$ → Constant orbit

$B\rho = P/e \Rightarrow B$ → Variable magnetic field

6.) Momentum Compaction Factor: α_p

particle with a displacement x to the design orbit
→ path length dl ...



$$\frac{dl}{ds} = \frac{\rho + x}{\rho}$$

$$\rightarrow \quad dl = \left(1 + \frac{x}{\rho(s)}\right) ds$$

circumference of an off-energy closed orbit

$$l_{\Delta E} = \oint dl = \oint \left(1 + \frac{x_{\Delta E}}{\rho(s)}\right) ds$$

remember: $x_{\Delta E}(s) = D(s)\frac{\Delta p}{p}$

$$\delta l_{\Delta E} = \frac{\Delta p}{p} \oint \left(\frac{D(s)}{\rho(s)}\right) ds$$

* The lengthening of the orbit for off-momentum particles is given by the dispersion function and the bending radius.

Definition: $\frac{\delta l_\varepsilon}{L} = \alpha_p \frac{\Delta p}{p}$

$$\rightarrow \alpha_p = \frac{1}{L} \oint \left(\frac{D(s)}{\rho(s)}\right) ds$$

For first estimates assume: $\frac{1}{\rho} = const.$

$$\int_{dipoles} D(s)\, ds \approx l_{\Sigma(dipoles)} \cdot \langle D \rangle_{dipole}$$

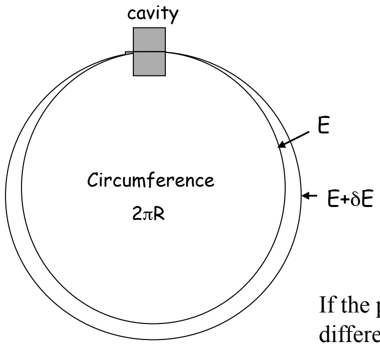
$$\alpha_p = \frac{1}{L}\, l_{\Sigma(dipoles)} \cdot \langle D \rangle \frac{1}{\rho} = \frac{1}{L}\, 2\pi\rho \cdot \langle D \rangle \frac{1}{\rho} \quad \rightarrow \quad \alpha_p \approx \frac{2\pi}{L} \langle D \rangle \approx \frac{\langle D \rangle}{R}$$

Assume: $v \approx c$

$$\rightarrow \quad \frac{\delta T}{T} = \frac{\delta l_\varepsilon}{L} = \alpha_p \frac{\Delta p}{p}$$

α_p combines via the dispersion function the momentum spread with the longitudinal motion of the particle.

7.) Dispersion Effects in a Synchrotron



If a particle is slightly shifted in momentum it will have a different orbit:

$$\alpha = \frac{p}{R}\frac{dR}{dp}$$

This is the "momentum compaction" generated by the bending field.

If the particle is shifted in momentum it will have also a different velocity. As a result of both effects **the revolution frequency changes:**

$$\eta = \frac{p}{f_r}\frac{df_r}{dp}$$

p=particle momentum

R=synchrotron physical radius

f_r=revolution frequency

Dispersion Effects in a Synchrotron

$$\eta = \frac{p}{f_r}\frac{df_r}{dp} \qquad f_r = \frac{\beta c}{2\pi R} \quad \Rightarrow \quad \frac{df_r}{f_r} = \frac{d\beta}{\beta} - \frac{dR}{R} \qquad \longrightarrow \frac{dR}{R} = \alpha\frac{dp}{p}$$

$$p = mv = \beta\gamma\frac{E_0}{c} \quad \Rightarrow \quad \frac{dp}{p} = \frac{d\beta}{\beta} + \frac{d(1-\beta^2)^{-\frac{1}{2}}}{(1-\beta^2)^{-\frac{1}{2}}} = (1-\beta^2)^{-1}\frac{d\beta}{\beta} \qquad \longrightarrow \frac{d\beta}{\beta} = \frac{1}{\gamma^2}\frac{dp}{p}$$

$$\frac{df_r}{f_r} = \left(\frac{1}{\gamma^2} - \alpha\right)\frac{dp}{p} \quad \longrightarrow \quad \eta = \frac{1}{\gamma^2} - \alpha$$

The change of revolution frequency depends on the particle energy γ and changes sign during acceleration.

Particles get faster in the beginning – and arrive earlier at the cavity: classic regime

Particles travel at v =c and get more massive – and arrive later at the cavity: relativistic regime

boundary between the two regimes: no frequency dependence on dp/p, η =0 "transition energy"

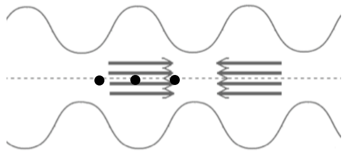
$$\gamma_{tr} = \frac{1}{\sqrt{\alpha}}$$

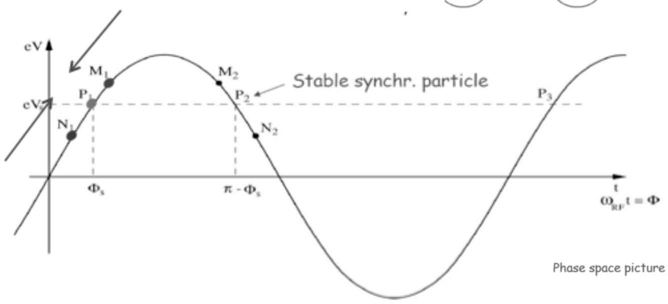
8.) The Acceleration for Δp/p≠0
"Phase Focusing" below transition

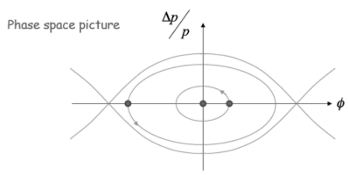
ideal particle •

particle with Δp/p > 0 • **faster**

particle with Δp/p < 0 • **slower**



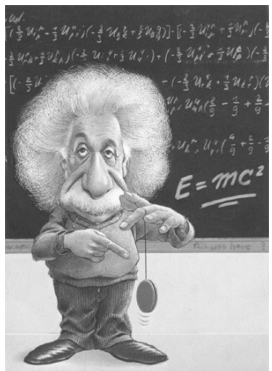


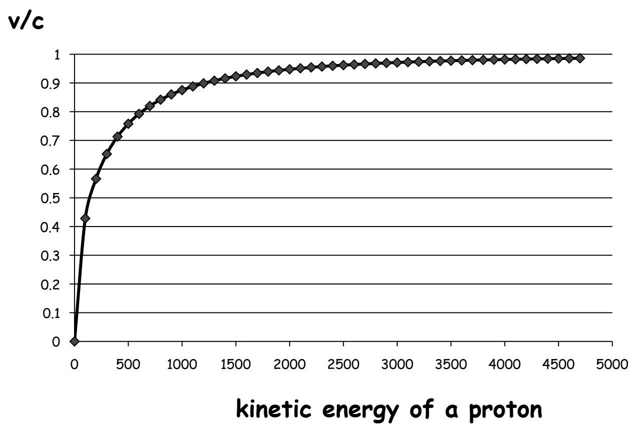


Focussing effect in the longitudinal direction
keeping the particles close together
... forming a "bunch"

... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow \frac{v}{c} = \sqrt{1-\frac{mc^2}{E^2}}$$





... some when the particles do not get faster anymore

.... but heavier !

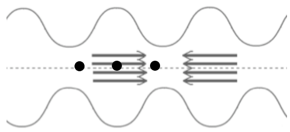
9.) The Acceleration for Δp/p≠0

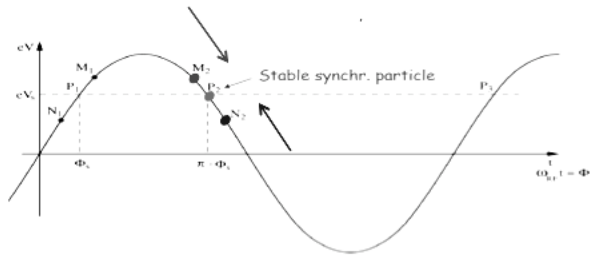
"Phase Focusing" above transition

ideal particle •

particle with Δp/p > 0 • heavier

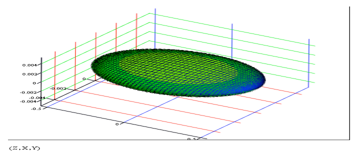
particle with Δp/p < 0 • lighter



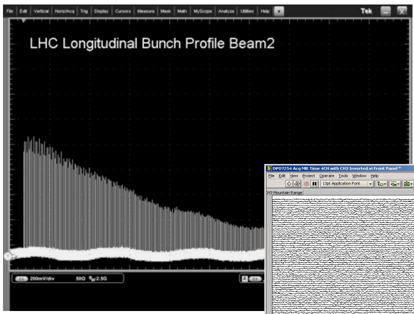


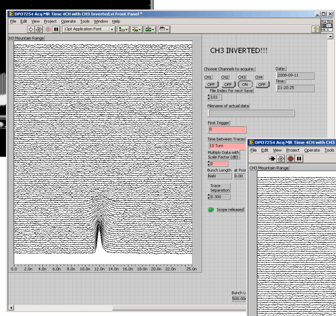
oscillation frequency ≈ some Hz

LHC Commissioning: RF

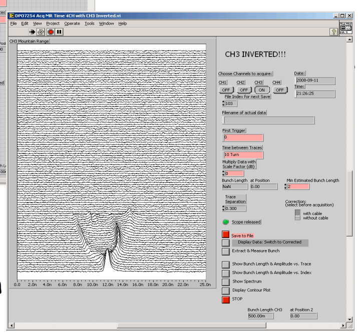


a proton bunch: focused longitudinal by the RF field





RF off



RF on,
wrong phase condition

... and how do we accelerate now ???
with the dipole magnets !

Energy ramping is simply obtained by varying the B field:

$$p = eB\rho \quad \Rightarrow \quad \frac{dp}{dt} = e\rho \dot{B} \quad \Rightarrow (\Delta p)_{turn} = e\rho \dot{B} T_r = \frac{2\pi\, e\rho R\dot{B}}{v}$$

Energy Gain per turn: $E^2 = E_0^2 + p^2 c^2 \quad \Rightarrow \quad \Delta E = v\Delta p$

$$\Delta E_{turn} = \Delta W_{turn} = 2\pi e\rho R\dot{B} = e\hat{V}\sin\phi_s$$

* The energy gain depends on the rate of change of the dipole field
* The number of stable synchronous particles is equal to the harmonic number h. They are equally spaced along the circumference.
* Each synchronous particle satifies the relation p = eBρ. They have the nominal energy and follow the nominal trajectory.

10.) Longitudinal Dynamics: synchrotron motion

We have to follow two coupled variables:

* the energy gained by the particle
* and the RF phase experienced by the same particle.

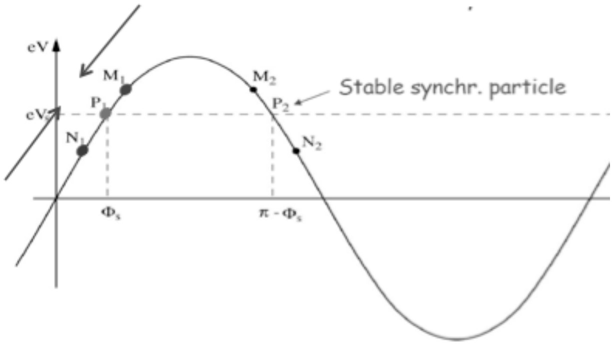
Since there is a well defined synchronous particle which has always the same phase ϕ_s, and the nominal energy E_s, it is sufficient and elegant to follow other particles with respect to that particle.

We will introduce the following relative variables:

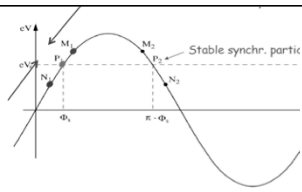
revolution frequency	:	$\Delta f_r = f_r - f_{rs}$
particle RF phase	:	$\Delta\phi = \phi - \phi_s$
particle momentum	:	$\Delta p = p - p_s$
particle energy	:	$\Delta E = E - E_s$
azimuth angle	:	$\Delta\theta = \theta - \theta_s$

The Equation of Motion:

Energy-Phase Relations in a Synchrotron
energy offset ←→ phase change



Equation of Motion:



Relation between momentum difference and difference in revolution frequency:

$$\frac{df_r}{f_r} = \left(\frac{1}{\gamma^2} - \alpha\right)\frac{dp}{p}$$

which translates into difference in revolution time:

$$\frac{dT}{T_0} = \left(\alpha - \frac{1}{\gamma^2}\right)\frac{dp}{p}$$

A particle with higher momentum travels faster (in the classical regime)

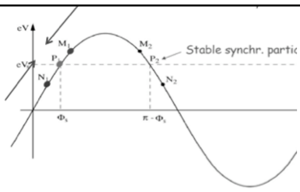
The result is a difference in phase at the cavity

$$\Delta\psi = 2\pi\frac{\Delta T}{T_{rf}} = \omega_{rf} * \Delta T$$
$$= h * \omega_0 * \Delta T = h * 2\pi\frac{\Delta T}{T_0}$$
$$= h * 2\pi\left(\alpha - \frac{1}{\gamma^2}\right)\frac{dp}{p}$$
$$= \frac{h * 2\pi}{\beta^2}\left(\alpha - \frac{1}{\gamma^2}\right)\frac{dE}{E}$$

The RF frequency has to be a integer multiple of the revolution frequency, "h" called harmonic number

difference in energy and difference in phase are related via the momentum compaction

Equation of Motion:



$$\Delta\psi = \frac{h*2\pi}{\beta^2}\left(\alpha - \frac{1}{\gamma^2}\right)\frac{dE}{E}$$

differentiate to time

(1) $$\Delta\dot{\psi} = \frac{\Delta\psi}{T_0} = \frac{h*2\pi}{\beta^2 T_0}\left(\alpha - \frac{1}{\gamma^2}\right)\frac{dE}{E}$$

rate of change of the phase difference wrt to the ideal particle

the energy change is given by the RF system:

$$\Delta E = e * U_0(\sin(\psi_s + \Delta\psi) - \sin\psi_s)$$

$$\sin(\psi_s + \Delta\psi) - \sin\psi_s = \underbrace{\sin\psi_s \cos\Delta\psi}_{\approx 1} - \cos\psi_s \underbrace{\sin\Delta\psi}_{\Delta\psi} - \sin\psi_s$$

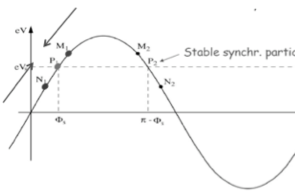
and the phase difference determines the rate of energy change per turn

$$\Delta\dot{E} = e * \frac{U_0}{T_0}\Delta\psi \cos\psi_s$$

differentiate a second time

(2) $$\Delta\ddot{E} = e * \frac{U_0}{T_0}\Delta\dot{\psi} \cos\psi_s$$

Equation of Motion:



(1) $$\Delta\dot{\psi} = \frac{\Delta\psi}{T_0} = \frac{h*2\pi}{\beta^2 T_0}\left(\alpha - \frac{1}{\gamma^2}\right)\frac{dE}{E}$$

(2) $$\Delta\ddot{E} = e * \frac{U_0}{T_0}\Delta\dot{\psi} \cos\psi_s$$

put (1) into (2) et c'est ca Equation of Motion in Phase Space E-ψ:

$$\Delta\ddot{E} = e * \frac{U_0}{T_0}\frac{2\pi h}{\beta^2 T_0}\left(\alpha - \frac{1}{\gamma^2}\right)\frac{dE}{E}\cos\psi_s$$

Definition:

$$\Omega = \omega_0 * \sqrt{\frac{-eU_0 h \cos\psi_s}{2\pi\beta^2 E}\left(\alpha - \frac{1}{\gamma^2}\right)}$$

$$\Delta\ddot{E} + \Omega^2 \Delta E = 0$$

We get a differential equation that describes the difference in energy of a particle to the ideal (i.e. synchronous) particle under the influence of the phaes focusing effect of our sinusoidal RF function.
And it is a harmonic oscillation !!!
The oscillation frequency Ω is called synchrotron frequency and usually in the range of some Hz ... kHz.

Small Amplitude Oscillations: phase stability

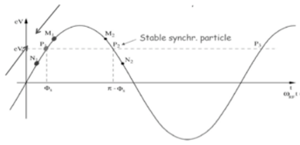
We get - in equivalent way - the harmonic oscillation of the particle phase with the oscillation frequency

$$\Delta\ddot{\psi} + \Omega_s^2 \Delta\psi = 0 \qquad \Omega_s = \omega_0 * \sqrt{\frac{eU_0 h \cos\psi_s}{2\pi\beta^2 E}\eta} \qquad \textit{remember} \quad \eta = \frac{1}{\gamma^2} - \alpha$$

Stability condition: Ω_s real $\quad \Omega_s^2 > 0$

$\gamma < \gamma_{tr} \quad \eta > 0 \quad 0 < \phi_s < \pi/2$

$\gamma > \gamma_{tr} \quad \eta < 0 \quad \pi/2 < \phi_s < \pi$

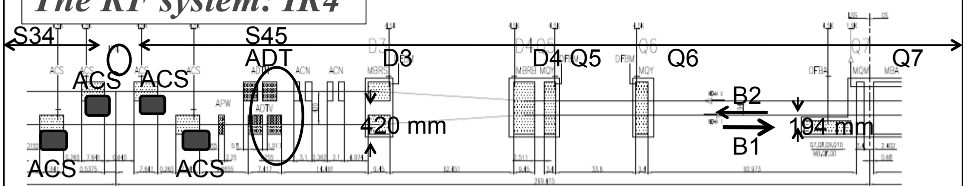


And we will find this situation
"h"-times in the machine

LHC:
35640 Possible Bunch Positions ("buckets")
2808 Bunches

oscillation frequency depends on
**** the square root***
**** of an electrical potential***
-> weak force <-> small frequncy

The RF system: IR4

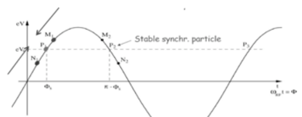


4xFour-cavity cryo module 400 MHz, 16 MV/beam
Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

Bunch length (4σ)	***ns***	***1.06***
Energy spread (2σ)	***10^{-3}***	***0.22***
Synchr. rad. loss/ turn	***keV***	***7***
Synchr. rad. power	***kW***	***3.6***
RF frequency	***MHz***	***400***
Harmonic number		***35640***
RF voltage/beam	***MV***	***16***
Energy gain/turn	***keV***	***485***
Synchrotron frequency	***Hz***	***23.0***

(small) ... Synchrotron Oscillations in Energy and Phase

$$\Delta\ddot{\psi} + \Omega_s^2\Delta\psi = 0$$



Ansatz: $\Delta\psi = \Delta\psi_{max} * \cos(\Omega_s t)$

take the first derivative and put it into the first energy-phase relation $\Delta\dot{\psi} = \frac{h * 2\pi}{\beta^2 T_0}\left(\alpha - \frac{1}{\gamma^2}\right)\frac{dE}{E}$

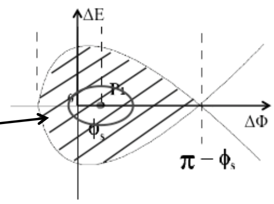
$$\frac{d(\Delta\psi)}{dt} = -\Delta\psi_{max} * \sin(\Omega_s t) * \Omega_s$$

to get the energy oscillations $\Delta E = \underbrace{\frac{\beta^2 T_0 \Omega_s \Delta\psi_{max} E_s}{2\pi h \eta}}_{\Delta E_{max}} \sin(\Omega_s t)$

$$\Delta E = \Delta E_{max} * \sin(\Omega_s t)$$

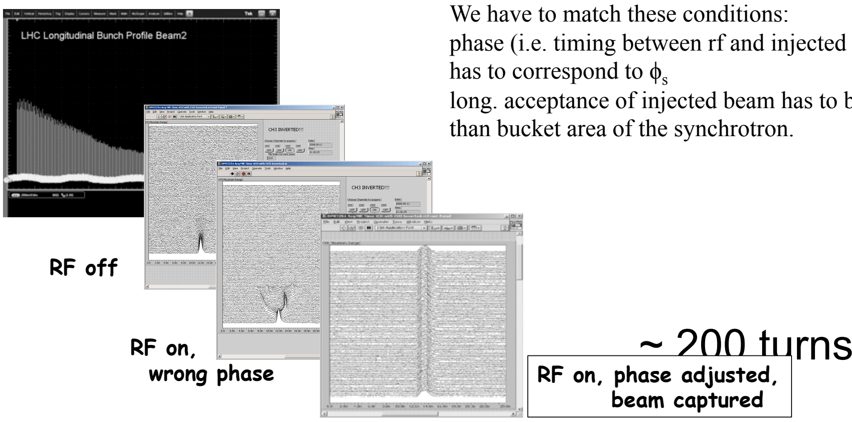
which defines an ellipse in phase space Δψ, ΔE:

$$\left(\frac{\Delta\psi}{\Delta\psi_{max}}\right)^2 + \left(\frac{\Delta E}{\Delta E_{max}}\right)^2 = 1$$



LHC Commissioning: RF

We have to match these conditions:
phase (i.e. timing between rf and injected bunch) has to correspond to ϕ_s
long. acceptance of injected beam has to be smaller than bucket area of the synchrotron.



max stable energy: set $\phi = \phi_s$ and calculate ΔE

$$(\Delta E_{max})_{sep} = \sqrt{\frac{p_s v_s e U_0}{2\pi h \eta_s}} * \sqrt{|4\cos\phi_s - (2\pi - 4\phi_s)\sin\phi_s|}$$

LHC injection:
acceptance: 1.4eVs
long emittance: 1.0 eVs

Than'x

Appendix: Relativistic Relations

court. Chris Prior, Trinity College / CAS

	$\frac{\Delta\beta}{\beta}$	$\frac{\Delta p}{p}$	$\frac{\Delta T}{T}$	$\frac{\Delta E}{E} = \frac{\Delta\gamma}{\gamma}$
$\frac{\Delta\beta}{\beta} =$	$\frac{\Delta\beta}{\beta}$	$\frac{1}{\gamma^2}\frac{\Delta p}{p}$ $\frac{\Delta p}{p} - \frac{\Delta\gamma}{\gamma}$	$\frac{1}{\gamma(\gamma+1)}\frac{\Delta T}{T}$	$\frac{1}{\beta^2\gamma^2}\frac{\Delta\gamma}{\gamma}$ $\frac{1}{\gamma^2-1}\frac{\Delta\gamma}{\gamma}$
$\frac{\Delta p}{p} =$	$\gamma^2\frac{\Delta\beta}{\beta}$	$\frac{\Delta p}{p}$	$\frac{\gamma}{\gamma+1}\frac{\Delta T}{T}$	$\frac{1}{\beta^2}\frac{\Delta\gamma}{\gamma}$
$\frac{\Delta T}{T} =$	$\gamma(\gamma+1)\frac{\Delta\beta}{\beta}$	$\left(1+\frac{1}{\gamma}\right)\frac{\Delta p}{p}$	$\frac{\Delta T}{T}$	$\frac{\gamma}{\gamma-1}\frac{\Delta\gamma}{\gamma}$
$\frac{\Delta E}{E} =$	$(\beta\gamma)^2\frac{\Delta\beta}{\beta}$	$\beta^2\frac{\Delta p}{p}$	$\left(1-\frac{1}{\gamma}\right)\frac{\Delta T}{T}$	$\frac{\Delta\gamma}{\gamma}$
$\frac{\Delta\gamma}{\gamma} =$	$(\gamma^2-1)\frac{\Delta\beta}{\beta}$	$\frac{\Delta p}{p} - \frac{\Delta\beta}{\beta}$		