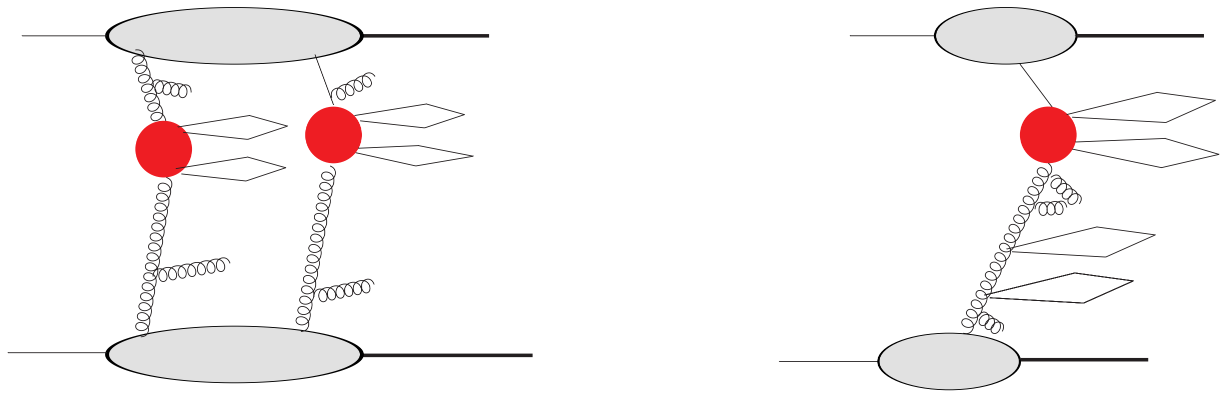
Workshop on Multiple Parton Interactions
Tel Aviv University, October 2012

Multi-parton interactions in the small-x region

F. Hautmann (Oxford)

Thanks for discussion and collaboration to
H. Jung, A. Grebenyuk, M. Hentschinski,
K. Kutak, A. Knutsson, P. Katsas

• Multi-parton interactions increasingly important as parton densities grow with energy



Multi-jet production by (left) multiple parton collisions; (right) single parton collision.

• Effective picture of parton density evolution based on collinear DGLAP for inclusive observables

• MPI contribute primarily to highly differential cross section, probing detailed distribution of the final states produced by parton evolution. This is not expected to be reliably represented when longitudinal momentum fractions are small and densities increase.

• How do high-energy corrections to parton shower evolution affect treatment of MPI

OUTLINE

- Influence of parton shower evolution on MPI

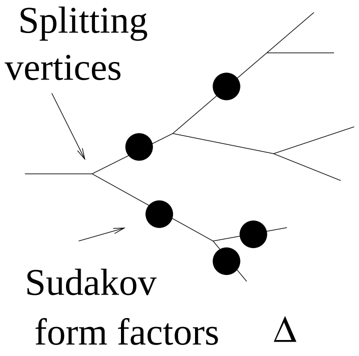
- Multi-jet final states, energy flow observables

- Mini-jets and the inelastic pp cross section

I. FROM QCD TO MONTE CARLO EVENT GENERATORS:
PROGRESS IN BRANCHING TECHNIQUES

- Factorizability of QCD x-sections $\longrightarrow$ probabilistic branching picture

♢ QCD evolution by "parton showering" methods:

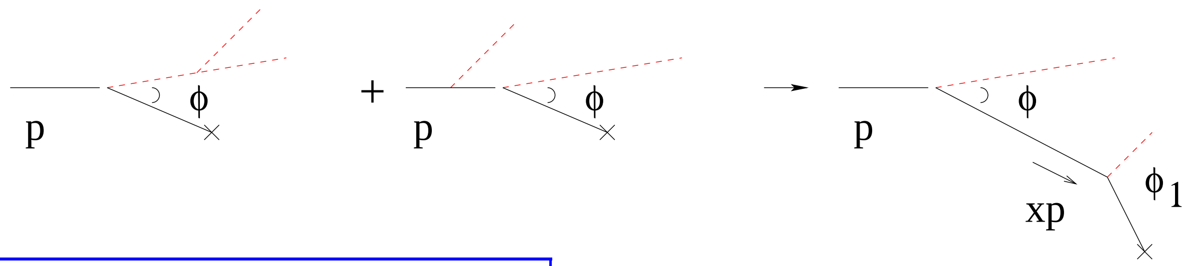


$$d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \ \alpha_S(q^2) \ P(z) \ \Delta(q^2, q_0^2)$$

$\hookrightarrow$ collinear, incoherent emission

♢ Soft emission $\longrightarrow$ interferences $\longrightarrow$ ordering in decay angles:

$\hookrightarrow$ gluon coherence for $x \sim 1$

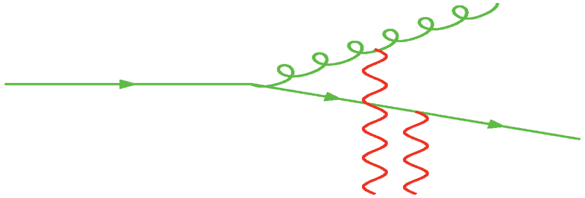


♢ Gluon coherence for $x \ll 1$ $\Rightarrow$ corrections to angular ordering:

$\hookrightarrow$ MC based on $k_\perp$-dependent unintegrated pdfs and MEs

SOFT-GLUON COHERENCE

▷ soft gluons radiated over long times $\longrightarrow$ quantum interferences



- Factorization in soft limit: *[J.C. Taylor, 1980; Gribov-Low (QED)]*

$$|M_{n+1}^{a_1 \ldots a_n a}(p_1, p_n, q)\rangle = \mathbf{J}^a |M_n^{a_1 \ldots a_n}(p_1, p_n)\rangle \ , \quad \mathbf{J}^{a\mu} = \sum_i \mathbf{Q}_i^a \, \frac{p_i^\mu}{p_i \cdot q}$$

interference terms ↓

$$d\sigma_{n+1} = d\sigma_n \, \frac{d^3 q}{(q^0)^3} \, \sum_{i,j} \mathbf{Q}_i \cdot \mathbf{Q}_j \; w_{ij} \quad , \qquad w_{ij} = \frac{(q^0)^2 \; p_i \cdot p_j}{(p_i \cdot q)(p_j \cdot q)}$$

— not positive definite, non-Markov..?

$\rightarrow$ spoils probabilistic picture? NO, owing to soft-gluon coherence $\hookrightarrow$

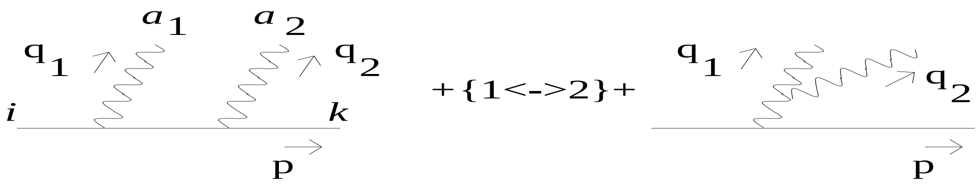
Dokshitzer, Khoze, Mueller and Troian, RMP (1988)

Webber, Ann. Rev. Nucl. Part. Sci. (1986)

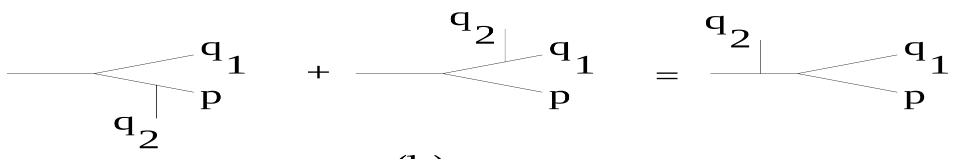
MULTIPLE EMISSION

- 2 soft gluon emission: q_1, q_2 with $q_2^0 \ll q_1^0$

$$\mathbf{J}_1^{\mu a_1} = \mathbf{Q}_p^{a_1} \frac{p^\mu}{p \cdot q_1} \quad , \quad \mathbf{J}_2^{\mu a_2} = \mathbf{Q}_p^{a_2} \frac{p^\mu}{p \cdot q_2} + \mathbf{Q}_{q_1}^{a_2} \frac{q_1^\mu}{q_1 \cdot q_2}$$



(a)

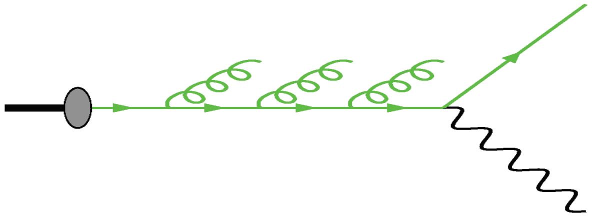


(b)

$$\begin{aligned} \mathcal{M}_{ki}^{a_1 a_2} &= g_s^2 \, \langle a_1 \; k | \, \mathbf{J}_2 \cdot \varepsilon_2 \, | a' \; i' \rangle \, \langle \; i' \; | \mathbf{J}_1 \cdot \varepsilon_1 | \; i \; \rangle \\ &= g_s^2 \, \frac{p \cdot \varepsilon_1}{p \cdot q_1} \left(\frac{p \cdot \varepsilon_2}{p \cdot q_2} \, t^{a_2} t^{a_1} + \frac{q_1 \cdot \varepsilon_2}{q_1 \cdot q_2} \, [t^{a_1}, t^{a_2}] \right)_{ki} \end{aligned}$$

- small angle: bremsstrahlung cones
- large angle ($\theta_{pq_2} \gg \theta_{pq_1}$): sees total charge $\mathbf{Q}_p + \mathbf{Q}_{q_1}$

COHERENCE IN HIGH-ENERGY LIMIT



multi-scale: $s = q_1^2 \gg \cdots \gg q_n^2 \gg \Lambda^2$

[e.g.: LHC final states with multi-jets]

$\Downarrow$

▷ internal emissions non-negligible

▷ current also factorizable at high-energy: *[Ciafaloni 1998; 1988]*

$$\left|M^{(n+1)}(k,p)\right|^2 = \left\{ [M^{(n)}(k+q,p)]^\dagger \, [\mathbf{J}^{(R)}]^2 \, M^{(n)}(k+q,p) - [M^{(n)}(k,p)]^\dagger \, [\mathbf{J}^{(V)}]^2 \, M^{(n)}(k,p) \right\} \ .$$

BUT... ▷

... • $\mathbf{J}$ depends on total transverse momentum transmitted

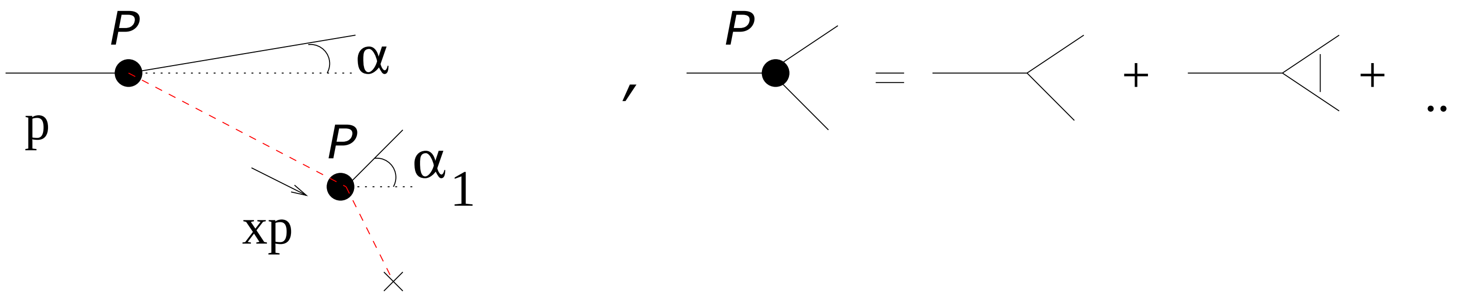
$\Rightarrow$ matrix elements and pdf at fixed $k_\perp$ ("unintegrated")

• virtual corrections not fully represented by Δ form factor

$\Rightarrow$ modified branching probability $P(z, k_\perp)$

K$_\perp$-DEPENDENT PARTON BRANCHING

$$\mathcal{G}(x,k_T,\mu) = \mathcal{G}_0(x,k_T,\mu) + \int \frac{dz}{z} \int \frac{dq^2}{q^2}\, \Theta(\mu - zq) \times \underbrace{\Delta(\mu,zq)}_{\text{Sudakov}} \ \underbrace{\mathcal{P}(z,q,k_T)}_{\text{unintegr. splitting}} \ \mathcal{G}(\frac{x}{z}, k_T + (1-z)q, q)$$

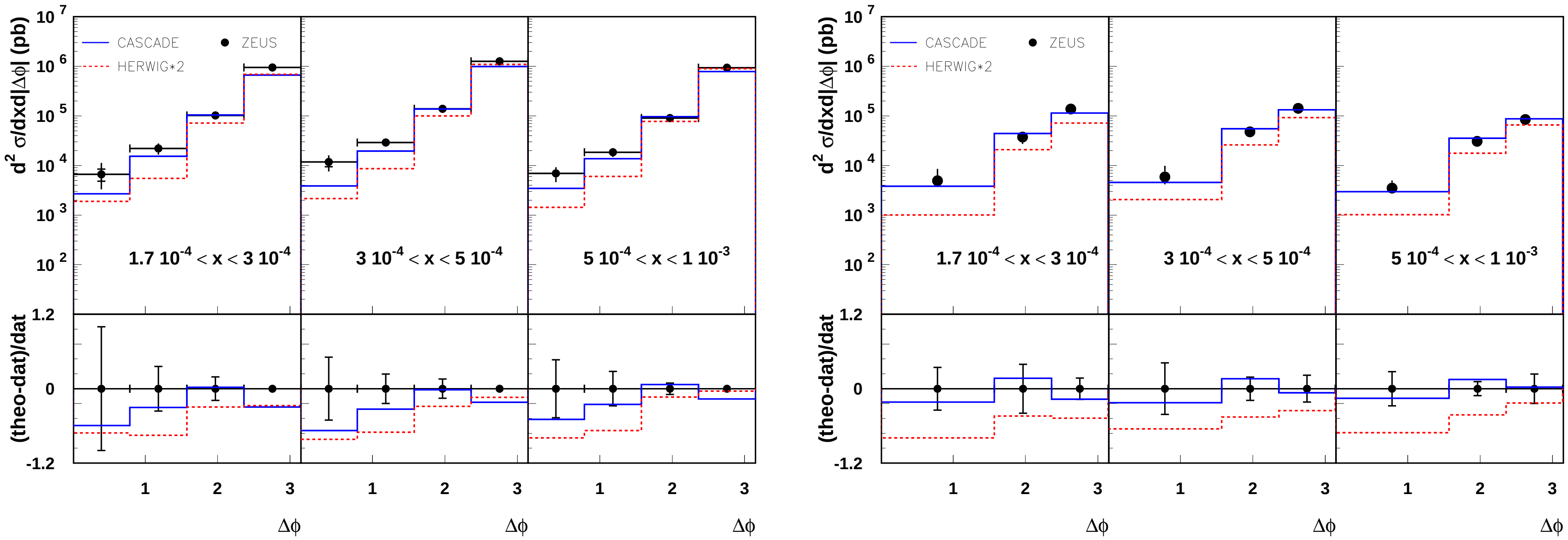


(left) Coherent radiation in the space-like parton shower for $x \ll 1$;
(right) the unintegrated splitting function $\mathcal{P}$, including small-x virtual corrections.

$$\alpha/x > \alpha_1 > \alpha \quad (\text{small} - \text{x} \ \ \text{coherence} \ \ \text{region})$$

- Monte Carlo event generators: Cascade, Ldcmc, Dipsy
  - how does k$_\perp$-branching affect analysis of MPI

NO-MPI CASE: ANGULAR JET CORRELATIONS IN EP FROM $K_\perp$-SHOWER (CASCADE) AND COLLINEAR-SHOWER (HERWIG)

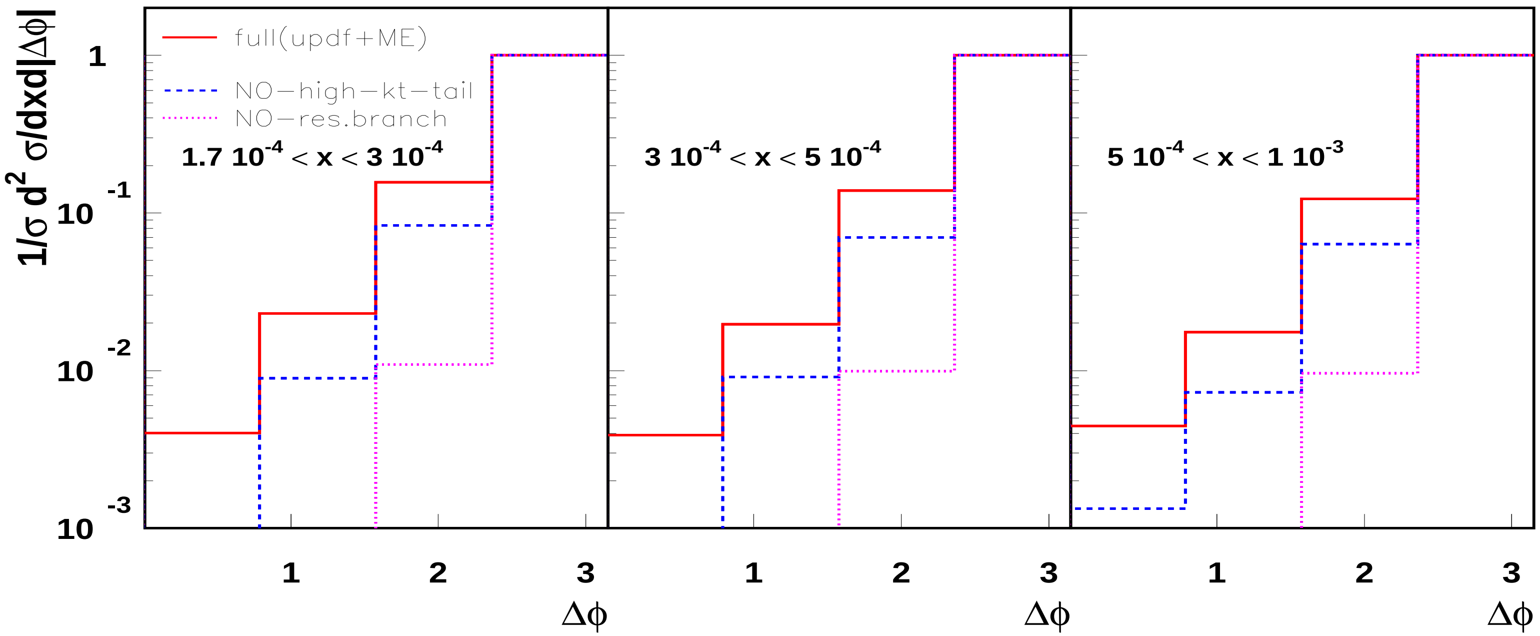


(left) di-jet cross section; (right) three-jet cross section

Jung & H, JHEP 0810 (2008) 113

- quantitative effects of small-x coherence sizeable
- largest differences at small $\Delta\phi$
- good description of shapes by $k_\perp$-shower
- HERWIG normalized to 2-jet region by K-factor

Normalize to the back-to-back cross section:



updf $\oplus$ ME

- - - updf $\oplus$ ME$_{collin.}$: $\mathcal{M} \to \mathcal{M}_{collin.}(k_T) = \mathcal{M}(0_\perp)\ \Theta(\mu - k_T)$

· · · · · · no resolved branching : $\mathcal{A} \to \mathcal{A}_{no-res.}(x, k_T, \mu) = \mathcal{A}_0(x, k_T, Q_0)\ \Delta(\mu, Q_0)$

▷ high-k$_\perp$, coherent effect essential for correlation at small $\Delta\phi$

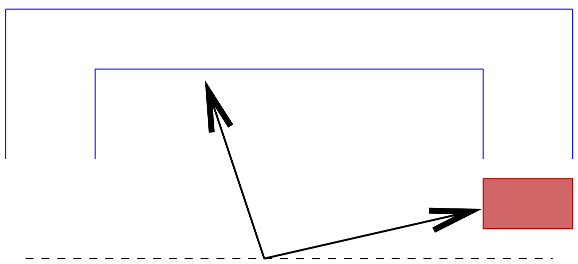
cfr. other approaches (e.g., KMR):

u-pdf but no ME correction

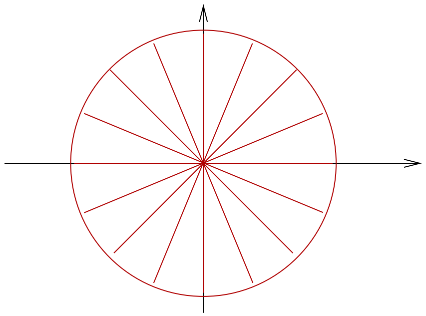
II. MULTIPLE JETS AT THE LHC: ASSOCIATED FORWARD AND CENTRAL JETS

- polar angles small but far enough from beam axis
  - measure correlations in azimuth, rapidity, p_T

$$p_\perp \gtrsim 20 \text{ GeV} , \ \Delta\eta \gtrsim 4 \div 6$$



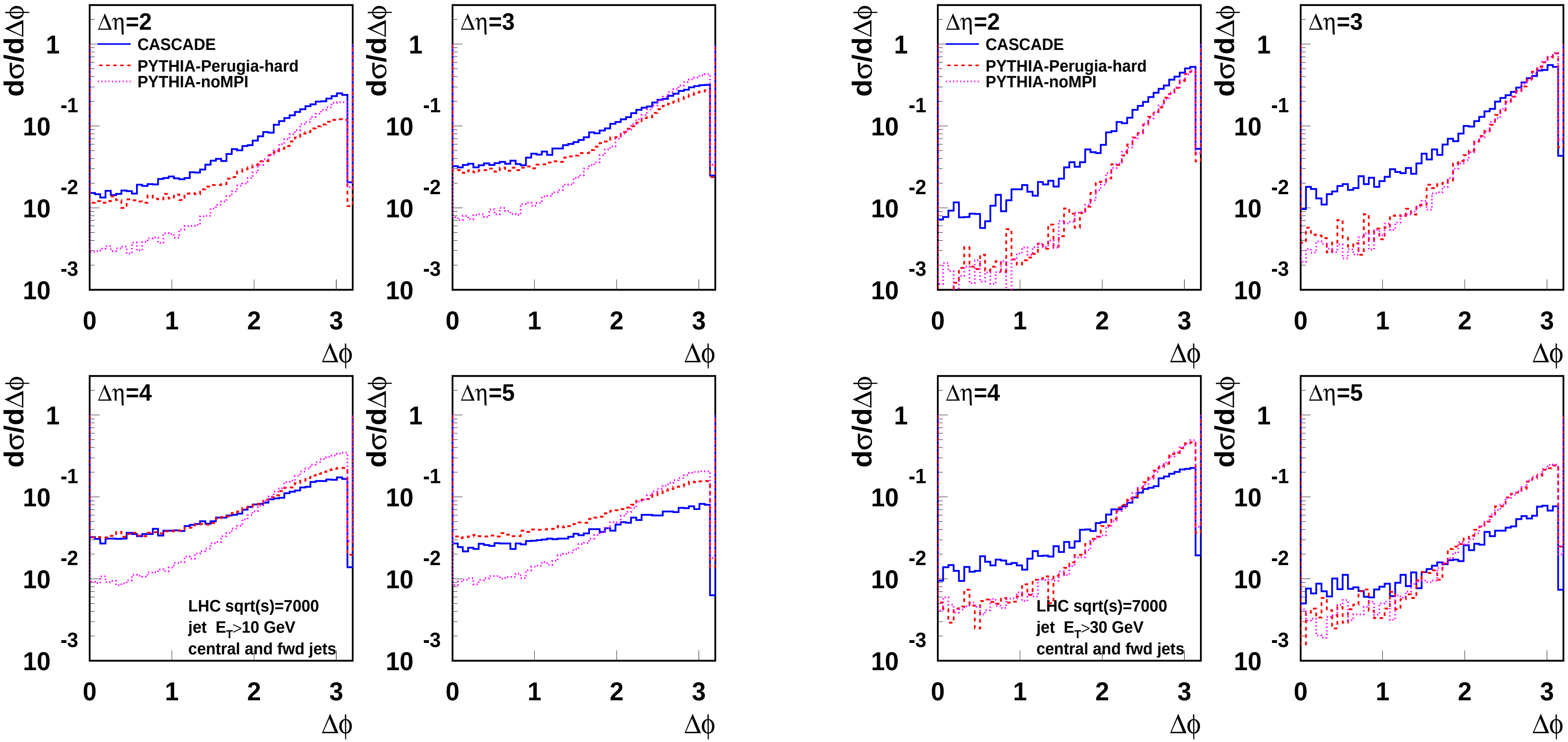
central + forward detectors



azimuthal plane

Cross section as a function of the azimuthal difference $\Delta\phi$ between central and forward jet for different rapidity separations

[Deak et al., arXiv:1012.6037]

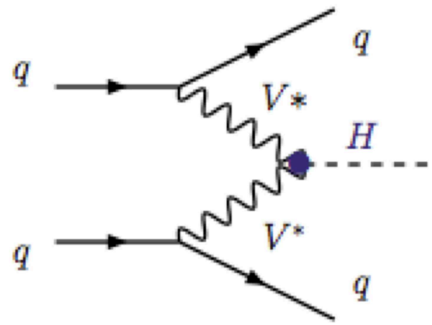


(left) low E_T: decorrelation increasing with $\Delta\eta$ from non-collinear corrections (CASCADE) and from MPI (PYTHIA); (right) high E_T: MPI effect dies out

Extension to forward-backward kinematics

♢ search for Mueller-Navelet effects

♢ background to searches in vector boson fusion channels



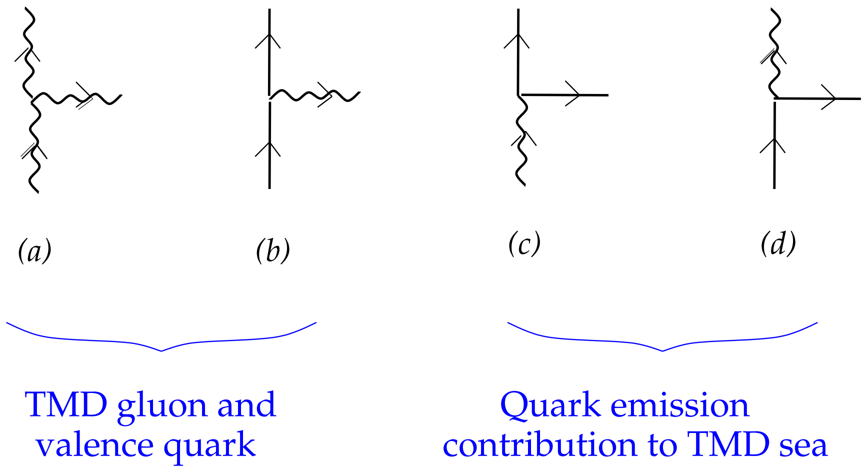
♢ large-Δy dijet data poorly understood

[CMS, arXiv:1204.0696]

[ATLAS, arXiv:1107.1641]

Beyond quenched approximation: unintegrated quark evolution

[Hentschinski, Jung & H, arXiv:1205.1759]



- sea: flavor-singlet evolution coupled to gluons at small x via

$$\mathcal{P}_{g\to q}(z;q,k) = P_{qg,\mathrm{DGLAP}}(z)\ \left(1+\sum_{n=0}^{\infty}\ b_n(z)(k^2/q^2)^n\right)$$

all b_n known; $\mathcal{P}_{g\to q}$ computed in closed form (positive-definite)

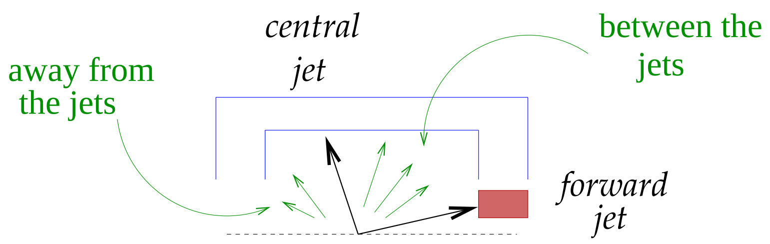
in *[Catani & H, 1994; Ciafaloni et al., 2005-2006]* by small-x factorization

- valence: independent evolution (dominated by soft gluons x $\to$ 1)

ENERGY FLOW OBSERVABLES

Transverse energy flow as a function of rapidity and azimuthal angle

$$1 < \eta_c < 2 \ , \qquad -5 < \eta_f < -4$$



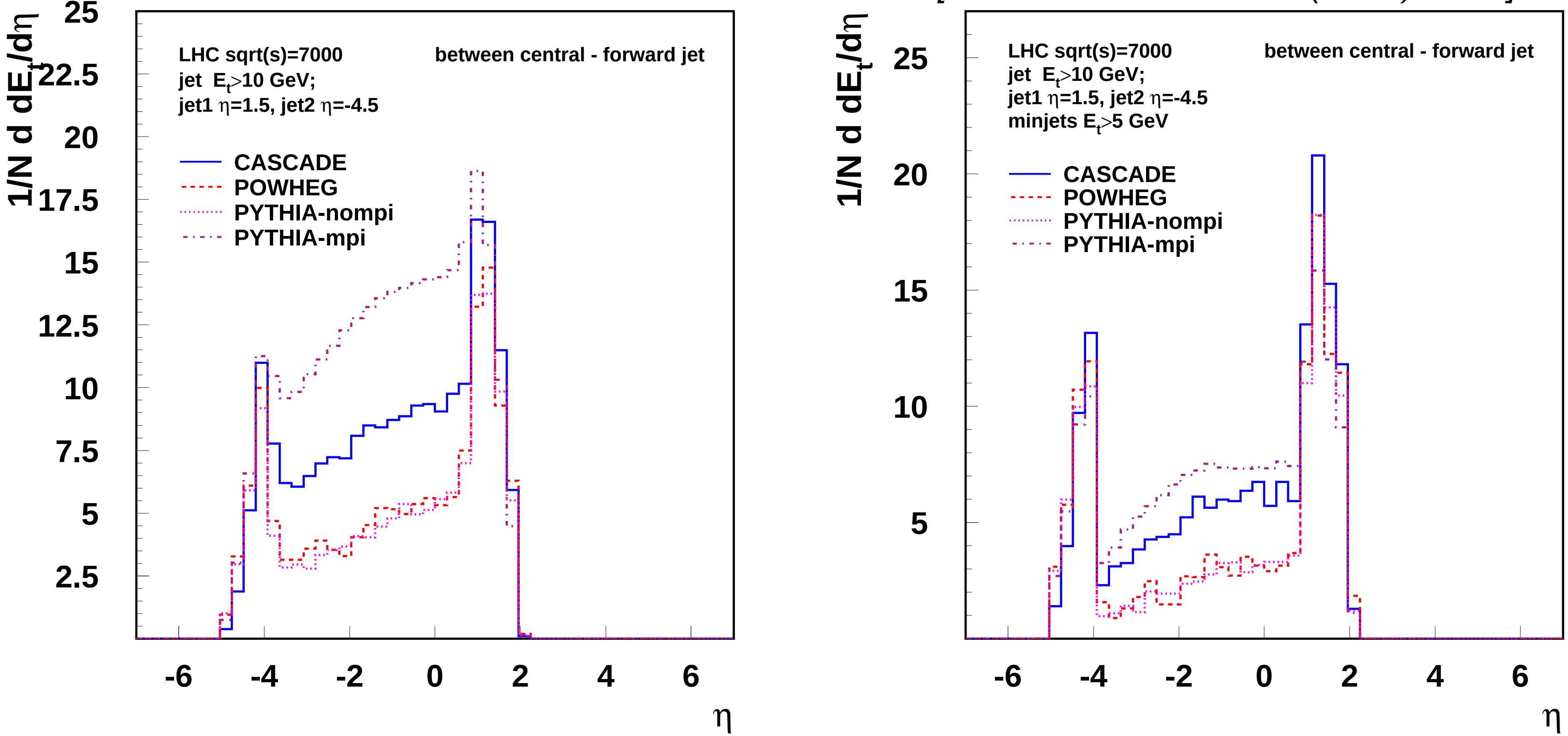
$$\frac{dE_\perp}{d\eta} = \frac{1}{\sigma} \int dq_\perp \ q_\perp \ \frac{d\sigma}{dq_\perp \ d\eta}$$

"Minijet" energy flow

- merge particles into jets via jet algorithm
- construct energy flow from jets with $q_\perp > q_0$
- $q_0 = \mathcal{O}(\text{a few GeV})$ feasible at the LHC

Transverse energy flow in the inter-jet region

[Deak et al., EPJC 72 (2012) 1982]

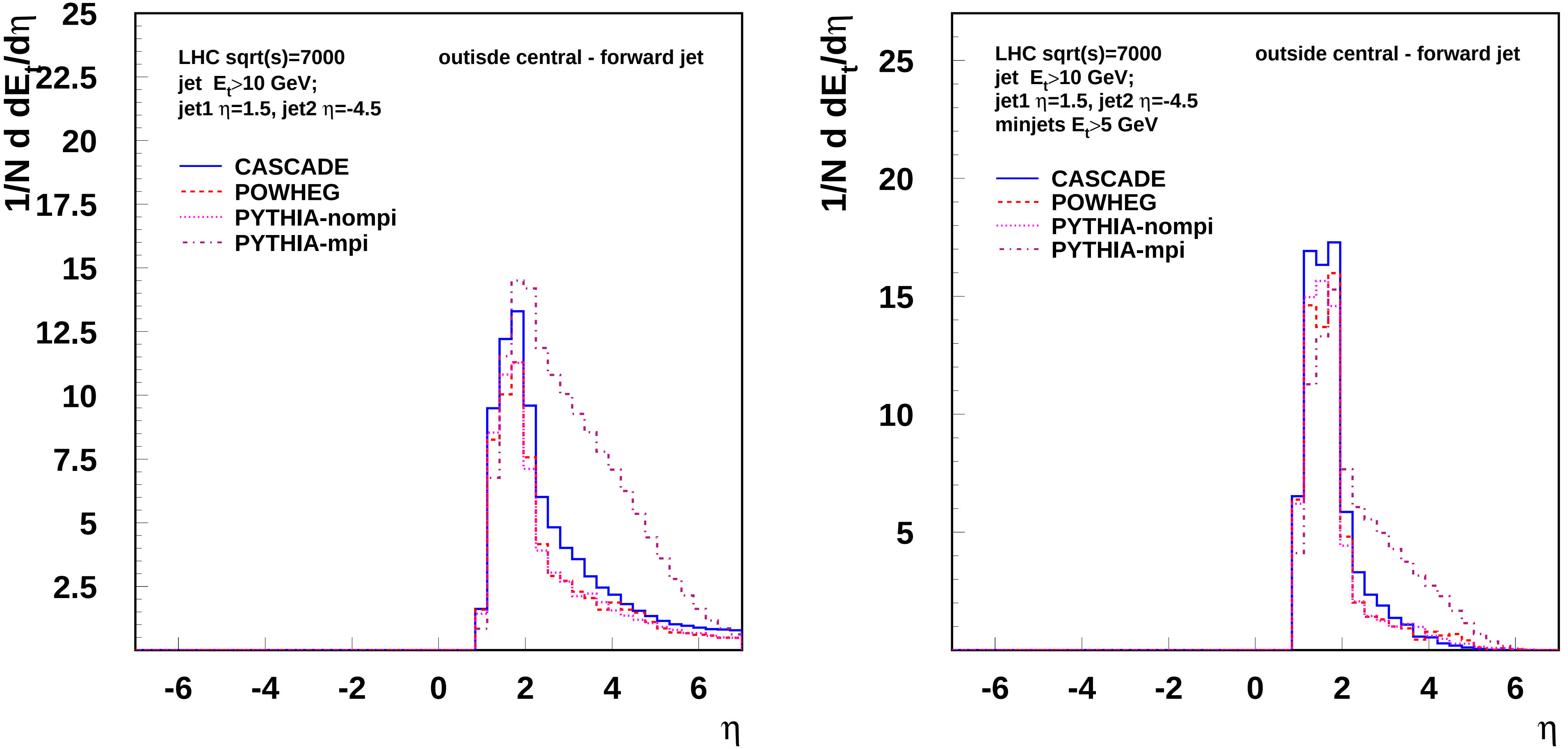


(left) particle flow; (right) minijet flow

▷ higher mini-jet activity in the inter-jet region from corrections to collinear ordering and from MPI (tune Z1)

▷ little effect from NLO hard correction in POWHEG

Transverse energy flow in the outside region

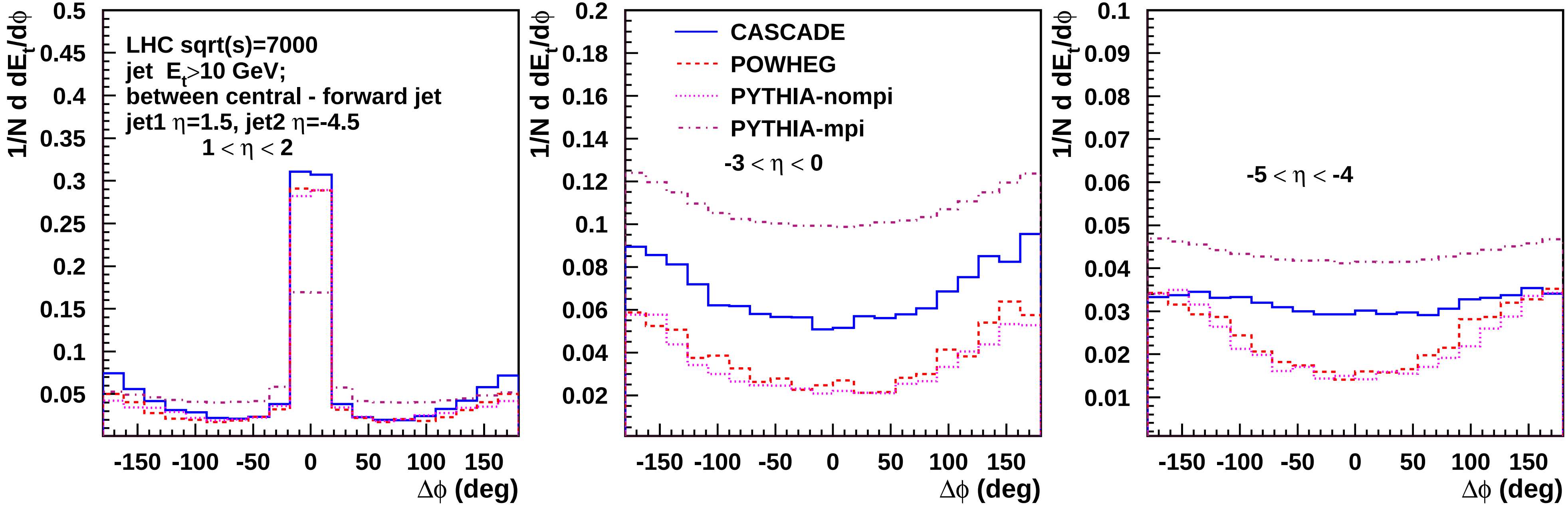


▷ at large (opposite) rapidities, full branching well approximated by collinear ordering

▷ higher energy flow only from multiple interactions

Azimuthal dependence of transverse energy flow

(left) central-jet; (middle) intermediate; (right) forward-jet rapidities



- more pronounced flattening of the $\Delta\phi$ distribution from CASCADE and PYTHIA-mpi (tune Z1) compared to POWHEG and PYTHIA-nompi

$\diamondsuit$ Particle spectra most commonly used

$\diamondsuit$ Energy flow due to minijets ($q_T > q_0, \ e.g. \ q_0 = 5$ GeV)

has different advantages:

- IR sensitivity controllable by jet clustering
- observables applicable uniformly at low and high transverse momenta (*e.g.* multiplicity distributions)

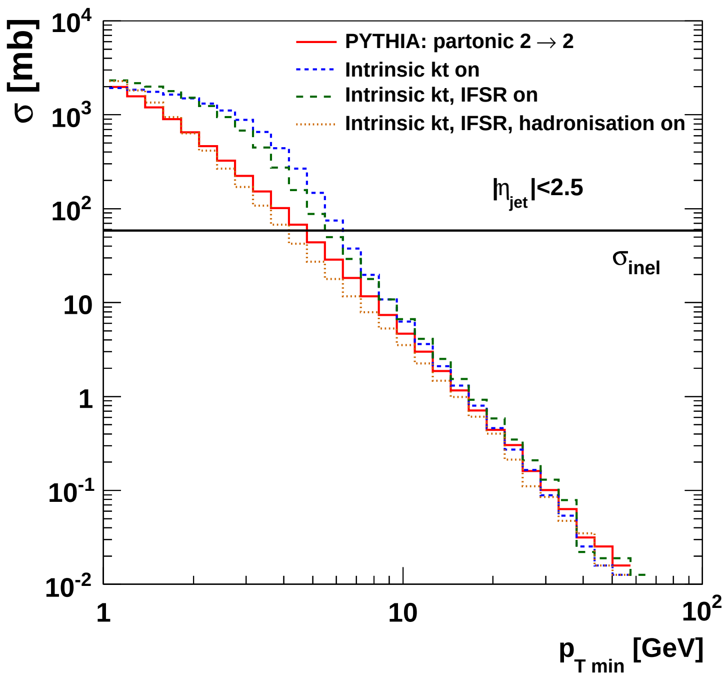
III. Mini-jets and the inelastic pp cross section

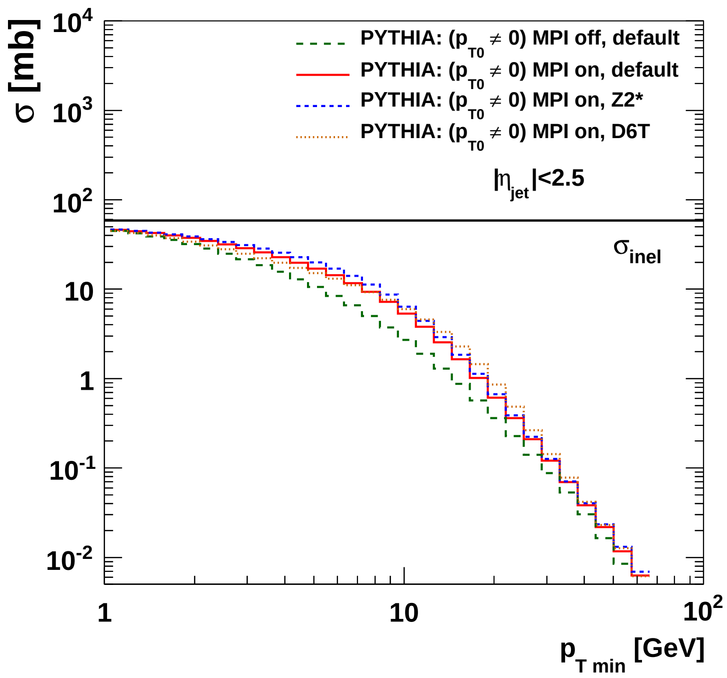
- Extend central jet measurements to lower $p_\perp$

$\Rightarrow$ visible jet cross section sensitive to bound from inelastic σ_{pp}

[ATLAS Coll., Nature Commun. 2 (2011) 46

CMS Coll., CMS PAS QCD-11-002]





(Left) cross section from purely partonic $2{\to}2$ process, including intrinsic k_t-effects, initial and final state parton showers (IFSR), hadronization;

(Right) result of applying $p_{T0} \neq 0$ and MPI with different UE tunes of PYTHIA.

[Grebenyuk et al., arXiv:1209.6265]

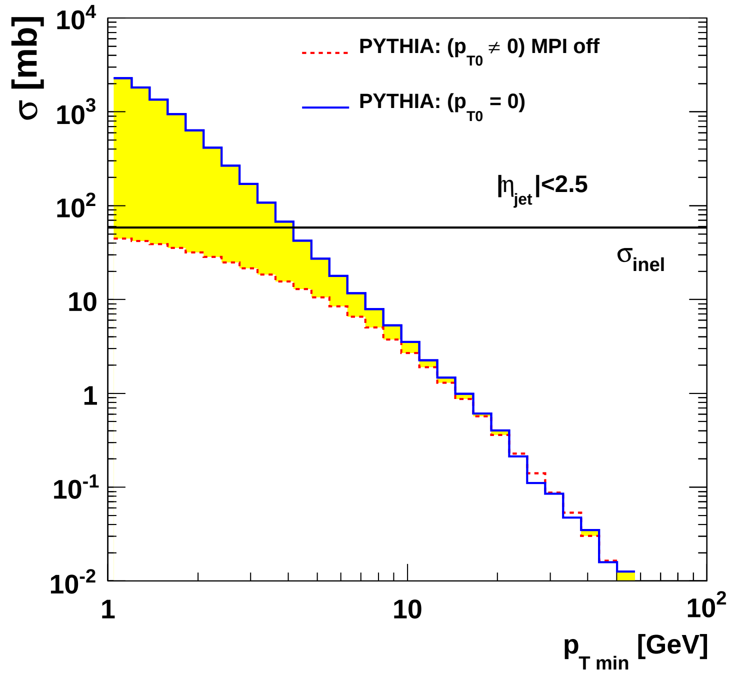
- low-p_T model in collinear framework (PYTHIA):

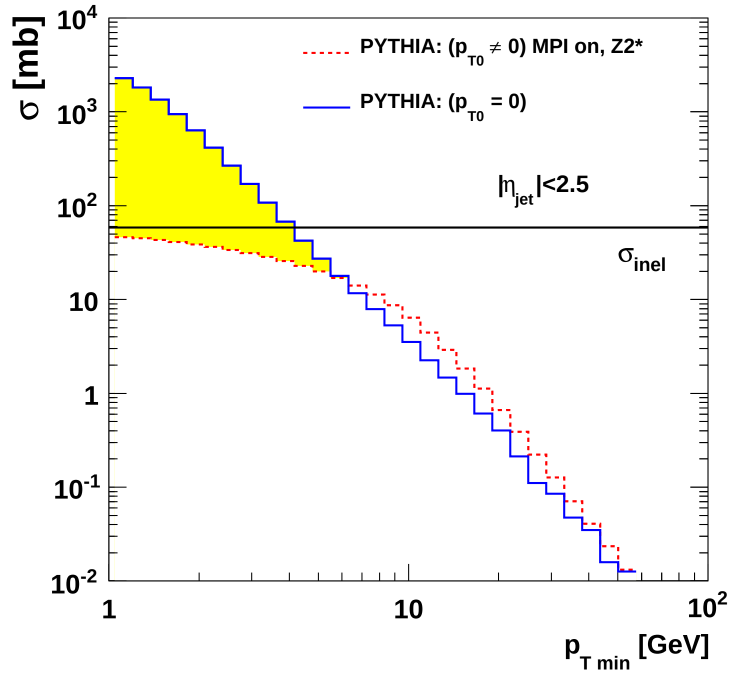
$$\sigma \to \sigma \times \frac{\alpha_s^2(p_{T0}^2 + p_T^2)}{\alpha_s^2(p_T^2)} \frac{p_T^4}{(p_{T0}^2 + p_T^2)^2}$$

- k_T factorized: low-p_T behavior results from

— ME dependence (standard low-p_T rise for $k_T \ll p_T$, slower rise for $k_T \simeq p_T$)

— unintegrated pdf (suppression of the low-k_T region)



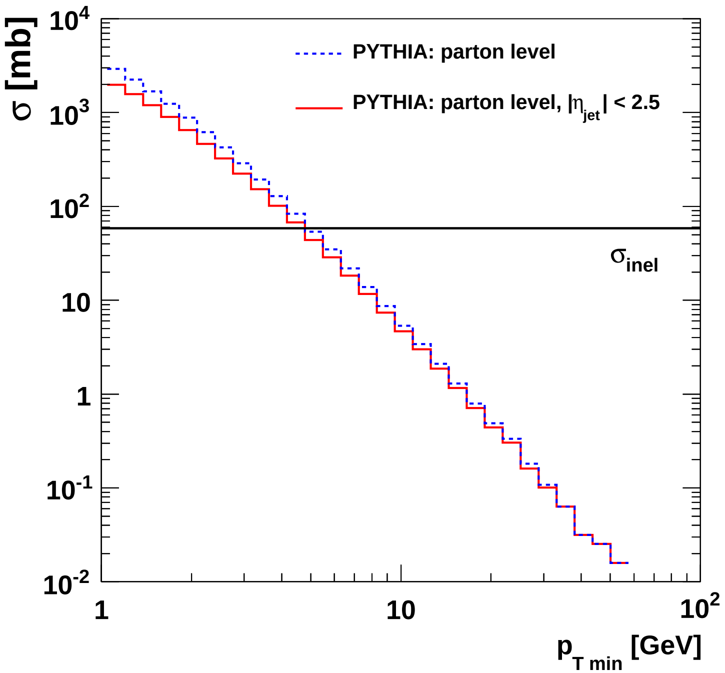


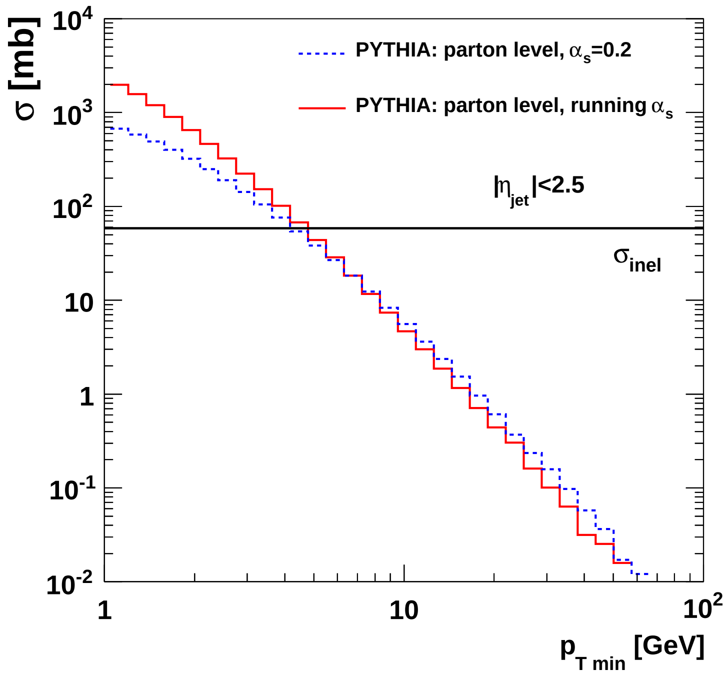
Left: without MPI. Right: including MPI

[Grebenyuk et al., arXiv:1209.6265]

• low p_T $\Rightarrow$ jets constructed from charged tracks $\Rightarrow$ *visible* cross section

• comparison with inelastic $\sigma(pp)$ within acceptance — no extrapolation needed

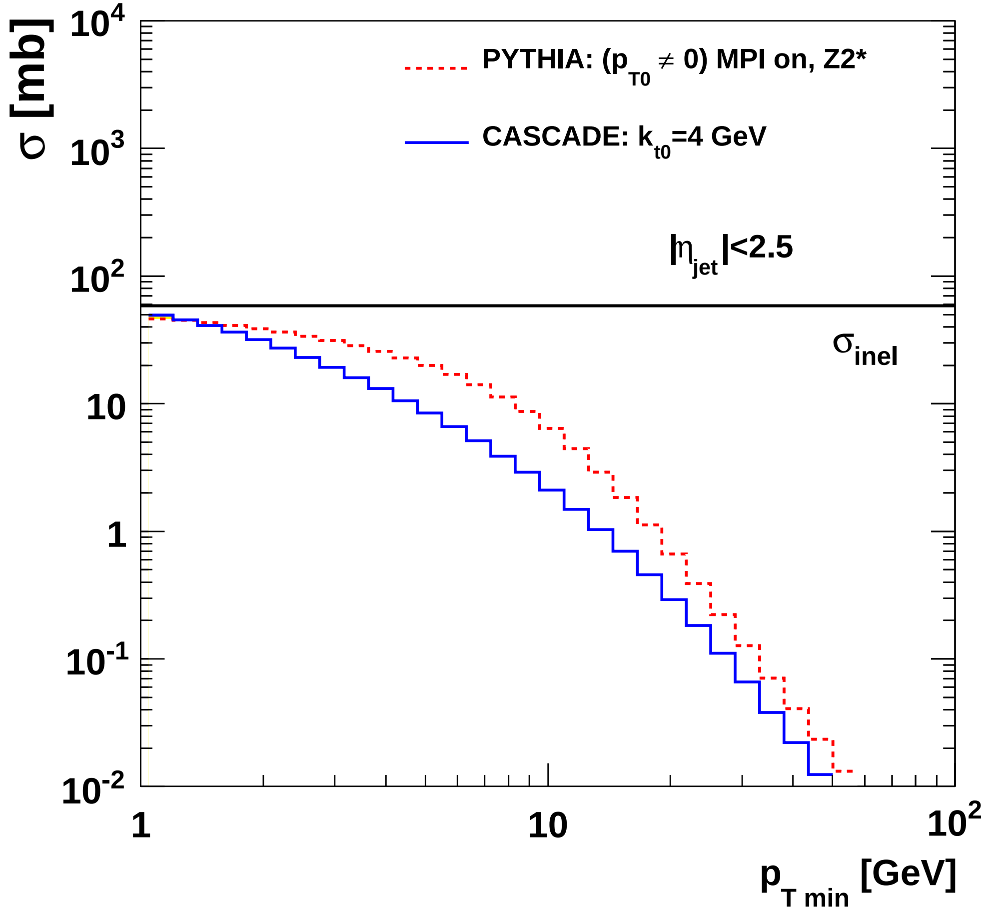




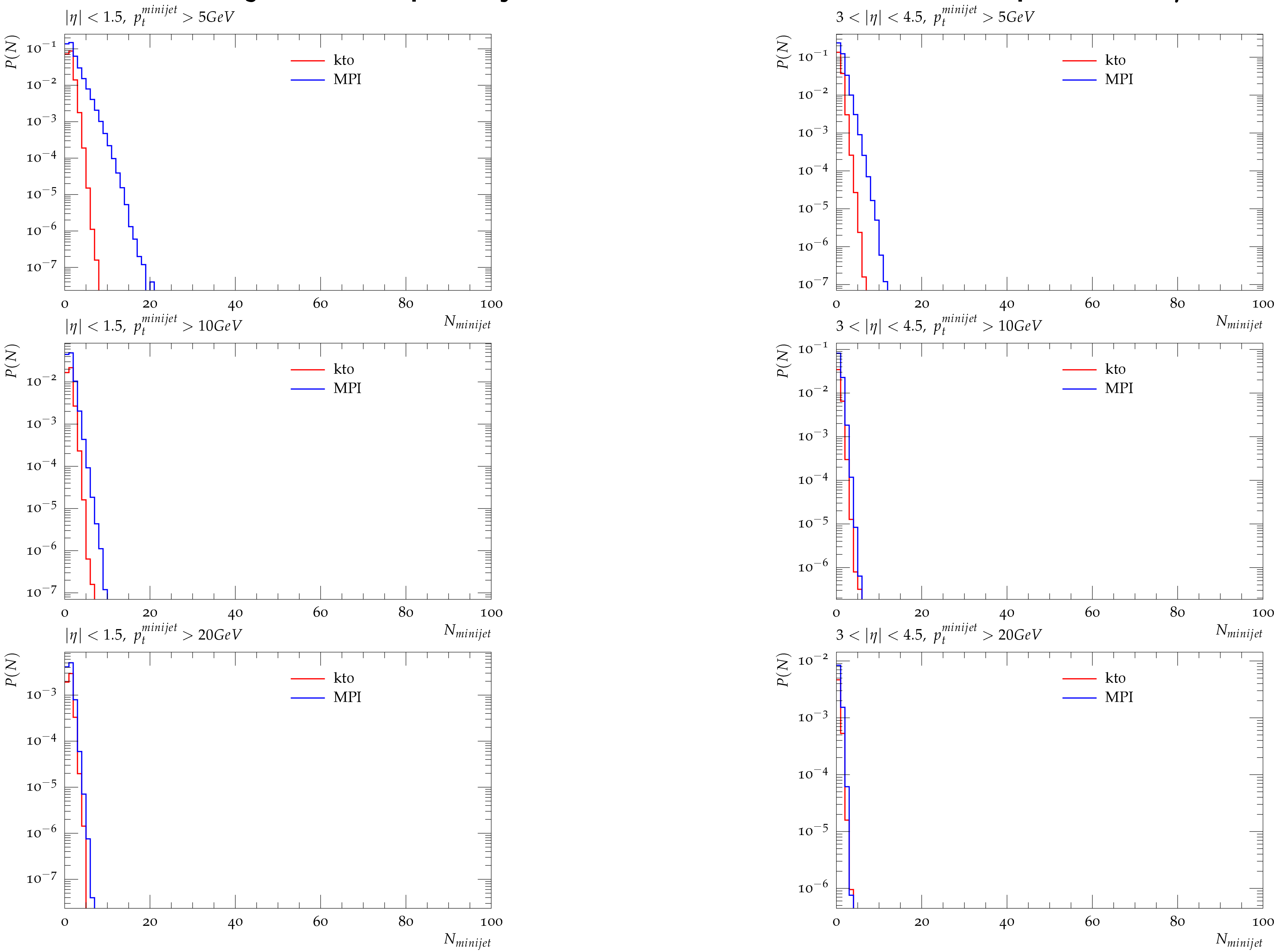
• no large effect from infrared behavior of the QCD coupling in the p_T region where jet cross section approaches the inelastic bound

♠ even though at weak coupling, dynamical effects slowing down the rise of the cross section can involve strong fields and nonperturbative physics

- Low transverse momentum cross section from k_T-shower and PYTHIA MPI model



• Mini-jet multiplicity distributions for different p_T and η



- besides cross sections, measure minijet multiplicity distributions for different transverse momenta $q_T > q_0$:
  e.g. $q_0 =$ 2 GeV, 5 GeV, 10 GeV, 20 GeV

- MPI contribution to high multiplicities for low q_0;
  how does it vary with increasing q_0

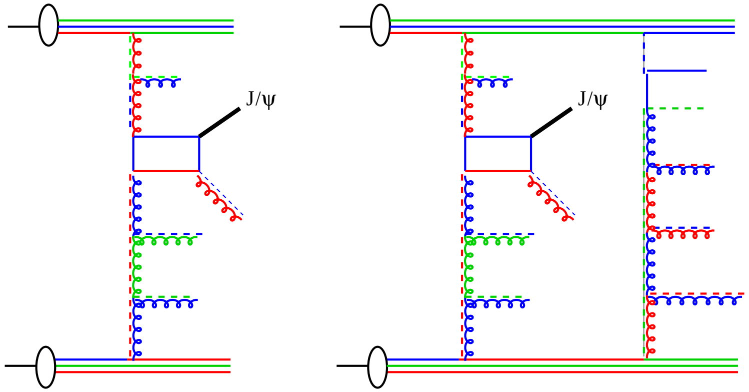
$\Rightarrow$ probe separately contribution from different
regions in impact parameter
using (mini)jet multiplicities uniformly from low to high p-transverse

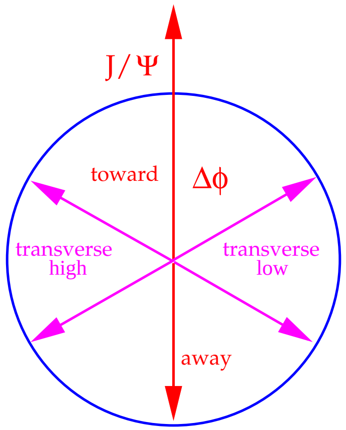
HEAVY FLAVOR + JETS

Example: J/ψ and associated mini-jet multiplicities

▷ underlying event analysis using gluonic probe [cfr. V-boson + jets]

▷ perturbative calculation down to $p_\perp$ of order m_ψ





▷ See also: J/ψ vs. charged particle multiplicity *[Portebeuf & Granier, arXiv:1012.0719]*

[ALICE Coll., arXiv:1202.2816]

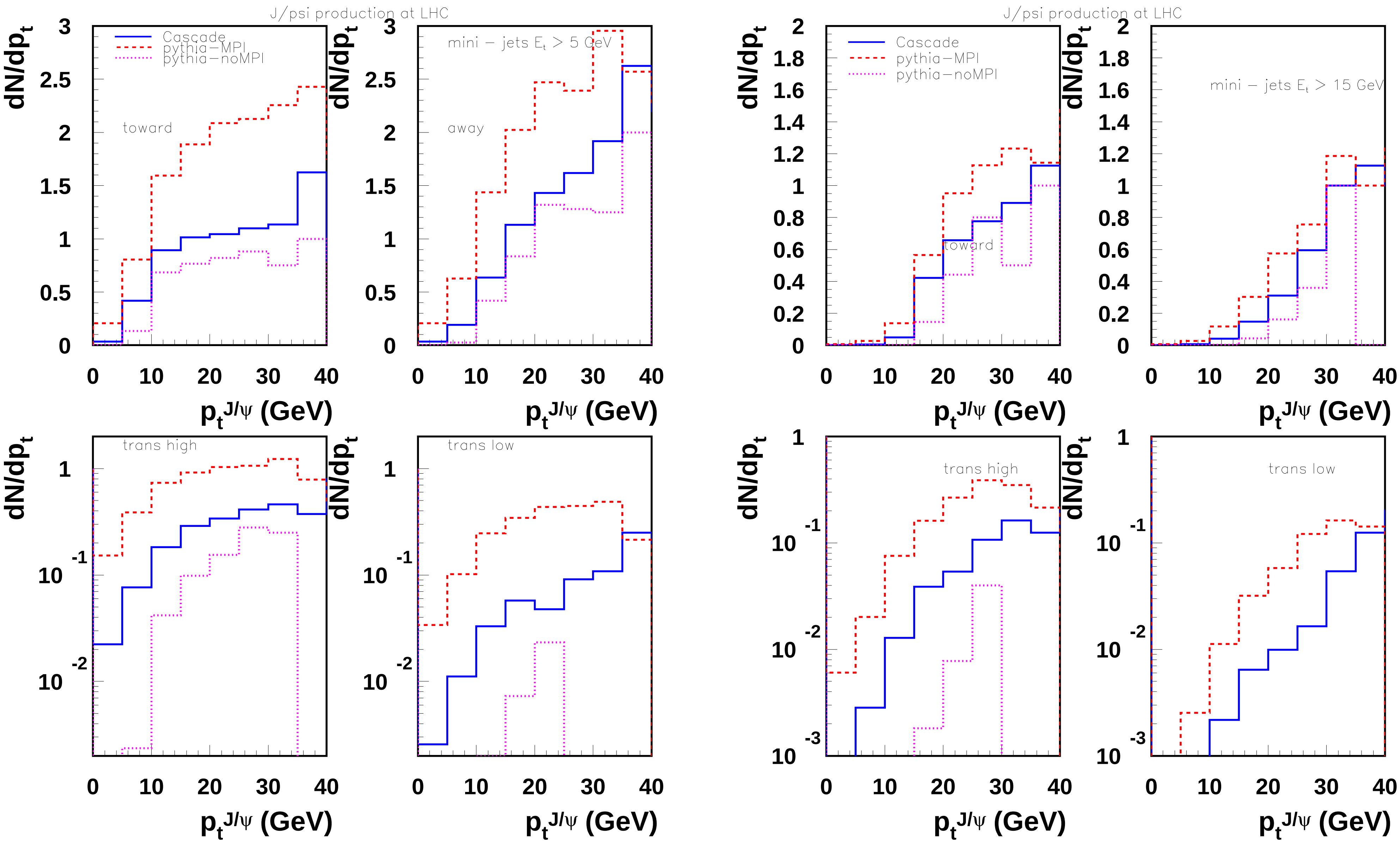
▷ J/ψ pairs as a probe of DPI *[Kom, Kulesza & Stirling, arXiv:1105.4186]*

[Baranov, Snigirev & Zotov, arXiv:1105.6276]

[LHCb Coll., arXiv:1109.0963]

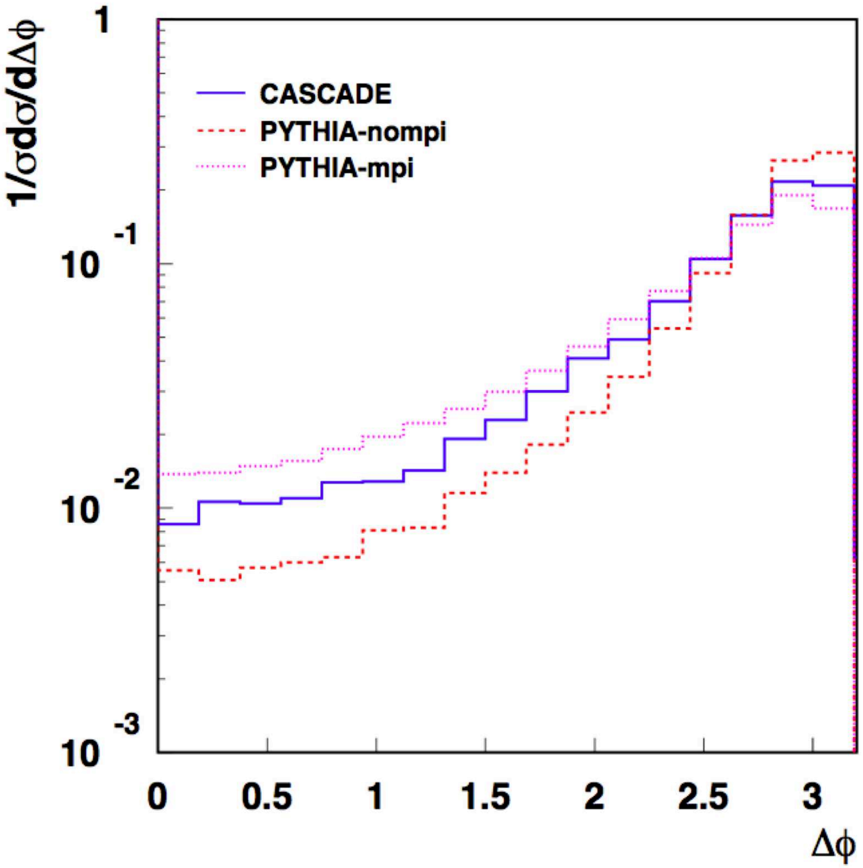
Mini-jets associated with J/ψ

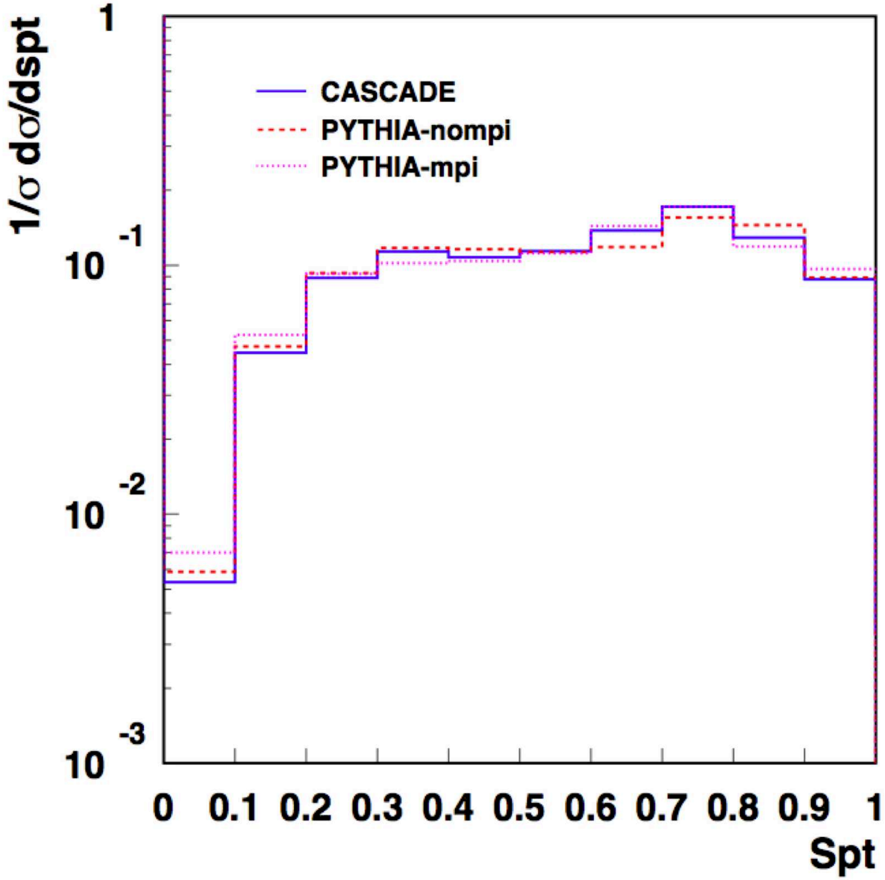
[Jung et al., in progress]



(left) $E_t > 5$ GeV; (right) $E_t > 15$ GeV

$J/\psi + 3$ jets


1/σdσ/dΔφ
CASCADE
PYTHIA-nompi
PYTHIA-mpi
Δφ


1/σ dσ/dspt
CASCADE
PYTHIA-nompi
PYTHIA-mpi
Spt

CONCLUSION

• MPI increasingly important as parton densities grow with energy

▷ sensitive to detailed structure of final states produced by shower evolution

⇒ what level accuracy required in parton branching algorithms?

⇒ amount of MPI reduced by inclusion of non-collinear-ordered effects to showers?

• J/ψ associated multiplicities probe gluonic jets at low but perturbative $p_\perp$

▷ complementary to UE / MPI studies in V-boson + jets

• Measure visible jet cross section down to $p_T \sim$ a few GeV using tracker and (mini-)jet multiplicity distributions per rapidity bin from low to high p_T

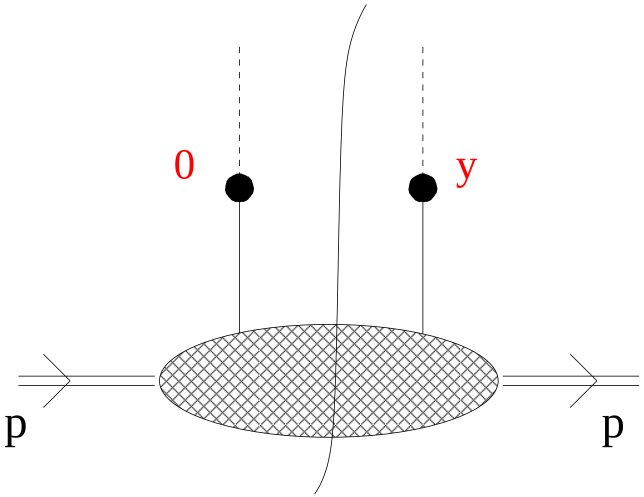
⇒ how does MPI contribution to high multiplicities change with p_T

⇒ p_{Tmin} dependence and inelastic pp cross section

weak-coupling but nonperturbative QCD physics

EXTRA SLIDES

UNINTEGRATED PARTON DISTRIBUTIONS



$p = (p^+ , m^2 / 2 p^+ , 0_\perp)$

$$\widetilde{f}(y) = \langle \; P \mid \overline{\psi}(y) \; V_y^\dagger(n) \; \gamma^+ \; V_0(n) \; \psi(0) \mid P \; \rangle \quad , \qquad y = (0, y^-, y_\perp)$$

$V_y(n) = \mathcal{P} \exp \left(i g_s \int_0^\infty d\tau \; n \cdot A(y + \tau \; n) \right)$ eikonal Wilson line in direction n

- works at tree level [Mulders, 2002; Belitsky et al., 2003]
- subtler at level of radiative corrections [Collins & Zu; H; Cherednikov et al.]

  $\hookrightarrow x \to 1 \Rightarrow$ explicit regularization method (unlike inclusive case)
- non-abelian Coulomb phase $\to$ spectator effects possibly non-decoupl.

  [Mulders, Bomhof; Collins, Qiu; Brodsky et al]

FULL TMD FACTORIZATION IS YET TO BE ACHIEVED

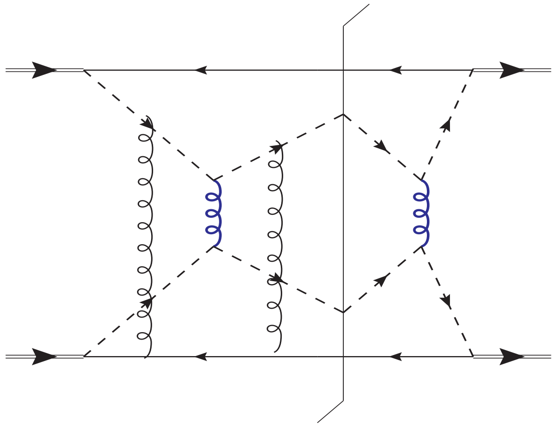
Mulders & Rogers, arXiv:1102.4569; arXiv:1001.2977; Xiao & Yuan, arXiv:1003.0482

• soft gluon exchange with spectator partons

Mert Aybat & Sterman, PLB671 (2009) 46

Boer, Brodsky & Hwang, PRD 67 (2003) 054003

⇒ factorization breaking in higher loops?



Collins, arXiv:0708.4410

Vogelsang and Yuan, arXiv:0708.4398

Bomhof and Mulders, arXiv:0709.1390

♢ likely suppressed for small-x, small-$\Delta\phi$

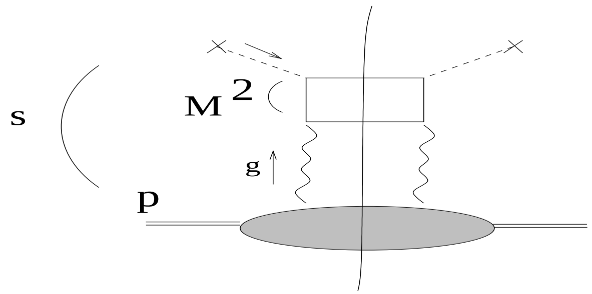
♢ could affect physical picture near large x, back-to-back region

• Note: Coulomb/radiative mixing terms also appear to break coherence in di-jet cross sections with gap in rapidity *[Forshaw & Seymour, arXiv:0901.3037]*

TMD FACTORIZATION AT SMALL X

A physical probe: *(in analogy with DIS/inclusive pdfs)*

TMD pdf factorization from heavy quark photo-production in high-energy limit:



♢ single gluon polarization dominates $s \gg M^2 \gg \Lambda^2_{\rm QCD}$

↪ gauge invariance rescued (despite gluon off-shell)

[Lipatov; Ciafaloni; Catani, H; ...]

♢ energy evolution equations / corrections down by $1/\ln s$ rather than $1/Q$

↪ BFKL (+ its variants)

♢ Note: it works to arbitrarily high $k_\perp$ in the UV ⇒

- suitable for simulations of jet physics at the LHC
- well-defined summation of higher-order radiative corrections

Forward jets:

- High-energy factorization at fixed transverse momentum

$$\frac{d\sigma}{dQ_t^2 d\varphi} = \sum_a \int \phi_{a/A} \otimes \frac{d\widehat{\sigma}}{dQ_t^2 d\varphi} \otimes \phi_{g^*/B}$$

▷ needed to resum consistently both logs of rapidity and logs of hard scale *Deak, Jung, Kutak & H, JHEP 09 (2009) 121*

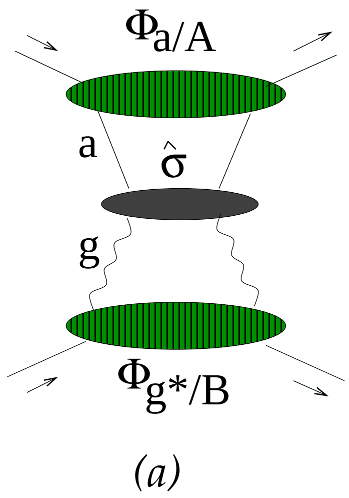


Figure 1: Factorized structure of the cross section.

♢ ϕ_a near-collinear, large-x; ϕ_{g^*} $k_\perp$-dependent, small-x

♢ $\widehat{\sigma}$ off-shell (but gauge-invariant) continuation of hard-scattering matrix elements *[Catani et al., 1991; Ciafaloni, 1998]*

PERTURBATIVE PROPERTIES

• Matrix elements factorize for high energy
not only in collinear region but also at finite angle

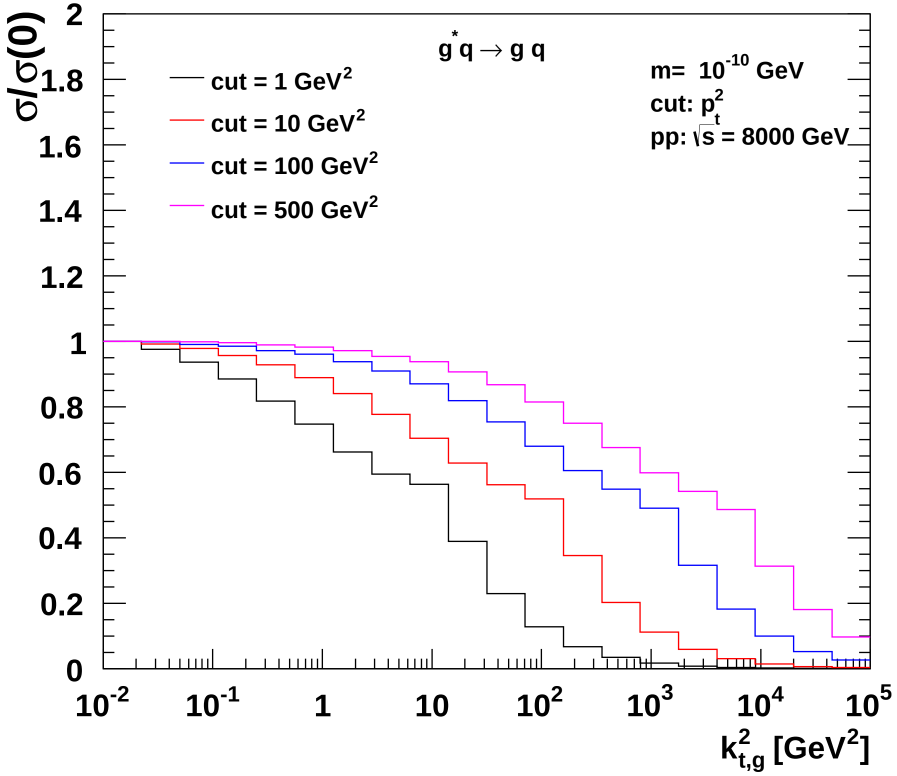
• Couple to $k_\perp$-dependent parton showers
⇒ *coherence* from gluon emission across large rapidity intervals (not small-angle)

• For inclusive quantities (e.g., hard-scattering coefficient functions) return
small-x limit of perturbative results at LO, NLO, NNLO, . . . in α_S

[*Avsar & Collins, arXiv:1209.1675*:
critical survey of published formulae]

HIGH-ENERGY FACTORIZED MATRIX ELEMENTS IN FULLY EXCLUSIVE FORM

Deak, Jung, Kutak & H, JHEP 09 (2009) 121



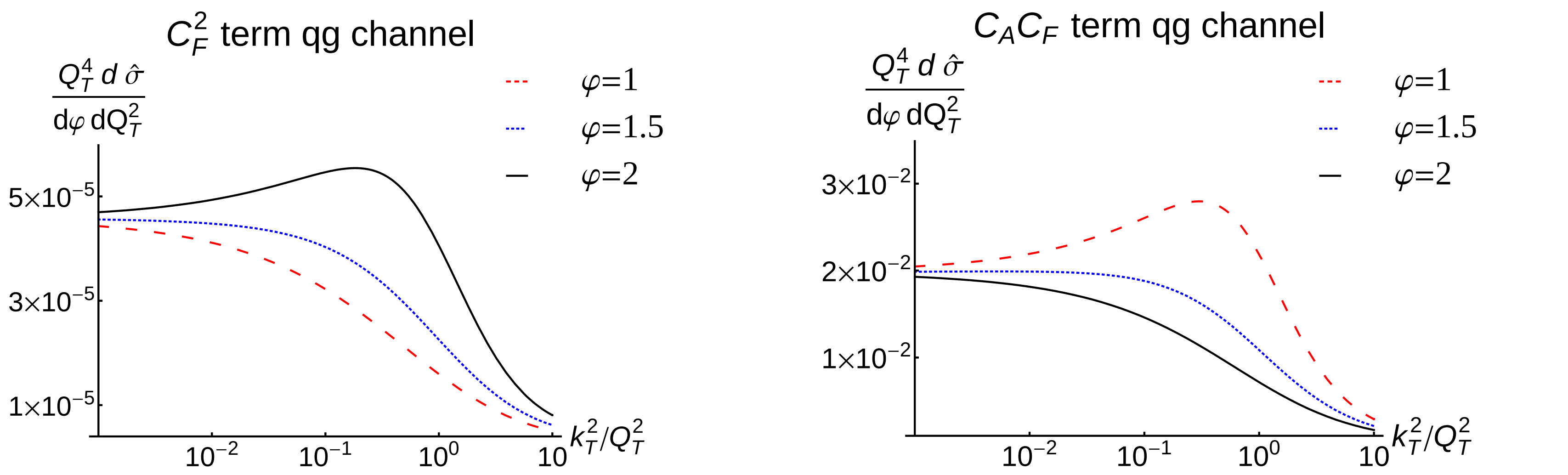
$$cf. \quad \mathrm{ME}_{\mathrm{collinear}} = \Theta(k_t < \mu) \quad , \qquad \mu \sim p_t^{(cut)}$$

FULLY EXCLUSIVE MATRIX ELEMENTS: BEHAVIOR AT LARGE $K_\perp$

Deak, Jung, Kutak & H, JHEP 09 (2009) 121

Q_t = final-state transverse energy (in terms of two leading jets p_t and y)

k_t = transverse momentum carried away by extra jets



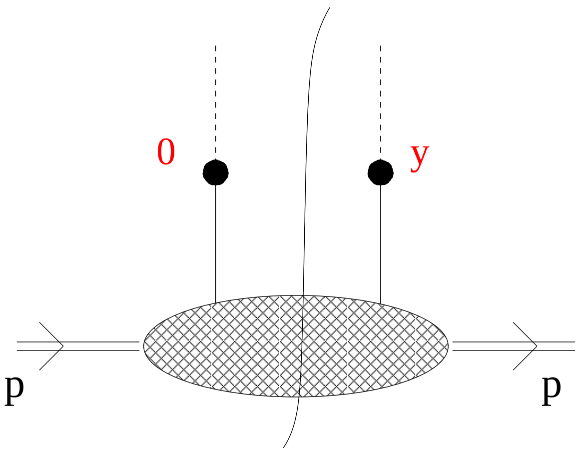
• Matrix elements factorize for high energy

not only in collinear region but also at finite angle

• Couple to $k_\perp$-dependent parton showers

⇒ *coherence* from gluon emission across large rapidity intervals (not small-angle)

UNINTEGRATED (OR TRANSVERSE MOMENTUM DEPENDENT) PARTON DISTRIBUTIONS



$p = (p^+ , m^2 / 2p^+ , 0_\perp)$

$$\widetilde{f}(y) = \langle \, P \mid \overline{\psi}(y) \; V_y^\dagger(n) \; \gamma^+ \; V_0(n) \; \psi(0) \mid P \, \rangle \quad , \qquad y = (0, y^-, y_\perp)$$

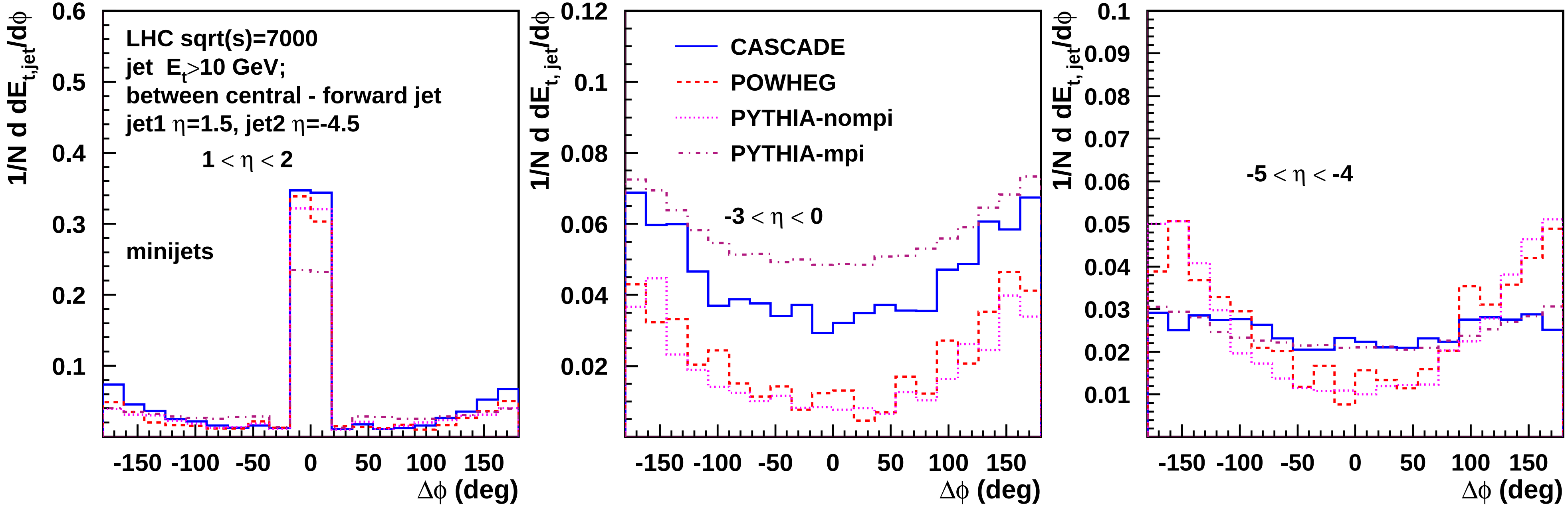
$$V_y(n) = \mathcal{P} \exp \left(i g_s \int_0^\infty d\tau \; n \cdot A(y + \tau \; n) \right)$$

correlation of parton fields ('dressed' with gauge links) at distances y, $y_\perp \neq 0$

- Sudakov region $\Rightarrow$ resummation $\alpha_S^n \ln^k (M/q_T)$
- high energy region $\Rightarrow$ resummation $\alpha_S^n \ln^k (\sqrt{s}/E_T)$

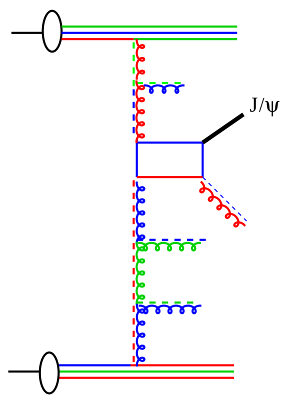
Azimuthal dependence of transverse energy flow

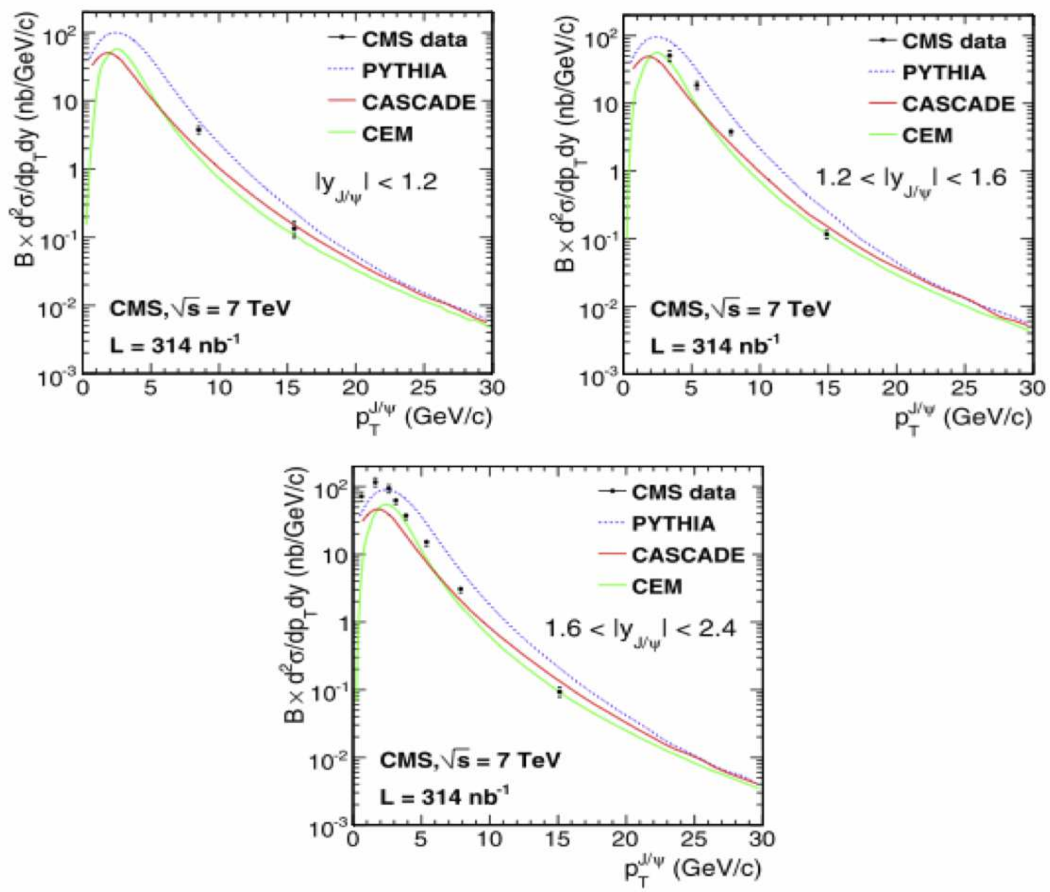
(left) central-jet; (middle) intermediate; (right) forward-jet rapidities



- mini-jet energy flow

Inclusive J/ψ spectra: comparison with CMS measurement





CMS Coll, Eur.Phys.J. C71 (2011) 1575

Charged particle multiplicity associated with J/ψ

[Jung & H, in progress]