Multi parton interactions in small x physics

Leonid Frankfurt

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Challenging questions.

The dependence of MPI on energy.

Puzzles of small x physics and their plausible resolution.

Outli

- Introduction .
- Lessons from small x physics at FNAL and HERA

  Violation of QCD factorization theorems in the central collision in the small x physics phenomena.

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- Specific features of MPI at LHC

Lessons from FNAL and HERA and challenging questions

1. Pattern of non perturbative high energy QCD .

i. Total cross sections of hadron-hadron collisions are increasing with energy as $\sigma_{tot} \propto s^{\lambda}$. $\lambda \approx 0.08 - 0.01$

ii. Partial waves for pp collision at central impact parameters are close to 1 i.e. close to the regime of complete absorption.

iii. The ratio $\sigma_{el}(pp)/\sigma_{tot}(pp) \approx 0.3$ at LHC and decreases with energy. This number is significantly smaller than 0.5 expected within the regime of complete absorbtion.

iv. The slope of t dependence of elastic cross section of pp collisions- radius of a hadron is increasing with energy.

v. Total cross section is dominated by peripheral collisions, by Pomeron physics.

$$\sigma_{tot} = 2\int Imf(b,s)d^2b \qquad Imf \leq 1$$

Hard QCD interaction is small at fixed target energies but more rapidly increasing with energy than soft QCD and repeats discussed above pattern given by non perturbative QCD.

Structure functions of a proton.

Structure functions of a target are more rapidly increasing with energy: xD(x,Q^2)=$\propto s^{\lambda(Q^2)}$This is numerical fit to data, to pQCD formulae.

$$\lambda(Q^2 \approx few\ GeV^2) = 0.2\ and\ increasing\ with\ Q^2.$$

Account of conservation of probability does not preclude an increase with energy of parton distributions forever similar to the total cross sections of hadronic collisions. To explain this behaviour one needs distribution over impact parameters.

Gluon distribution in impact parameter space.

It follows from QCD factorization theorem that:

$$d\sigma(\gamma + p \rightarrow J/\psi + p) = d\sigma(t=0)/dt F_{2g}^2(t, Q^2, s)$$

$$F_{2g}(b^2, Q^2, s) = \int d^2q \; expi(q_t b) F_{2g}(-q_t^2)$$

The dependence on t is contained in the overlapping integral between wave functions of initial and final nucleon. At large Q form factor F should become independent on Q because large Q squizes vector meson. This squizing was clearly observed at HERA for the elastic electro production of J/\psi ,rho mesons etc. At small Q the dependences on t for the elastic production of different vector mesons are different.

$$F_{2g}(t) = 1/(m_g^2 - t)^2$$

or F=exp(Bt/2)

F(b,s) describes parton distribution in impact parameter space . Data can be parametrized as

$$F = 1(1 - t/m_g^2)^2$$

This parametrization at not too small x follows from the same approaches as for electromagnetic form factors of a nucleon. At very large t (-t>16 GeV^2) perturbative tail in the nucleon wave function excluded from above formulae will become important: $F = \alpha_s(t)^2/t^2$. m_g^2 should decrease with decrease of x because of increased role of pion field of and of appearance of additional degrees of freedom.

Data are often parametrized as F=exp(Bt/2).

Cross sections of MPI are controlled by different GPD: $G(x_1,x_2,Delta,Q^2)$.

Uncorrelated parton approximation (mean field approximation): $G(x_1,x_2,Delta,Q^2)=G(x\_1,Delta,Q^2)$ $G(x_2, Delta,Q^2) = G(x_1,0,Q^2)\ G(x_2,Q^2)\ F^2(Delta)$ For $Delta^2>>m^2_g$ instead of F^2 perturbative tail: c $alpha_s$/ $Delta^4$. With decrease of x correlation function should more rapidly decrease with Delta because of increased role of other quark-gluon components in the nucleon wave function. Shrinking with energy of diffractive peak in exclusive processes is one of small x phenomena.

Thus parton distribution within a nucleon in impact parameter space is rapidly decreasing with impact parameter b. At large b $F(b) \propto exp(-m_g b)$ with $m_g \approx 1 GeV$ or $F \propto exp(-b^2/B)$ with $B \approx 4 GeV^2$

Knowledge of impact parameter distribution allows to impose probability conservation and to evaluate parton distributions within a nucleon at small x.

Total cross sections, structure functions are rapidly increasing with energy forever -no slowing down.

This is because probability conservation stops increase of structure functions at given impact parameter only.

For the illustration let us consider model: dipole -nucleon scattering and use pQCD evaluation as input. Probability conservation gives

$$Imf(b, s, Q^2) \leq 1$$

Within the LT approximation of pQCD partial amplitude can be parametrized as

$$Imf(b, \ldots) = (c/Q^2)s^{\lambda} \exp -(\mu b)$$

Thus for large s partial amplitude is 1 for $b \leq b_{max}$

Thus we obtain:

$$xD(x,Q^2) \propto \int Im f(b,..) d^2b =$$

$$c_1 \ln^2(x_0/x) + c_2 (x_0/x)^\lambda$$

First term gives Froisart limit familiar from hadronic collisions. Thus at the energies achievable at accelerators parton distribution is increasing with energy since dependence of both terms on collision energy is similar. More accurate analysis shows that actually first term is $xD \propto \ln^3(x_0/x)$ follows from the singular properties of photon wave function.

Analysis of data on diffraction in DIS indicates that gluon distribution at small impact parameters is close to the boundary dictated by the conservation of probability at $x \sim 10^{(-4)}$.

Radius of a nucleon increases rapidly with collision energy.

$$\sigma_{tot}/2 \geq \sigma_{el}$$

$$d\sigma_{el}/dt = \sigma_{tot}^2 F_{2g}^2(t)/16\pi$$

$$1 \geq \sigma_{tot} \int F_{2g}^2(t,\ldots)dt/8\pi$$

Since total cross section is rapidly increasing with energy this inequality requires increase with energy of the slope of dependence of form factor F on t. Distinctive property of black disc regime is that near black disc limit wave function of dipole experiences radical transformation leading to increasing with energy the number of degrees of freedom.

Violation of leading twist approximation in the vicinity of black disc limit.

Amplitudes of hard exclusive processes are higher twist effects. In this case technologies based on QCD factorization theorems are applicable and proved. Using QCD factorization theorems it is easy to evaluate Q and x dependencies of HT effects-mostly from multiple ladders . This is helpful since MPI are higher twist effects. The cross section of n dijet production at small x is roughly

$$\sigma_n = (c_1/Q^2)(Q_0^2/Q^2)^{(n-1)}[\alpha_s(Q^2)]^n(x_0/x)^{n\lambda}$$

At sufficiently small x near BDR series loose property of asymptotic series and become divergent-beyound the radius of convergency since larger n more rapid increase with energies.This means that series should be regularized to define how to treat singularity . Example is

$$\sum_n z^n = 1/1+z$$

Radius of convergency is determined by singularity at z=-1

Few signals for BDR.

New QCD regime should reveal itself mostly in the pp, pA collisions at central impact parameters. It gives small contribution into parton pdfs since they are integral over all impact parameters. On the contrary in the hard pp,pA collisions, in MPI its contribution should be enhanced since these processes are dominated by collisions at central impact parameters.

Using a hard process in pp, pA collisions as a trigger it is legitimate to ask question on the distribution of leading hadrons in such inelastic events. Black disc regime would reveal itself through causality,energy-momentum conservation.

The contribution of planar diagrams at large energies is zero-prove by deformation of contour of integration over diffractively produced mass. (S.Mandelstam1963,V.Gribov1965) Another prove follows from the dominance of inelastic processes and from the fact that amplitudes of high energy processes are predominantly imaginary. Therefore planar diagrams in QCD in difference from non relativistic quantum mechanics violate strongly energy-momentum conservation and causality . Non planar diagrams guarantee that energy of projectile is divided between its constituents long before the collision. Each ladder depends on the fraction of energy of projectile carried by interacting parton. This is why eikonal approximation is inapplicable in QCD as the quantum field theory. Instead one may use Gribov-Glauber approximation . The special difference is for the scattering at central impact parameters. Near BDR several ladders become important . The probability of increase with energy of the number of ladders is increasing . Therefore energy per ladder is decreasing with energy. This leads to the suppression of the yield of leading hadrons as compared to the regime of moderate x. This is phenomenon of fractional energy losses.

Distinctive phenomena

Violation of QCD factorization theorems.

Violation of scale and conformal invariance.

Appearance of new invariant mode describing motion in impact parameter-new Goldstone boson? In the model for soft QCD interactions considered by Markesini, Chiafaloni, Parisi it was really Goldstone boson. Similar situation in hard QCD.

Dipole looses its meaning because of revealing of infinite number of degrees of freedom which interact through NLT terms.

Small x MPI

Kinematical domain which can be probed by MPI is till $x=10^{(-3,-4)}$ i.e. close to the kinematics achieved at HERA for the exclusive processes which are HT effects. So certain characteristic features should be the same.

$$G_T(x_1,x_2,\Delta,Q^2)c(x_1)^{(\lambda)}(x_2)^{(\lambda)}$$