CAS April-May 2013 Erice, Italy

AC/RF SUPERCONDUCTIVITY

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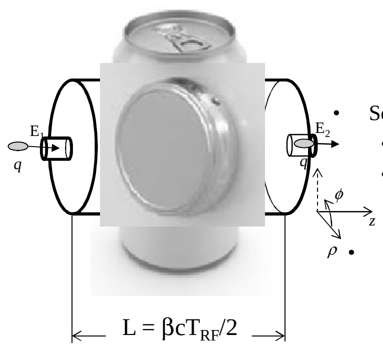
Outline

- Introduction to RF Cavities
- Electrodynamics of normal-conductors
- Electrodynamics of superconductors
  - Surface impedance
    - Two-fluid model and BCS theory
- Residual resistance
- Superheating field
- Field dependence of the surface resistance due to thermal feedback

RF Cavities

- Devices that store e.m. energy and transfer it to a charged particle beam



- E.m. field in the cavity is solution to the wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix}\mathbf{E}\\ \mathbf{H}\end{Bmatrix} = 0 \qquad \hat{n}\times\mathbf{E} = 0, \quad \hat{n}\cdot\mathbf{H} = 0$$

- Solutions are two family of modes with different eigenfrequencies
  - TE_{mnp} modes have only transverse electric fields
  - TM_{mnp} modes have only transverse magnetic fields (but longitudinal component for E)
- Accelerating mode: TM_{010}

$$E_z = E_0 J_0\left(\frac{2.405\rho}{R}\right) e^{-i\omega t}$$

$$H_\phi = -i\frac{E_0}{\eta} J_1\left(\frac{2.405\rho}{R}\right) e^{-i\omega t}$$

$$\omega_{010} = \frac{2.405c}{R} \qquad \eta = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

Example: 2 GHz cavity and speed of light e^- $\rightarrow$ L = 7.5 cm, R = 5.7 cm

Figures of merit (1)

- *What is the energy gained by the particle?*
- Let's assume a relativistic e^-
- Integrate the E-field at the particle position as it traverses the cavity

$$V_c = \left|\int_0^d E_z(\rho = 0, z) e^{i\omega_0 z/c} dz\right| = \frac{2}{\pi} E_0 L$$

- We can define the **accelerating field** as: $E_{acc} = \frac{V_c}{L} = \frac{2}{\pi} E_0$
- Important for the cavity performance are the ratios of the peak surface fields to the E_{acc}. Ideally, these should be small to limit losses and other troubles at high fields

$$E_p / E_{acc} = \frac{\pi}{2} = 1.6$$

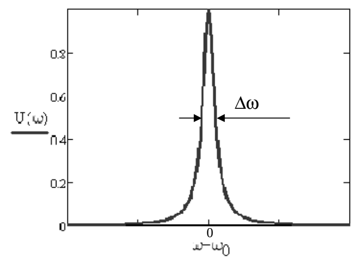
$$H_p / E_{acc} = \frac{\pi}{2}\frac{J_1(1.84)}{\eta} = 2430\frac{\mathrm{A/m}}{\mathrm{MV/m}} = 30.5\frac{\mathrm{Oe}}{\mathrm{MV/m}}$$

Figures of merit (2)

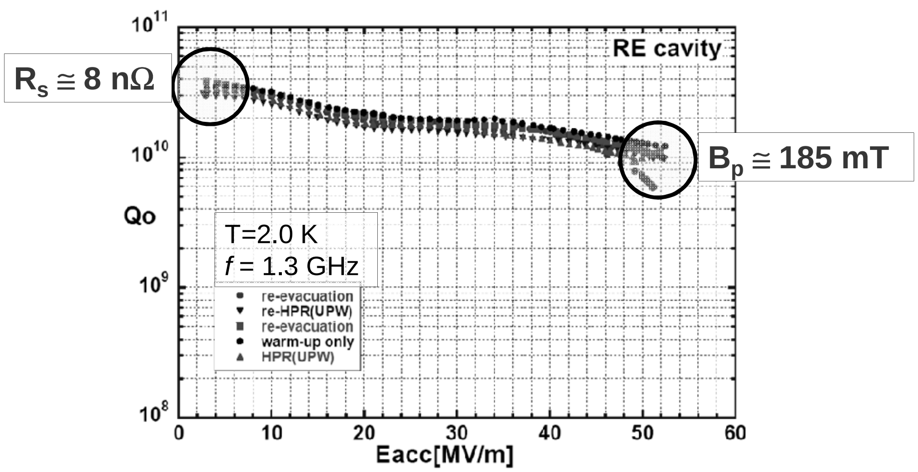
- The power dissipated in the cavity wall due to Joule heating is given by: $P_c = \frac{1}{2}\mathrm{Re}\left\{\int_V \mathbf{J}\cdot\mathbf{E}\, dV\right\} = \frac{1}{2}R_s\int_S |H|^2 ds$
- The energy stored in the cavity is given by: $U = \frac{1}{2}\mu_0\int_{\mathbf{V}} |\mathbf{H}|^2 dv$
- The cavity **quality factor** is defined as:

$$Q_0 = \frac{\omega_0 U}{P_c} = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 ds}{R_S \int_S |\mathbf{H}|^2 ds} = \frac{\omega_0}{\Delta\omega}$$

= G



SRF cavity performance



F. Furuta et al., Proc. EPAC'06, p. 750

Surface Impedance

- The electromagnetic response of a metal, whether normal or superconducting, is described by a complex surface impedance:

$$Z_s = \frac{|E_\parallel|}{\int_0^\infty J(x)dx} = \frac{E_\parallel}{H_\parallel} = R_s + iX_s$$

surface reactance

surface resistance

For a good conductor and $\omega < \sim 10^{16}$ Hz $\frac{\partial D}{\partial t} \approx 0 \longrightarrow \nabla \times H = J$

- The impedance of vacuum is: $Z_0 = \left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2} \simeq 377\Omega$

For accelerator applications, the rate of oscillation of the e.m. field falls in the radio-frequency (RF) range (3 kHz – 300 GHz)

Electrodynamics of normal conductors

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0} \qquad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \cdot B = 0 \qquad \nabla \times H = J + \frac{\partial D}{\partial t}$$

Maxwell's equations

$+$

$$D = \varepsilon_0 \varepsilon \; E$$

$$B = \mu_0 \mu \; H$$

$$J = f\,(E)$$

(linear and isotropic) material's equations

- From Drude's model ("nearly free electrons"):

$E = E_0 e^{i\omega t}$ $\qquad \frac{\partial J}{\partial t} + \frac{J}{\tau} = \frac{ne^2}{m} E$ $\qquad \tau = l/v_F \approx 10^{-14}$ s is the electrons' scattering time

$$J = \underbrace{\frac{ne^2}{m\tau}}_{\sigma} \frac{1}{(1+i\omega\tau)} E = \sigma E$$

Ohm's law, <u>local</u> relation between J and E

$\omega\tau \ll 1$ at RF frequencies

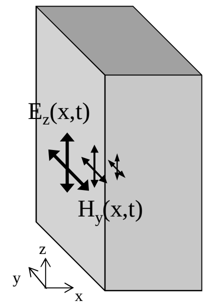
Surface impedance of normal conductors

- From previous slide you obtain: $\nabla^2 H = i\sigma\mu_0\mu\omega H$

- Solution (semi-infinite slab): $H_y = H_0 e^{-x/\delta} e^{-ix/\delta}$

$$E_z = -\frac{(1+i)}{\sigma\delta} H_y$$

skin depth: $\boxed{\delta = \sqrt{\frac{2}{\mu_0\mu\sigma\omega}}}$



$$Z_s = \frac{|E_z(0)|}{H_y(0)} = \frac{1+i}{\sigma\delta} \qquad \boxed{R_s = X_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\mu_0\mu\omega}{2\sigma}}}$$

Example: Cu at 1.5 GHz, 300 K ($\sigma = 5.8\times10^7$ 1/Ωm, $\mu_0=1.26\times10^{-6}$ Vs/Am, $\mu=1$)

⇨ $\delta = 1.7$ µm, $R_s = 10$ mΩ

What happens at low temperature?

- $\sigma(T)$ increases, δ decreases ➡ The skin depth (the distance over which fields vary) can become less than the mean free path of the electrons (the distance they travel before being scattered) ➡ $J(x) \neq \sigma E(x)$

- Introduce a new relationship where J is related to E over a volume of the size of the mean free path (l)

$$\vec{J}(\vec{r},t) = \frac{3\sigma}{4\pi l}\int_V d\vec{r}\,' \frac{\vec{R}\left[\vec{R}\cdot\vec{E}(\vec{r}\,',t-\vec{R}/v_F)\right]}{R^4} e^{-R/l} \quad \text{with } \vec{R} = \vec{r}\,' - \vec{r}$$

Effective conductivity $\sigma_{eff} \approx \frac{\delta}{l}\sigma = \frac{\delta}{l}\frac{ne^2 l}{m v_F}$

$= \tau$

Contrary to the DC case higher purity (longer l) does not increase the conductivity → anomalous skin effect

So, how good is Cu at low T?

$$R_s(l \to \infty) = \left[\sqrt{3}\pi \left(\frac{\mu_0}{4\pi} \right)^2 \right]^{1/3} \omega^{2/3} \left(\frac{l}{\sigma} \right)^{1/3}$$

"Extreme" anomalous limit
(OK for Cu in RF and low T)

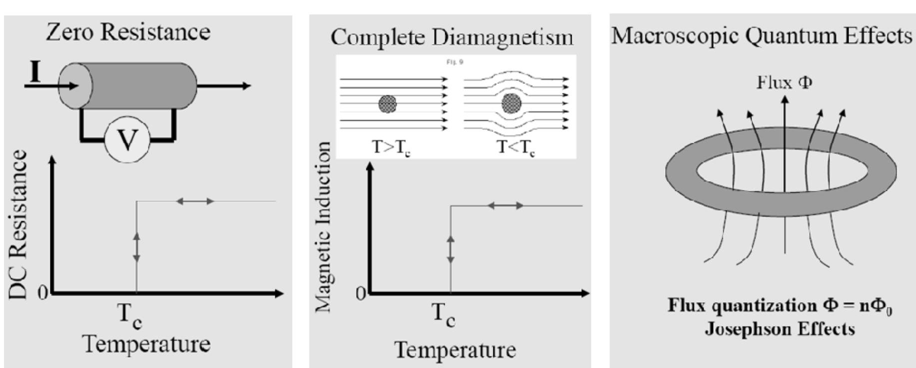
$l/\sigma = 6.8 \times 10^{-16}\ \Omega \cdot \mathrm{m}^2$ for Cu

$$\frac{R_s(4.2\ \mathrm{K}, 1.5\ \mathrm{GHz})}{R_s(300\ \mathrm{K}, 1.5\ \mathrm{GHz})} \approx 0.14$$

Does not compensate for the refrigerator efficiency!!!

Superconductivity - remainder

The 3 Hallmarks of Superconductivity



London equations - remainder

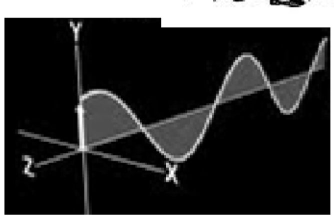
$$\frac{dJ_s}{dt} = \frac{1}{\mu_0 \lambda_L^2} E$$

- **E**=0: J_s goes on forever
- **E** is required to maintain an AC current

$$\nabla \times J_s = -\frac{1}{\mu_0 \lambda_L^2} B$$

- **B** is the source of J_s
- Spontaneus flux exclusion

Enter AC Superconductivity



Bad news: $R_s > 0$

- In AC fields, the time-dependent magnetic field in the penetration depth will generate an electric field:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

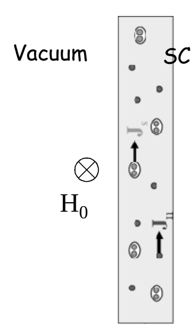
- Because Cooper pairs have inertia (mass=$2m_e$) they cannot completely shield nc electrons from this E-field $\rightarrow R_s > 0$

So, how does R_s for a superconductor compare to that of a normal conductor?

Two-fluid model

- Gorter and Casimir (1934) two-fluid model: charge carriers are divided in two subsystems, superconducting carriers of density $\mathbf{n_s}$ and normal electrons of density $\mathbf{n_n}$.
- The superconducting carriers are the Cooper pairs (1956) with charge -2e and mass 2m
- The normal current $\mathbf{J_n}$ and the supercurrent $\mathbf{J_s}$ are assumed to flow in parallel. J_s flows with no resistance.

$$J = J_n + J_s$$

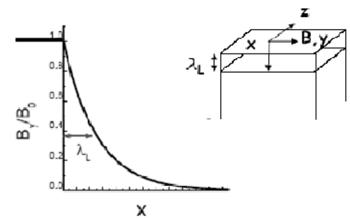


Electrodynamics of superconductors **(at low field)**

- London equations:

$$\frac{\partial}{\partial t}\vec{J}_{\mathrm{s}} = \frac{\vec{E}}{\mu_0 \lambda_L^2}$$

$$\nabla \times \vec{J}_{\mathrm{s}} = -\frac{1}{\lambda_{\mathrm{L}}^2}\vec{H}$$



$$\boxed{\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}}$$ London penetration depth

- Currents and magnetic fields in superconductors can exist only within a layer of thickness λ_L)

Note: $\vec{J}_{\mathrm{s}} = -\frac{1}{\lambda_{\mathrm{L}}^2}\vec{A}.$ Local condition between current and field. Valid if $\xi_0 << \lambda_L$ or $l << \lambda_L$

Surface impedance of superconductors

$$\frac{\partial}{\partial t}\vec{J}_{\mathrm{s}} = \frac{\vec{E}}{\mu_0 \lambda_L^2} \Rightarrow J_s = -i\underbrace{\frac{1}{\omega\mu_0\lambda_L^2}}_{=\sigma_2}E \Rightarrow J = J_n + J_s = \underbrace{(\sigma_1 - i\sigma_2)}_{\sigma}E$$

$$\sigma_1 = \sigma_n = \frac{n_n e^2 \tau}{m}, \quad \sigma_2 = \frac{n_s e^2}{m\omega}$$

- Electrodynamics of sc is the same as nc, only that we have to change $\sigma \rightarrow \sigma_1 - i\,\sigma_2$

- Penetration depth: $\delta = \sqrt{\frac{2}{\mu_0 \sigma \omega}} = \frac{1}{\sqrt{\mu_0 \omega \sigma_2}}\sqrt{\frac{2i}{1 - i\,\sigma_1/\sigma_2}} \cong (1+i)\lambda_L\left(1 + i\frac{\sigma_1}{2\sigma_2}\right)$

$\sigma_1 << \sigma_2$ for sc at $T<<T_c$

$$H_y = H_0 \exp\left(-\frac{(1+i)}{\delta}x\right) \Rightarrow \boxed{H_y = H_0 e^{-\frac{x}{\lambda_L}} e^{-i\frac{x}{\lambda_L}\frac{\sigma_1}{2\sigma_2}}}$$

For Nb, λ_L = 36 nm, compared to δ = 1.7 μm for Cu at 1.5 GHz

Surface impedance of superconductors

$$Z_s = \sqrt{\frac{i\omega\mu_0}{\sigma}} = \sqrt{\frac{\omega\mu_0}{2\sigma_1}}\left(\varphi_- + i\varphi_+\right)$$

$$\varphi_\pm^2 = \frac{y}{1+y^2}\left(\sqrt{1+y^2} \pm 1\right) \qquad y = \frac{\sigma_1}{\sigma_2}$$

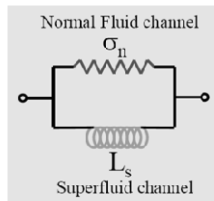
For a sc $\sigma_1 << \sigma_2 \to y << 1$ ➡ $\varphi_- = \sqrt{\frac{y^3}{2}}, \quad \varphi_+ = \sqrt{2y}$

$$Z_s = R_s + iX_s$$

$$X_s = \omega \underbrace{\mu_0 \lambda_L}_{L_s}$$

L_s: kinetic inductance

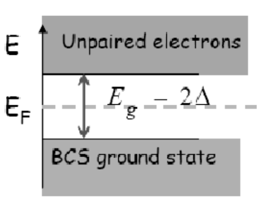
$$R_s = \frac{1}{2}\mu_0^2\omega^2\sigma_1\lambda_L^3$$

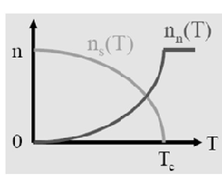


Surface resistance of superconductor

$$R_s = \frac{1}{2}\mu_0^2\omega^2\sigma_1\lambda_L^3$$

- $R_s \propto \sigma_1 \propto l \to$ longer m.f.p (higher conductivity) of unpaired e^- results in higher R_s !
- $R_s \propto \omega^2 \to$ use low-frequency cavities to reduce power dissipation
- Temperature dependence:





$n_s(T) \propto 1-(T/T_c)^4$

$\sigma_1(T) \propto n_n(T) \propto e^{-\Delta/k_B T}$

Unpaired electrons are created by the thermal breakup of Cooper pairs

$$R_s \propto \omega^2\lambda_L^3\, l \exp\left(-\Delta/k_B T\right) \qquad T < T_c/2$$

Material purity dependence of R_s

- If $\xi_0 >> \lambda_L$ and $l >> \lambda_L$, the local relation between current and field is not valid anymore (similarly to anomalous skin effect in normal conductors)

Pippard (1953): $$\vec{J}_s(\vec{r}) = \frac{3}{4\pi\xi_0\lambda_L^2}\int_V \frac{\vec{R}\vec{R}\cdot\vec{A}(\vec{r}')e^{-R/\xi}}{R^4}d\vec{r}' \qquad \frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{l}$$

- The dependence of the penetration depth on l is approximated as $\lambda(l) \approx \lambda_L\sqrt{1+\frac{\xi_0}{l}}$

$$\Rightarrow \quad R_s \propto \left(1+\frac{\xi_0}{l}\right)^{3/2} l \quad \Rightarrow \quad \begin{array}{ll} R_s \propto l & \text{if } l >> \xi_0 \text{ ("clean" limit)} \\ R_s \propto l^{-1/2} & \text{if } l << \xi_0 \text{ ("dirty" limit)} \end{array}$$

R_s has a minimum for $l = \xi_0/2$

BCS surface resistance (1)

- From BCS theory of sc, Mattis and Bardeen (1958) have derived a non-local equation between the total current

ca q)

$$\mathrm{Re}\{\mathrm{K(q)}\} = \frac{3}{\hbar \mathrm{v}_0 \lambda_{\mathrm{L}0}^2 \mathrm{q}} \times$$
$$\left\{ \int_{\max\{\Delta-\hbar\omega,-\Delta\}}^{\Delta} [1-2f(E+\hbar\omega)]\{\frac{E^2+\Delta^2+\hbar\omega E}{\sqrt{\Delta^2-E^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}} R(a_2,a_1+b) + S(a_2,a_1+b)\}dE \right.$$
$$+ \frac{1}{2}\int_{\Delta-\hbar\omega}^{-\Delta}[1-2f(E+\hbar\omega)]\{[g(E)+1]S(a^-,b)-[g(E)-1]S(a^+,b)\}dE$$
$$- \int_{\Delta}^{\infty}[1-f(E)-f(E+\hbar\omega)][g(E)-1]S(a^+,b)dE$$
$$\left. + \int_{\Delta}^{\infty}[f(E)-f(E+\hbar\omega)][g(E)+1]S(a^-,b)dE \right\}$$

$$\mathrm{Im}\{\mathrm{K(q)}\} = \frac{3}{\hbar \mathrm{v}_0 \lambda_{\mathrm{L}0}^2 \mathrm{q}} \times$$
$$\left\{ -\frac{1}{2}\int_{\Delta-\hbar\omega}^{-\Delta}[1-2f(E+\hbar\omega)]\{[g(E)+1]R(a^-,b)+[g(E)-1]R(a^+,b)\}dE \right.$$
$$\left. + \int_{\Delta}^{\infty}[f(E)-f(E+\hbar\omega)]\{[g(E)+1]R(a^-,b)+[g(E)-1]R(a^+,b)\}dE \right\}$$

BCS surface resistance (2)

- There are numerical codes (Halbritter (1970)) to calculate R_{BCS} as a function of ω, T and material parameters (ξ_0, λ_L, T_c, Δ, l)
- For example, check http://www.lepp.cornell.edu/~liepe/webpage/researchsrimp.html
- A go … $\omega < \Delta/\hbar$ is:

R_B … $C_1 = 2.246$

Let's run som … n, $\sigma_n = 3.3\times10^8$ 1/Ωm,
$\Delta/k_BT_c = 1.85$, …

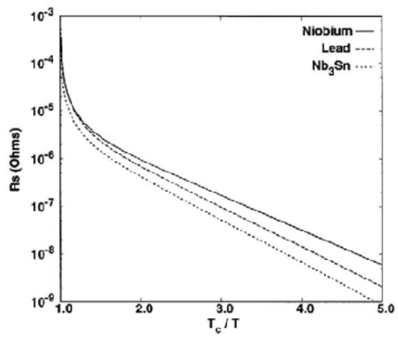


Figure 4.5: Theoretical surface resistance at 1.5 GHz of lead, niobium and Nb_3Sn as calculated from program [94]. The values given in Table 4.1 were used for the material parameters.

Time to celebrate !

Not so fast…

- Refrigeration isn't free:

Carnot efficiency: η_C = 2 K/(300 K – 2 K) = 0.007

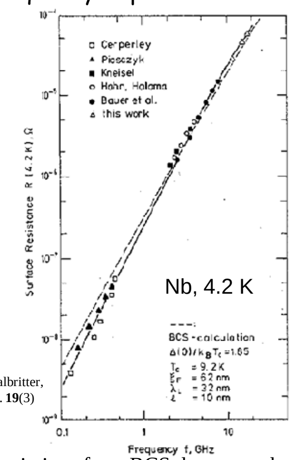
Technical efficiency of cryoplant: $\eta_T \cong 0.2$

➡ Total efficiency: $\eta_{tot} \cong 0.0014 \cong 1/700$

➡ Power reduction from Cu(300K) to Nb(2K) is $\cong$ **10^3**

Experimental results

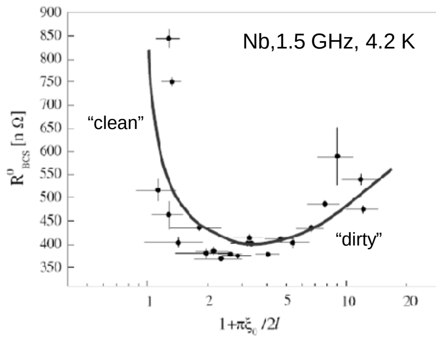
Frequency dependence



A. Phillip and J. Halbritter, IEEE Trans. Magn. **19**(3) (1983) 999.

- Small deviations from BCS theory can be explained by strong coupling effects, anisotropic energy gap in the presence of impurity scattering or by inhomogeneities

Dependence on material purity



C. Benvenuti et al., Physica C **316** (1999) 153.

- Nb films sputtered on Cu
- By changing the sputtering species, the mean free path was varied

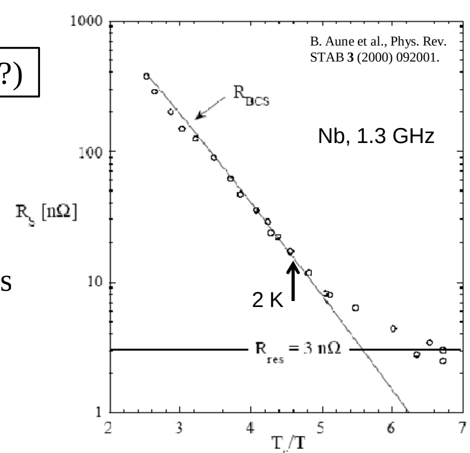
R_{BCS} can be optimized by tuning the density of impurities at the cavity surface.

Residual resistance

$R_s = R_{BCS}(\omega, T, \Delta, T_c, \lambda_L, \xi_0, l) + R_{res}(?)$

Possible contributions to R_{res}:

- Trapped magnetic field
- Normal conducting precipitates
- Grain boundaries
- Interface losses
- Subgap states

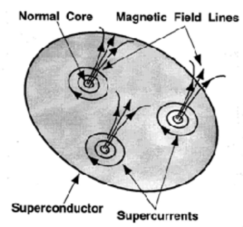


For Nb, R_{res} (~1-10 nΩ) dominates R_s at low frequency (f < ~750 MHz) and low temperature (T < ~2.1 K)

Possible contributions to R_{res} in Nb (1)

- Trapped magnetic field

In technical materials, the Meissner effect is incomplete when cooling below T_c in the presence of a residual magnetic field due to pinning



Fluxoids: normal conducting core of area ~$\pi\xi_0^2$ and normal-state surface resistance, R_n

$$R_{res} = N\frac{\pi\xi_0^2}{A}R_n$$

$$\frac{N}{A} = \mu_0 H_{ext}/\Phi_0 \quad \text{if 100\% flux pinning}$$

$$= \frac{H_{ext}\pi\xi_0^2\mu_0}{\Phi_0}R_n = \frac{H_{ext}}{2H_{c2}}R_n \qquad H_{c2} = \frac{\Phi_0}{2\pi\mu_0\xi_0^2}$$

$$\boxed{R_{res} \cong 0.2(\text{n}\Omega)H_{ext}(\text{mOe})\sqrt{f(\text{GHz})}}$$

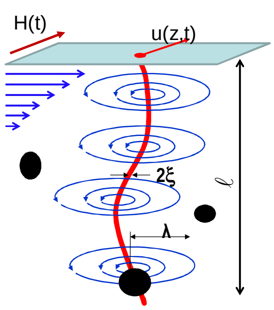
R_{res} due to Earth's field (~500 mG) at 1.5 GHz: ~120 nΩ (~6×R_{BCS}(2K))

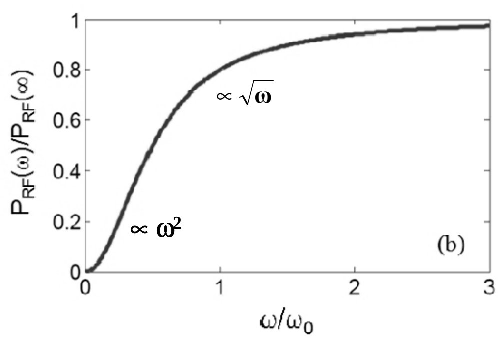
⇨ Apply magnetic shielding around cavities

Possible contributions to R_{res} in Nb (1)

- Trapped magnetic field

Including the oscillatory motion of a fluxoid due to the Lorentz force:





A. Gurevich and G. Ciovati, Phys. Rev. B. **87,** 054502 (2013)

In the frequency limit where only the tip of the fluxoid vibrates:

$$R_{res} = \frac{H_{ext}}{H_c}\frac{R_n}{\sqrt{g}}$$

$$g = \frac{1}{\Gamma^2}\ln(\Gamma\kappa_{GL}) + \frac{1}{2} \qquad \Gamma = \frac{\lambda_c}{\lambda} \quad \text{Anisotropy parameter}$$

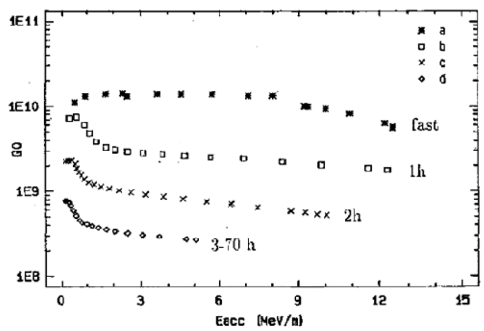
For Nb: ρ_n~5×10^{-10} Ωm, H_c=2000 Oe, 2g=1

$$\boxed{R_{res} \cong 1(\text{n}\Omega)H_{ext}(\text{mOe})\sqrt{f(\text{GHz})}}$$

Possible contributions to R_{res} in Nb (2)

- Normal conducting precipitates

If the bulk H content in high-purity (RRR~300) Nb is > ~5 wt.ppm, precipitation of normal-conducting NbH_x islands occurs at the surface if the cooldown rate is < ~1 K/min in the region 75-150 K



Nb cavities are heat treated at 600 – 800 °C in a UHV furnace to degas H

B. Aune et al., Proc. 1990 LINAC Conf. (1990) 253.

Possible contributions to R_{res} in Nb (3)

- Grain boundaries ⟶ Results are still inconclusive
- Subgap states
- Interface losses ⟶ Results are still inconclusive

Tunneling measurements show that the BCS singularity in the electronic density of states is smeared out and subgap states with finite $N(\varepsilon)$ appear at energies below Δ.

Phenomenological formula [Dynes (1978)]:

$$N(\epsilon) = \mathrm{Re}\frac{N_n(\epsilon - i\gamma)}{\sqrt{(\epsilon - i\gamma)^2 - \Delta^2}}$$

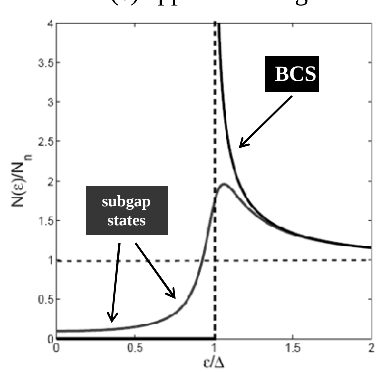
γ: damping parameter

Finite density of states at the Fermi level:

$$N(0) \simeq \gamma N_n/\Delta$$

Residual resistance ($\sigma_1 \rightarrow \sigma_n\gamma/\Delta$):

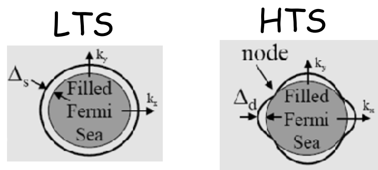
$$R_{res} \approx \mu_0^2 \omega^2 \lambda^3 \sigma_n \gamma/\Delta$$

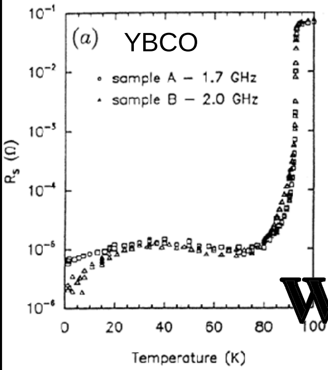


R_{res} ~10 nΩ at 1.5 GHz for $\gamma/\Delta = 10^{-3}$

A. Gurevich, Rev. Accel. Sci. Tech. **5,** 119 (2012)

About HTS…





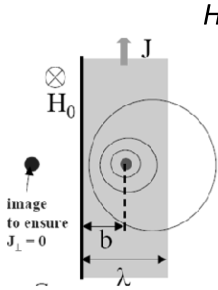
Hein M A 1996 *Studies in High Temperature Superconductors* vol 18 ed A Narlikar (Nova Science Publishers) pp 141–216

- HTS materials have nodes in the energy gap. This leads to **power-law** behavior of $\lambda(T)$ and $R_s(T)$ and **high** residual losses
- $\xi \sim 1 - 2$ nm ($\ll \lambda$) $\rightarrow$ superconducting pairing is easily disrupted by defects (cracks, grain boundaries)
- "Granular" superconductors: high grain boundary resistance contributing to R_{res}

We are stuck with LHe!

Surface barrier

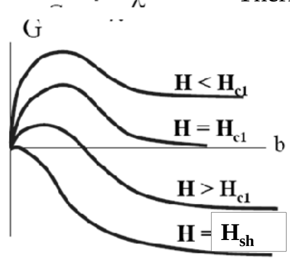
How do vortices get in a superconductor?



Two forces acting on the vortex at the surface:

- Meissner currents push the vortex in the bulk
- Attraction to the antivortex image pushes the vortex out

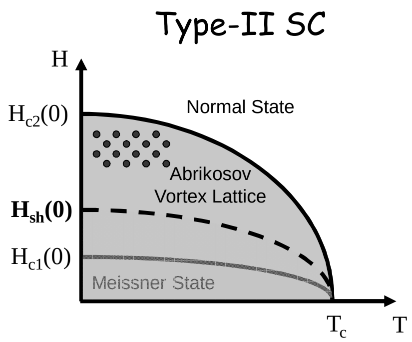
Thermodynamic potential $G(b)$ as a function of the position b:



$$G(b) = \phi_0[\underset{\text{Meissner}}{H_0 e^{-b/\lambda}} - \underset{\text{Image}}{H_v(2b)} + H_{c1} - H_0]$$

- Vortices have to overcome the surface barrier even at $H > H_{c1}$
- Surface barrier disappears only at $H = H_{sh}$
- Surface barrier is reduced by defects

What is the highest RF field applicable to a superconductor?



- Penetration and oscillation of vortices under the RF field gives rise to strong dissipation and the surface resistance of the order of R_s in the normal state
- the Meissner state can remain metastable at higher fields, $H > H_{c1}$ up to the superheating field H_{sh} at which the Bean-Livingston surface barrier for penetration of vortices disappears and the Meissner state becomes unstable

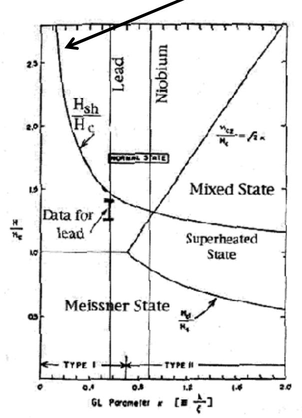
H_{sh} is the maximum magnetic field at which a type-II superconductor can remain in a true non-dissipative state not altered by dissipative motion of vortices.

At $H = H_{sh}$ the screening surface current reaches the depairing value $J_d = n_s e\Delta/p_F$

Superheating field: theory

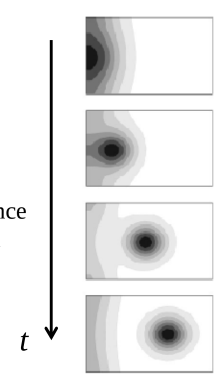
- Calculation of $H_{sh}(\kappa)$ from Ginzburg-Landau theory ($T \approx T_c$) [Matricon and Saint-James (1967)]:

$$H_{sh} \approx 1.2 H_c, \quad \kappa \cong 1$$
$$H_{sh} \approx 0.745 H_c, \quad \kappa >> 1$$



H. Padamsee et al., *RF Superconductivity for Accelerators* (Wiley&Sons, 1998)

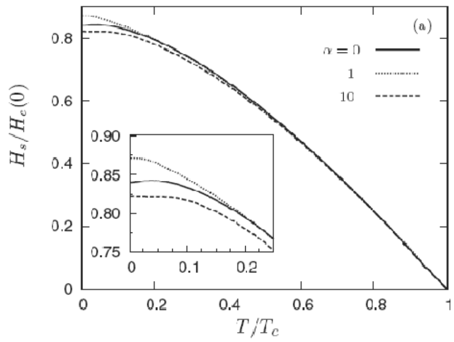
Time evolution of the spatial pattern of the order parameter in a small region around the boundary where a vortex entrance is taking place, calculated from time-dependent GL-equations.



A. D. Hernandez and D. Dominguez, Phys. Rev. B **65**, 144529 (2002)

Superheating field: theory

- Calculation of $H_{sh}(T, l)$ for $\kappa >> 1$ from Eilenberger equations ($0<T<T_c$) [Pei-Jen Lin and Gurevich (2012)]:



F. Pei-Jen Lin and A. Gurevich, Phys. Rev. B **85**, 054513 (2012)

$\alpha = \pi\xi_0/l$ Impurity scattering parameter

$$\boxed{H_{sh} \approx 0.845 H_c}$$

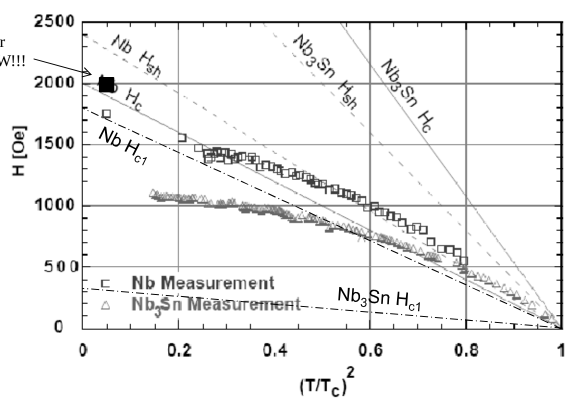
$$H_{sh}(T) \cong H_{sh}(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

- Weak dependence of H_{sh} on non-magnetic impurities

Superheating field: experimental results

- Use high-power (~1 MW) and short (~100 μs) RF pulses to achieve the metastable state before other loss mechanisms kick-in

Highest H_p ever measured in CW!!!

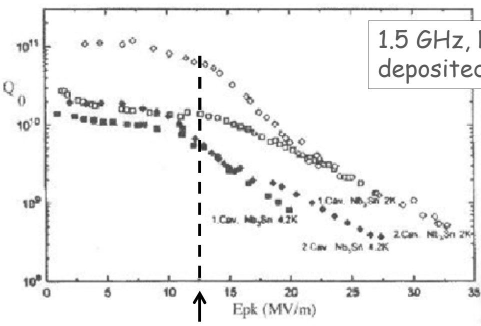


T. Hays and H. Padamsee, Proc. 1997 SRF Workshop, Abano Terme, Italy, p. 789 (1997) .

- RF magnetic fields higher than H_{c1} have been measured in both Nb and Nb_3Sn cavities. However max H_{RF} in Nb_3Sn is << predicted H_{sh}…

However…

- In "real" surfaces, the surface barrier can be easily suppressed locally by "defects" such as roughness or impurities, so that vortices may enter the sc already at $H_{RF} \approx H_{c1}$

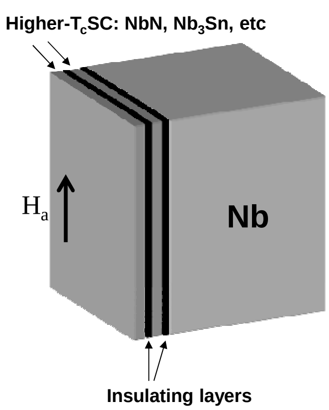


G. Muller et al., Proc. 1996 EPAC, Spain, p. 2085 (1996)

$H_p \sim 30$ mT $\sim H_{c1}$

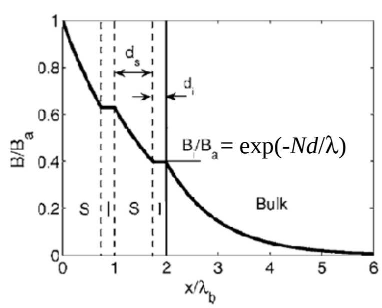
Multilayer Films

- If H_{c1} is indeed a major limit for RF application of high-κ materials, a possible solution consists of S-I-S multilayers [Gurevich (2006)]:



Suppression of vortex penetration due to the enhancement of H_{c1} in a thin film with d < λ [Abrikosov, (1964)]

$$H_{c1} = \frac{2\phi_0}{\pi d^2}\left(\ln\frac{d}{\xi} - 0.07\right)$$



A. Gurevich, Appl. Phys. Lett. **88**, 012511 (2006)

Alternatives to Nb

Material	T_c (K)	H_c [T]	H_{c1} [mT]	H_{c2} [T]	$\lambda(0)$ [nm]	Δ [meV]
Nb	9.2	0.2	170	0.4	40	1.5
$B_{0.6}K_{0.4}BiO_3$	31	~0.44	30	30	160	4.4
Nb_3Sn	18	~0.5	40	30	85	3.1
NbN	16.2	~0.23	20	15	200	2.6
MgB_2	40	~0.32	20-60	3.5-60	140	2.3; 7.1
$Ba_{0.6}K_{0.4}Fe_2As_2$	38	~0.5	20	>100	200	>5.2

Example: 4 layers 30 nm thick of Nb_3Sn on Nb $\rightarrow$ H_a up to ~400 mT [~$H_{sh}(Nb_3Sn)$] with H_i ~ 100 mT << H_{sh}(Nb)

- Global surface resistance: $R_s = \left(1 - e^{-2Nd/\lambda}\right) R_0 + e^{-2Nd/\lambda} R_b$

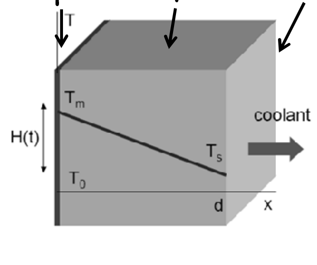
$R_0^{Nb3Sn}(2\text{ K}) \cong 0.1\, R_b^{Nb}(2\text{ K}) \rightarrow R_s \approx 0.15\, R_b$

Global thermal instability

- The exponential temperature dependence of $R_s(T)$ provides a strong positive feedback between RF Joule power and heat transport to the coolant $\rightarrow$ thermal instability above the breakdown field H_b

κ: thermal conductivity
h_K: Kapitza conductance

$$\frac{1}{2} R_s(T_m) H_0^2 = \int_{T_m}^{T_s} \kappa(T)\, dT = h_K(T_s, T_0)(T_s - T_0)$$



$T_m - T_0 << T_0$

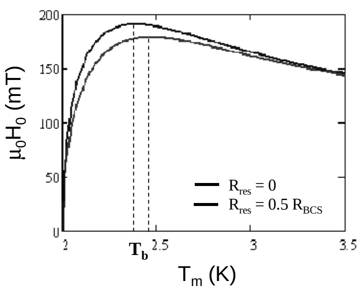
$$\frac{1}{2} R_s(T_m) H_0^2 = \frac{h_K \kappa}{\kappa + d h_K}(T_m - T_0)$$

= α, total thermal conductance

A. Gurevich, Physica C **441**, 38 (2006)

Uniform thermal breakdown field

$$R_s(T_m) \cong \frac{A\omega^2}{T_m} e^{-\Delta/kT_m} + R_{res} \qquad H_0^2 = \frac{2\kappa h_K T_m (T_m - T_0)}{(\kappa + h_K d)\left[A\omega^2 \exp(-\Delta/kT_m) + R_{res} T_m\right]}$$



d = 3 mm
f = 1.5 GHz
R_{BCS}(2 K) = 20 nΩ
h_K = 5 kW/m² K
κ = 10 W/m K
T_0 = 2 K
Δ/k = 17.1 K

$H_b = \max H_0(T_m) = H_0(T_b)$

$T_b - T_0 \approx T_0^2/\Delta T_c$ = 0.23 K for Nb

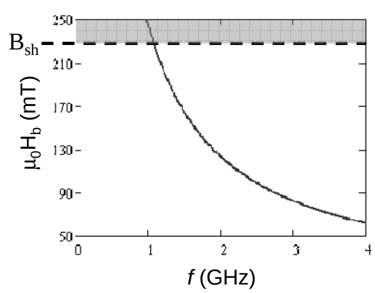
$R_{res} \cong 0$

← $\exp(-\Delta T_c/T_b) \approx \exp(-\Delta T_c/T_0 + 1)$

$$\boxed{H_b = \left[\frac{2kT_0^2 h_K \kappa}{(\kappa + d h_K)\Delta e R_{BCS}}\right]^{1/2}}$$

Uniform thermal breakdown field

Nb, 2 K, same parameters as previous slide



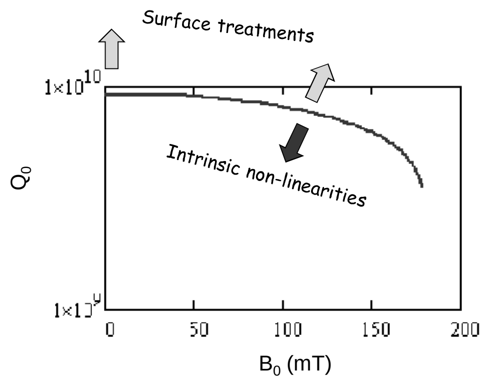
- Higher frequencies not only reduce the cavity Q_0 (↑ R_s) but also the breakdown field

In case of multilayers the thermal conductance is: $\alpha = \dfrac{h_K}{1 + h_K\left[\dfrac{d}{\kappa} + N\left(\dfrac{d_i}{\kappa_i} + \dfrac{d_s}{\kappa_s}\right)\right]}$

Nb_3Sn coating with Nd_s = 100 nm, $\kappa_s = 10^{-2}$ W/m K
Insulating Al_2O_3 layers, Nd_i = 10 nm, κ_i = 0.3 W/m K
$d_i/\kappa_i = 1/300(d_s/\kappa_s)$ → Insulating layers are negligible
$d/\kappa = 3Nd_s/\kappa_s$ → TFML adds ~30% to the thermal resistance of the Nb shell

$Q_0(B_0)$ curve

- Because of the $T_m(H_0)$ dependence, R_s acquires a H_0-dependence



Nb, 2 K, 1.5 GHz, G=280 Ω, R_s and thermal parameters as in previous slides

Summary

- Unlike in the DC case, dissipation occurs in SC in RF because of the inertia of Cooper-pairs
- The surface resistance can be easily understood in terms of a two-fluid model and is due to the interaction of the E-field (decaying from the surface) with thermally excited normal electrons
- $R_s = R_{BCS} + R_{res}$
  - R_{res}:
    - Residual DC magnetic field
    - Normal-conducting precipitates (NbH)
    - …
  - R_{BCS}:
    - Increases quadratically with frequency
    - Decreases exponentially with temperature
    - Has a minimum as a function of material purity
- The maximum theoretical RF field on the surface of a SC is the superheating field ≈ thermodynamic critical field
- Multilayer films may be a practical way to reach H_{sh} in SC with higher T_c than Nb
- Thermal feedback couples R_s at low field to the breakdown field

Acknowledgments & References

- Inspiration and material for this lecture was taken from earlier ones from: Prof. A. Gurevich, ODU; Prof. Steven M. Anlage, UMD; Prof. J. Knobloch, BESSY; Prof. H. Padamsee, Cornell U.

- **Tutorials** on SRF can be found on the webpages of SRF Workshops: http://accelconf.web.cern.ch/accelconf/
- Recommended **references**:

  - M. Tinkham, *Introduction to Superconductivity*, McGraw-Hill, New York, 2nd edition, 1996
  - H. Padamsee, J. Knobloch and T. Hays, *RF Superconductivity for Accelerators*, J. Wiley & Sons, New York, 1998
  - H. Padamsee, *RF Superconductivity. Science, Technology, and Applications*, Wiley-VCH, Weinheim, 2009
  - A. Gurevich, "Superconducting Radio-Frequency Fundamentals for Particle Accelerators", *Rev. Accel. Sci. Tech*. **5**, 119 (2012)