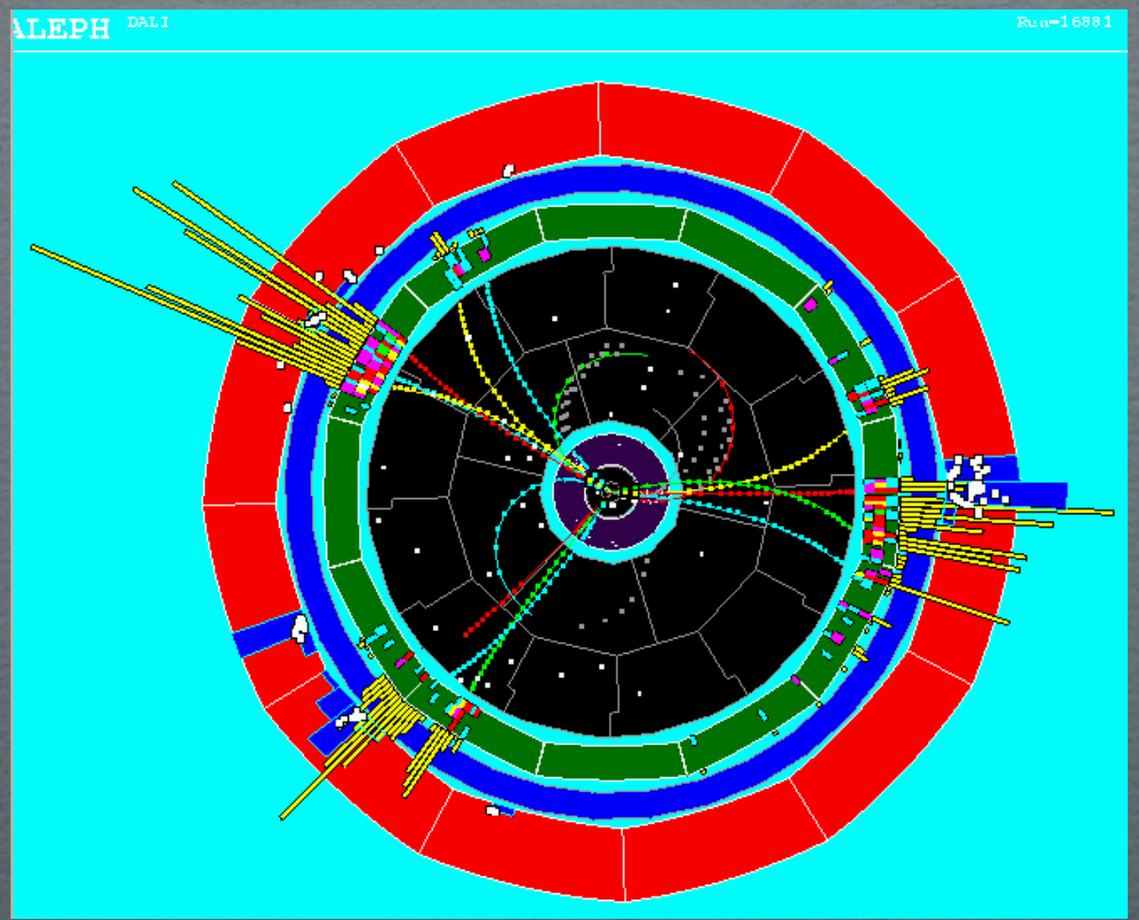
Jet extraction for precision physics



Andrea Banfi
University of Freiburg

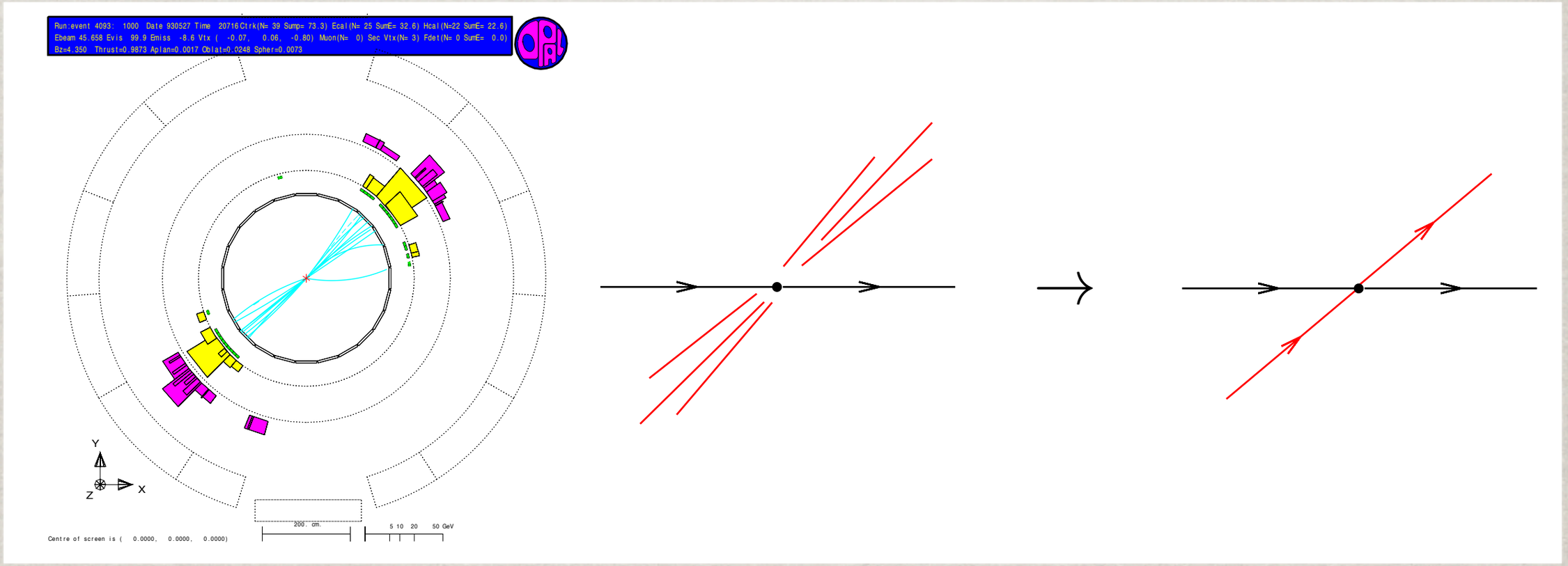
Outline

- Definitions of jets
  - Computing jet observables in perturbative QCD
  - Cone and sequential jet algorithms
- Jets at hadron colliders
  - Impact of hadronisation and underlying event
  - Jet areas and pile-up subtraction
- Jet observables for precision studies
  - Exclusive jet cross sections
  - Dijet azimuthal decorrelations

Why jets?

- Quarks and gluons, the basic objects entering the QCD Lagrangian, are never observed into detectors

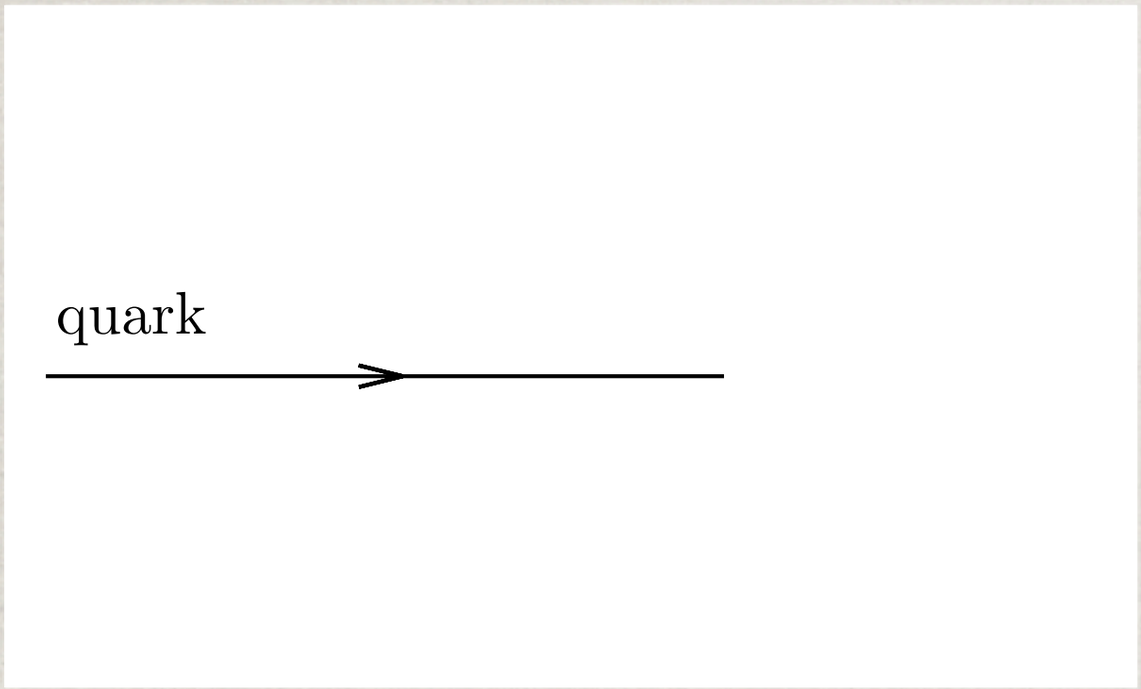
- QCD final states involve highly collimated sprays of energetic hadrons, a.k.a. jets



- Jets are the footprints of partons (quarks and gluons) in the detector $\Rightarrow$ ideally 1 jet $\sim$ 1 parton

Formation of a jet

- The structure of jets reflects the properties of QCD radiation



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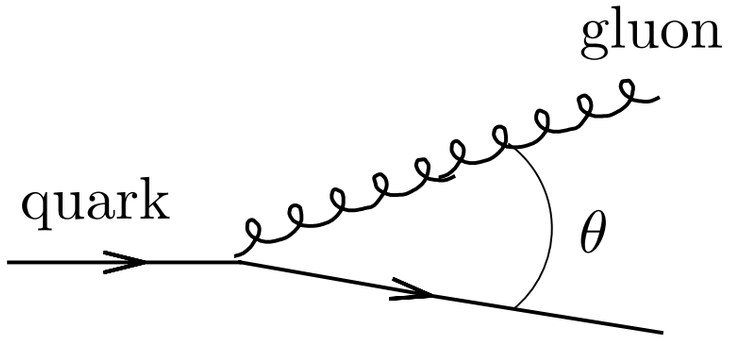

- Gluon emission

$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s(E\theta) \gg 1$$

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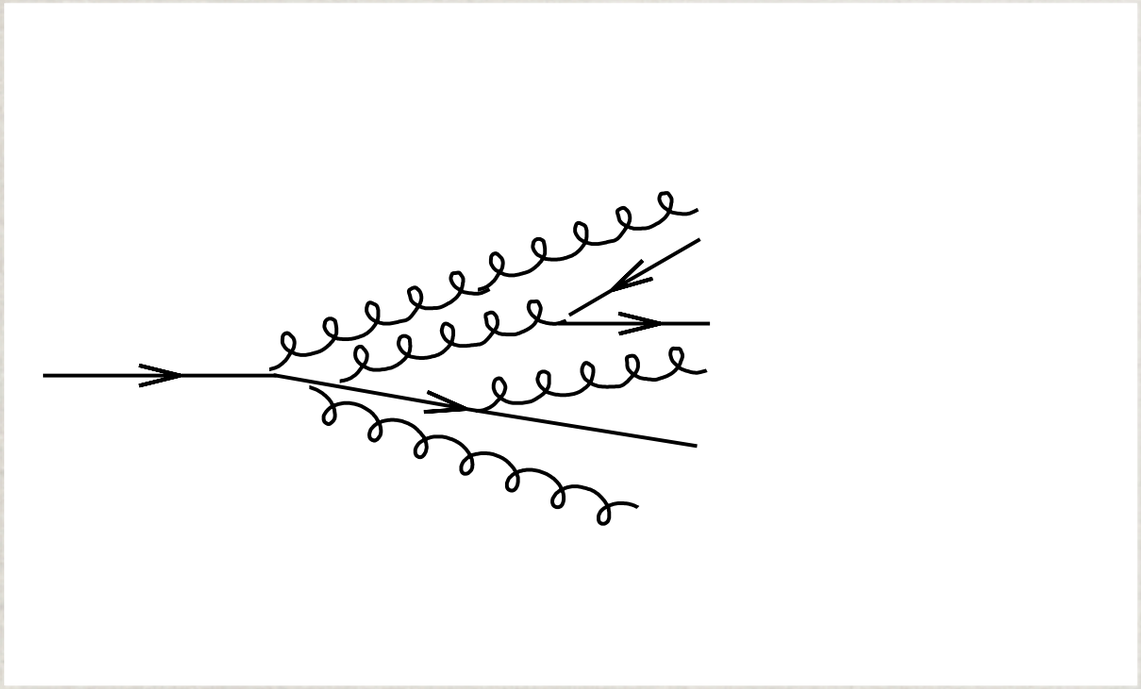

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- High probability of emitting soft ($E \to 0$) and collinear ($\theta \to 0$) gluons (parton shower)

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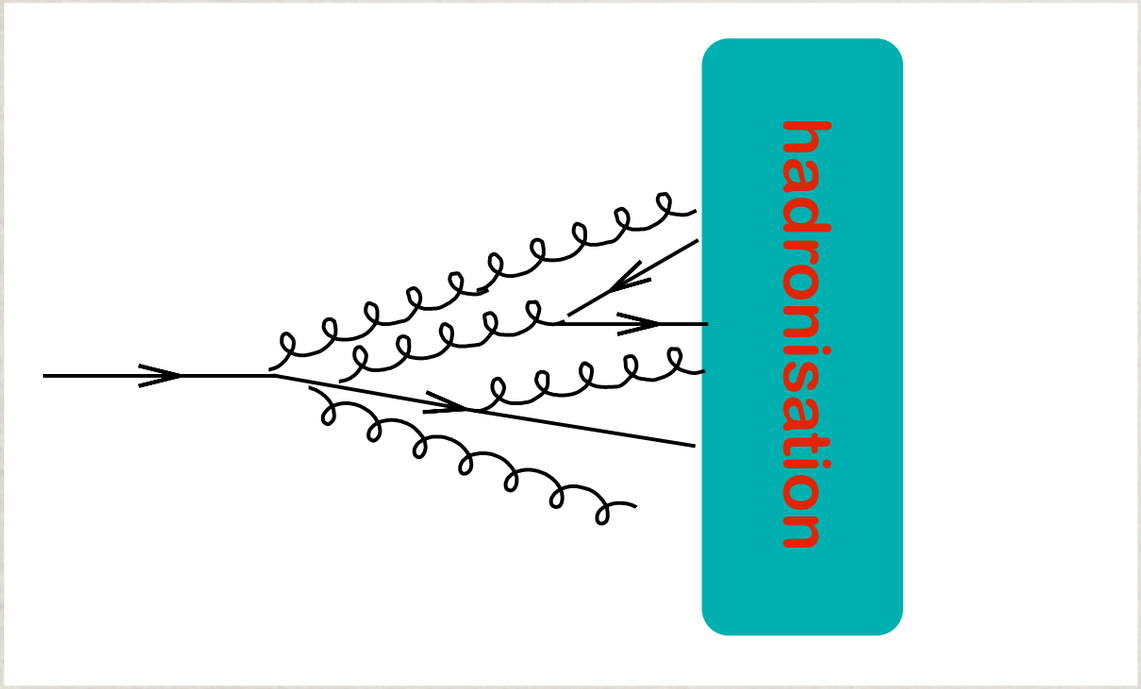

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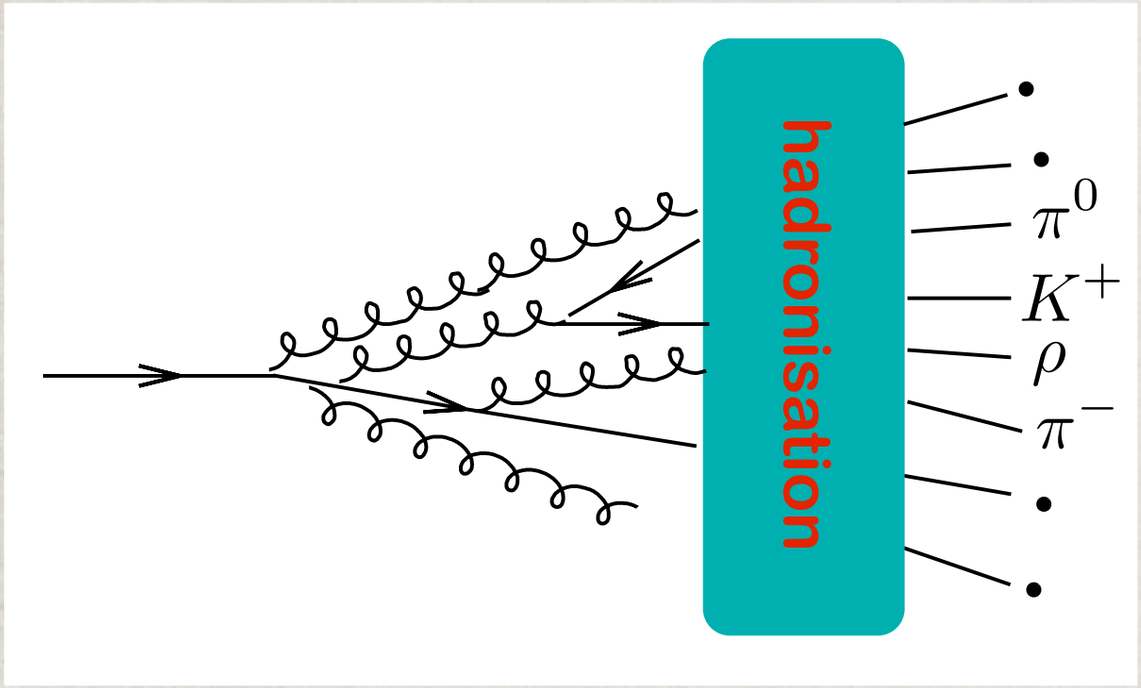

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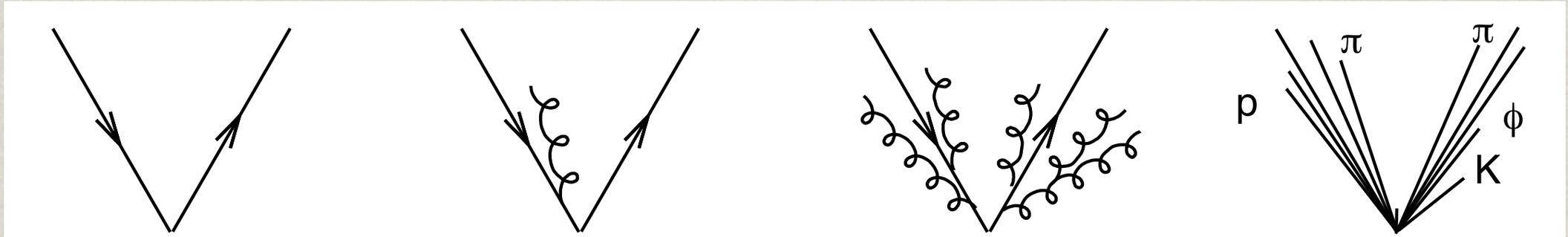
Jet algorithms

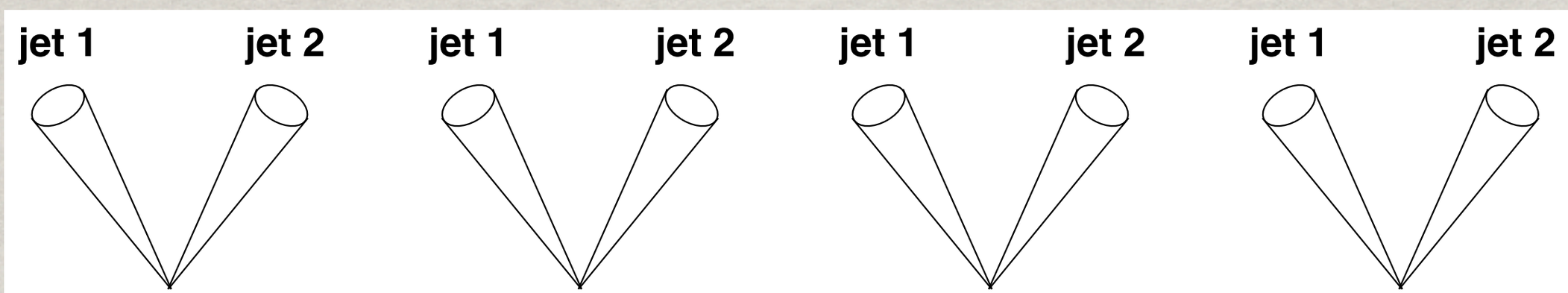
- A jet algorithm is a procedure to map a set of objects (quarks and gluons, hadrons, calorimetric towers, etc.) onto a simpler set of jets

- A good jet definition should be resilient to QCD effects (energetic gluon emissions, parton shower, hadronisation)



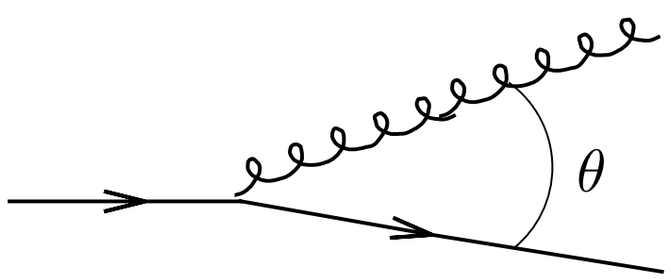
LO partons | NLO partons | parton shower | hadrons

Jet | algo | Jet | algo | Jet | algo | Jet | algo

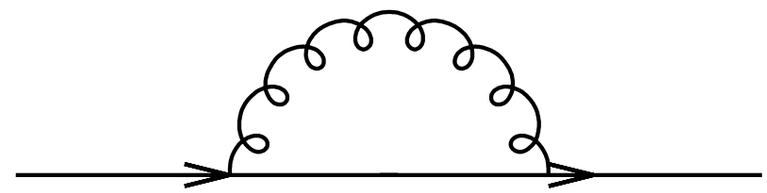


Infrared and collinear safety

- The probability for emission of a soft ($E \to 0$) and collinear ($\theta \to 0$) gluon is infinite



$+\infty$



$-\infty$

- However, soft and collinear gluons can also be reabsorbed (they are virtual, quantum fluctuations) with a negative infinite probability

- If jet definitions are insensitive to soft and collinear gluons, real and virtual infinities cancel in actual observables, which can be described using the language of quarks and gluons

- This property is called infrared and collinear safety

Snowmass accord

- In early '90s the "Snowmass accord" was elaborated, stating desirable properties of jet algorithms

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in theoretical calculation;
3. Defined at any order of perturbation theory
4. Yields finite cross sections at any order in perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronisation

[3] J. E. Huth et al., "Toward a standardization of jet definitions", FNAL-C-90-249-E

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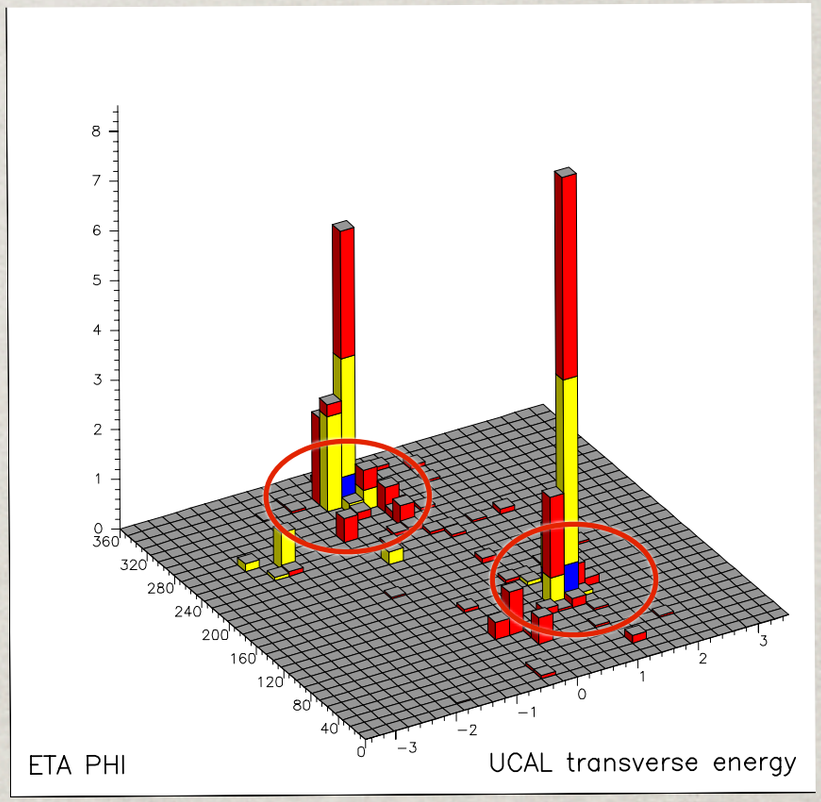
- Infrared and collinear (IRC) safety is crucial to guarantee 4 and 5

- Hadronisation effects for IRC safe observables are power-suppressed in the jet energy (or jet p_t) $\Rightarrow$ in the high-energy limit we recover the property 1 jet $\sim$ 1 parton

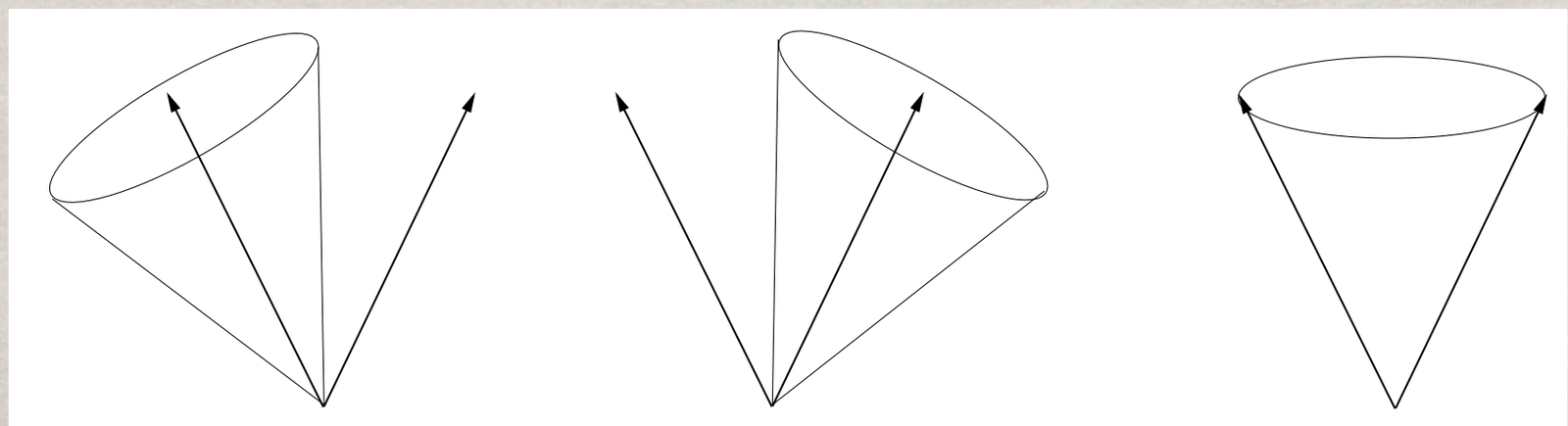
Cone algorithms

- A natural way of thinking of a jet is a cone in which a lot of energy is deposited ⇒ useful to subtract uniform backgrounds (e.g. pile-up)



- Mathematically, a cone algorithm is a procedure to find all stable cones, containing particles whose distance between their c.o.m is less than R

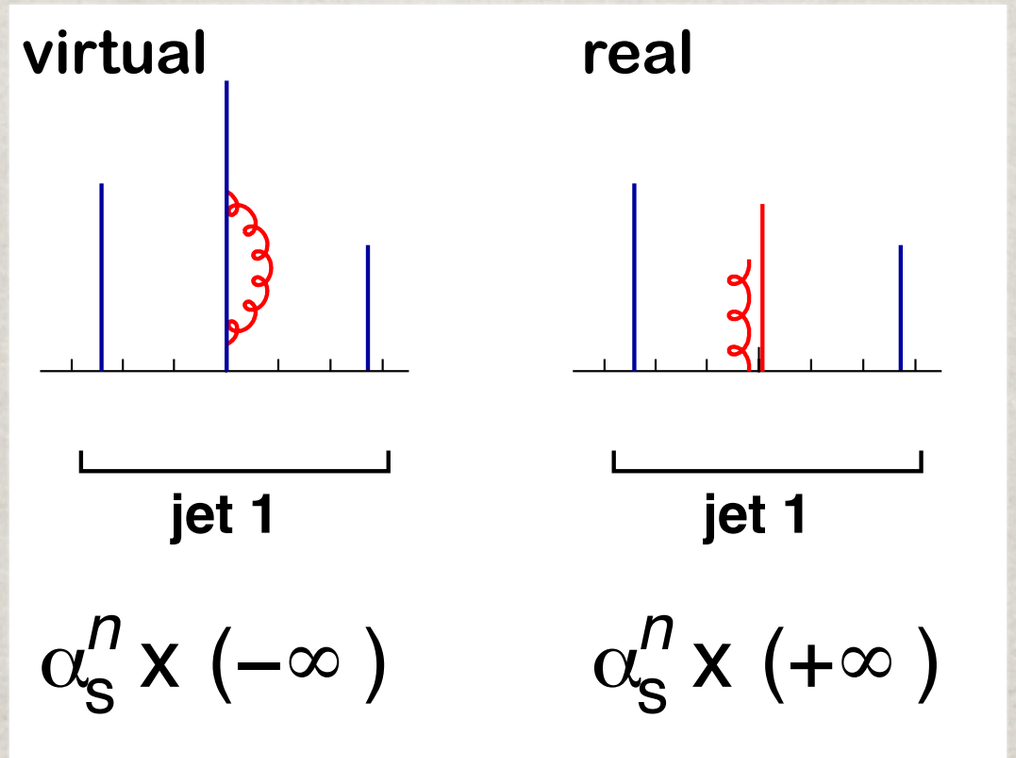
- Examples of stable cones



Cone IRC safety issues (I)

- IRC safety requires that the stable cones do not change after the addition of an extra-soft particle or a collinear splitting

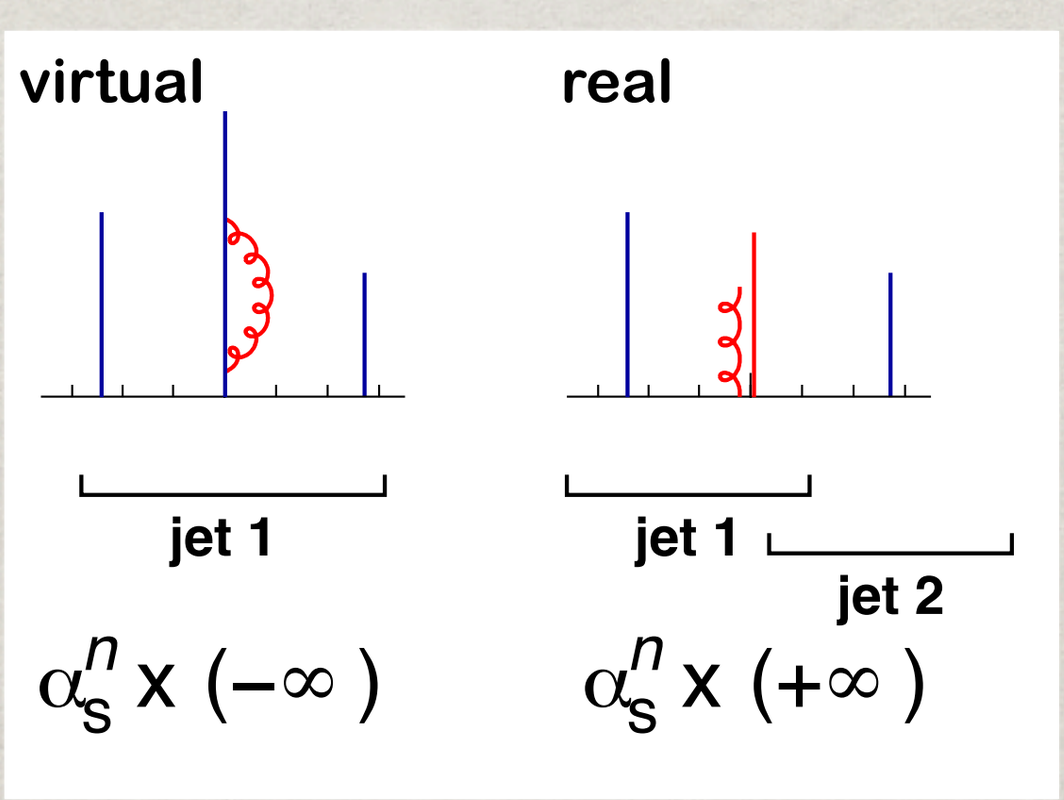
- Outcome of IRC safe procedures: real and virtual contributions give the same stable cones



Cone IRC safety issues (II)

- IRC safety requires that the stable cones do not change after the addition of an extra-soft particle or a collinear splitting

- Example of IRC unsafe safe procedures: draw cones starting from the hardest particle in the event ⇒ real and virtual corrections do not give the same stable cones



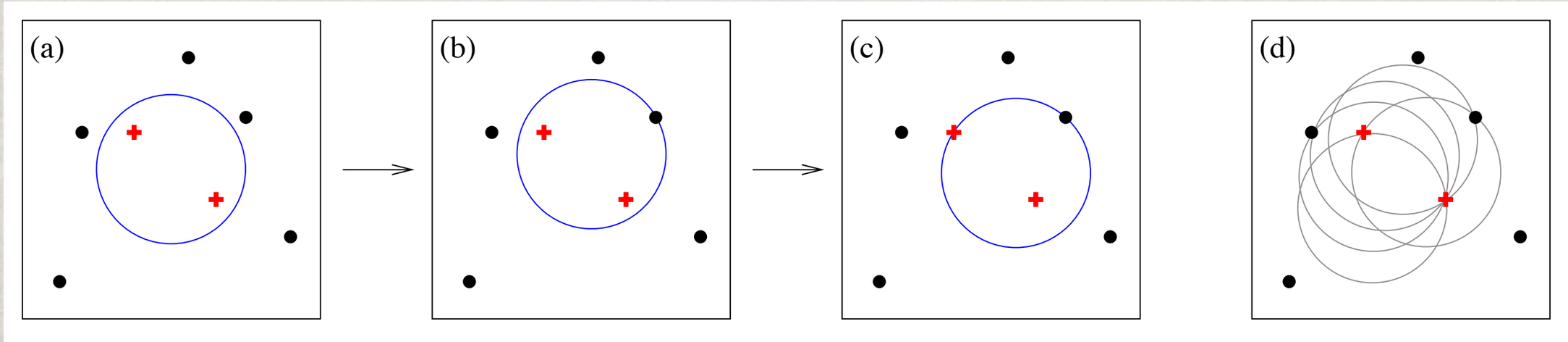
- The hardest particle is a collinear unsafe concept!

Seedless cones

- The final solution to the IRC safety problems of cone algorithms is a procedure to find all stable cones

- This is the Seedless Infrared Safe Cone algorithm (SISCone), which finds all stable cones by using the methods of computational geometry

[Salam Soyez '07]



- However, there is a surprising and simpler way to obtain circular cones: this is done with a sequential algorithm

Sequential algorithms (I)

- Sequential jet algorithms use an iterative procedure to map all particles into jets. Here is their "modern" inclusive version

  - For any pair of particles p_i, p_j ,find the minimum $d_{\min}$ of the distances

$$d_{ij} = \frac{\min\{k_{ti}^{2p}, k_{tj}^{2p}\}}{R^2}\Delta R_{ij}^2, \quad d_{iB} = k_{ti}^{2p}, \quad d_{jB} = k_{tj}^{2p}$$

  - If $d_{\min} = d_{iB}$, p_i is considered as a jet and is removed from the list of particles, the same happens to p_j if $d_{\min} = d_{jB}$

  - If $d_{\min} = d_{ij}$, particles p_i and p_j are merged together to form a jet, for instance by defining $p_{\text{jet}} = p_i + p_j$ (E-scheme)

- The procedure is repeated until no (pseudo)-particles are left

Sequential algorithms (II)

- According to the parameter p, we distinguish three algorithms

$$d_{ij} = \frac{\min\{k_{ti}^{2p}, k_{tj}^{2p}\}}{R^2}\Delta R_{ij}^2, \quad d_{iB} = k_{ti}^{2p}, \quad d_{jB} = k_{tj}^{2p}$$

  - $p = 1$, k_t algorithm: its distance measure reflects soft and collinear emission probability [Catani Dokshitzer Seymour Webber '93, Ellis Soper '93]

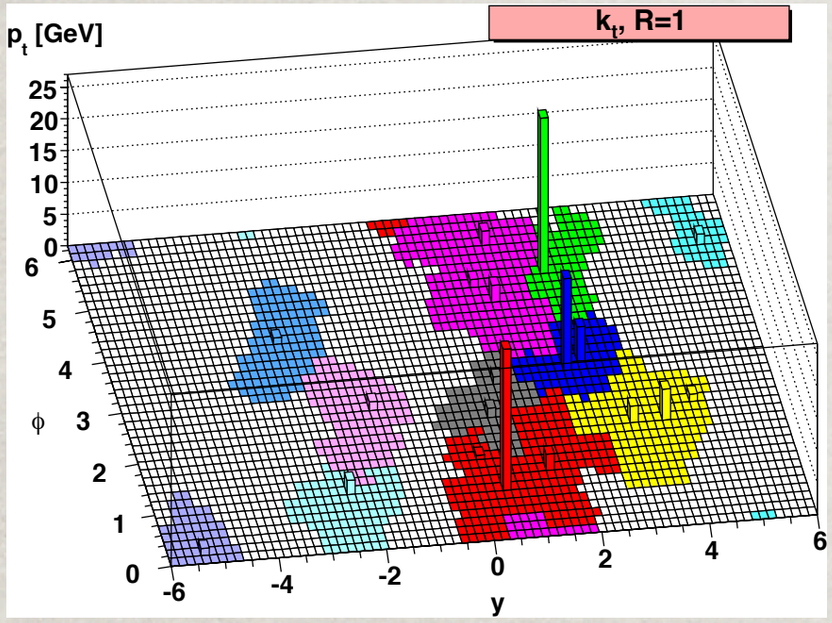
  - $p = 0$, Cambridge/Aachen: reverts the angular ordered pattern of QCD multiple soft and collinear emissions [Dokshitzer Leder Moretti Webber '97, Wobisch Wengler '99]

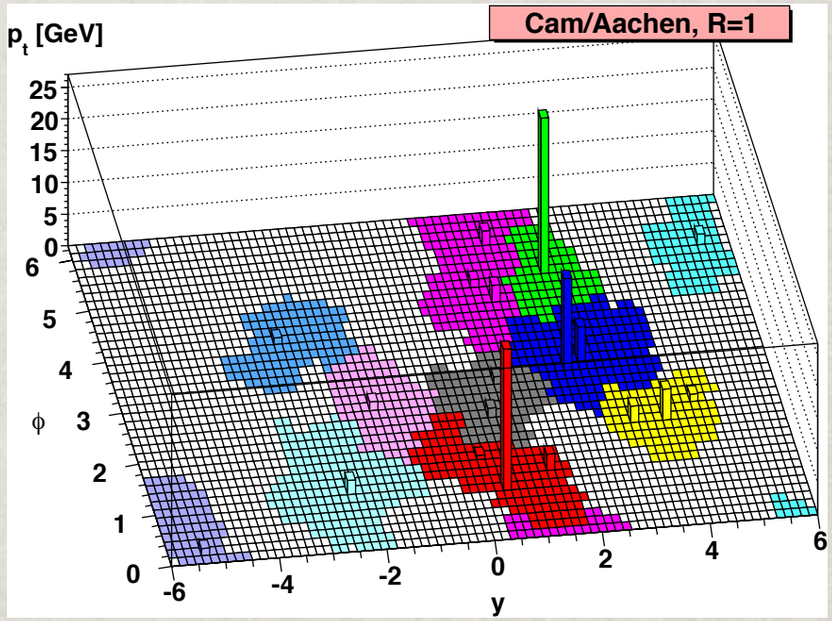
  - $p = -1$, anti-k_t algorithm: not linked with QCD radiation, clusters particles around the hardest one [Cacciari Salam Soyez '08]

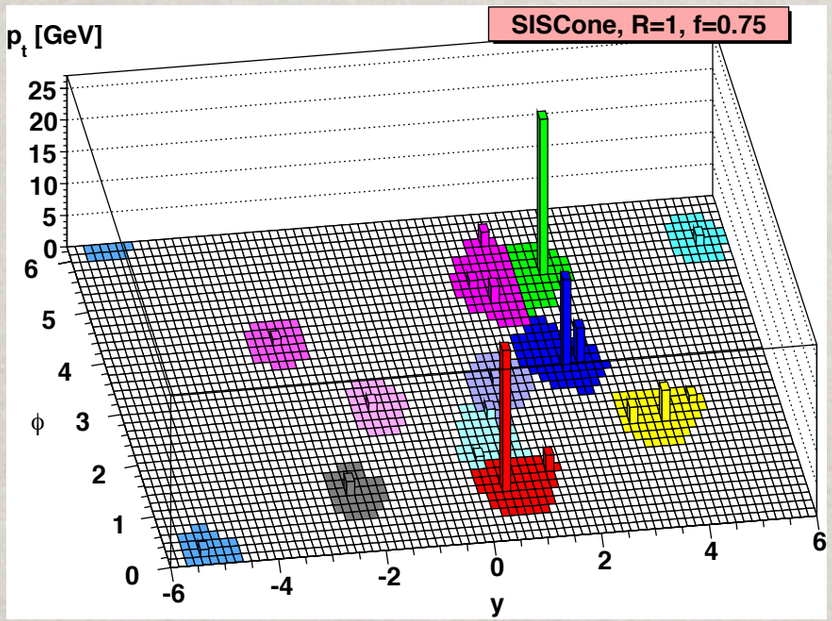
- Note: as for SISCone, what the IRC safe is the number of jets with $p_t > p_{\mathrm{t,min}}$, not the total number of jets (there can be any number of infinitely soft jets)

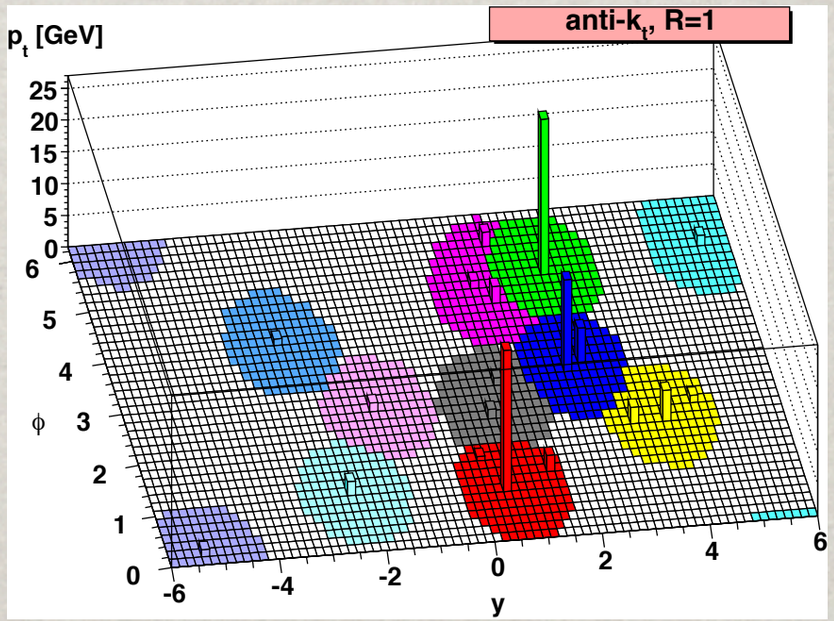
Geometric areas of jets

- It is interesting to look at the "jet boundary", determined by the particles included in each jet, for each IRC safe algorithm







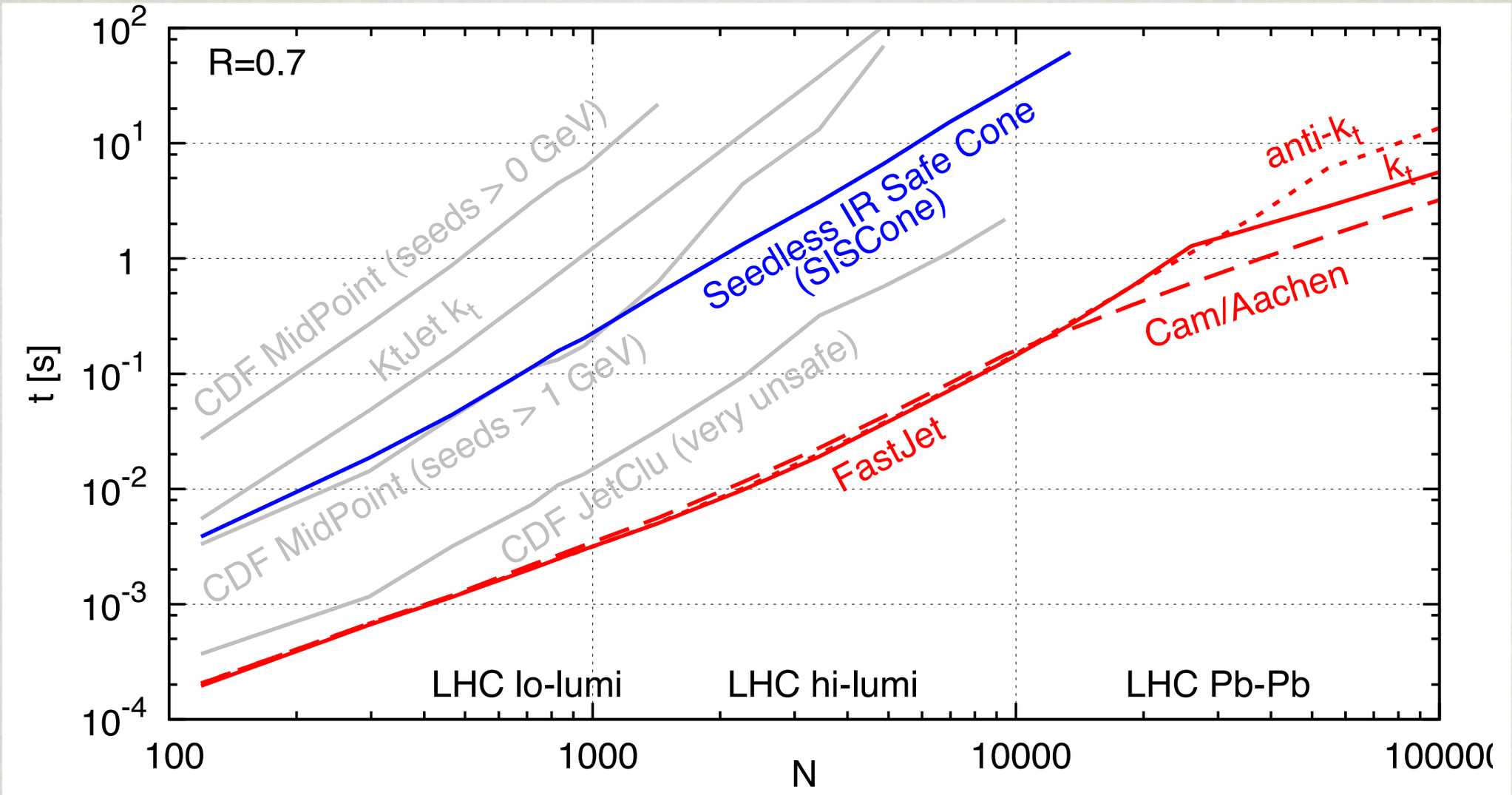


- It is a sequential algorithm (the anti-k_t) that gives us circular cones!

Modern jet algorithms

- We have a variety IRC safe jet algorithms for hadron collisions, and a fast implementation

[Cacciari Salam Soyez, http://fastjet.hepforge.org]

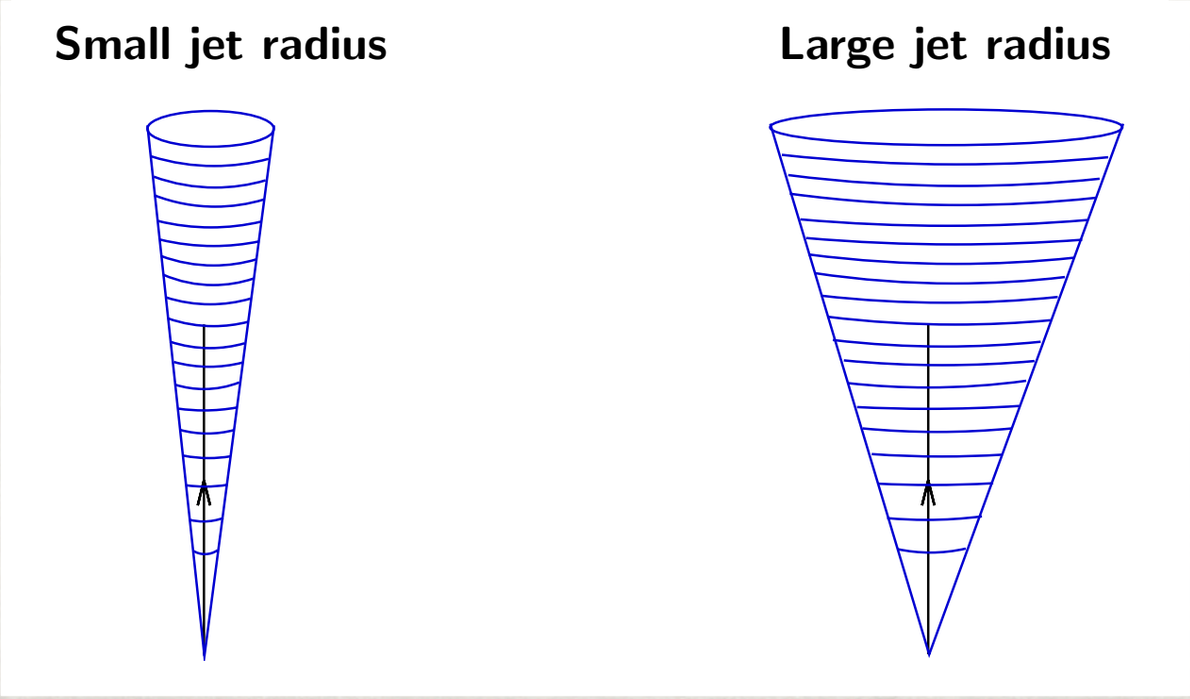


- Relevant properties of IRC safe jets (cross sections, mass, etc.) can be calculated in terms of quarks and gluons (i.e. in perturbative QCD)

Role of jet radius (LO)

- The jet radius R determines how a jet is related to its parent parton

[Dasgupta Magnea Salam '08]

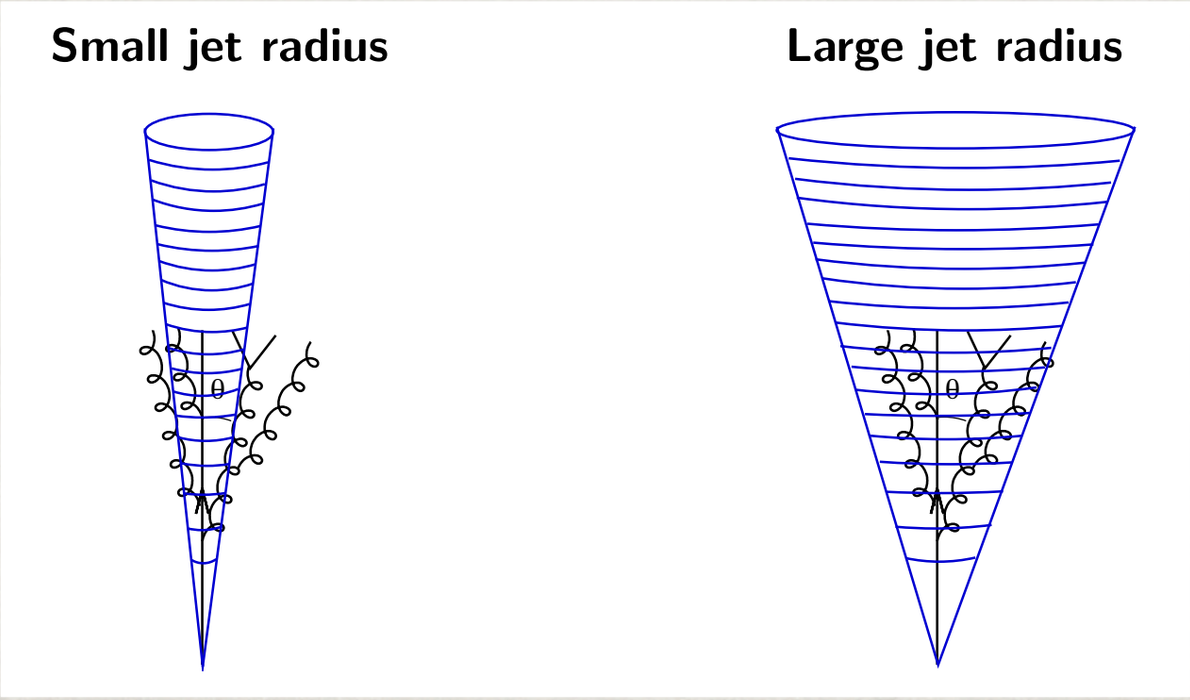


- LO = single parton: jet radius is irrelevant

Role of jet radius (PT)

- The jet radius R determines how a jet is related to its parent parton

[Dasgupta Magnea Salam '08]



- Beyond LO: large jet radius better (captures more QCD radiation)

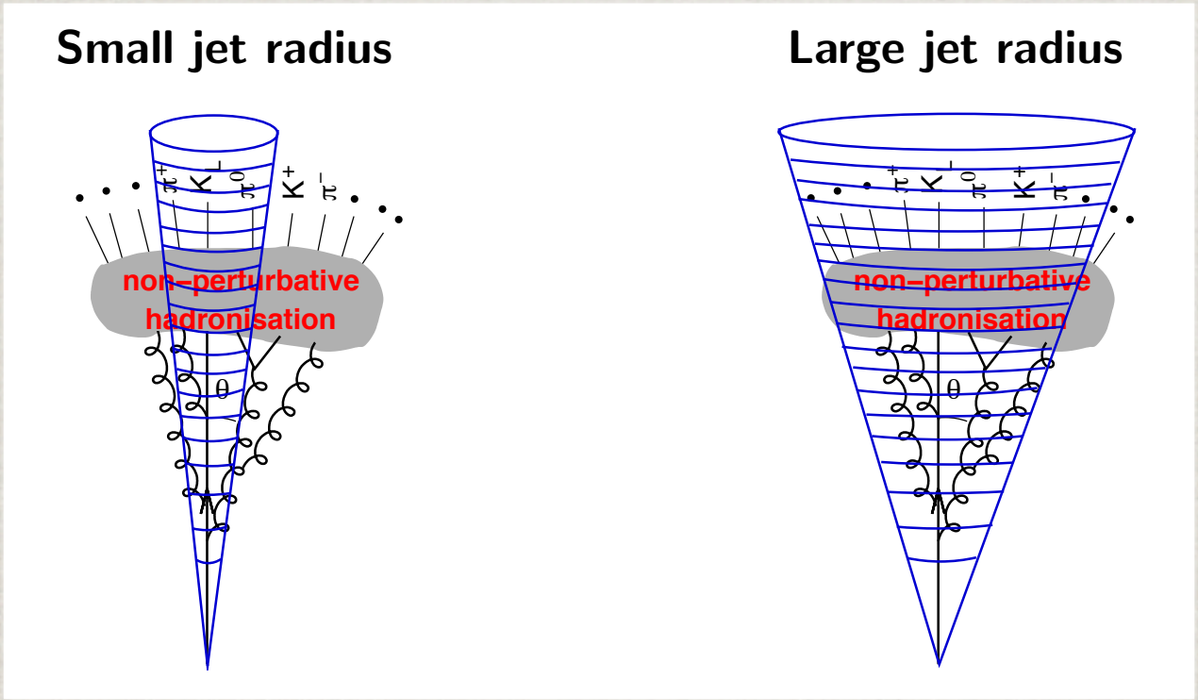
$$\langle \delta p_t \rangle_{\rm PT} \sim -\alpha_s \, p_t \, \ln \frac{1}{R} \times \begin{cases} C_F & \text{quarks} \\ C_A + 0.1\, n_f & \text{gluons} \end{cases}$$

- The $\ln R$ originates from the collinear divergence of QCD radiation

Role of jet radius (NP)

- The jet radius R determines how a jet is related to its parent parton

[Dasgupta Magnea Salam '08]



- Hadronisation: large jet radius better (captures more hadrons)

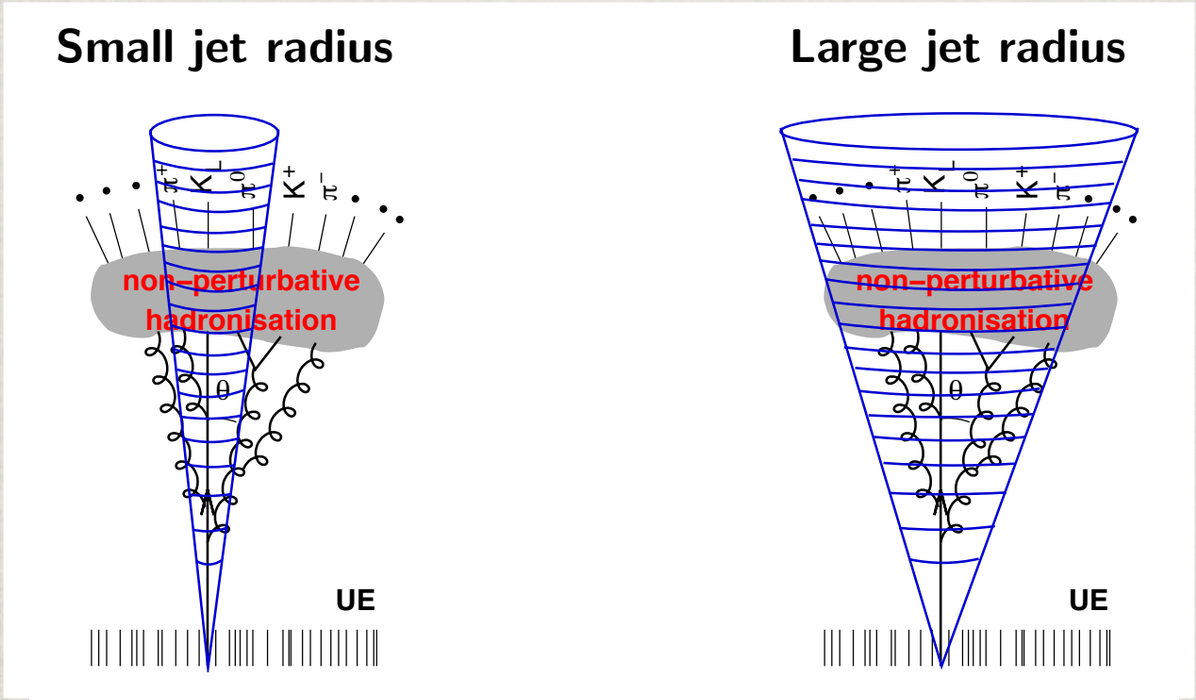
$$\langle \delta p_t \rangle_{\rm h} \sim -\frac{0.4{\rm GeV}}{R} \begin{cases} C_F & \text{quarks} \\ C_A & \text{gluons} \end{cases}$$

- The $1/R$ behaviour is again induced by the QCD collinear singularity

Role of jet radius (UE)

- The jet radius R determines how a jet is related to its parent parton

[Dasgupta Magnea Salam '08]



- Underlying event = uniform background in rapidity and azimuth.
  Large radius worse (capture more "soft junk")

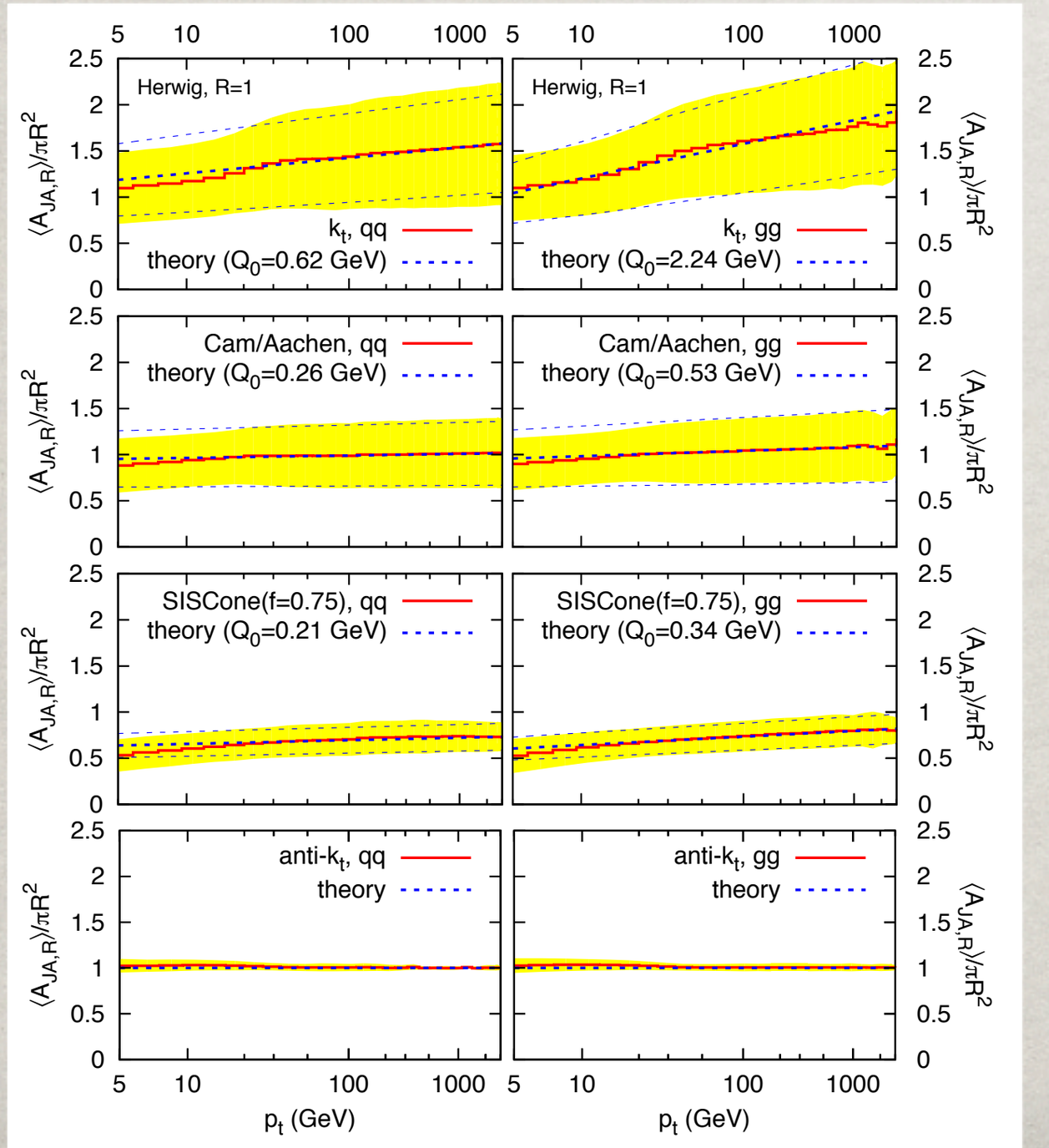
$$\langle \delta p_t \rangle_{\mathrm{UE}} \simeq 1.2\,\mathrm{GeV} \times (\pi R^2) \qquad \sqrt{s} = 7\,\mathrm{TeV}$$

- Fragmentation and UE effects have opposite behaviour $\Rightarrow$ it is always possible to find an "optimal" jet radius!

Jet areas

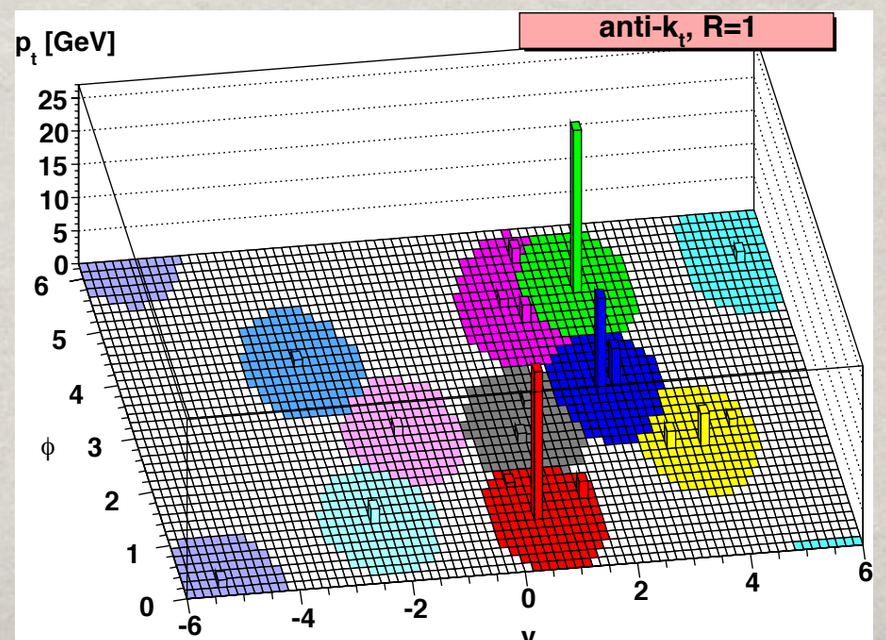
- Sensitivity of a jet algorithm to a "noise" uniform in rapidity and azimuth can be mathematically defined ⇒ jet area

[Cacciari Salam Soyez '08]



- Interplay between UE and QCD radiation ⇒ the jet area depends on the jet transverse momentum

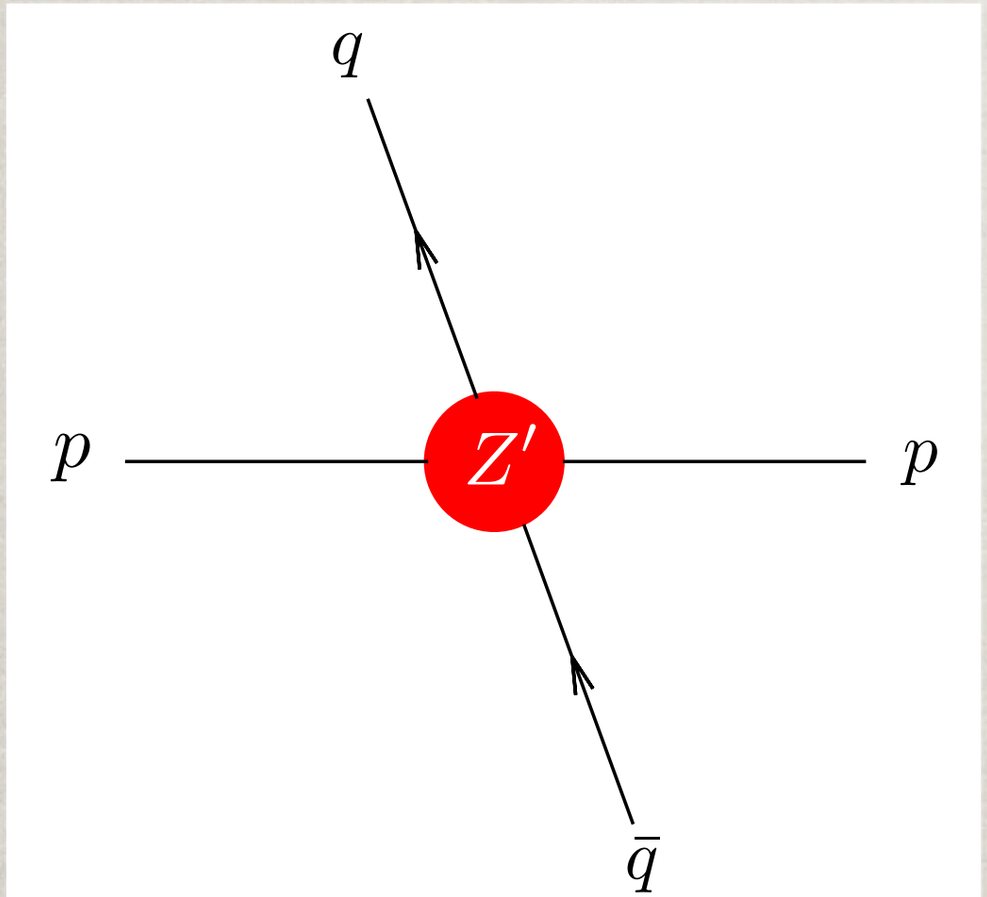
- Only the area of anti-k_t jets equals their geometric area

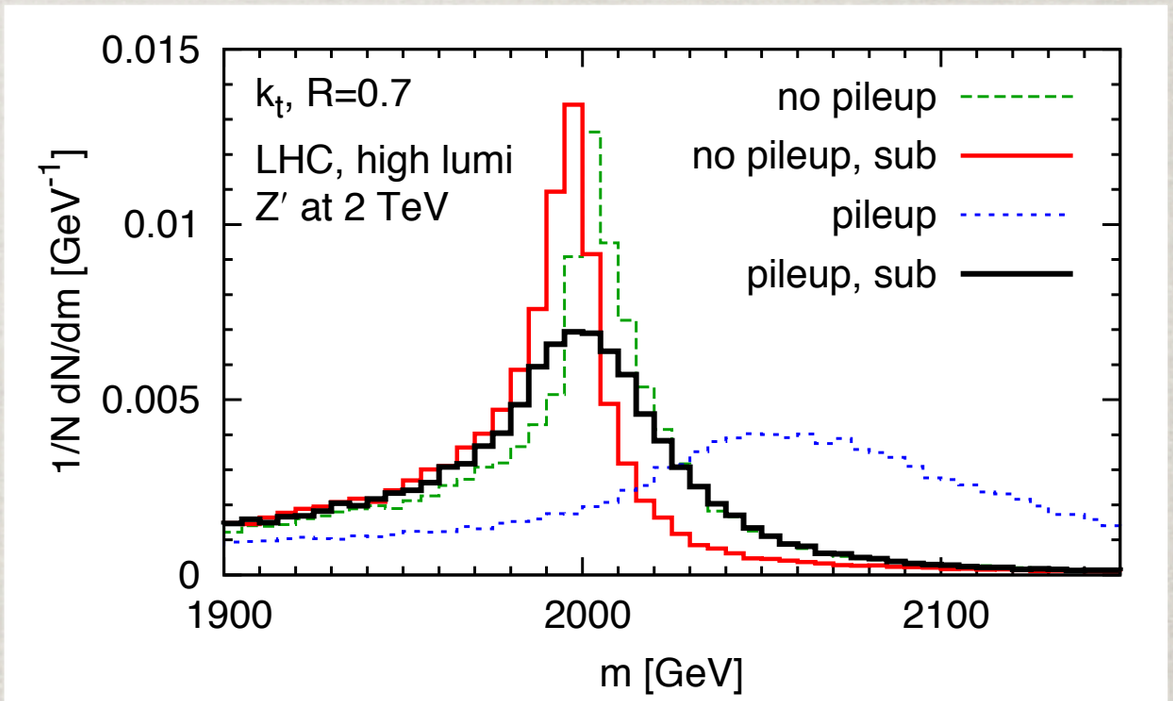


Pile-up subtraction

- A quantitative definition of the sensitivity of a jet algorithm to a uniform "noise" makes it possible to perform pile-up/UE subtraction on an event-by-event basis ⇒ crucial for resonance reconstruction

[Cacciari Salam Soyez '08]

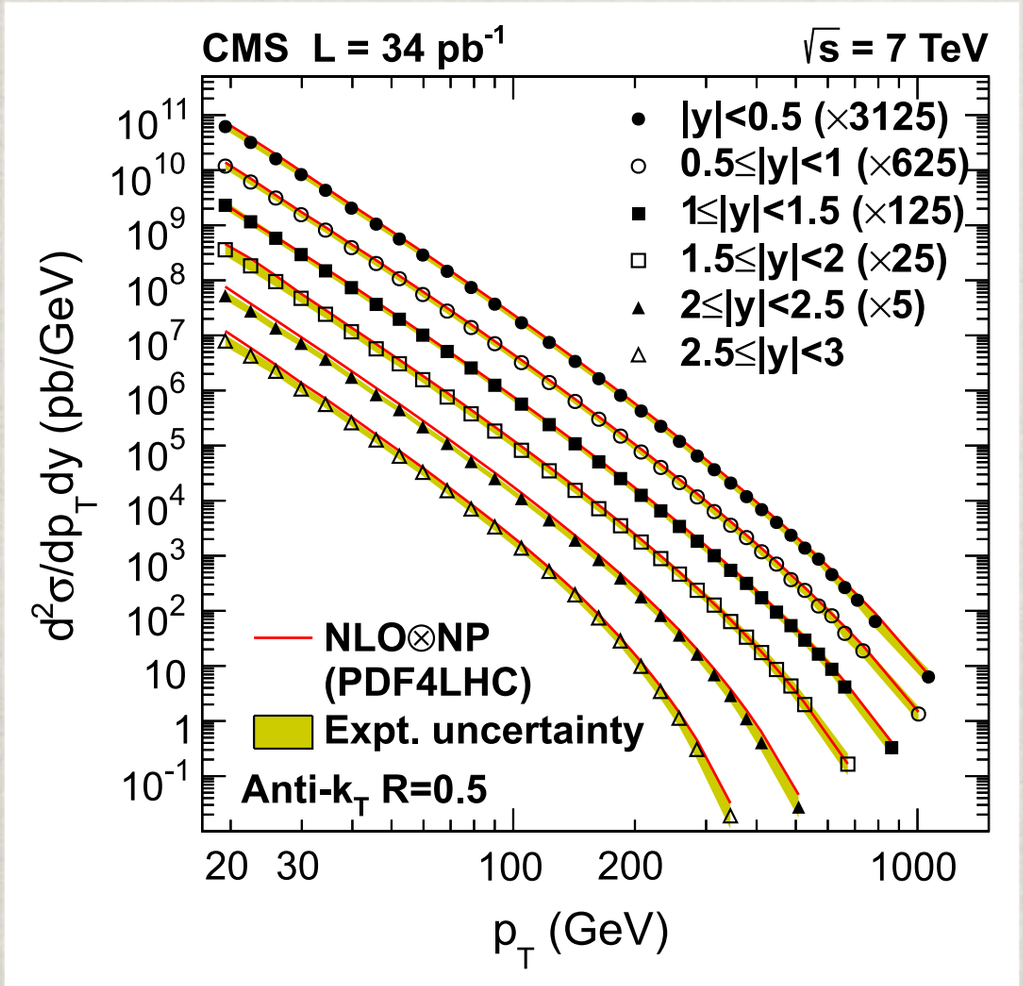


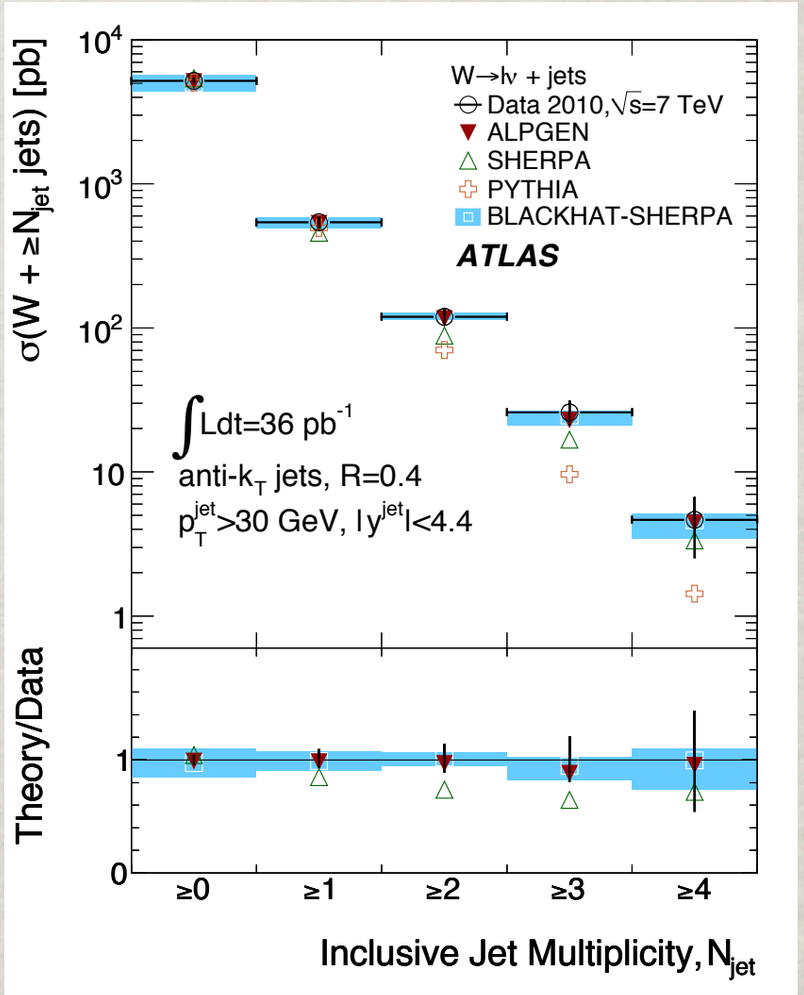


- Pile-up subtraction opens also the possibility to understand the properties of jets in terms of quarks and gluons only

Jets at fixed order QCD...

- Cross sections for hard, well separated jets, can be perfectly described in terms of few, energetic partons (fixed order PT theory)



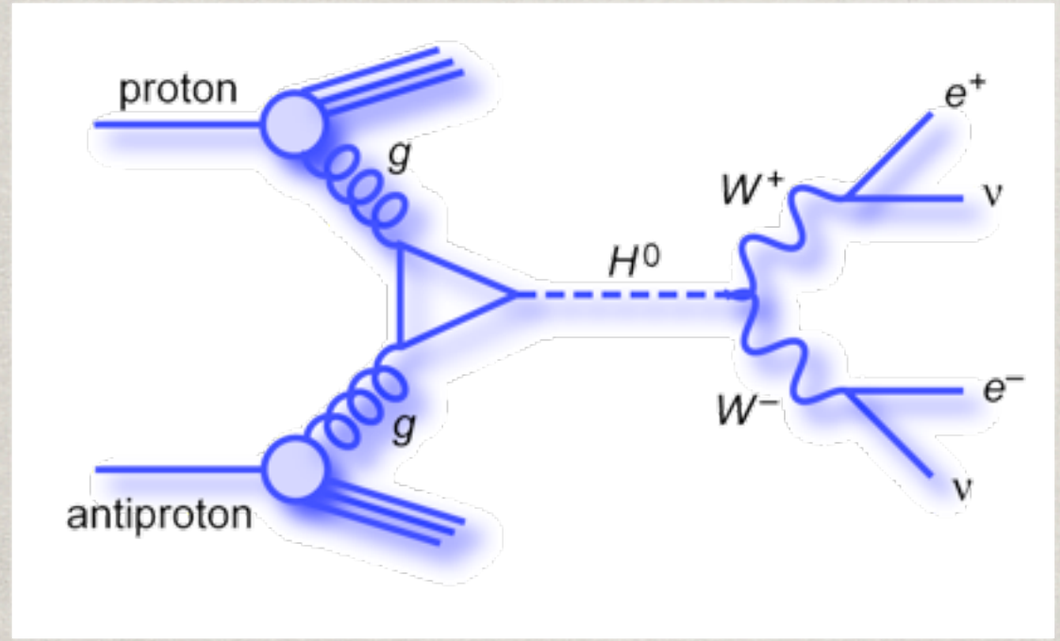


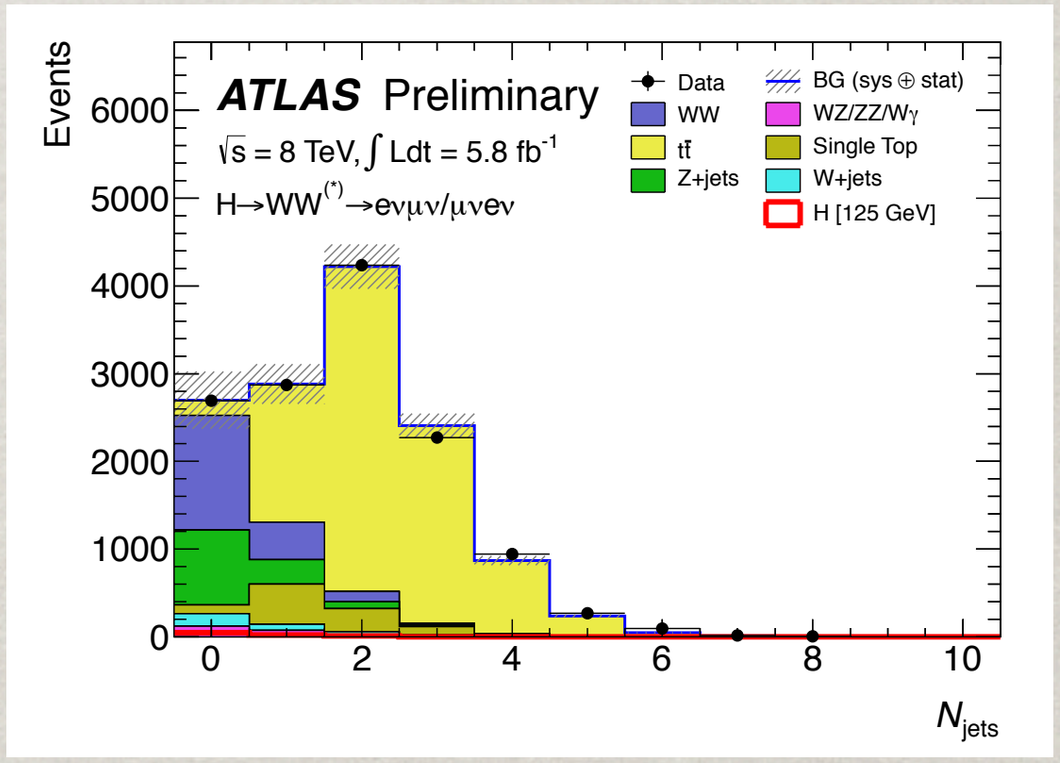
- Once non-perturbative effects like hadronisation and underlying event are accounted for with Monte Carlo event generators, description of existing data seems very good over a wide range of scales

… AND AT ALL ORDERS

- For some jet-observables, it is also possible to predict their structure for an arbitrary number of soft and collinear emissions

- This is the case of the 0-jet (all jets have $p_{\mathrm{t,jet}} < p_{\mathrm{t,veto}}$) cross section in Higgs productions, used to suppressed top background in $H \to WW$

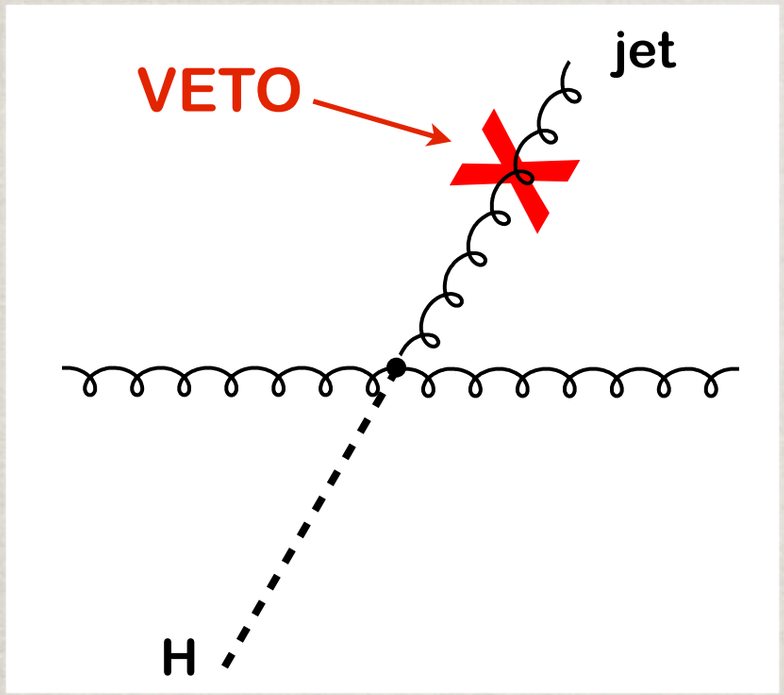




- The smaller $p_{\mathrm{t,veto}}$, the closer we are to exclusive Higgs production

All-order 0-jet cross section

- The 0-jet cross section $\Sigma^{(J)}(p_{\mathrm{t,veto}})$ has different behaviours according to the value of $p_{\mathrm{t,veto}}$ with respect to m_H



- $p_{\mathrm{t,veto}} \sim m_H$: veto one hard emission

$$\Sigma^{(J)}(p_{\mathrm{t,veto}}) \sim \mathcal{L}_{gg}(m_H) \times (1 + \alpha_s)$$

All-order 0-jet cross section

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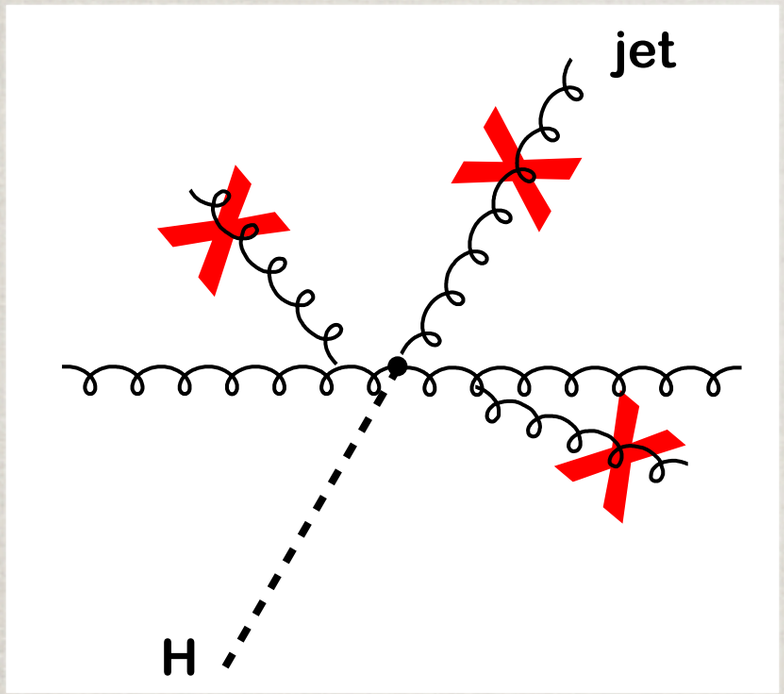
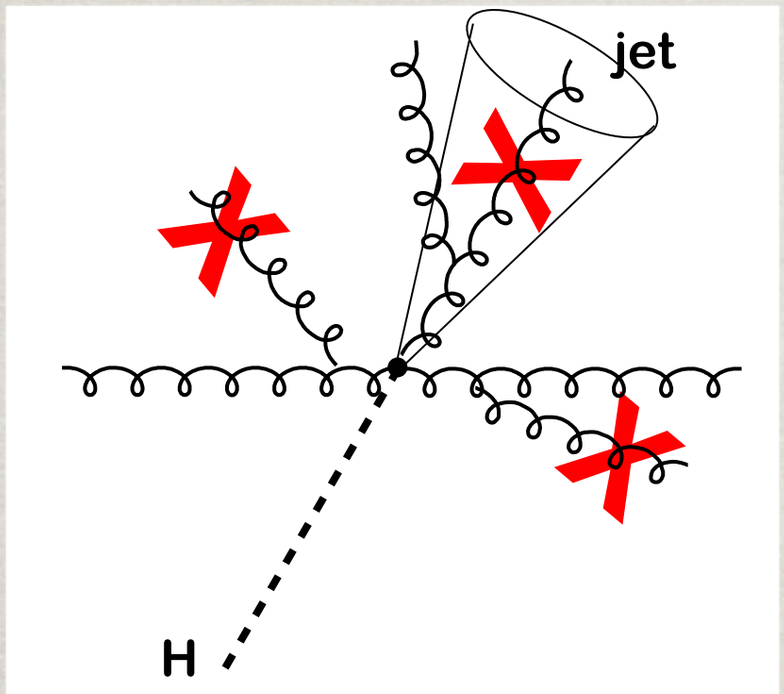


- $p_{\mathrm{t,veto}} \sim m_H$: veto one hard emission
- $\alpha_s \ln(m_H/p_{\mathrm{t,veto}}) \sim 1$: veto multiple soft-collinear emissions

$$\Sigma^{(J)}(p_{\mathrm{t,veto}}) \sim \mathcal{L}_{gg}(p_{\mathrm{t,veto}}) \times e^{\alpha_s L^2 + \alpha_s L}$$

All-order 0-jet cross section

- The 0-jet cross section $\Sigma^{(J)}(p_{\mathrm{t,veto}})$ has different behaviours according to the value of $p_{\mathrm{t,veto}}$ with respect to m_H



- $p_{\mathrm{t,veto}} \sim m_H$: veto one hard emission
- $\alpha_s \ln(m_H/p_{\mathrm{t,veto}}) \sim 1$: veto multiple soft-collinear emissions
- $\alpha_s \ln(m_H/p_{\mathrm{t,veto}}) \sim 1$: secondary gluon splitting is unlikely $\mathcal{O}(\alpha_s(p_{\mathrm{t,veto}}))$

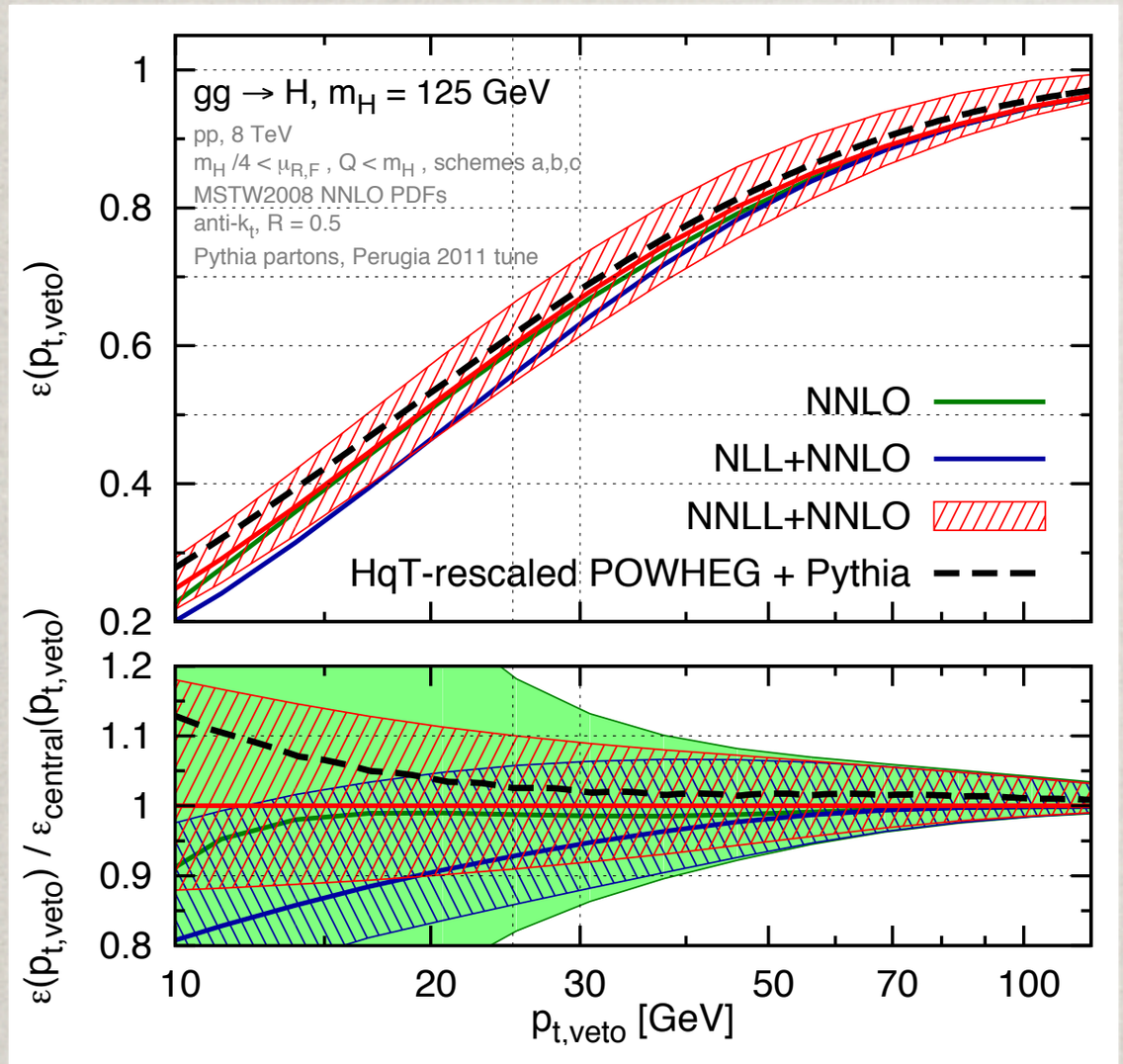
$$\Sigma^{(J)}(p_{\mathrm{t,veto}}) \sim \mathcal{L}_{gg}(p_{\mathrm{t,veto}}) \times e^{\alpha_s L^2 + \alpha_s L}\,(1 + \alpha_s(p_{\mathrm{t,veto}}))$$

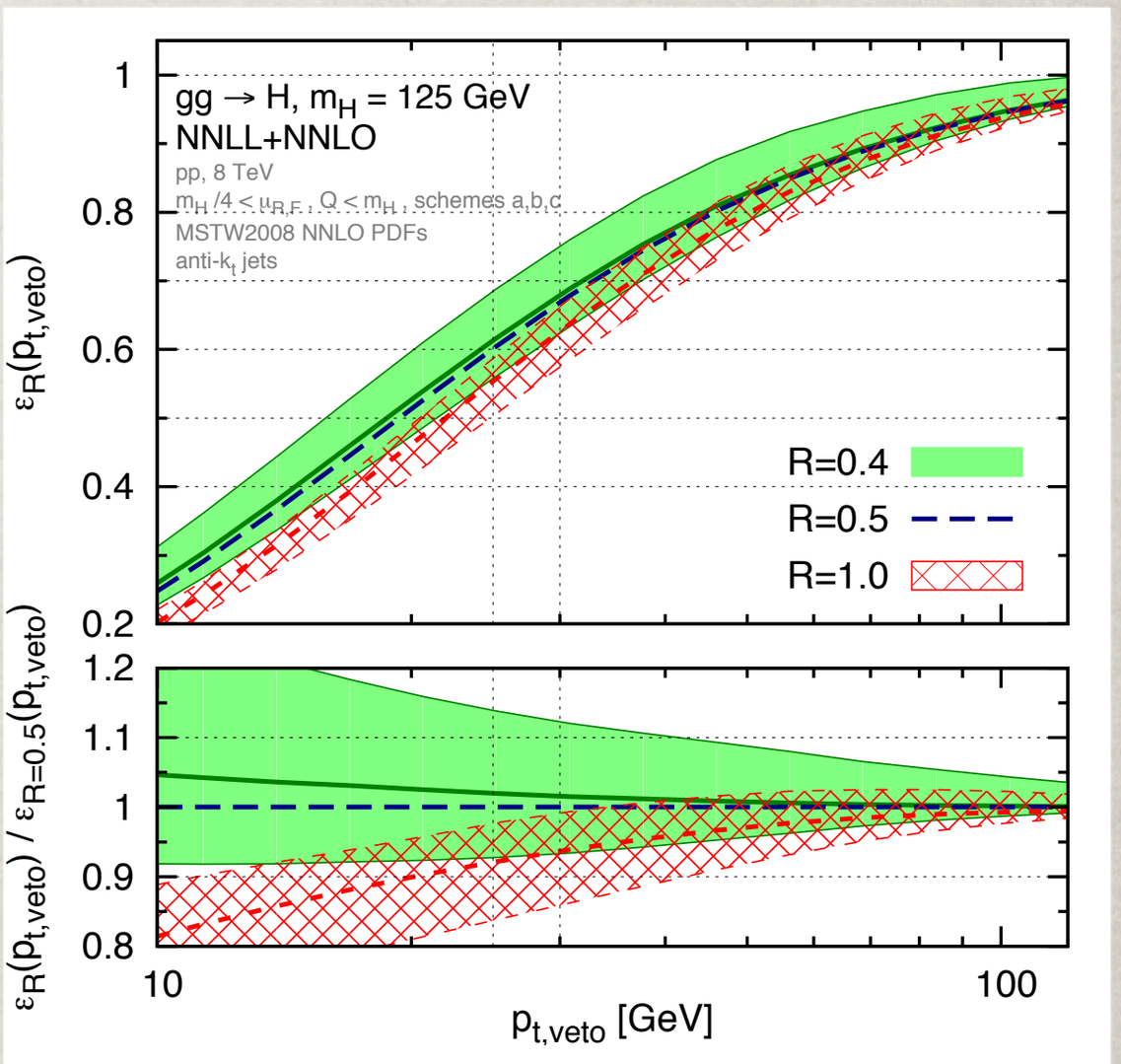
- Note: only the small correction from a single secondary gluon splitting is sensitive to the jet definition and the exact jet boundary

Precision predictions for jet-veto

- There exist very precise theoretical predictions for the jet-veto efficiency $\epsilon(p_{\mathrm{t,veto}}) = \Sigma^{(J)}(p_{\mathrm{t,veto}})/\sigma_{\mathrm{tot}}$

[AB Salam Monni Zanderighi '12]



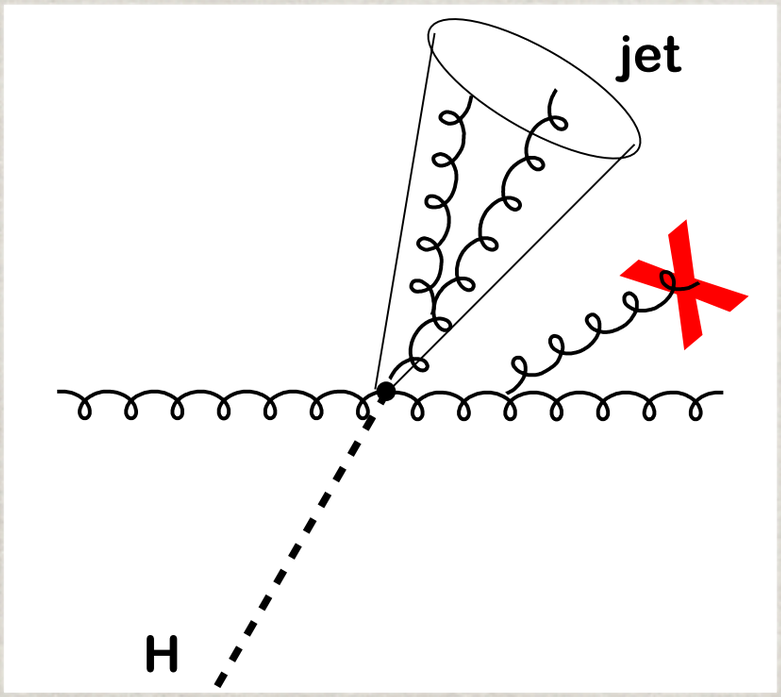


- Note the dependence on the jet radius, the larger the radius, the smaller the uncertainty (less radiation escapes from the leading jet)

When jet boundary is important

- The 0-jet cross section has the special property that is sensitive to soft and collinear gluons everywhere in the phase space

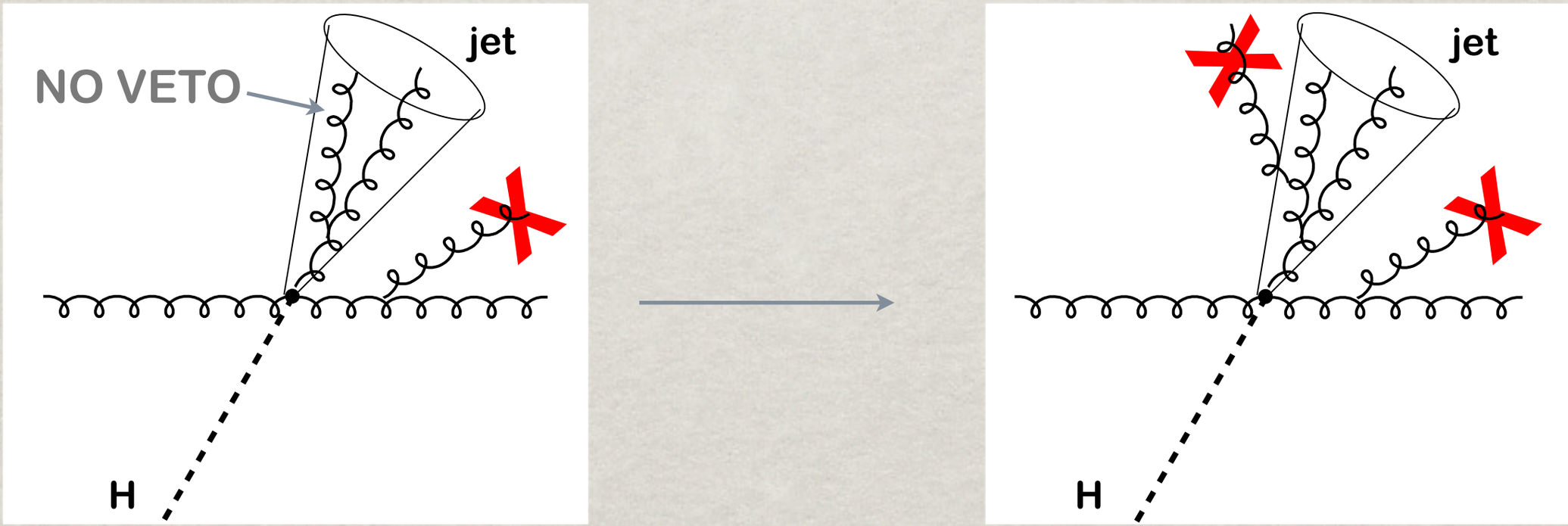
- This is not the case for the 1-jet exclusive cross sections, which is insensitive to emissions inside the tagged jet



When jet boundary is important

- The 0-jet cross section has the special property that is sensitive to soft and collinear gluons everywhere in the phase space

- This is not the case for the 1-jet exclusive cross section ($p_{\mathrm{t,veto}} \ll p_{\mathrm{t,jet}}$) which is insensitive to emissions inside the tagged jet



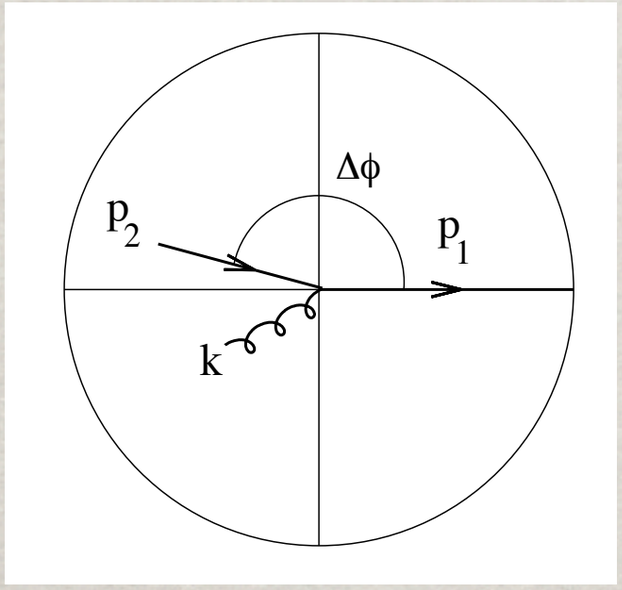
- A gluon splitting close to the jet boundary gives a large contribution (of order $\alpha_s L \sim 1$) $\Rightarrow$ needed all-order treatment available only for large-N_c , bad for precision

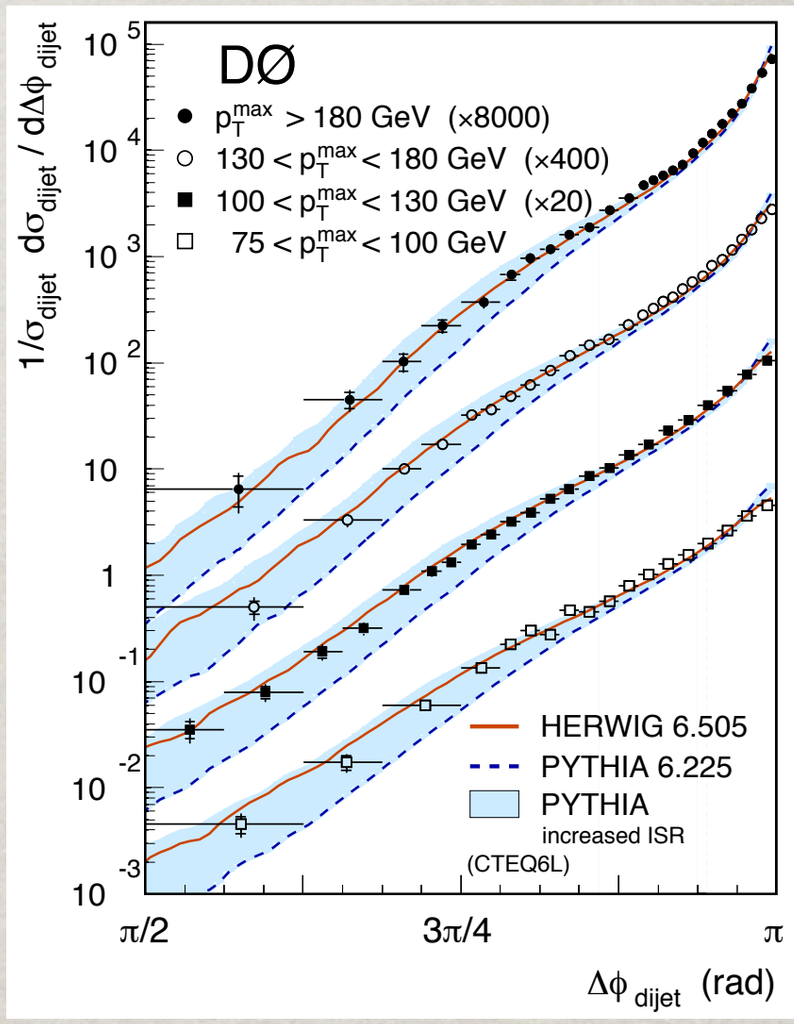
[Dasgupta Salam '01, AB Dasgupta '05]

Azimuthal dijet correlations (I)

- A multi-jet observable that might be interesting to predict with high accuracy is dijet azimuthal decorrelations

- For single parton-parton scattering, most events are around $\Delta\phi \sim \pi$, where multiple soft-collinear emissions become important

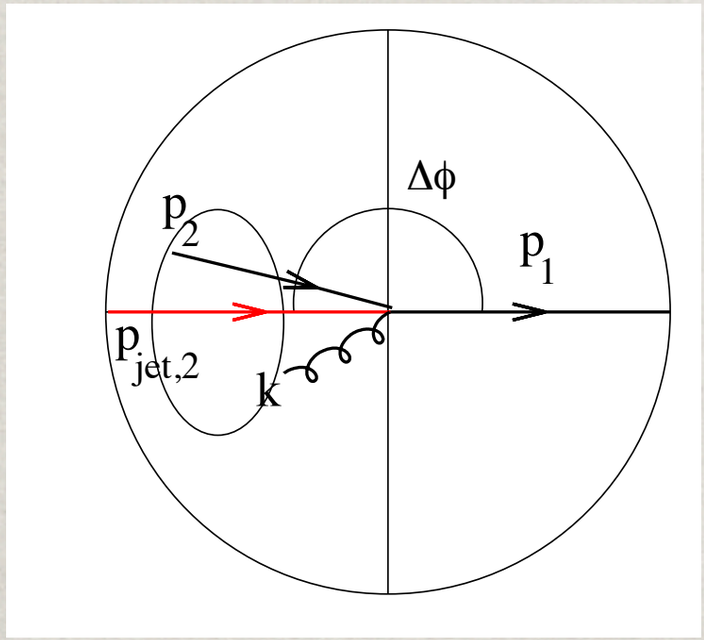




- In currently used jet algorithm, azimuthal decorrelations are insensitive to emissions inside the jets ⇒ all-order sensitivity to soft gluons close to the jet boundary!

Azimuthal dijet decorrelations (II)

- Sensitivity to radiation inside the jets can be achieved by using a different recombination scheme, the p_t-weighted scheme



$$\phi_{\mathrm{jet},2} = \frac{p_{t2}\,\phi_2 + k_t\,\phi}{p_{t2} + k_t}$$

$$|\pi - \Delta\phi| \simeq \frac{k_t}{p_{t1}} |\sin^3\phi|$$

- With this recombination scheme dijet azimuthal decorrelations can be predicted at all order in QCD

[AB Dasgupta '08]

- Note: how "good" an observable is can nowadays be checked using the numerical program CAESAR, available on demand

[AB Salam Zanderighi '05, http://qcd-caesar.org]

Conclusions

- There exist a variety of infrared and collinear safe jet algorithm for hadron collisions, and a fast numerical implementation

[http://fastjet.hepforge.org]

- The k_t, anti-k_t , Cambridge-Aachen and SISCone algorithms have well understood behaviour under perturbative and non-perturbative fragmentation, as well as underlying event/pile-up

- Precision studies with high-p_t , well-separated jets, for these algorithms, can be reliably performed in fixed-order QCD

- All-order treatment of multiple soft-collinear effects, needed for more exclusive observables (e.g. jet-veto efficiencies), is available only for a restricted class of jet observables, but reached accuracy is very high

- Improving observable's definitions is in general a non-trivial task, but is possible in many relevant cases (e.g. dijet azimuthal decorrelations)