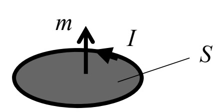
Superconductor dynamics

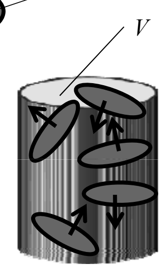
Fedor Gömöry
Institute of Electrical Engineering
Slovak Academy of Sciences
Dubravska cesta 9, 84101 Bratislava, Slovakia
elekgomo@savba.sk www.elu.sav.sk

Some useful formulas:



magnetic moment of a current loop

$$\vec{m} = I\vec{S} \quad [\mathrm{Am}^2]$$



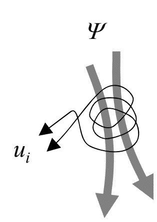
magnetization of a sample $\vec{M} = \frac{\sum \vec{m}}{V}$ [A/m]

alternative (preferred in SC community) $\vec{M} = \mu_0 \frac{\sum \vec{m}}{V}$ **[T]**

Measurable quantities:

magnetic field B [T] – *Hall probe, NMR*

voltage from a pick-up coil [V]



$$u_i = -\frac{\mathrm{d}\Psi}{\mathrm{d}t} \approx -N\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -NS\frac{\mathrm{d}\bar{B}}{\mathrm{d}t}$$

linked magnetic flux (Ψ), *number of turns* (N), *area of single turn* (S), *average of magnetic field* ($\bar{B}$)

Outline:

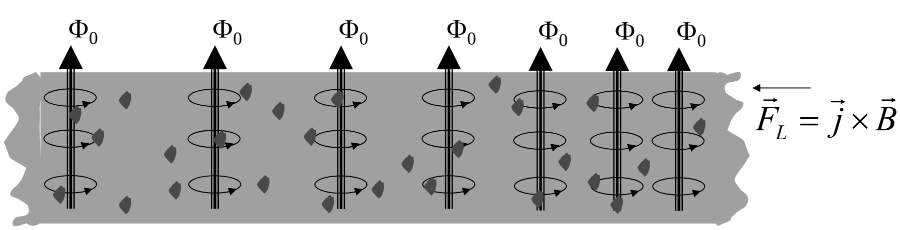
1. Hard superconductor in varying magnetic field
2. Magnetization currents: Flux pinning
   Coupling currents
3. Possibilities for reduction of magnetization currents
4. Methods to measure magnetization and AC loss

Superconductors used in magnets - what is essential?

pinning of magnetic flux

mechanism(s) hindering the change of magnetic field distribution

type II. superconductor with flux pinning = hard superconductor



pinning of flux quanta

gradient in the flux density $\frac{\partial B_z}{\partial x} = -\mu_0 j_y$

distribution persists in static regime (DC field), but would require a work to be changed

=> dissipation in dynamic regime

(repulsive) interaction of flux quanta
=> flux line lattice

$\Phi_0 = 2\times10^{-15}$ Vs

$B = \frac{\Phi_0}{a^2}$

$a = \sqrt{\frac{\Phi_0}{B}}$

summation of microscopic pinning forces
+ elasticity of the flux line lattice
= macroscopic pinning force density F_p [N/m^3]

$B = 1$ T;

$a = 45$ nm

macroscopic behavior described by the critical state model [Bean 1964]:
local density of electrical current in hard superconductor is either 0 or the critical current density, j_c

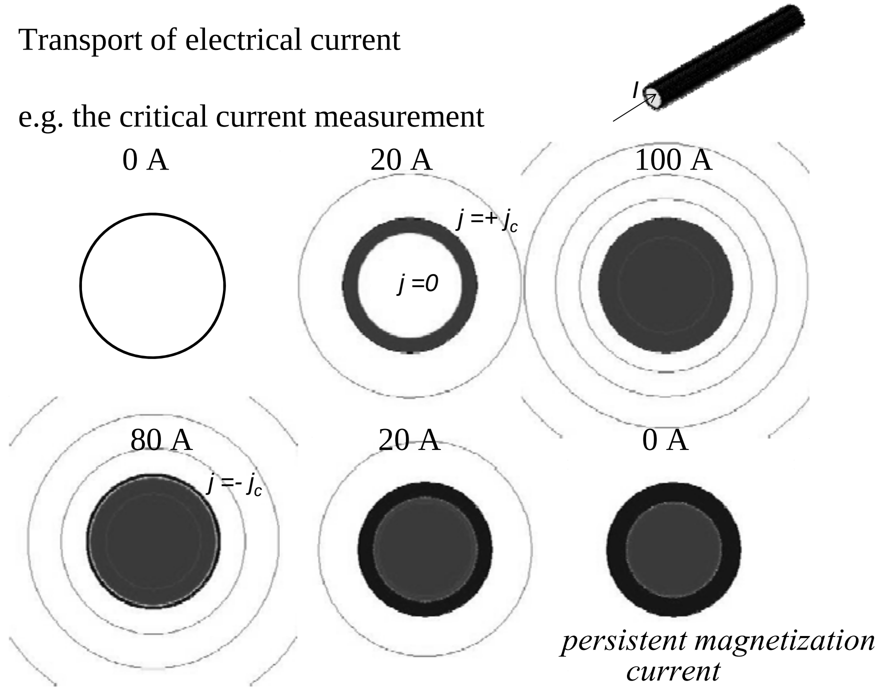
in the simplest version (first approximation) j_c =const.

Outline:

1. Hard superconductor in varying magnetic field
2. Magnetization currents: Flux pinning, Coupling currents
3. Possibilities for reduction of magnetization currents
4. Methods to measure magnetization and AC loss

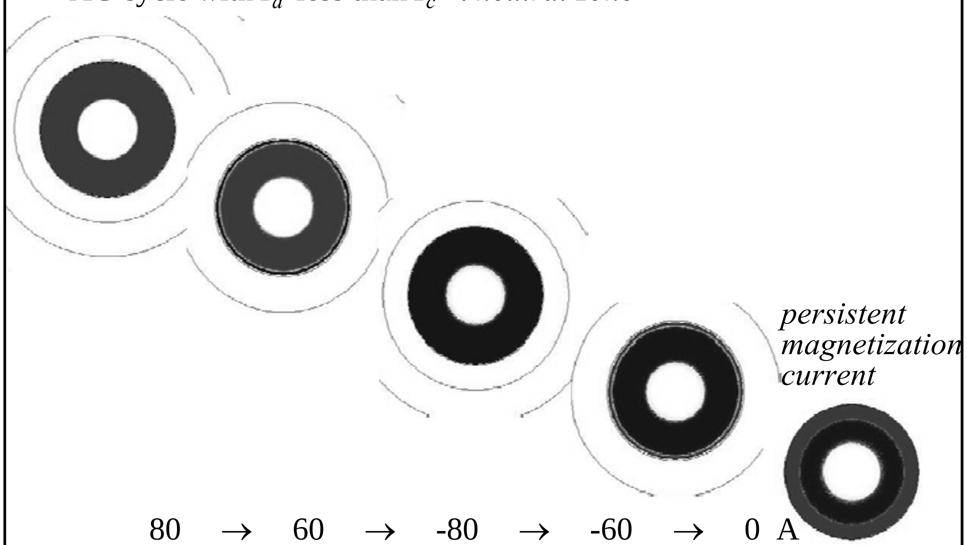
Transport of electrical current

e.g. the critical current measurement



Transport of electrical current

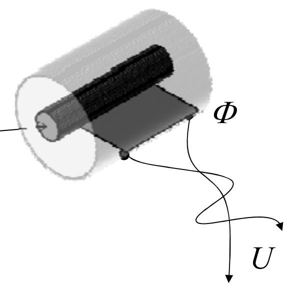
AC cycle with I_a less than I_c : *neutral zone*



AC transport in hard superconductor is not dissipation-less (AC loss)

$$Q = \int_T IUdt = -\int Id\Phi$$

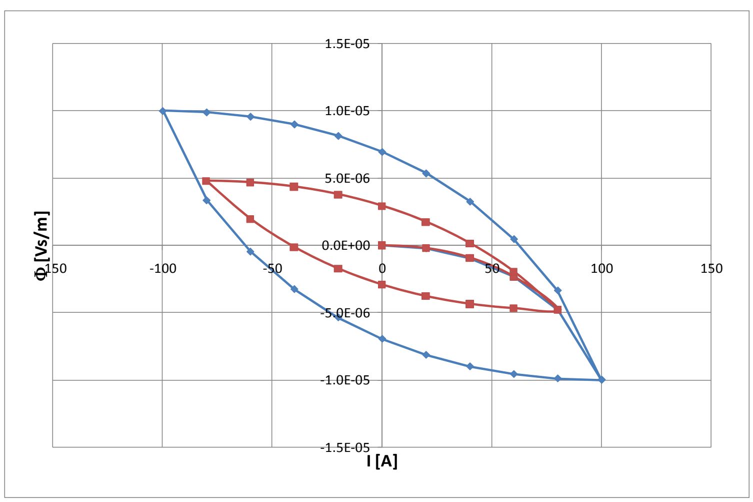
neutral zone:
$j = 0$, $E = 0$



$$U = -\frac{\partial \Phi}{\partial t}$$

check for hysteresis in I *vs.* Φ plot

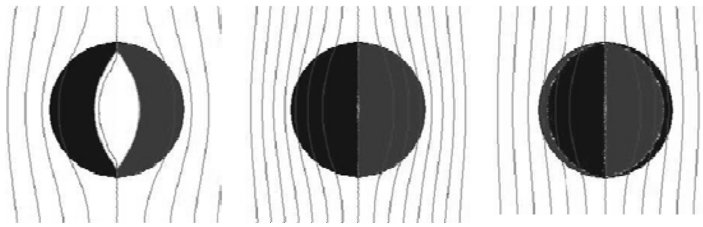
AC transport loss in hard superconductor



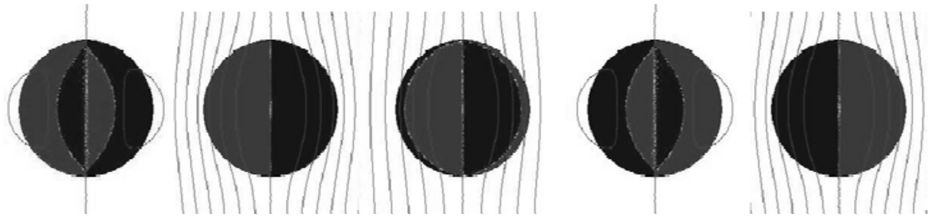
hysteresis $\rightarrow$ dissipation $\rightarrow$ **AC loss**

Hard superconductor in changing magnetic field

0 → 30 → 50 → 40 mT

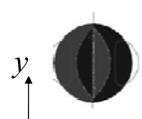


→ 0 → -50 → -40 → 0 → 50 mT



Hard superconductor in changing magnetic field

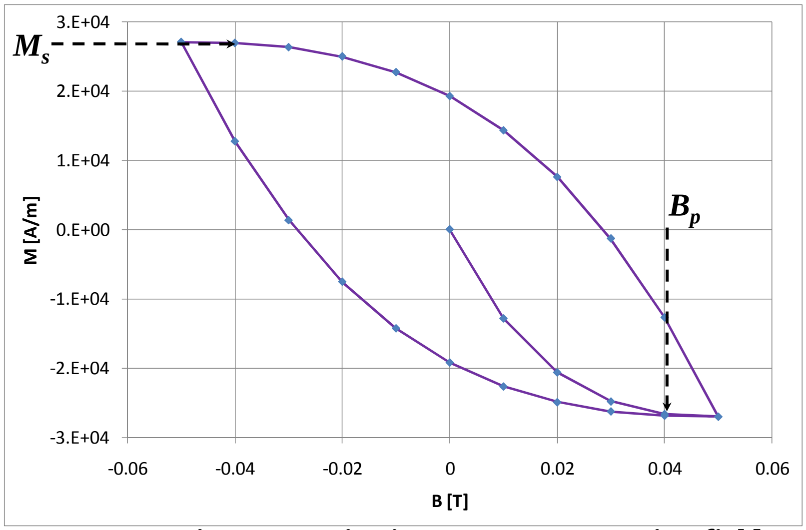
dissipation because of flux pinning



volume loss density Q [J/m^3]
$$\frac{Q}{V} = \oint B_a \mathrm{d}M$$

magnetization:
$$M = \int_S -x.j(x,y)\mathrm{d}x\mathrm{d}y$$

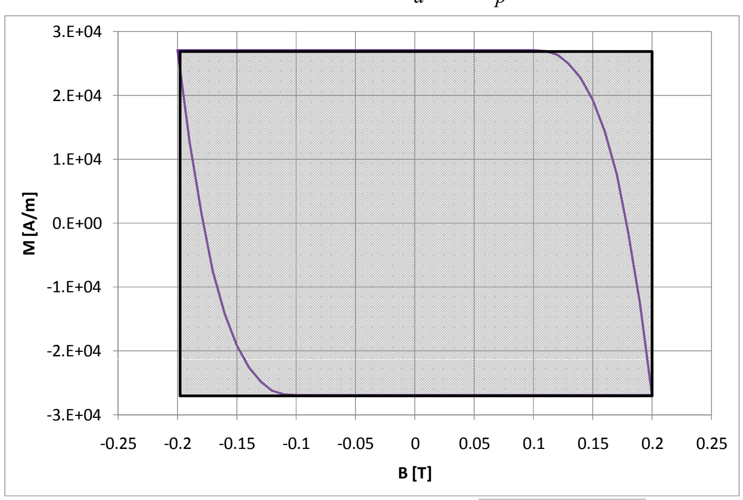
Round wire from hard superconductor in changing magnetic field



M_s saturation magnetization, B_p penetration field

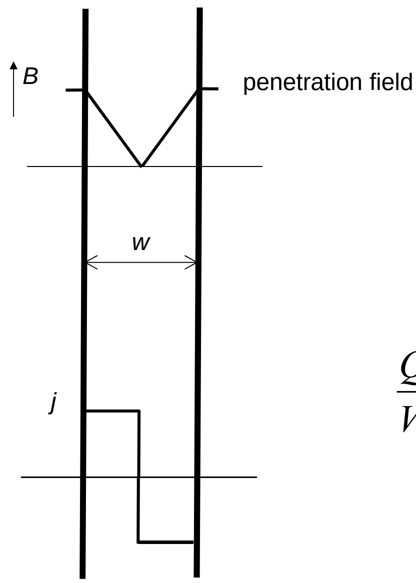
Round wire from hard superconductor in changing magnetic field

estimation of AC loss at $B_a >> B_p$



$$\frac{Q}{V} \approx 4 B_a M_s$$

Slab in parallel magnetic field – analytical solution



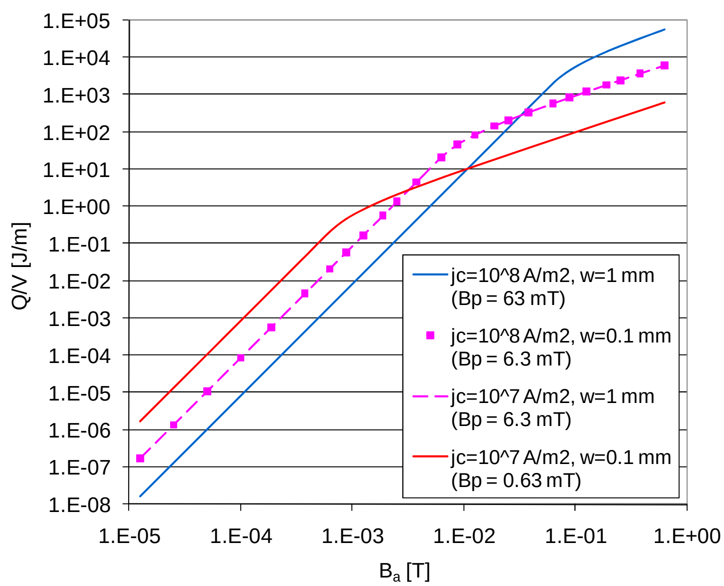
$$B_p = \mu_0 j_c \frac{w}{2}$$

$$M_s = j_c \frac{w}{4} = \frac{B_p}{2\mu_0}$$

$$\frac{Q}{V} = \frac{1}{\mu_0}\begin{cases} \dfrac{2}{3}\dfrac{B_a^3}{B_p} \\ 2B_pB_a - \dfrac{4}{3}B_p^2 \end{cases}$$

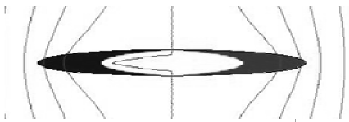
$$Q \approx 4B_aM_s$$

Slab in parallel magnetic field – analytical solution

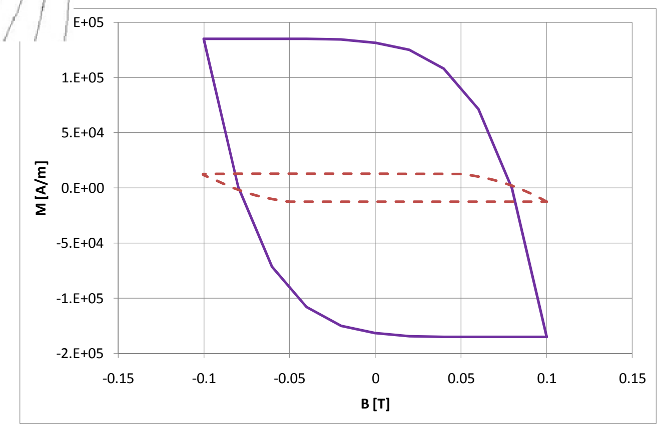


$$Q \approx 4B_aM_s$$

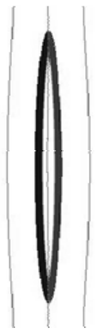
effect of the field orientation



perpendicular field

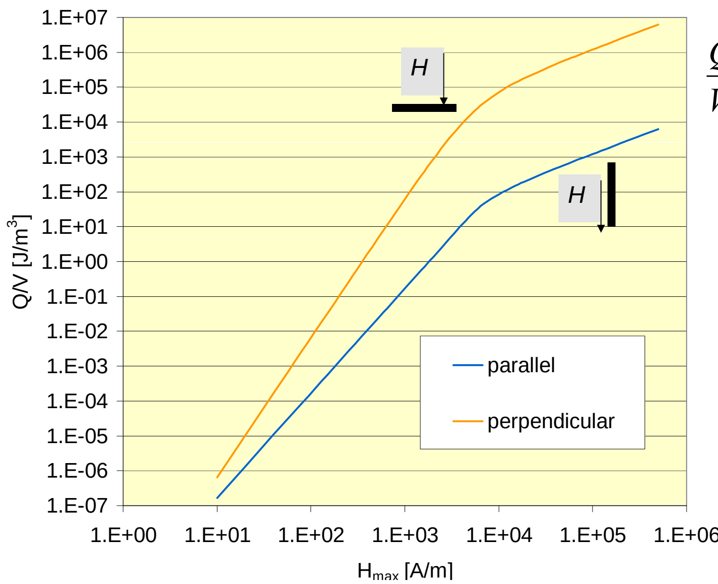


parallel field



$$\frac{Q}{V} \approx 4 B_a M_s$$

Magnetization loss in strip with aspect ratio 1:1000

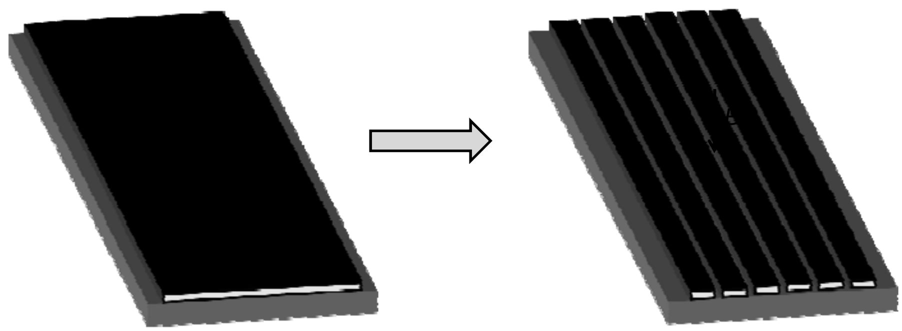


$$\frac{Q}{V} \approx 4 B_a M_s$$

in the case of flat wire or cable the orientation is not a free parameter

= reduction of the width

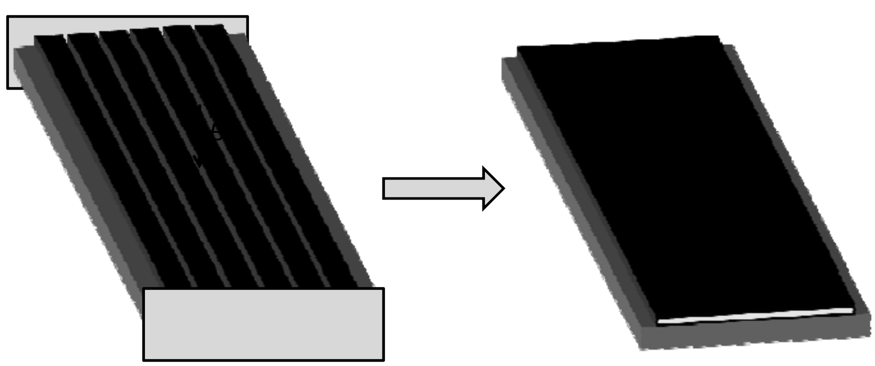
e.g. striation of CC tapes



~ 6 times lower magnetization

striation of CC tapes

but in operation the filaments are connected at magnet terminations

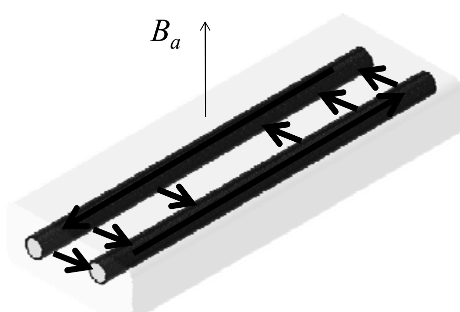


coupling currents will appear

Outline:

1. Hard superconductor in varying magnetic field
2. Magnetization currents: Flux pinning
   Coupling currents
3. Possibilities for reduction of magnetization currents
4. Methods to measure magnetization and AC loss

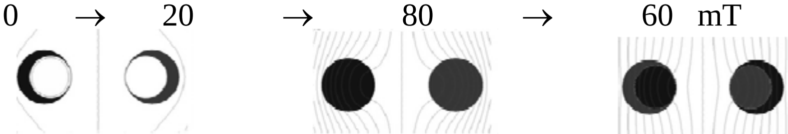
Two parallel superconducting wires in metallic matrix



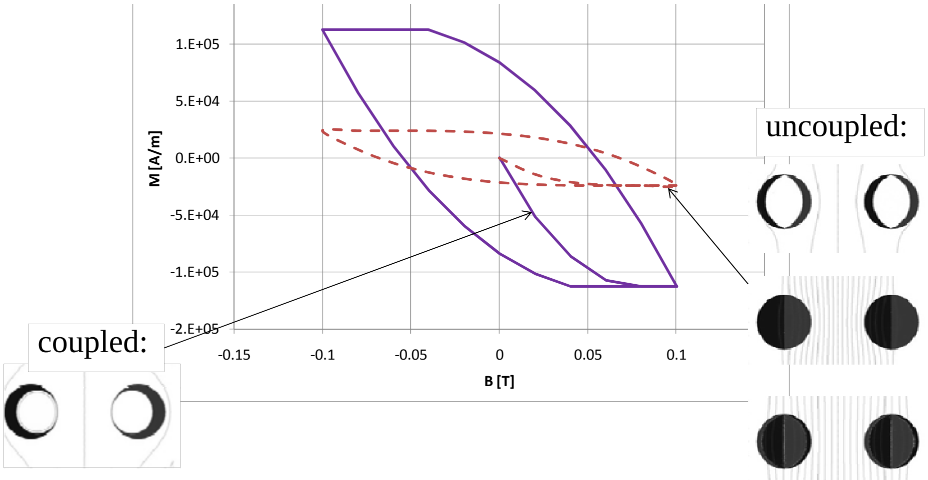
coupling currents

in the case of a perfect coupling:

0 → 20 → 80 → 60 mT

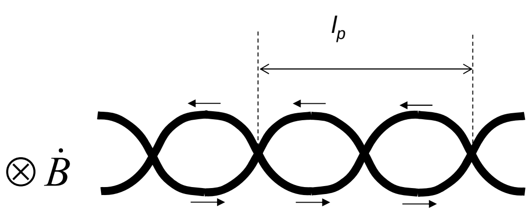


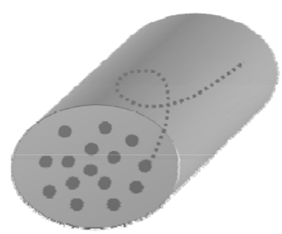
Magnetization of two parallel wires

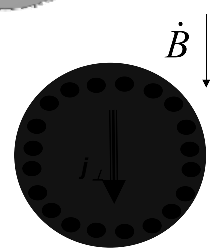


we need to reduce the coupling currents !

Composite wires – twisted filaments







good interfaces $\rho_t = \rho_m \dfrac{1-\lambda}{1+\lambda}$

bad interfaces $\rho_t = \rho_m \dfrac{1+\lambda}{1-\lambda}$

$$\lambda = \frac{S_{SC}}{S_m}$$

$$j_\perp = \frac{l_p \dot{B}}{2\pi\rho_t}$$

Composite wires – twisted filaments

coupling currents (partially) screen the applied field

$$B_i = B - \tau \dot{B}$$

τ - time constant of magnetic flux diffusion

$$\tau = \frac{\mu_0}{2\rho_t}\left(\frac{l_p}{2\pi}\right)^2$$

A.Campbell (1982) Cryogenics 22 3
K. Kwasnitza, S. Clerc (1994) Physica C 233 423
K. Kwasnitza, S. Clerc, R. Flukiger, Y. Huang (1999) Cryogenics 39 829

$$\frac{Q}{V} = \frac{B_{\max}^2}{\mu_0}\frac{2\pi\omega\tau}{1+\omega^2\tau^2}$$

round wire

$$\frac{Q}{V} = \frac{B_{\max}^2}{\mu_0}\frac{\chi_0\pi\omega\tau}{1+\omega^2\tau^2}$$

flat wire

$$\tau = \tau_{round}\frac{\chi_0}{2}A$$

Outline:

1. Hard superconductor in varying magnetic field
2. Magnetization currents: Flux pinning
   Coupling currents
3. Possibilities for reduction of magnetization currents
4. Methods to measure magnetization and AC loss

Persistent currents:

at large fields proportional to $B_p \sim j_c\, w$ ← width of superconductor (perpendicular to the applied magnetic field)

= magnetization reduction by either lower j_c or reduced w

lowering of j_c would mean more superconducting material required to transport the same current

thus only plausible way is the reduction of w

Coupling currents:

at low frequencies proportional to the time constant of magnetic flux diffusion

$$\tau = \frac{\mu_0}{2\rho_t}\left(\frac{l_p}{2\pi}\right)^2$$

l_p – transposition length

ρ_t – effective resistivity

= filaments (in single tape) or strands (in a cable) should be transposed

= low loss requires high inter-filament or inter-strand resistivity

but good stability needs the opposite

Outline:

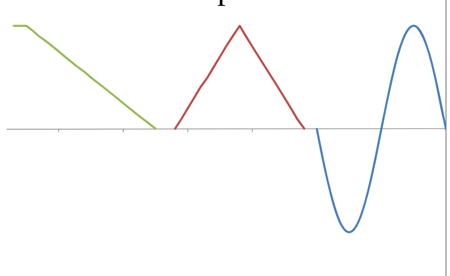
1. Hard superconductor in varying magnetic field
2. Magnetization currents: Flux pinning
   Coupling currents
3. Possibilities for reduction of magnetization currents
4. Methods to measure magnetization and AC loss

Different methods necessary to investigate

- Wire (strand, tape)
- Cable
- Magnet

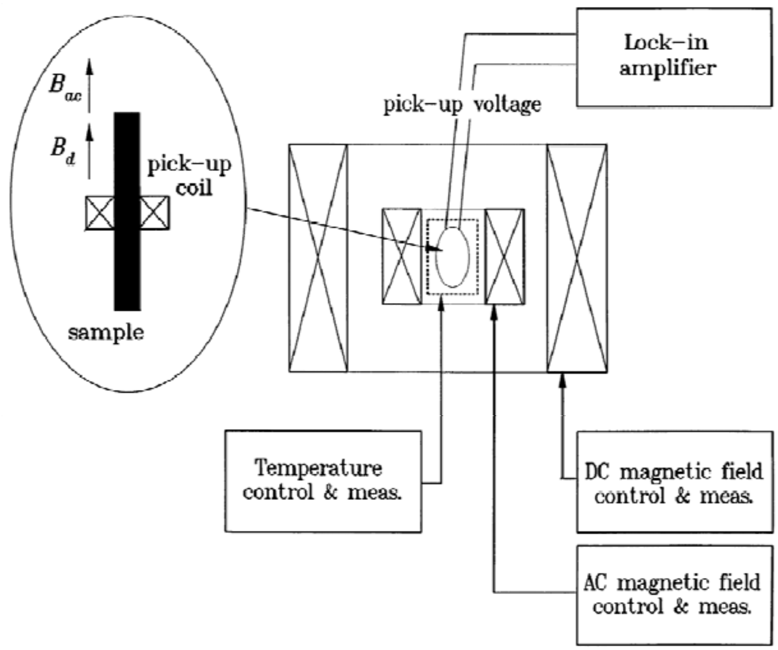
shape of the excitation field (current) pulse

transition unipolar harmonic



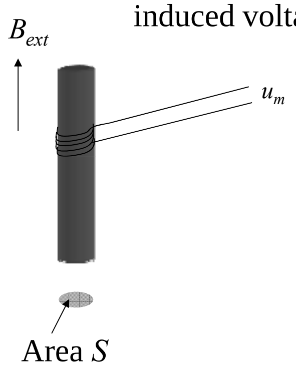
relevant information can be achieved in harmonic regime

final testing necessary in actual regime ideal magnetization loss measurement:



pick-up coil wrapped around the sample

induced voltage $u_m(t)$ in one turn:



$$u_m(t) = -\frac{\mathrm{d}\phi_m(t)}{\mathrm{d}t} = -S\frac{\mathrm{d}\overline{B}(t)}{\mathrm{d}t}$$

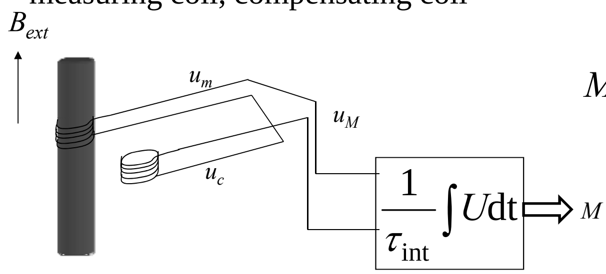
$$\overline{B}(t) = \frac{1}{S}\int_S B_{\mathrm{int}}(t)\mathrm{d}S = B_{ext}(t) + M(t)$$

$$u_m(t) = -S\left[\frac{dB_{ext}(t)}{dt} + \frac{dM(t)}{dt}\right]$$

pick-up coil voltage processed by integration
either numerical or by an electronic integrator:

$$M(t) = -\frac{1}{S}\int u_m(t)\mathrm{d}t - B_{ext}(t)$$

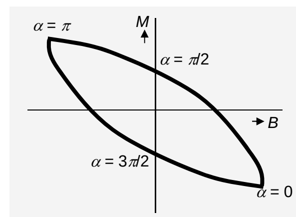
Method 1: double pick-up coil system with an electronic integrator : measuring coil, compensating coil



$$M(t) = -\frac{1}{S}\int u_M(t)\mathrm{d}t$$

AC loss in one magnetization cycle [J/m^3]:

$$Q = \frac{1}{\mu_0}\oint B\mathrm{d}M = \frac{1}{\mu_0}\int_0^T B(t)\frac{\mathrm{d}M}{\mathrm{d}t}\mathrm{d}t$$



Harmonic AC excitation – use of complex susceptibilities

$$B_{ext}(t) = B_a \cos \omega t$$

$$M(t) = B_a \sum_{n=1}^{\infty}\left(\chi_n'\cos n\omega t + \chi_n''\sin n\omega t\right)$$

fundamental component $n = 1$

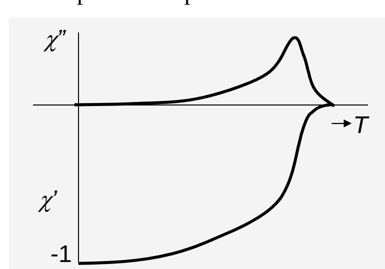
AC loss per cycle

$$W_q = -\pi\chi''\frac{B_a^2}{\mu_0}$$

energy of magnetic shielding

$$W_m = \chi'\frac{B_a^2}{2\mu_0}$$

Temperature dependence:



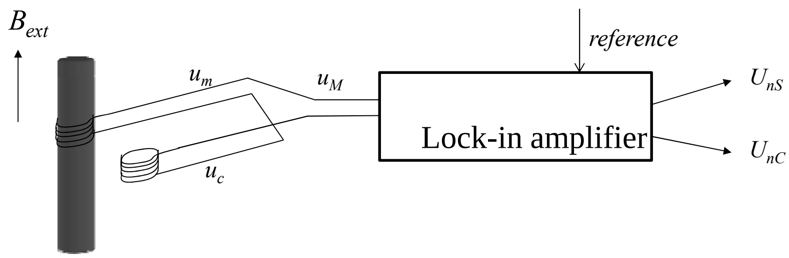
Method 2: Lock-in amplifier
– phase sensitive analysis of voltage signal spectrum
in-phase and out-of-phase signals

$$B_{ext} = B_a \cos \omega t$$

$$U_{nS} = \frac{1}{\pi} \int_0^{2\pi} u_M(t) \sin n\omega t \, \mathrm{d}\omega t$$

$$U_{nC} = \frac{1}{\pi} \int_0^{2\pi} u_M(t) \cos n\omega t \, \mathrm{d}\omega t$$

reference signal necessary to set the
frequency
phase
taken from the current energizing the
AC field coil



Method 2: Lock-in amplifier – only at harmonic AC excitation

$$B_{ext} = B_a \cos \omega t$$

$$u_M(t) = S\omega B_a \left[\sin \omega t + \sum_{n=1}^{\infty} n\left(\chi_n' \sin n\omega t - \chi_n'' \cos n\omega t\right) - \sin \omega t \right]$$

empty coil — sample magnetization — compensation coil

fundamental susceptibility

$$\chi' = \frac{U_{1S}}{S\omega B_a} = \frac{U_{1S}}{U_N}$$

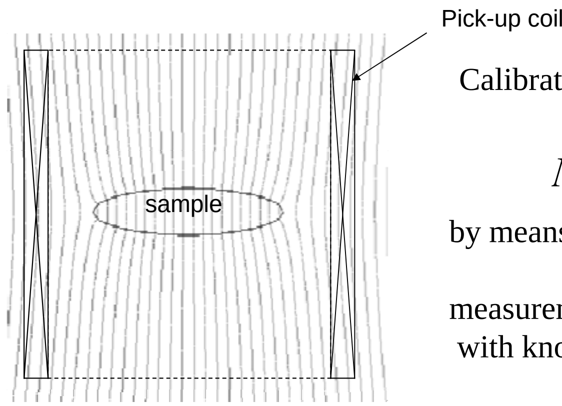
$$\chi'' = \frac{-U_{1C}}{S\omega B_a} = \frac{-U_{1C}}{U_N}$$

higher harmonic susceptibilities

$$\chi_n' = \frac{U_{nS}}{nU_N}$$

$$\chi_n'' = \frac{U_{nC}}{nU_N}$$

Real magnetization loss measurement:



Calibration necessary

$$M = C\int u\mathrm{d}t$$

by means of:

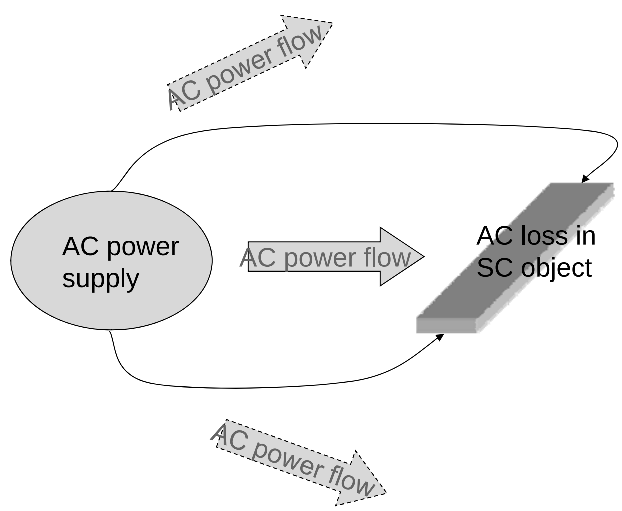
measurement on a sample with known properrties

calibration coil

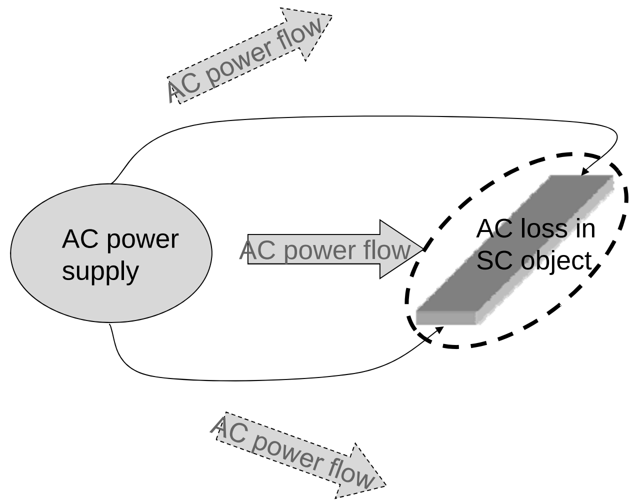
numerical calculation

…

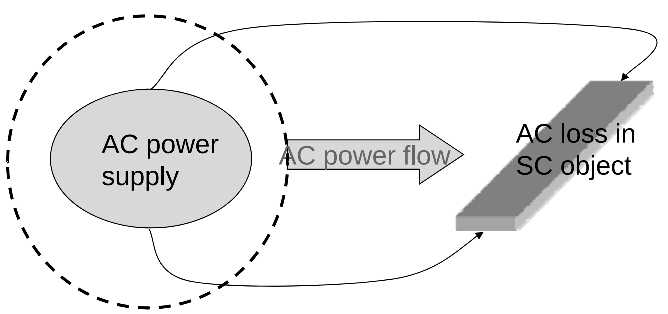
AC loss can be determined from the balance of energy flows



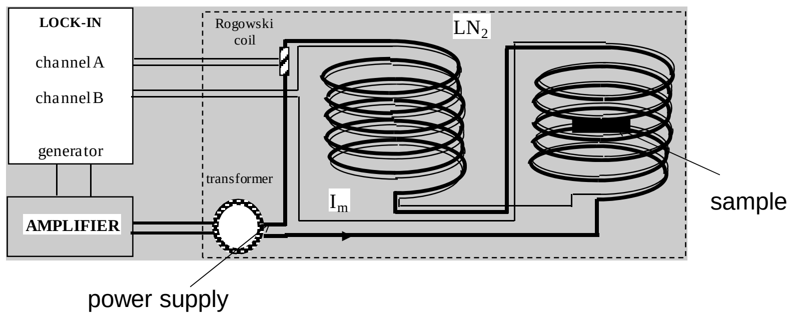
Solution 1- detection of power flow to the sample



Solution 2- elimination of parasitic power flows



Loss measurement from the side of AC power supply:



$$P_{sample} = I_m U_B$$

Loss measurement from the side of AC power supply:

Ψ(I) hysteresis loop registration for superconducting magnet (Wilson 1969)