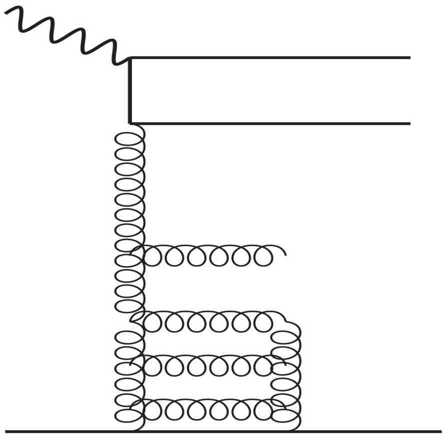
Triple pomeron vertex in double logarithmic limit

Krzysztof Kutak

The triple pomeron vertex -momentum space

The LO momentum space expression for the 2 → 4 Vertex derived in connection with gamma + q → (q q + n gluons) + q

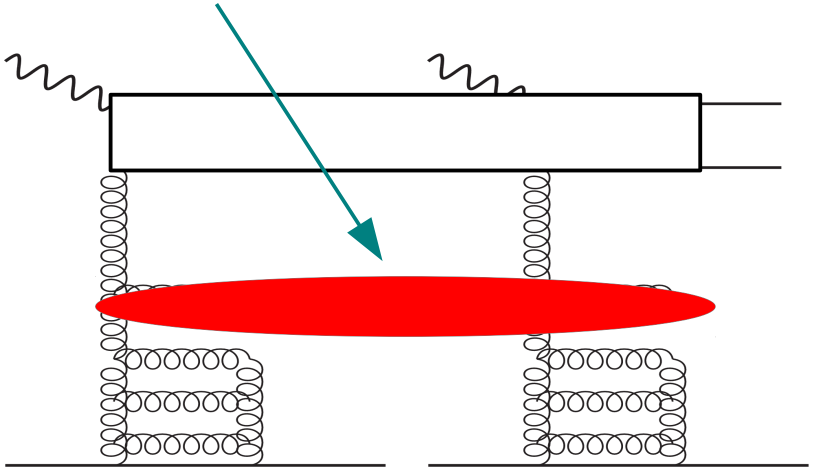


Gribov,Levin, Ruskin, Phys.Rept. 100 (1983) 1-150

Only connected pieces

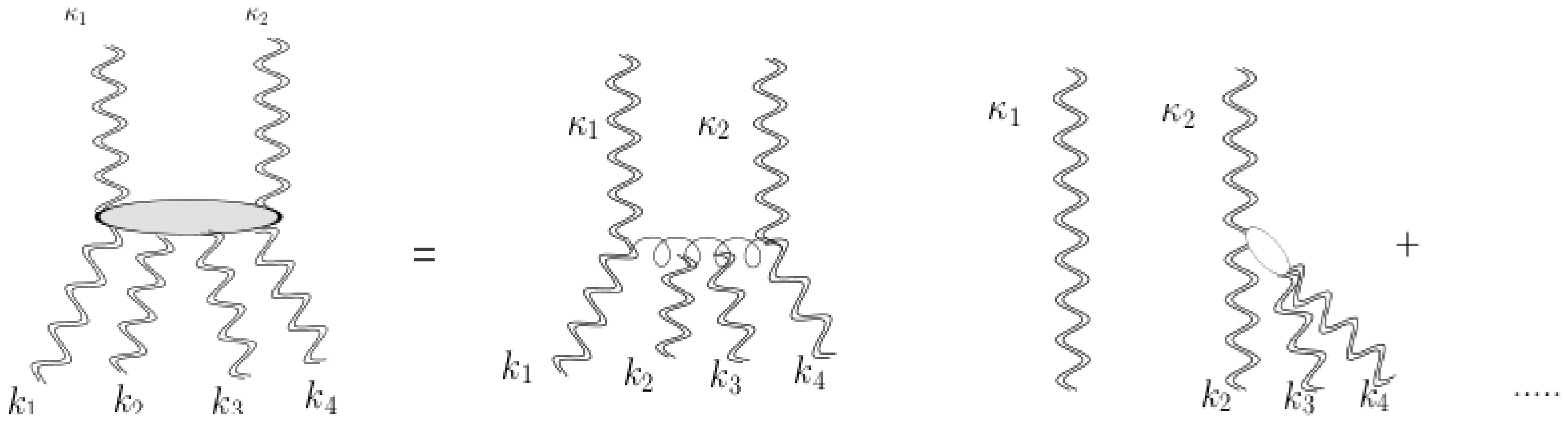
Bartels, Wusthoff Z.Phys. C66 (1995) 157-180

Connected and disconnected



The study lead to the so called Diffractive Triple Pomeron Vertex and to the Triple Pomeron Vertex

The triple pomeron vertex in the momentum space



$$\mathcal{V}_{a_1'a_2';a_1a_2a_3a_4}(\kappa_1,\kappa_2;k_1,k_2,k_3,k_4) = \frac{\sqrt{2}\pi\delta_{a_1'a_2'}}{\sqrt{N_c^2-1}}\Big[\delta_{a_1a_2}\delta_{a_3a_4}V(1,2,3,4)$$

$$+\delta_{a_1a_3}\delta_{a_2a_4}V(1,3,2,4)+\delta_{a_1a_4}\delta_{a_2a_3}V(1,2,3,4)\Big]$$

- Zero if any momenta is zero.
  - Conformal invariant w.r.t Mobius transformations
    - Anticollinear pole dominates

Bartels, Wusthoff Z.Phys. C66 (1995) 157-180

The triple pomeron vertex -momentum space

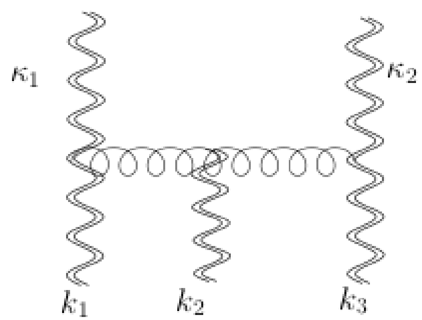
Vertex can be expressed in term of a building block called a G function

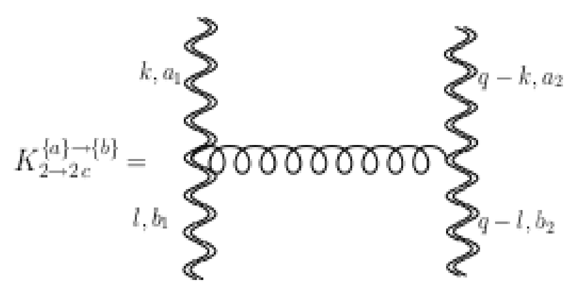
$$V(\boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = \frac{1}{2} g^4 \Big[G(\boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2, \mathbf{k}_1, \mathbf{k}_2 + \mathbf{k}_3, \mathbf{k}_4) + ... \Big]$$

$$G(\boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = G_1(\boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + G_2(\boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

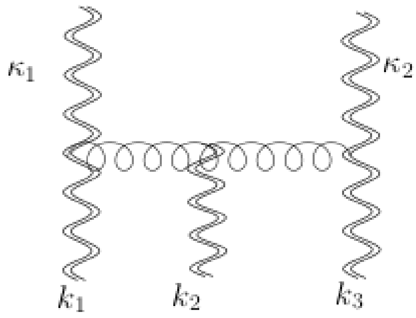
The relation with the BFKL is

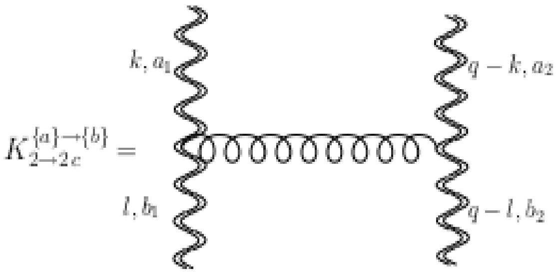
$$G(\boldsymbol{\kappa}_1, -\boldsymbol{\kappa}_1, \mathbf{k}_1, -, -\mathbf{k}_1) \rightarrow K_{BFKL}(\boldsymbol{\kappa}_1, -\boldsymbol{\kappa}_1; \mathbf{k}_1, -\mathbf{k}_1)$$





Structure of the vertex

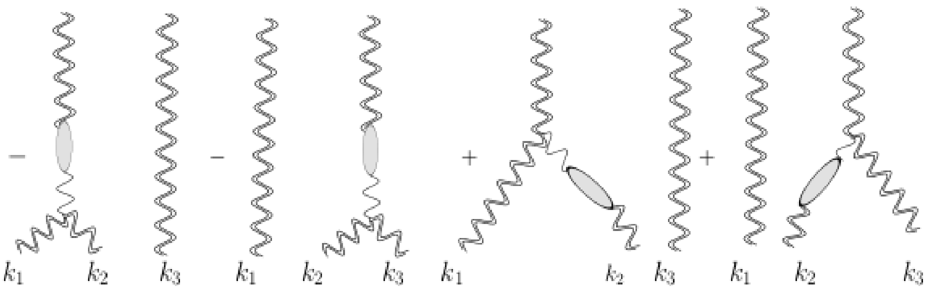


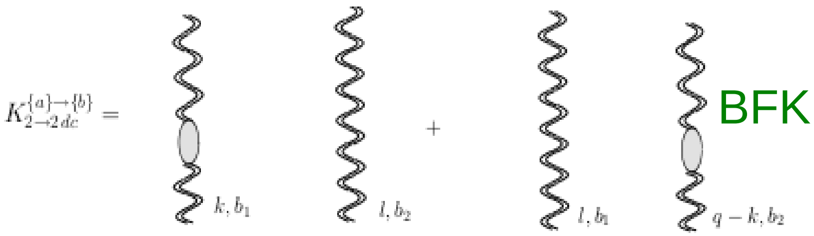


BFKL real

connected piece

$$G_1(\boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{(\mathbf{k}_2+\mathbf{k}_3)^2\boldsymbol{\kappa}_1^2}{(\boldsymbol{\kappa}_1-\mathbf{k}_1)^2} + \frac{(\mathbf{k}_1+\mathbf{k}_2)^2\boldsymbol{\kappa}_2^2}{(\boldsymbol{\kappa}_2-\mathbf{k}_3)^2} - \frac{\mathbf{k}_2^2\boldsymbol{\kappa}_1^2\boldsymbol{\kappa}_2^2}{(\boldsymbol{\kappa}_1-\mathbf{k}_1)^2(\boldsymbol{\kappa}_2-\mathbf{k}_3)^2} - (\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)^2$$



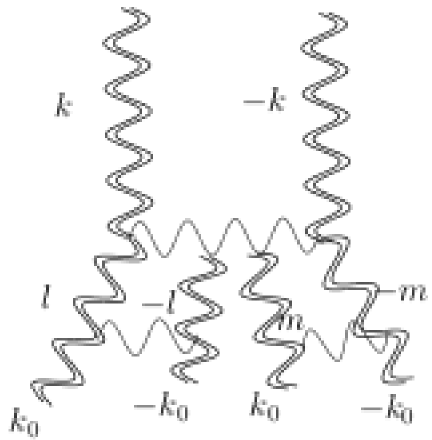


BFKL virt.

disconnected piece

$$G_2(\boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \quad = \quad -\frac{\boldsymbol{\kappa}_1^2\boldsymbol{\kappa}_2^2}{N_c}\Big([\omega(\mathbf{k}_2)-\omega(\mathbf{k}_2+\mathbf{k}_3)]\delta^{(2)}(\boldsymbol{\kappa}_1-\mathbf{k}_1) + [\omega(\mathbf{k}_2)-\omega(\mathbf{k}_1+\mathbf{k}_2)]\delta^{(2)}(\boldsymbol{\kappa}_1-\mathbf{k}_3)\Big)$$

Collinear limit of the TPV – leading Nc



largest

smallest

Bartels, Kutak Eur.Phys.J. C53 (2008) 533-548

$$\mathcal{V}_{LON_c}^{r\{a'\}\{b\}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)^{\tau=4} = \delta^{a'_1,a'_2}\delta^{b_1,b_2}\delta^{b_3,b_4}\frac{\sqrt{2}\pi}{N_c^2-1}\frac{g^4}{2}2\mathbf{k}^2\frac{4(-\mathbf{k}_1\cdot\mathbf{k}_2\,\mathbf{k}_3\cdot\mathbf{k}_4+\mathbf{k}_1\cdot\mathbf{k}_4\,\mathbf{k}_2\cdot\mathbf{k}_3+\mathbf{k}_1\cdot\mathbf{k}_3\,\mathbf{k}_2\cdot\mathbf{k}_4)}{\mathbf{k}^4}$$

$$\mathbf{k}_1=-\mathbf{k}_2,\quad \mathbf{k}_3=-\mathbf{k}_4$$

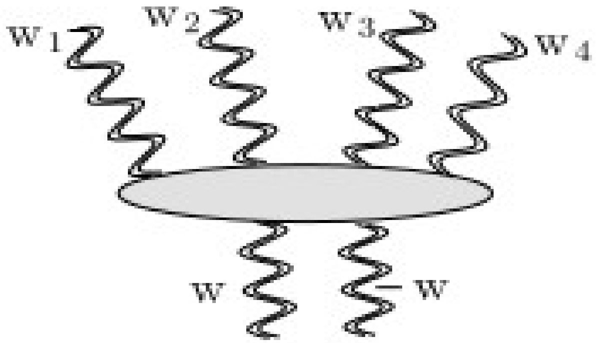
$$K(\mathbf{q}_1,\mathbf{q}_2;\mathbf{k}_1,\mathbf{k}_2) = -N_c g^2\left[(\mathbf{k}_1+\mathbf{k}_2)^2-\frac{\mathbf{q}_2^2\mathbf{k}_1^2}{(\mathbf{k}_2-\mathbf{q}_2)^2}-\frac{\mathbf{q}_1^2\mathbf{k}_2^2}{(\mathbf{k}_1-\mathbf{q}_1)^2}\right]$$

$$\mathbf{k}_1=\mathbf{l},\ \mathbf{k}_2=-\mathbf{l},\ \mathbf{k}_3=\mathbf{m},\ \mathbf{k}_4=-\mathbf{m}$$

$$(K\,K)\otimes\mathcal{V}_{LON_c}^{r\{a'\}\{b\}\,\tau=4} = \delta^{a'_1,a'_2}\delta^{b_1,b_2}\delta^{b_3,b_4}\frac{\sqrt{2}\pi}{N_c^2-1}N_c^2 2\mathbf{k}^2\frac{g^8}{2}\cdot$$

$$\int_{\mathbf{k}_0^2}^{\mathbf{k}^2}\frac{d^2\mathbf{l}}{(2\pi)^3}\int_{\mathbf{k}_0^2}^{\mathbf{k}^2}\frac{d^2\mathbf{m}}{(2\pi)^3}\frac{2\mathbf{k}_0^2}{\mathbf{l}^4}\frac{2\mathbf{k}_0^2}{\mathbf{m}^4}\frac{4(2(\mathbf{l}\cdot\mathbf{m})^2-\mathbf{l}^2\mathbf{m}^2)}{\mathbf{k}^4} \quad =0$$

Antocollinear limit



$$|\mathbf{w}| \ll |\mathbf{w}_1|, |\mathbf{w}_2|, |\mathbf{w}_3|, |\mathbf{w}_4|.$$

$$\mathcal{V}^{\mathrm{r}\{a'\}}_{\mathrm{LO}N_c}(\mathbf{p}, -\mathbf{p}-\mathbf{r}, \mathbf{q}, -\mathbf{q}+\mathbf{r})^{\mathrm{leading}} = \delta^{a'_1,a'_2}\delta^{b_1,b_2}\delta^{b_3,b_4}\frac{\sqrt{2}\pi}{N_c^2-1}\frac{g^4}{2}2\mathbf{w}^2$$

The TPV has non zero anticollinear limit. This property prevents the diffusion to small Momenta when the TPV is inserted in the BK equation.

All order result - connected part

First we split the connected part of G function into master integrals

$$G_1(\mathbf{l},\mathbf{m}) = \frac{\mathbf{k}^2\mathbf{l}^2}{(\mathbf{k}-\mathbf{l})^2} + \frac{\mathbf{k}^2\mathbf{m}^2}{(\mathbf{k}+\mathbf{m})^2} - \frac{\mathbf{k}^4(\mathbf{l}+\mathbf{m})^2}{(\mathbf{k}-\mathbf{l})^2(\mathbf{k}+\mathbf{m})^2}$$

$$I_B = \frac{1}{(2\pi)^2}\int_0^{2\pi} d\alpha \int_0^{2\pi} d\beta\, B = \frac{\mathbf{k}^2\mathbf{m}^2}{|\mathbf{m}^2-\mathbf{k}^2|}$$

$$I_A = \frac{1}{(2\pi)^2}\int_0^{2\pi} d\alpha \int_0^{2\pi} d\beta\, A = \frac{\mathbf{k}^2\mathbf{l}^2}{|\mathbf{l}^2-\mathbf{k}^2|}$$

$$I_{C_1} = \frac{1}{(2\pi)^2}\int_0^{2\pi} d\alpha d\beta \frac{\mathbf{k}^4\mathbf{l}^2}{(\mathbf{k}^2-2\mathbf{l}\mathbf{k}\cos(\alpha+\beta)+\mathbf{l}^2)(\mathbf{k}^2+2\mathbf{m}\mathbf{k}\cos\beta+\mathbf{m}^2)}$$

$$I_{C_1} = \frac{\mathbf{l}^2\mathbf{k}^4}{|\mathbf{l}^2-\mathbf{k}^2||\mathbf{m}^2-\mathbf{k}^2|}$$

$$I_{C_2} = \frac{1}{(2\pi)^2}\int_0^{2\pi} d\alpha d\beta \frac{2\mathbf{k}^4|\mathbf{l}||\mathbf{m}|\cos\alpha}{(\mathbf{k}^2-2\mathbf{l}\mathbf{k}\cos(\alpha+\beta)+\mathbf{l}^2)(\mathbf{k}^2+2\mathbf{m}\mathbf{k}\cos\beta+\mathbf{m}^2)}$$

$$I_{C_2} = \frac{-8\mathbf{l}^2\mathbf{m}^2\mathbf{k}^6}{|\mathbf{l}^2-\mathbf{k}^2||\mathbf{m}^2-\mathbf{k}^2|(\mathbf{l}^2+\mathbf{k}^2+|\mathbf{l}^2-\mathbf{k}^2|)(\mathbf{m}^2+\mathbf{k}^2+|\mathbf{m}^2-\mathbf{k}^2|)}$$

$$I_{C_3} = \frac{1}{(2\pi)^2}\int_0^{2\pi} d\alpha d\beta \frac{\mathbf{k}^4\mathbf{m}^2}{(\mathbf{k}^2-2\mathbf{l}\mathbf{k}\cos(\alpha+\beta)+\mathbf{l}^2)(\mathbf{k}^2+2\mathbf{m}\mathbf{k}\cos\beta+\mathbf{m}^2)}$$

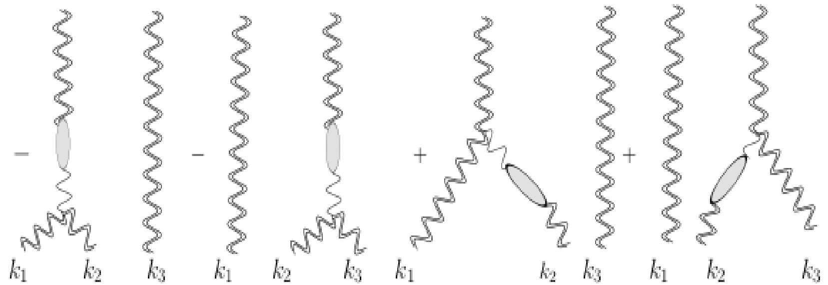
$$I_{C_3} = \frac{\mathbf{m}^2\mathbf{k}^4}{|\mathbf{l}^2-\mathbf{k}^2||\mathbf{m}^2-\mathbf{k}^2|}$$

$$\sum_{A,..,C_3} I = \frac{2\mathbf{l}^2\mathbf{m}^2\mathbf{k}^2 - 2\mathbf{l}^2\mathbf{k}^4 - 2\mathbf{b}^2\mathbf{k}^4 + 2\mathbf{k}^6}{(\mathbf{l}^2-\mathbf{k}^2)(\mathbf{m}^2-\mathbf{k}^2)} = 2\mathbf{k}^2$$

$$\frac{1}{(2\pi)^2}\int_0^{2\pi} d\alpha d\beta G_1(\mathbf{l},\mathbf{m}) = 2\mathbf{k}^2\theta(\mathbf{l}^2-\mathbf{k}^2)\theta(\mathbf{m}^2-\mathbf{k}^2)$$

All order result - disconnected part

$$G_2(\mathbf{l},\mathbf{m}) = -\mathbf{k}^4 \frac{1}{8\pi^2}\left(\ln\frac{\mathbf{l}^2}{(\mathbf{l}+\mathbf{m})^2}\delta^{(2)}(\mathbf{l}-\mathbf{k}) + \ln\frac{\mathbf{m}^2}{(\mathbf{l}+\mathbf{m})^2}\delta^{(2)}(\mathbf{m}-\mathbf{k})\right)$$



$$I_D = \frac{1}{(2\pi)^2}\int_0^{2\pi} d\alpha d\beta \ln\frac{\mathbf{l}^2}{\mathbf{l}^2+\mathbf{m}^2+2\mathbf{l}\mathbf{m}\cos\alpha} = \ln\frac{\mathbf{l}^2}{\mathbf{m}^2}$$

$$I_D = \frac{1}{(2\pi)^2}\int_0^{2\pi} d\alpha d\beta \ln\frac{\mathbf{m}^2}{\mathbf{l}^2+\mathbf{m}^2+2\mathbf{l}\mathbf{m}\cos\alpha} = \ln\frac{\mathbf{m}^2}{\mathbf{l}^2}$$

$$G_2(\mathbf{l},\mathbf{m}) = -\mathbf{k}^4 \frac{1}{8\pi^2}\left(\ln\frac{\mathbf{l}^2}{\mathbf{m}^2}\theta(\mathbf{m}^2-\mathbf{l}^2)\delta^{(2)}(\mathbf{l}-\mathbf{k}) + \ln\frac{\mathbf{m}^2}{\mathbf{l}^2}\theta(\mathbf{l}^2-\mathbf{m}^2)\delta^{(2)}(\mathbf{m}-\mathbf{k})\right)$$

All order result

Average the leading in Nc part of the vertex over angles

$$\frac{1}{(2\pi)^2}\int_0^{2\pi} d\alpha d\beta V(\mathbf{k},-\mathbf{k};\mathbf{l},-\mathbf{l},\mathbf{m},-\mathbf{m}) = 4\frac{g^4}{2}\Big[2\mathbf{k}^2\theta(\mathbf{l}^2-\mathbf{k}^2)\theta(\mathbf{m}^2-\mathbf{k}^2)$$
$$+\frac{1}{8\pi^2}\left(\ln\left(\frac{\mathbf{l}^2}{\mathbf{m}^2}\right)\delta^{(2)}(\mathbf{l}-\mathbf{k})\theta(\mathbf{m}^2-\mathbf{l}^2)+\ln\left(\frac{\mathbf{m}^2}{\mathbf{l}^2}\right)\delta^{(2)}(\mathbf{m}-\mathbf{k})\theta(\mathbf{l}^2-\mathbf{m}^2)\right)\Big].$$

- θ functions indicate that vertex contributes when the momenta on the projectail side is smaller than the momenta on the target side. Gluons fuse when they can resolve pomeron from above.
  - Explains suppression of diffusion of BFKL
    - Suggest that higher twist (collinear) is not relevant in DLLA
      - Finite Nc has been investigated and there is nonvanishing contribution

$$\mathcal{V}_{subN_c}^{r\{a'\},\{b\}}(\mathbf{l},\mathbf{m},-\mathbf{l},-\mathbf{m})^{\tau=4} = \delta^{a'_1 a'_2}\delta^{b_1,b_2}\delta^{b_3,b_4}\frac{\sqrt{2}\pi}{(N_c^2-1)^2}g^4\mathbf{k}^2\frac{4\mathbf{l}^2\mathbf{m}^2}{\mathbf{k}^4}$$

The nonlinear equation for unintegrated gluon density

Using the TPV one can derive a fan diagram equation directly in the momentum space

Gribov,Levin, Ruskin, Phys.Rept. 100 (1983) 1-150

Bartels, Kutak Eur.Phys.J. C53 (2008) 533-548

$$\frac{\partial \mathcal{F}(x,\mathbf{q},\mathbf{k})}{\partial \ln 1/x} = \int \frac{d^2\mathbf{l}}{(2\pi)^3} K(\mathbf{l},\mathbf{q}-\mathbf{l};\mathbf{k},\mathbf{q}-\mathbf{k}) \frac{\mathcal{F}(x,\mathbf{q},\mathbf{l})}{\mathbf{l}^2(\mathbf{q}-\mathbf{l})^2}$$

$$-\pi \int d^2\mathbf{r} \frac{d^2\mathbf{l}}{(2\pi)^3} \frac{d^2\mathbf{m}}{(2\pi)^3} V\left(\mathbf{k},-\mathbf{k}+\mathbf{q};\mathbf{l},-\mathbf{l}-\frac{\mathbf{q}}{2}+\mathbf{r},\mathbf{m},-\mathbf{m}-\frac{\mathbf{q}}{2}-\mathbf{r}\right) \times \frac{\mathcal{F}(x,\frac{\mathbf{q}}{2}+\mathbf{r},\mathbf{l})}{\mathbf{l}^2(-\mathbf{l}+\frac{\mathbf{q}}{2}+\mathbf{r})^2} \frac{\mathcal{F}(x,\frac{\mathbf{q}}{2}-\mathbf{r},\mathbf{m})}{\mathbf{m}^2(-\mathbf{m}+\frac{\mathbf{q}}{2}-\mathbf{r})^2}$$

The coupling with the target is via

$$F(\mathbf{r},R) = \frac{e^{\frac{-\mathbf{r}^2 R^2}{4}}}{2\pi}$$

One assumes a factorization

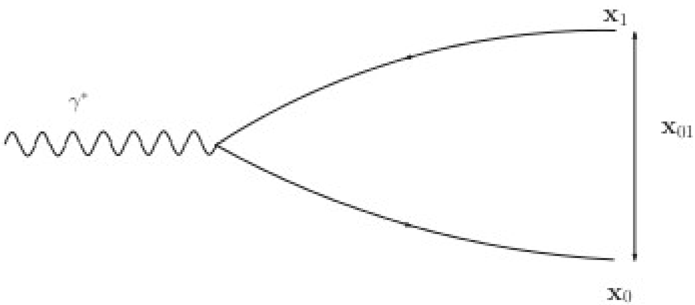
$$\mathcal{F}(x,\mathbf{r},\mathbf{k}) = \mathcal{F}(x,\mathbf{k})F(\mathbf{r},R)$$

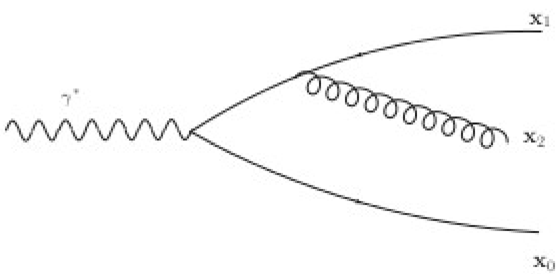
$$\frac{\partial f(x,\mathbf{k}^2)}{\partial \ln 1/x} = \frac{N_c\alpha_s}{\pi}\mathbf{k}^2 \int_0^\infty \frac{d\mathbf{l}^2}{\mathbf{l}^2}\left[\frac{f(x,\mathbf{l}^2)-f(x,\mathbf{k}^2)}{|\mathbf{k}^2-\mathbf{l}^2|} + \frac{f(x,\mathbf{k}^2)}{\sqrt{(4\mathbf{l}^4+\mathbf{k}^4)}}\right] - \frac{\alpha_s^2}{2R^2}\left\{2\mathbf{k}^2\left[\int_{\mathbf{k}^2}^\infty \frac{d\mathbf{l}^2}{\mathbf{l}^4} f(x,\mathbf{l}^2)\right]^2 + 2\, f(x,\mathbf{k}^2)\int_{\mathbf{k}^2}^\infty \frac{d\mathbf{l}^2}{\mathbf{l}^4}\ln\left(\frac{\mathbf{l}^2}{\mathbf{k}^2}\right) f(x,\mathbf{l}^2)\right\}$$

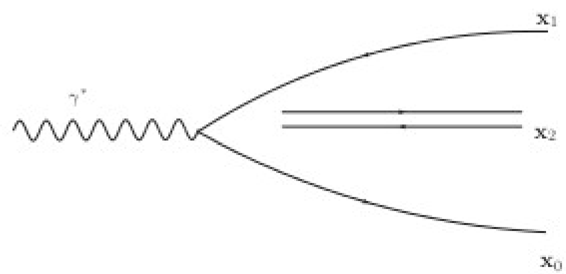
Kutak,Kwiecinski Eur.Phys.J.C29:521,2003

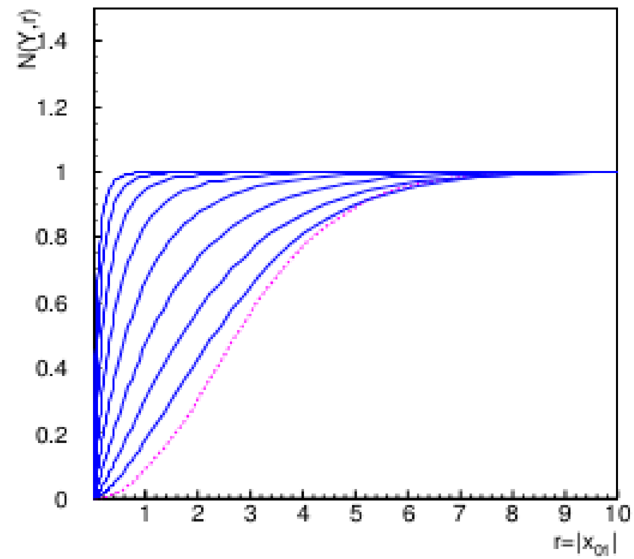
Target's radius

The BK equation in the coordinate space









The saturated dipole amplitude

From A. Stasto

$\mathbf{x}_{01} = \mathbf{x}_0 - \mathbf{x}_1$

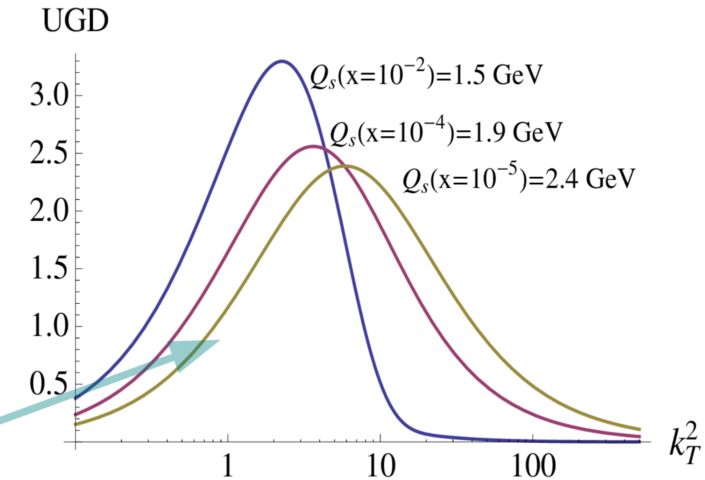
$\mathbf{b} = (\mathbf{x}_0 + \mathbf{x}_1)/2$

$$\frac{\partial N(\mathbf{x}_{01}, \mathbf{b}, Y)}{\partial Y} = \frac{\alpha_s N_c}{2\pi^2} \int d^2 x_2 \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{02}^2 \mathbf{x}_{12}^2} [N(\mathbf{x}_{02}, \mathbf{b} + \frac{1}{2}\mathbf{x}_{12}, Y) + N(\mathbf{x}_{12}, \mathbf{b} + \frac{1}{2}\mathbf{x}_{20}, Y)$$
$$- N(\mathbf{x}_{01}, \mathbf{b}, Y) - N(\mathbf{x}_{02}, \mathbf{b} + \frac{1}{2}\mathbf{x}_{21}, Y) N(\mathbf{x}_{12}, \mathbf{b} + \frac{1}{2}\mathbf{x}_{20}, Y)]$$

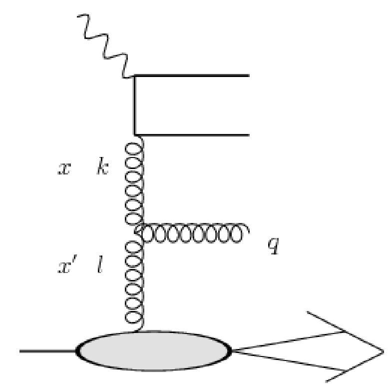
The BK equation in the momentum space

dipole density

$$\Phi(x,k^2)=\int\frac{d^2\mathbf{b}d^2\mathbf{r}}{2\pi}\exp(-i\mathbf{k}\cdot\mathbf{r})\frac{N(x,r,b)}{r^2}$$



$$\Phi(x,k^2)=\Phi_0(x,k^2)+\overline{\alpha}_s\int_{x/x_0}^{1}\frac{dz}{z}\int_0^\infty\frac{dl^2}{l^2}\left[\frac{l^2\Phi(x/z,l^2)-k^2\Phi(x/z,k^2)}{|k^2-l^2|}+\frac{k^2\Phi(x/z,k^2)}{\sqrt{(4l^4+k^4)}}\right]-\frac{\overline{\alpha}_s}{\pi R^2}\int_{x/x_0}^{1}\frac{dz}{z}\Phi^2(x/z,k^2)$$



gluon density

Solved by: Enberg; Golec-Biernat; Lublinsky; Soyez,...

$$\mathcal{F}(x,k^2)=\frac{N_c}{4\alpha_s\pi^2}k^2\nabla_k^2\Phi(x,k^2)$$

Leading order equation
not applicable at large k_t
Not obvious probabilistic interpretation
Might be difficult for MC

Kutak, Kwiecinski '03; Nikolaev, Scheffer '04

Final words and open questions

- *The TPV vertex derived for the lepton-proton scattering vanishes in the collinear limit*
  - *The main contribution is due to anticollinear pole*
    - *Probably the verex is not universal:* *Bartels, Ewerz, Mischler, Hentschinski '10*
      - *For hadronic impact factor new vertex emerges:* *Bartels, Motyka ' 07*
        the double log limit has not been taken