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# PYTHIA and DIPSY

## MPI, small $x$ , correlations, and diffraction

in  $pp$  collisions

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# Content

## 1. PYTHIA MC (T. Sjöstrand):

Very good description of inelastic reactions in DIS and  $pp$

Needs input structure functions determined by data

Simplified assumptions about correlations and diffraction

## 2. DIPSY (C. Flensburg, GG, L. Lönnblad):

Understand underlying dynamics in more detail  
(at the cost of lower precision)

Evolution of parton densities to small  $x$

Correlations and fluctuations

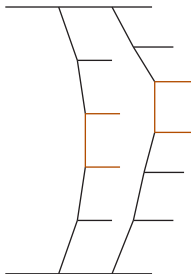
Diffraction

(Nucleus coll.)

## PYTHIA

Assume interaction dominated by perturbative parton-parton subcollisions

Color charge screened  $\Rightarrow$   
small  $p_{\perp}$  suppressed in hard subcollisions



$$\frac{d\hat{\sigma}}{dp_{\perp}^2} \propto \frac{\alpha_s^2(p_{\perp}^2)}{p_{\perp}^4} \rightarrow \frac{\alpha_s^2(p_{\perp 0}^2 + p_{\perp}^2)}{(p_{\perp 0}^2 + p_{\perp}^2)^2}$$

Higher energy  $\Rightarrow$  higher parton density  
 $\Rightarrow$  stronger screening

$$p_{\perp 0}(E_{\text{CM}}) = p_{\perp 0}^{\text{ref}} \times \left( \frac{E_{\text{CM}}}{E_{\text{CM}}^{\text{ref}}} \right)^{\epsilon}$$

## Matter profile in impact-parameter space

Simple Gaussian or more peaked variants

BFKL: random walk in transverse space:

*x*-dependent proton size

$$\rho(r, \mathbf{x}) \propto \frac{1}{a^3(\mathbf{x})} \exp\left(-\frac{r^2}{a^2(\mathbf{x})}\right) \quad \text{with} \quad a(\mathbf{x}) = a_0 \left(1 + a_1 \ln \frac{1}{\mathbf{x}}\right)$$

$a_1$  and  $a_0$  tuned to **rise** of  $\sigma_{\text{ND}}$

$a_1 \approx 0.15 \Rightarrow a_0 \approx \text{constant}$ , indep. of  $s$

(Corke-Sjöstrand, JHEP 05 (2011) 009)

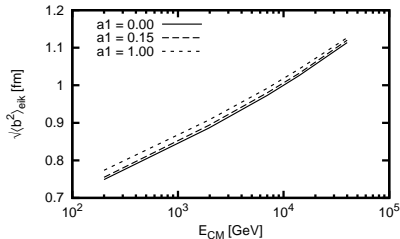
Convolution of two protons gives impact parameter shape

$$\tilde{O}(b; x_1, x_2) = \frac{1}{\pi} \frac{1}{a^2(x_1) + a^2(x_2)} \exp\left(-\frac{b^2}{a^2(x_1) + a^2(x_2)}\right)$$

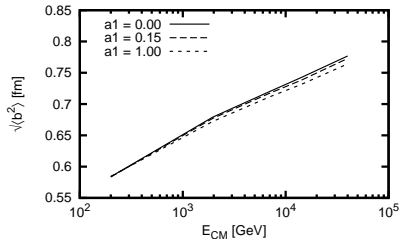
$\bar{n}(b)$  = average number of interactions for passage at  $b$

$$\bar{n}(b) = \sum_{i,j} \iiint dx_1 dx_2 dp_{\perp}^2 f_i(x_1, p_{\perp}^2) f_j(x_2, p_{\perp}^2) \left. \frac{d\hat{\sigma}_{ij}}{dp_{\perp}^2} \right|_{\text{reg}} \tilde{O}(b; x_1, x_2)$$

Gives  $\sqrt{\langle b^2 \rangle}$  for  $\sigma_{\text{ND}}$   
(b)



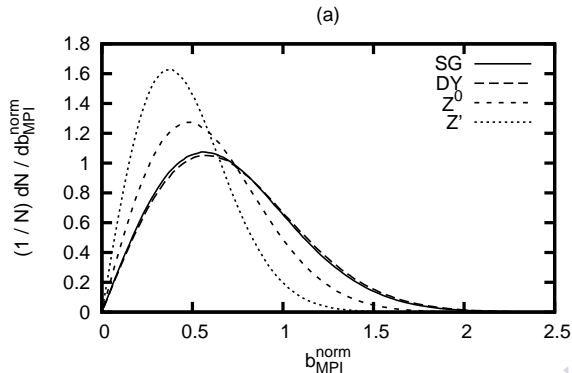
and for  $\sigma_{\text{hard}}$   
(a)



## Consequence:

collisions at large  $x$  will have to happen at small  $b$

⇒ further large-to-medium- $x$  MPIs are enhanced,  
while low- $x$  partons are so spread out that it plays less role.



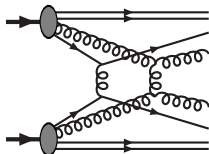
## Other features

### Further correlations

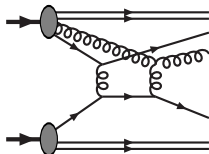
- ISR and MPI compete for beam momentum  $\rightarrow$  PDF rescaling
- + flavour effects (valence,  $q\bar{q}$  pair companions, ...)
- + correlated primordial  $k_{\perp}$  and colour in beam remnant

### Rescattering

Often  
assume  
that  
MPI =



... but  
should  
also  
include



Same order in  $\alpha_s$ ,  $\sim$  same propagators, but **small contribution to double scattering**

Not included in default version

## Interleaved evolution

Transverse-momentum-ordered parton showers for ISR, FSR, and MPI

$$\frac{d\mathcal{P}}{dp_{\perp}} = \left( \frac{d\mathcal{P}_{\text{MPI}}}{dp_{\perp}} + \sum \frac{d\mathcal{P}_{\text{ISR}}}{dp_{\perp}} + \sum \frac{d\mathcal{P}_{\text{FSR}}}{dp_{\perp}} \right) \times \exp \left( - \int_{p_{\perp}}^{p_{\perp, \text{max}}} \left( \frac{d\mathcal{P}_{\text{MPI}}}{dp'_{\perp}} + \sum \frac{d\mathcal{P}_{\text{ISR}}}{dp'_{\perp}} + \sum \frac{d\mathcal{P}_{\text{FSR}}}{dp'_{\perp}} \right) dp'_{\perp} \right)$$

Ordered in decreasing  $p_{\perp}$  using “Sudakov” trick.

Affects the number of secondary interactions

## Hard trigger

Theoretical trigger for hard interactions available in the MC



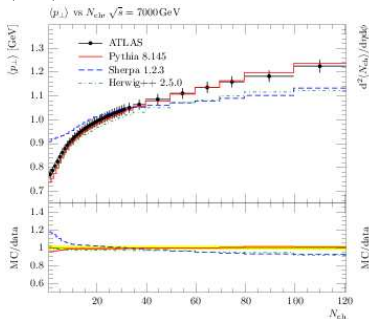
**Final states:** Add FSR and hadronization

Many partons close in space-time  $\Rightarrow$  colour rearrangement:

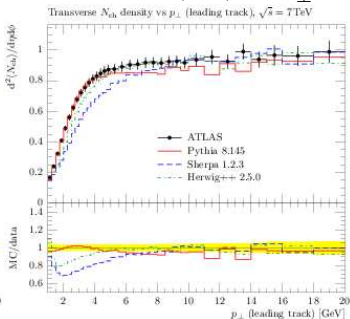
Reduction of total string length  $\Rightarrow$  steeper  $\langle p_{\perp} \rangle (n_{ch})$

Some results and comparisons with HERWIG++ and SHERPA:

$\langle P_{\perp} \rangle$  vs  $N_{ch}$



Transverse  $\langle N_{ch} \rangle$  vs  $p_{\perp}^{lead}$



(A. Buckley et al., Phys. Rep. 504 (2011) 145 (MCnet/11/01, arXiv:1101.2599[hep-ph]))

# DIPSY

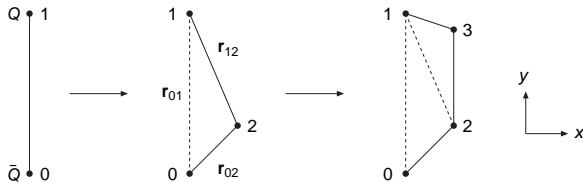
MC generator based on BFKL evolution and saturation

Based on Mueller's Dipol model:

LL BFKL evolution in transverse coordinate space

Colour charge always screened by accompanying anticharge

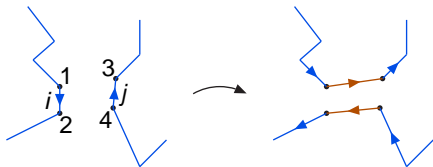
Gluon emission: dipole splits in two dipoles:



$$\text{Emission probability: } \frac{dP}{dy} = \frac{\bar{\alpha}}{2\pi} d^2\mathbf{r}_2 \frac{r_{01}^2}{r_{02}^2 r_{12}^2}$$

## Dipole-dipole scattering

Single gluon exchange  $\Rightarrow$  Colour reconnection  
between projectile and target



Born amplitude:

$$f_{ij} = \frac{\alpha_s^2}{2} \ln^2 \left( \frac{r_{13} r_{24}}{r_{14} r_{23}} \right)$$

Multiple interactions:

Stochastic process  $\Rightarrow$  Born ampl.  $F = \sum_{ij} f_{ij}$

**Unitarity:** Eikonal approx. in imp. parameter space

**Unitarized ampl.:**  $T = 1 - e^{-\sum f_{ij}}$  (neglecting fluctuations)

$$d\sigma_{el}/d^2b = T^2, \quad d\sigma_{tot}/d^2b = 2T$$

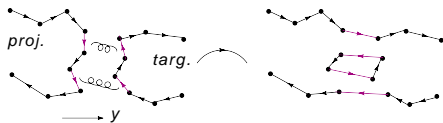
# The Lund cascade model, DIPSY

Includes:

- ▶ Important non-leading effects in BFKL evol.  
(most essential rel. to energy cons. and running  $\alpha_s$ )
- ▶ Saturation from pomeron loops in the evolution  
(Not included by Mueller or in BK)
- ▶ Confinement  $\Rightarrow$   $t$ -channel unitarity
- ▶ MC DIPSY  
gives also fluctuations and correlations
- ▶ Applicable to collisions between electrons, protons,  
and nuclei

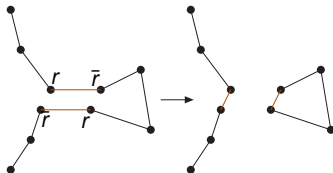
# Saturation

Multiple interactions  $\Rightarrow$  colour loops  $\sim$  pomeron loops



Multiple interaction in one frame  $\Rightarrow$  colour loop within evolution in another frame

Gluon scattering is colour suppressed compared to gluon emission  $\Rightarrow$  Loop formation related to identical colors.



Same colour  $\Rightarrow$  quadrupole

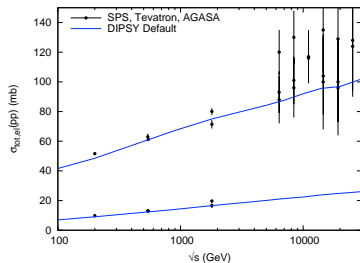
May be better described by recoupled smaller dipoles

$\Rightarrow$  smaller cross section:  
fixed resolution  $\Rightarrow$  effective  
 $2 \rightarrow 1$  and  $2 \rightarrow 0$  transitions

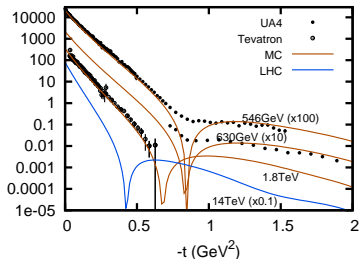
# $pp$ total and elastic cross sections

Initial proton wavefunction  $\sim$  three dipoles in a triangle

$\sigma_{tot}$  and  $\sigma_{el}$

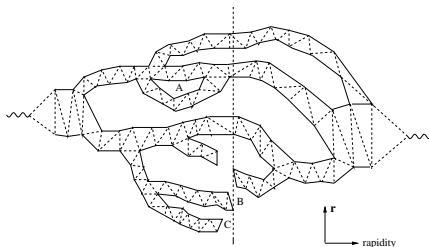


$d\sigma/dt$



## Exclusive final states

Schematic picture: BFKL is a stochastic process:  
independent dipole-dipole interactions



Non-interacting  
branches cannot  
come on shell.

To get final states:

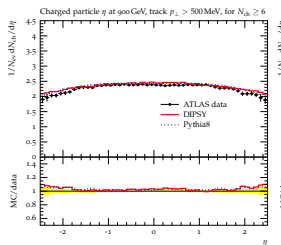
- Determine which dipoles interact
- Absorb non-interacting chains
- Determine final state radiation
- Hadronize

# Comparisons to ATLAS data

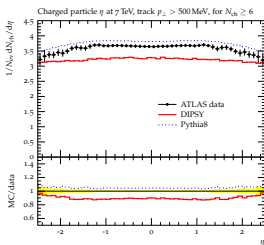
[JHEP 2011, arXiv:1103.4321]

## Min bias

$\eta$  distrib. charged particles  
0.9 TeV

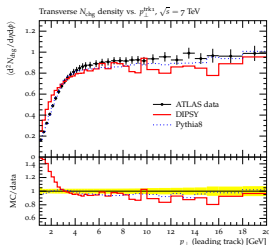


7 TeV



## Underlying event

$N_{ch}$  in transv. region  
vs  $p_{\perp}^{lead}$ , 7 TeV



Our aim to get dynamical insight, not to give precise predictions  
At present no quarks, only gluons



## Correlations. Double parton distributions

Define double parton distribution and imp. param. profile  $F$ :

$$\Gamma(x_1, x_2, b; Q_1^2, Q_2^2) \equiv D(x_1, Q_1^2) D(x_2, Q_2^2) F(b; x_1, x_2, Q_1^2, Q_2^2),$$

and effective cross section:  $\sigma_{(A,B)}^D \equiv \frac{1}{(1+\delta_{AB})} \frac{\sigma_A^S \sigma_B^S}{\sigma_{\text{eff}}}$

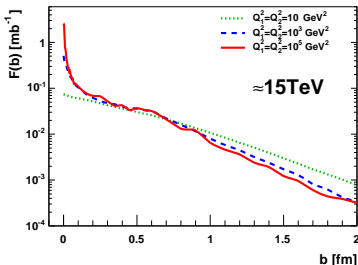
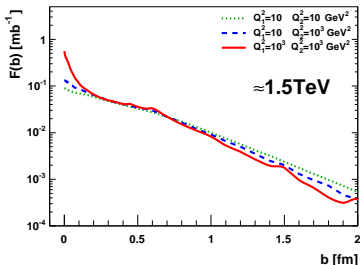
For double hard interactions at midrapidity this gives:

$$\sigma_{\text{eff}} = \left[ \int d^2b (F(b))^2 \right]^{-1}$$

$F$  and  $\sigma_{\text{eff}}$  often assumed to depend only weakly on  $x_i$  and  $Q_i^2$

# Correlation function $F(b)$

Depends on both  $x$  and  $Q^2$



Spike (hotspot) develops for small  $b$  at larger  $Q^2$

Fourier transform:  $D(x_1, x_2, Q_1^2, Q_2^2; \vec{\Delta})$  (Blok *et al.*)

Spike for small  $b \Rightarrow$  tail for large momentum imbalance  $\Delta$

[JHEP 2011, arXiv:1103.4320]

$\Rightarrow \sigma_{\text{eff}}$  depends strongly on  $Q^2$  for fixed  $\sqrt{s}$

$Q_1^2, Q_2^2$ [GeV <sup>2</sup> ], $x_1, x_2$				$\sigma_{\text{eff}}$ [mb]	$\int F$
1.5 TeV, midrapidity					
10	10	0.001	0.001	35.3	1.09
$10^3$	$10^3$	0.01	0.01	23.1	1.06
15 TeV, midrapidity					
10	10	0.0001	0.0001	40.4	1.11
$10^3$	$10^3$	0.001	0.001	26.3	1.07
$10^5$	$10^5$	0.01	0.01	19.6	1.03

Part of the correlations is due to fluctuations

No fluct.  $\Rightarrow \int d^2b F(b) = 1$ ; the MC gives  $\sim 1.1$

# Fluctuations cause Diffractive excitation

## Good–Walker formalism

Projectile with a **substructure**

The mass eigenstates,  $\Psi_k$ , can differ from the eigenstates of diffraction,  $\Phi_n$  (with eigenvalues  $T_n$ )

Elastic amplitude:  $\langle \Psi_{in} | T | \Psi_{in} \rangle = \langle T \rangle$ ,  $d\sigma_{el}/d^2b = \langle T \rangle^2$

Ampl. for transition to state  $\Psi_k$  given by  $\langle \Psi_k | T | \Psi_{in} \rangle$

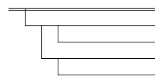
Total diffractive cross section (incl. elastic):

$$d\sigma_{diff}/d^2b = \sum_k \langle \Psi_{in} | T | \Psi_k \rangle \langle \Psi_k | T | \Psi_{in} \rangle = \langle T^2 \rangle$$

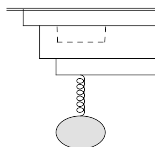
$$d\sigma_{diff\ ex}/d^2b = d\sigma_{diff} - d\sigma_{el} = \langle T^2 \rangle - \langle T \rangle^2 = V_T$$

## What are the diffractive eigenstates?

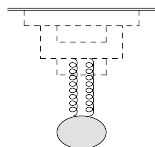
Parton cascades, which can come on shell through interaction with the target.



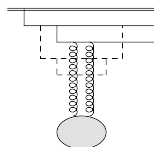
*Virtual cascade*  
a



*Inelastic int.*  
b



*Elastic scatt.*  
c



*Diffractive ex.*  
d

BFKL dynamics  $\Rightarrow$  Large fluctuations,

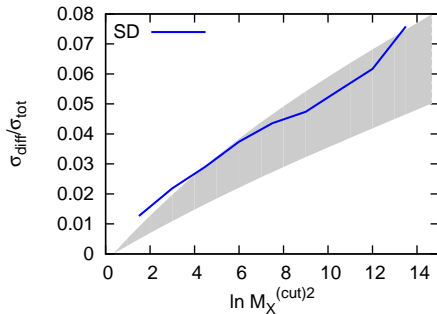
Continuous distrib. up to high masses

(Also Miettinen–Pumplin (1978), Hatta *et al.* (2006))

## DIPSY: $pp$ 1.8 TeV

Single diffractive cross section for  $M_X^2 < M_{max}^2$

Shaded area: Estimate of CDF result



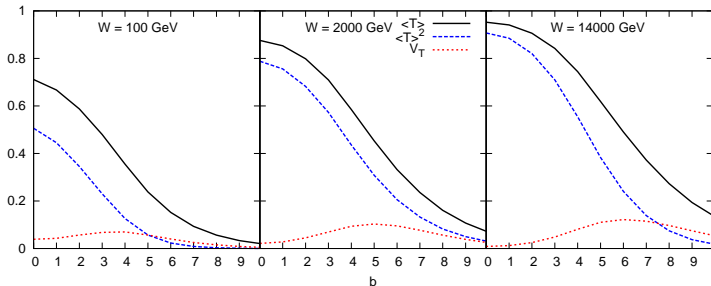
Note: Tuned only to  $\sigma_{tot}$  and  $\sigma_{el}$ . No new parameter

## Impact parameter profile

Saturation  $\Rightarrow$  Fluctuations suppressed in central collisions

Diff. excit. largest in a circular ring,

expanding to larger radius at higher energy



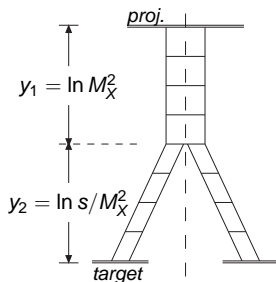
Factorization broken between  $pp$  and DIS

## Relation Good–Walker vs triple-pomeron

Claim: Good–Walker and Triple-pomeron describe the same dynamics

Triple-pomeron:

$$\frac{d\sigma_{SD}}{d \ln M^2} \sim g_{pP}^3 g_{3P} \left(\frac{s}{M^2}\right)^{2(\alpha_P-1)} (M^2)^{\alpha_P-1}$$



Due to the stochastic nature of the BFKL cascade:

⇒ # dipoles satisfy approx. KNO scaling:

$$\sigma^2 \approx \langle n \rangle^2$$

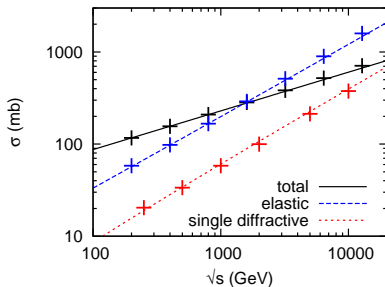
(For more details see arXiv:1206.1733)



# DIPSY results have the expected triple-regge form

*BARE* pomeron (Born amplitude without saturation effects)

Total, elastic and single diffractive cross sections



Triple-Regge fit with a single pomeron pole

$$\alpha(0) = 1.21, \quad \alpha' = 0.2 \text{ GeV}^{-2}$$

$$g_{pP}(t) = (5.6 \text{ GeV}^{-1}) e^{1.9t}, \quad g_{3P}(t) \approx 1 \text{ GeV}^{-1} \text{ (dep. on def.)}$$

## Exclusive final states in diffraction

If gap events are analogous to diffraction in optics  $\Rightarrow$

Diffraction excitation fundamentally a quantum effect

Different contributions interfere destructively,  
no probabilistic picture

Still, different components can be calculated in a MC,  
added with proper signs, and squared

Possible because opt. th.  $\Rightarrow$  all contributions real

[arXiv:1210.2407]

(Makes it also possible to take Fourier transform and get  $d\sigma/dt$ .

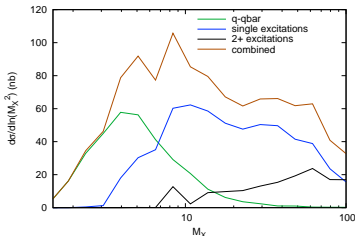
JHEP 1010, 014, arXiv:1004.5502)

# Early results for DIS ( $W = 120 \text{ GeV}$ , $Q^2 = 24 \text{ GeV}^2$ ):

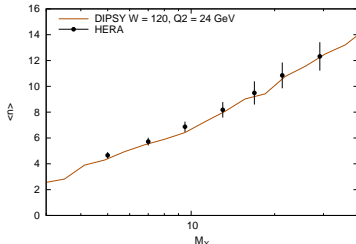
Distrib. in  $\ln M_X^2$

separated in **parton states** with  $q\bar{q} + 0, 1, \text{ and } \geq 2$  gluons

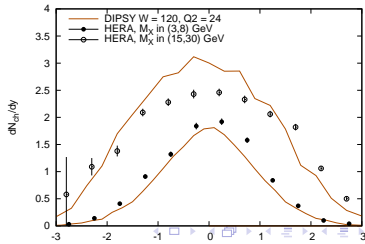
(cutoff for  $M_X > 50 \text{ GeV}$   
due to Lorentz frame used)



**Hadronic state:**  $\langle n_{ch} \rangle$  vs  $M_X$



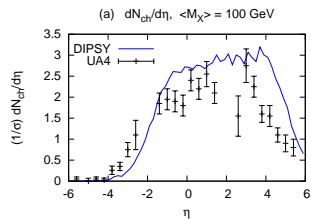
$dn_{ch}/d\eta$  in 2  $M_X$ -bins



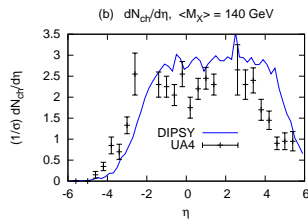
# $pp$ collisions:

Early result for  $dn_{ch}/d\eta$  at  $W = 546$  GeV,

$\langle M_X \rangle = 100$  GeV



$\langle M_X \rangle = 140$  GeV



Too hard for large  $\eta$ . Due to lack of quarks in proton wavefunction; no forward baryon

Has to be added in future improvements

Note: Based purely on fundamental QCD dynamics

# Conclusions:

## 1. Pythia

### Recent developments related to MPI:

- x-dependent proton size
- rescattering
- Two hard trigger processes in MPI
- MPI framework for hard diffraction
- Central diffraction

Generally very good agreement with data

## 2. DIPSY

Can data be understood from QCD evolution and saturation?

Model with no input pdf:s, no tunable soft cutoff for hard subcollisions, no input diffractive pdf:s or pomeron flux factors

- MC gives correlations, fluctuations, and expansion in  $b$ -space
- Double parton distr. strongly dependent on  $Q^2$  and  $x$
- Works well for inclusive observables
- Fair description of exclusive final states (also in diffraction)
- Diffraction described in Good–Walker formalism also for high masses
- Applicable for  $e$ ,  $p$ , and  $A$  collisions

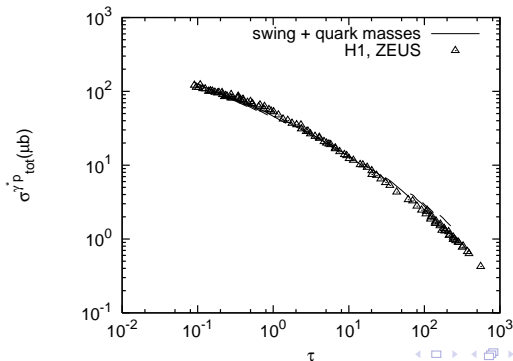
# Extra slides

## Structure functions

$$F_2(x, Q^2) \sim \gamma^* p \text{ cross section}$$

$\gamma^* \rightarrow q\bar{q}$  dipole wavefunction from QED

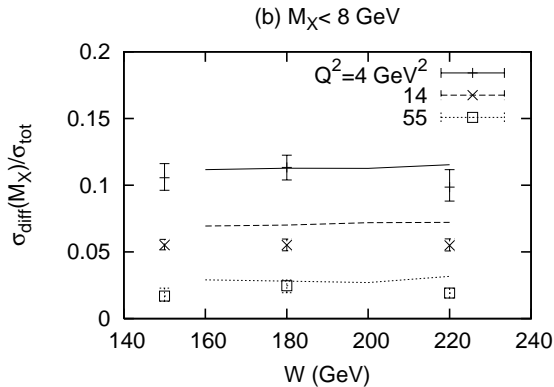
Satisfies geometric scaling.  $\tau = Q^2/Q_s^2(x)$ ,  $Q_s^2 \propto x^{-0.3}$





# Diffractive excitation in DIS

Example  $M_X < 8$  GeV,  $Q^2 = 4, 14, 55$  GeV<sup>2</sup>.



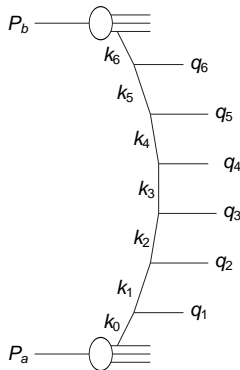
Data from Zeus

# Exclusive final states

BFKL: Inclusive

Exclusive: CCFM

In momentum space



**Inclusive:**

Cross section determined by “backbone” emissions, where  $k_{\perp}$  gets a big kick: “ $k_{\perp}$ -changing emissions”

(either  $k_{\perp i} \gg k_{\perp i-1}$ ;  $q_{\perp i} \simeq k_{\perp i}$   
 or  $k_{\perp i} \ll k_{\perp i-1}$ ;  $q_{\perp i} \simeq k_{\perp i-1}$ )  
 (Lund 1996, Salam 1999)

**Exclusive:**

Select backbone chains  
 Reabsorb remaining gluons  
 add FSR & hadronization

# Fluctuations

## Fluctuations modify unitarization effects

Born ampl.  $F \Rightarrow$  Unitarized ampl.  $T = 1 - e^{-F}$

$F$  fluctuates from event to event  $\Rightarrow$

Elastic ampl. given by  $\langle T \rangle = \langle 1 - e^{-F} \rangle \neq 1 - e^{-\langle F \rangle}$

Suppresses interaction for small  $b$ , and enhances it for larger  $b$ -values

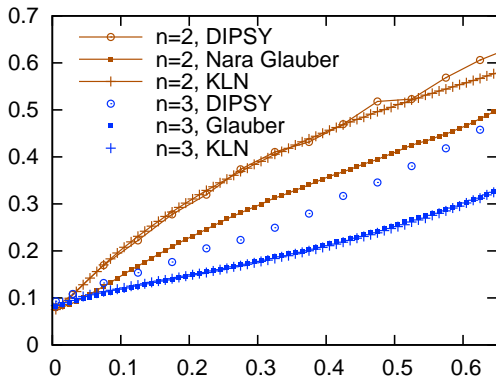
(Cf. “diffusive scaling”)

## Fluctuations give also odd eccentricity moments

triangular flow in  $pp$  and  $AA$  collisions

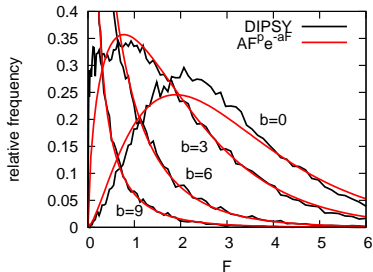
# Eccentricities in PbPb coll. at $\sqrt{s_{NN}} = 2.76$ GeV

for  $n = 2, 3$  from DIPSY, MC-KLM, and Glauber MC

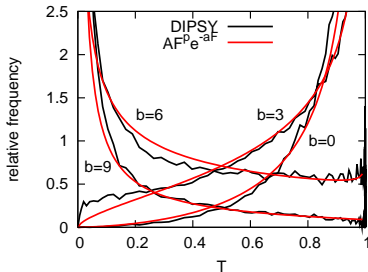


# Fluctuations in $pp$ amplitudes

Born ampl.  $F$   $W = 2 \text{ TeV}$



Unitarized ampl.  $T = 1 - e^{-F}$



Born approximation: large fluctuations

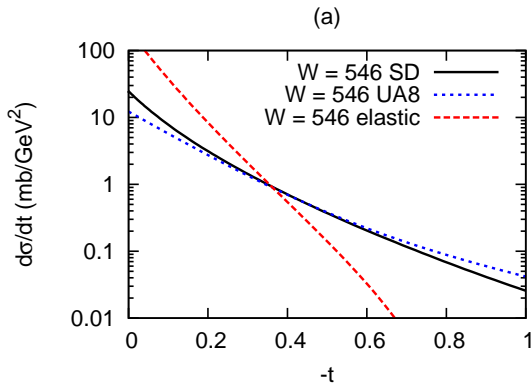
$\langle F \rangle$  is large  $\Rightarrow$  Unitarity effects important

$\sim$  enhanced diagrams in triple-regge formalism

Fluctuations strongly reduced for central collisions

# $t$ -dependence

Single diffractive and elastic cross sections



Agrees with fit to UA8 data

## Nonleading effects in the BFKL evolution

- ▶ Energy conservation ( $\sim$  non-sing. terms in  $P(z)$ )  
 small dipole — high  $p_{\perp} \sim 1/r$   
 Cascade ordered in  $p_{+}$   
 $\Rightarrow$  small dipoles suppressed for small  $\delta y$
- ▶ “Energy scale terms”  $\sim$  “consistency constraint”  
 $\Rightarrow$  Cascade ordered in  $p_{-}$   
 A single chain is left-right symmetric
- ▶ Running  $\alpha_s$

## Exclusive diffractive states

Diffraction is a quantum effect  $\Rightarrow$  interference is important  
 $\Rightarrow$  no probabilistic picture

But: positive and negative contributions to the amplitude can be generated by DIPSY, added, and squared.

### Toy model example

System with a valence particle, which can emit a single gluon

2 states: valence only  $\Psi_0 = |1, 0\rangle$

valence + gluon  $\Psi_1 = |1, 1\rangle$

Probability for emission:  $\beta^2$  prob. for no em.:  $\alpha^2 = 1 - \beta^2$

General state  $\Psi = a\Psi_0 + b\Psi_1 \equiv \begin{pmatrix} a \\ b \end{pmatrix}$



Assume an initial state  $\Psi$  which evolves to a cascade  $\Phi$  at the time of interaction with the target

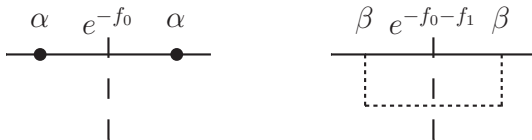
$$\Phi = U_{\text{evol}} \Psi$$

Evolution operator  $U_{\text{evol}}$  is a unitary matrix  $= \begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix}$

Eikonal interaction operator  $U_{\text{int}} = \begin{pmatrix} e^{-f_0} & 0 \\ 0 & e^{-f_0-f_1} \end{pmatrix}$

$$\Psi_{\text{out}} = S\Psi_{\text{in}} = U_{\text{evol}}^\dagger U_{\text{int}} U_{\text{evol}} \Psi_{\text{in}}$$

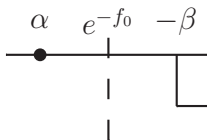
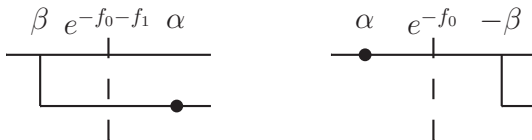
## Elastic scattering:



## Elastic amplitude

$$T_{11} = 1 - S_{11} = 1 - \alpha^2 e^{-f_0} - \beta^2 e^{-f_0-f_1} = \\ \alpha^2(1 - e^{-f_0}) + \beta^2(1 - e^{-f_0-f_1})$$

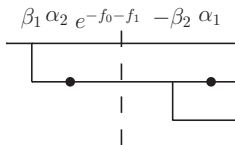
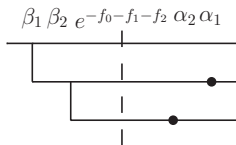
## Diffractive excitation:



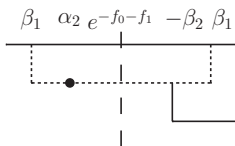
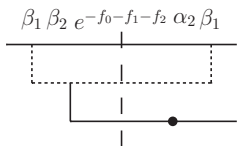
$$T_{21} = -S_{21} = -\alpha\beta e^{-f_0-f_1} - \alpha(-\beta)e^{-f_0} = \alpha\beta e^{-f_0}(1 - e^{-f_1})$$

## Cascade with 2 possible emissions

Ex.: Final state with both emissions



Final state with only the second emission



## Generalizations:

Continuous cascades

Independent gluon emissions  $\rightarrow$  dipole cascade

Include target cascade

## Calculations:

Collide many similar real cascades (emissions before and after interaction) which interfere

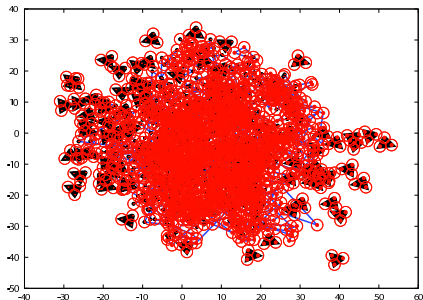
Collide with large no. of target cascades

Computationally demanding, but still possible in the MC

# Nucleus collisions

Gives full partonic picture:

Ex.:  $Pb - Pb$  200 GeV/N



Accounts for:

- saturation within the cascades,
- correlations and fluctuations (gives e.g. triangular flow),
- finite size effects

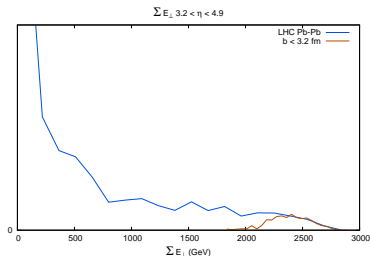
# The interaction gives a dense gluon soup

Independent FSR and hadronization  $\Rightarrow$  too many particles

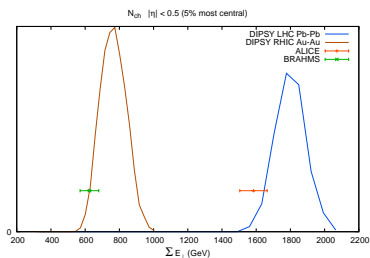
Toy model “thermalization”:

allow gluons within 1 fm to interact and reconnect

Forward  $E_{\perp}$  at LHC PbPb



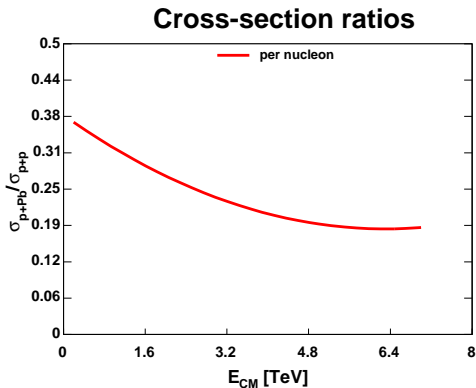
$n_{ch}$  in  $|\eta| < 0.5$  at RHIC and LHC



Future: Energy & momentum density as input for hydro expansion

# $pA$ collisions

Shadowing:  $R = \sigma_{pA}/A\sigma_{pp}$



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for  $n = 2, 3$  from DIPSY, MC-KLM, and Glauber MC

