

Key features of MPI Dynamics

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Multi-Parton Interactions

- potential source of extra “background” to multi-jet production
- potential source of extra information about structure of the proton

Obviously, MPI physics bound to be complicated.

Two approaches to attacking MPI problems :

a formal bookkeeping of all possible flavor, spin, color combinations that one may encounter when discussing multi-parton configurations

an approach aiming at predicting observable cross sections by exploiting limited information we have gained from previous life

IT IS THE SECOND ROOT THAT WE TOOK - AND KEEP TREADING -
WITH BORIS BLOK, LONYA FRANKFURT AND MARK STRIKMAN

- **Basic motivation:** search for adequate QCD means for describing MPI
- **Surprise :** search for QFT means, prior to QCD means !

What have we achieved ?

Understood the origin of the fake on-mass-shell singularities

Introduced Generalized Double Parton Distributions \mathcal{G}_2 GPD

Constructed the first model for \mathcal{G}_2 GPD (building up on HERA GPDs)

Introduced 3-parton collisions as an essential extra

Sorted out the confusing role of “2->4 parton collisions” (ain't any MPI)

Explained the Tevatron enhancement puzzle as being due to 3->4

Proposed a model for non-PT correlations at small x (inelastic diffraction)



The Four jet production at LHC and Tevatron in QCD

Phys. Rev. D83 : 071501, 2011; e-Print: arXiv:1009.2714 [hep-ph]

pQCD Physics of Multiparton Interactions

Eur.Phys.J. C72 (2012) 1963; e-Print: arXiv:1106.5533 [hep-ph]

Origins of Parton Correlations in Nucleon and Multi-Parton Collisions

PRL to appear; e-Print: arXiv:1206.5594 [hep-ph]

At high energies the rich QFT-structure of hadron constituents becomes visible.

To resolve the internal structure of interacting objects we probe them with large momentum transfer processes - “hard interactions”

$$\frac{d\sigma}{dQ^2} \propto \frac{1}{Q^4} \quad Q^2 \gg R^{-2}$$

Hadron structure matters little in such standard (small-cross section) hard interactions.

Colliding **hadrons** serve as sources of two colliding **partons**, in inclusive manner: $D_h^p(x, Q^2)$

The size - and the buildup - of the hadron manifests itself in **multi-parton collisions**

$$\frac{d\sigma}{dQ^2} \propto \frac{1}{Q^4} \times \frac{1}{R^2 Q^2}$$

This may seem a small (“higher twist”) correction to the total cross section. And so it is.

In specific circumstances turns out to be **significant** (comparable with the “Leading Twist”)

e.g., 4-jet production in the back-to-back kinematics

2-parton collision

The standard approach to the multi-jet production is the QCD improved parton model.

It is based on the assumption that the cross section of a hard hadron–hadron interaction is calculable in terms of the convolution of parton distributions within colliding hadrons with the cross section of a hard **two-parton collision**.



$$\sigma_2 = \int d^2\rho_1 d^2B \ f(x_1, \vec{\rho}_1, p^2) f(x_2, \vec{B} - \vec{\rho}_1, p^2) \frac{d\sigma^h}{d\hat{t}} d\hat{t}$$

parton probability density : $f(x, \vec{\rho}, p^2) = \psi^+(x, \vec{\rho}, p^2) \psi(x, \vec{\rho}, p^2)$

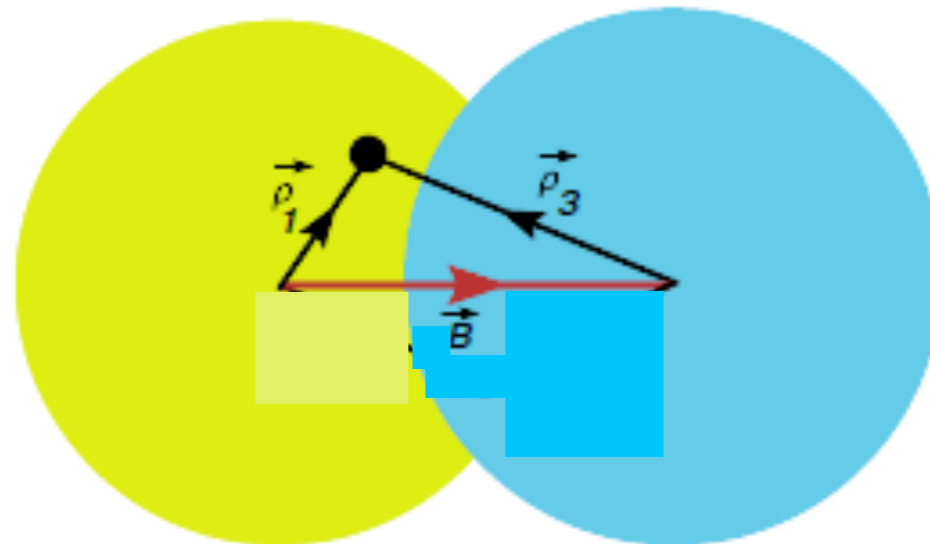
Result of the impact parameter integration - squaring of the amplitude in the momentum space:

$$\int \frac{d^2k_\perp}{(2\pi)^2} \psi(x, k_\perp) \int \frac{d^2k'_\perp}{(2\pi)^2} \psi^\dagger(x, k'_\perp) \times \int d^2\rho e^{i\vec{\rho} \cdot (\vec{k}_\perp - \vec{k}'_\perp)} = \int \frac{d^2k_\perp}{(2\pi)^2} \psi(x, k_\perp) \times \psi^\dagger(x, k_\perp)$$

An application of this picture to the processes with production of, e.g., **four jets** implies that all jets in the event are produced in a hard collision of **two** initial state partons.

Recent data of the CDF and D0 Collaborations provide evidence that there exists a kinematical domain where a more complicated mechanism becomes important :

double hard interaction
of two partons in one hadron
with two partons in the second hadron.



Let us see, what difference does it make to our formulae

Multi-parton wave function

$$\psi_n(x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, \dots) = \int \prod_{i=1}^{i=n} \frac{d^2 k_i}{(2\pi)^2} \exp(i \sum_{i=1}^{i=n} \vec{k}_i \vec{\rho}_i) \psi_n(x_1, \vec{k}_1, x_2, \vec{k}_2, \dots) (2\pi)^2 \delta(\sum \vec{k}_i)$$

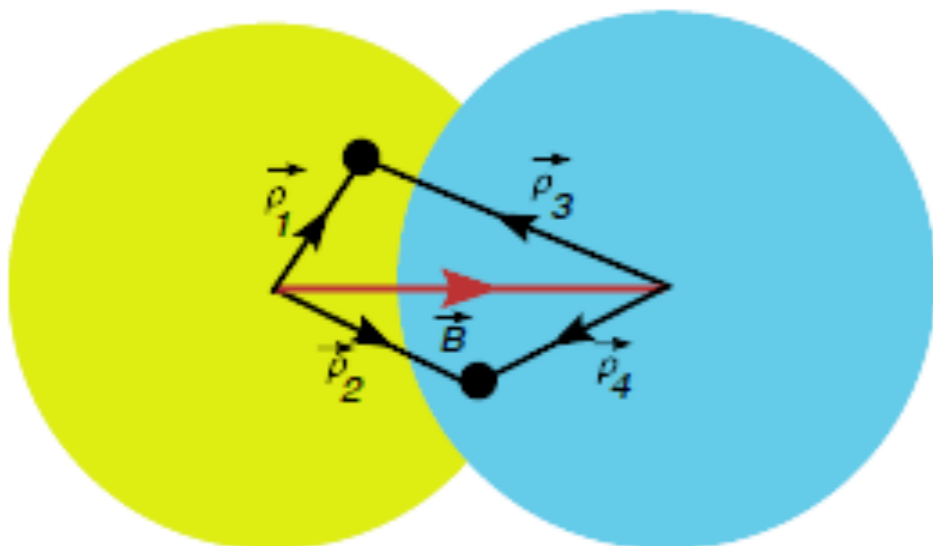
Inclusive 2-parton probability distribution in the impact parameter space :

$$D(x_1, x_2, \vec{\rho}_1, \vec{\rho}_2) = \sum_{n=3}^{n=\infty} \int \prod_{i \geq 3}^{i=n} [dx_i d^2 \rho_i] \psi_n(x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, \dots, x_i, \vec{\rho}_i, \dots) \psi_n^\dagger(x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, \dots, x_i, \vec{\rho}_i, \dots) \delta(\sum_{i=1}^{i=n} x_i \vec{\rho}_i)$$

Before:

Independent impact parameter integration \longrightarrow equality of parton momenta in ψ and ψ^\dagger

$$k_\perp = k'_\perp$$



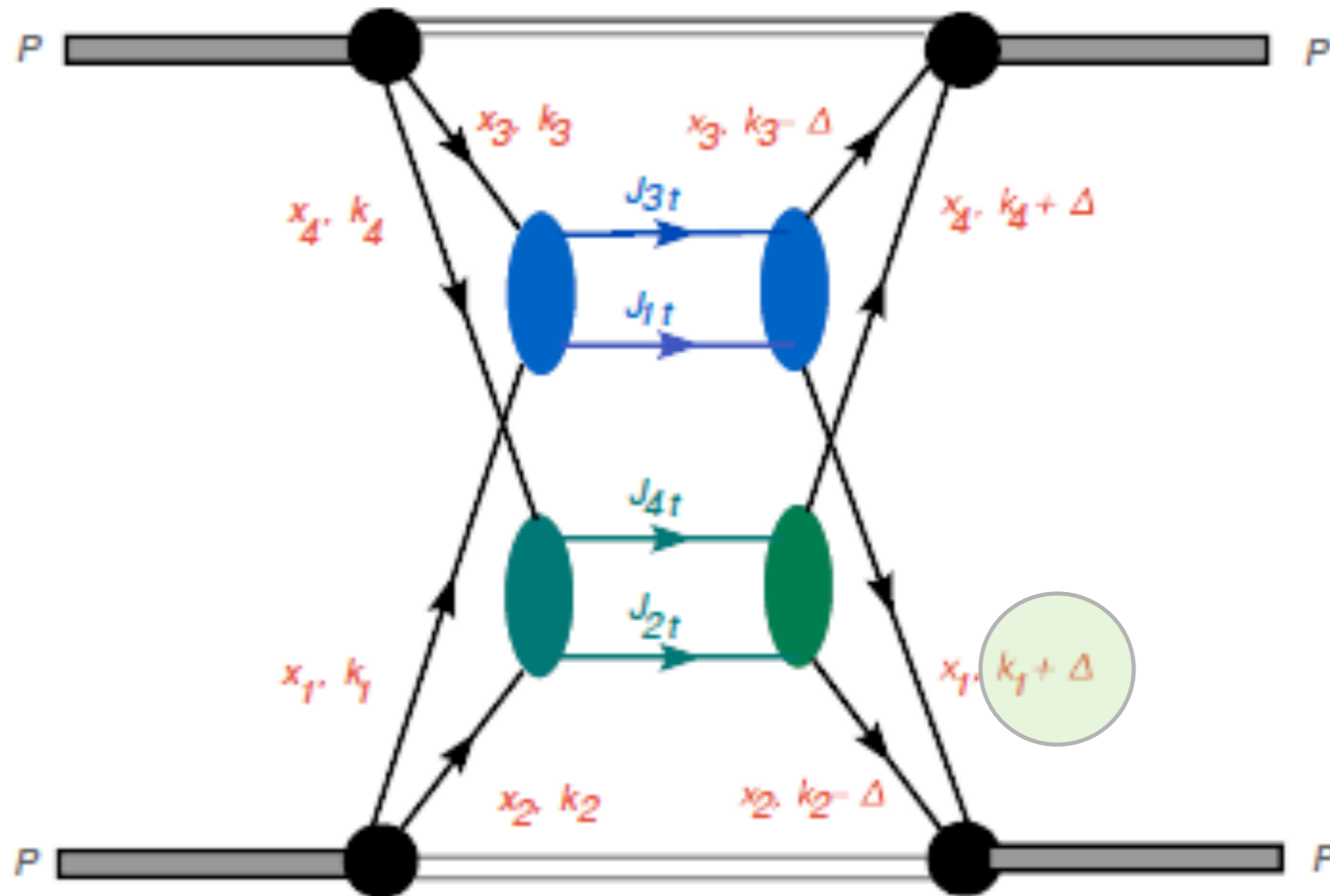
$$\rho_1 + \rho_2 \longrightarrow k'_1 - k_1 = -(k'_2 - k_2) \equiv \Delta$$

$$\rho_3 + \rho_4 \longrightarrow k'_3 - k_3 = -(k'_4 - k_4) \equiv \tilde{\Delta}$$

$$(\rho_1 - \rho_2) + (\rho_3 - \rho_4) \longrightarrow \Delta = -\tilde{\Delta}$$

$$\delta((\rho_1 - \rho_2) - (\rho_3 - \rho_4)) \longrightarrow \vec{\Delta} \text{ arbitrary}$$

4-parton collision

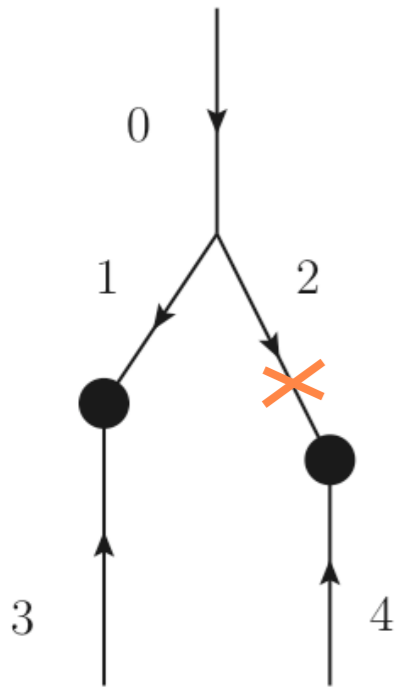


In order to be able to trace the *relative distance between the partons*, one has to use the mixed *longitudinal momentum – impact parameter* representation which, in the momentum language, reduces to introduction of a **mismatch** between the transverse momentum of the parton in the **amplitude** and that of the same parton in the **amplitude conjugated**.

We have examined the *transverse momentum* structure of the interaction amplitude

Now, have a look at the *longitudinal momenta* of participating partons ...

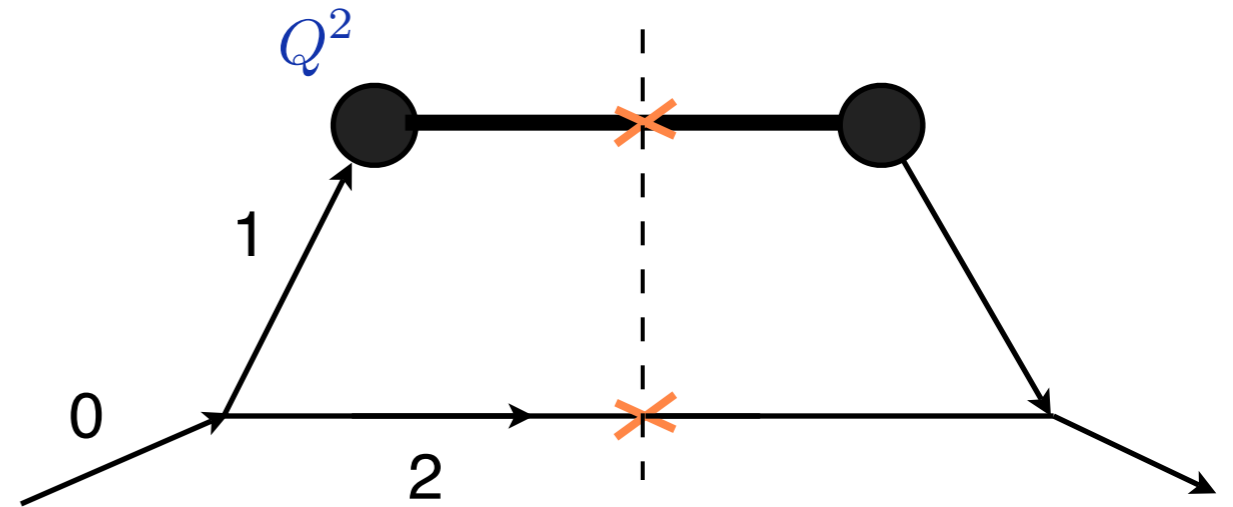
An underwater stone
of the MPI analysis



A tree Feynman diagram. Momenta of internal parton lines are fixed ... **not anymore**

Singularities in the physical region of parton momenta !

Return to a good old single hard interaction picture :



In DIS we trace the fate of **1** but *integrate* over “histories” of the accompanying parton **2**.

Now we want **#2** to enter 2nd hard interaction.

In the above picture it does it “*in the next room*”. Or on the Moon, for that matter ...

In fact, partons **3** and **4** *cannot be* represented by plainly independent *plane waves*: they belong to *one hadron*, and therefore, are *localized within the hadron pancake*...

Remedy: introduce wave packet smearing (longitudinal momentum fraction integral).

Importantly, this has to be done at the *amplitude level* !

$k_{3+} + k_{4+}$ fixed by hard scattering kinematics

$k_{3+} - k_{4+}$ arbitrary

The fake singularity disappears !

**Generalized
double parton
distributions**

4-parton cross section

$$\frac{1}{S} = \frac{\int \frac{d^2 \vec{\Delta}}{(2\pi)^2} D_a(x_1, x_2; \vec{\Delta}) D_b(x_3, x_4; -\vec{\Delta})}{D(x_1) D(x_2) D(x_3) D(x_4)}$$

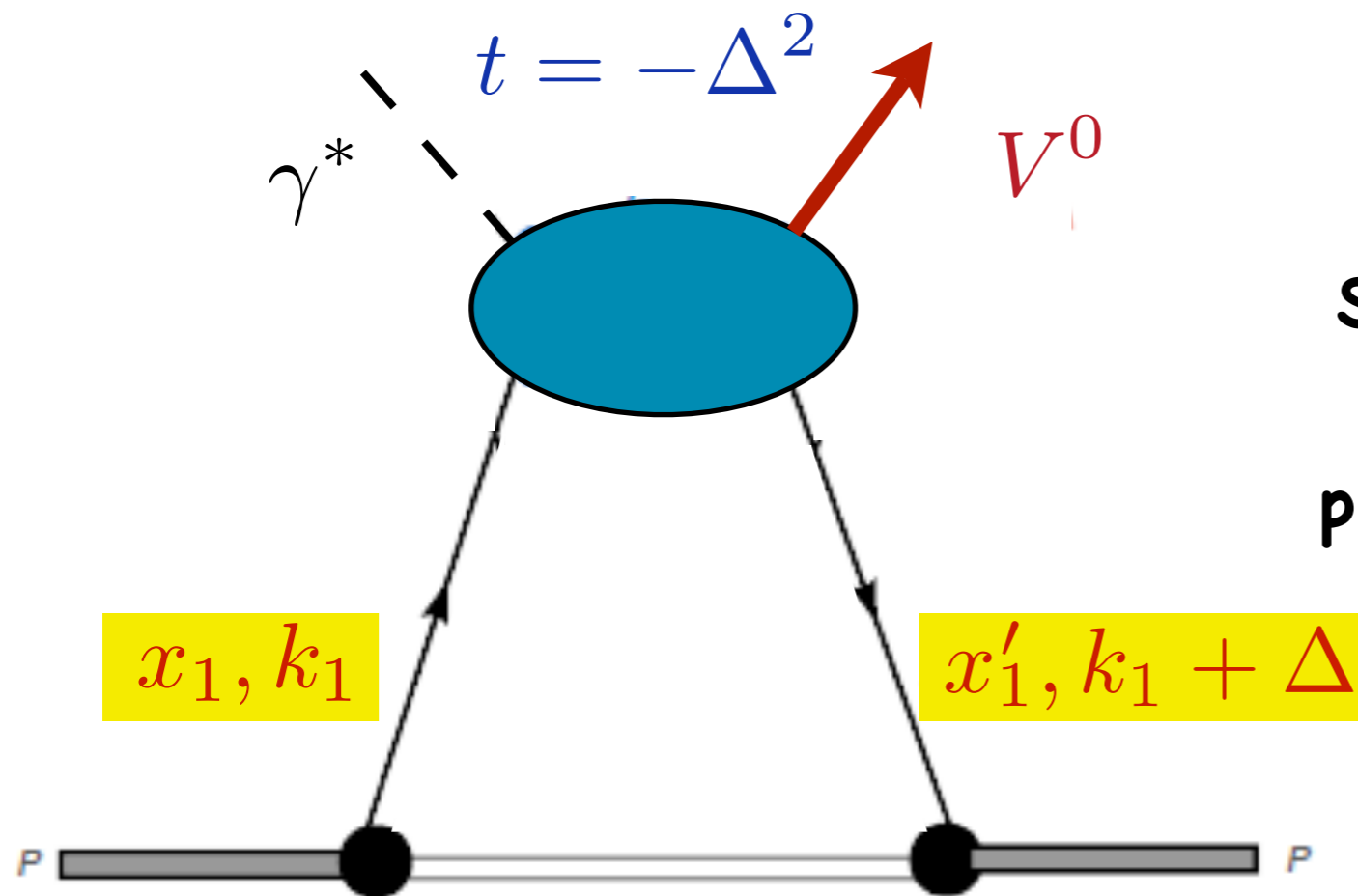
S - effective parton interaction area (**S** - *is NOT* a “cross section”!)

$$\frac{d\sigma(x_1, x_2, x_3, x_4)}{d\hat{t}_1 d\hat{t}_2} = \frac{d\sigma^{13}}{d\hat{t}_1} \frac{d\sigma^{24}}{d\hat{t}_2} \times \frac{1}{S}$$

D - *the generalized double parton distribution* - is a new object we know little about.

Can one model it, for lack of anything better ?

- **Failure of the independent parton approximation and a viable model for non-PT intra-hadron 2-parton correlations**



Such an amplitude describes
exclusive photo-(/electro-)
production of **vector mesons**
at HERA !

Note : the analogy is *imperfect*. **OK** for high enough energies: $A \simeq i \text{Im}A$
Imaginary part of the “skewed” amplitude vs. that of non-diagonal “elastic” transition ...

Generalized parton distribution :

$$G_N(x, Q^2, \vec{\Delta}) = G_N(x, Q^2) F_{2g}(\Delta)$$

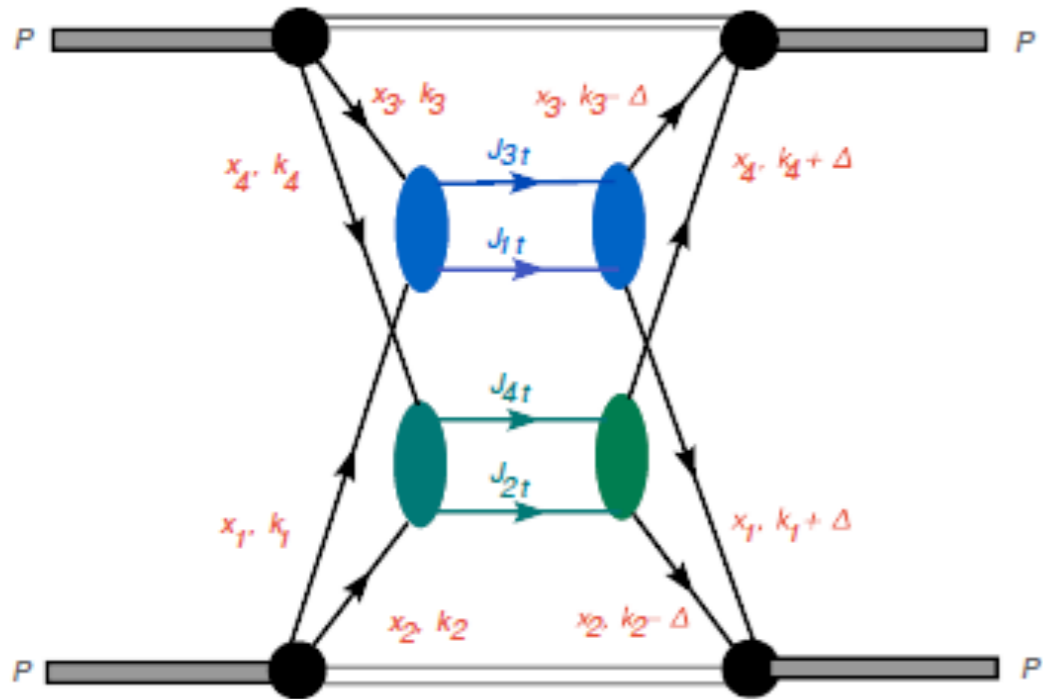
G - the usual 1-parton distribution (determining DIS structure functions)

F - the two-gluon form factor of the nucleon

the dipole fit :

$$F_{2g}(\Delta) \simeq \frac{1}{(1 + \Delta^2/m_g^2)^2}$$

$$m_g^2(x \sim 0.03, Q^2 \sim 3\text{GeV}^2) \simeq 1.1\text{GeV}^2$$



If partons were *uncorrelated*, we would write

$$D(x_1, x_2, p_1^2, p_2^2, \vec{\Delta}) = G(x_1, p_1^2, \vec{\Delta})G(x_2, p_2^2, \vec{\Delta})$$

and use the dipole fit to get the estimate

$$\frac{D(x_1, x_2, -\vec{\Delta})D(x_3, x_4, \vec{\Delta})}{D(x_1)D(x_2)D(x_3)D(x_4)} \simeq F_{2g}^4(\Delta)$$

The “interaction area” :

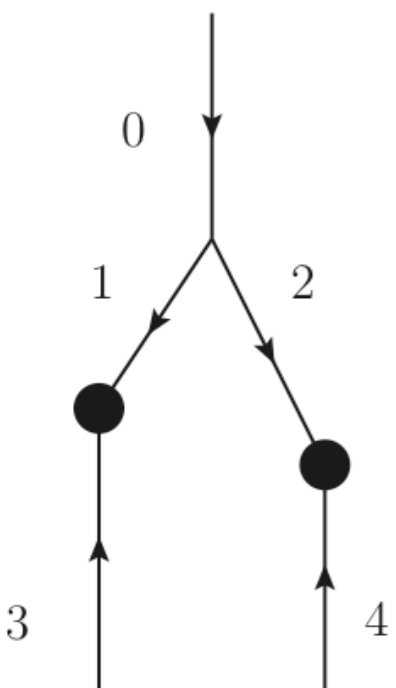
$$\longrightarrow \int \frac{d^2 \Delta}{(2\pi)^2} F_g^2(\Delta^2) \times F_g^2(\Delta^2) = \frac{m_g^2}{28\pi}$$

Another mechanism : 2 partons from a short-range PT correlation

No Δ —dependence from the upper side ! $\longrightarrow \int \frac{d^2 \Delta}{(2\pi)^2} F_g^2(\Delta^2) = \frac{m_g^2}{12\pi}$

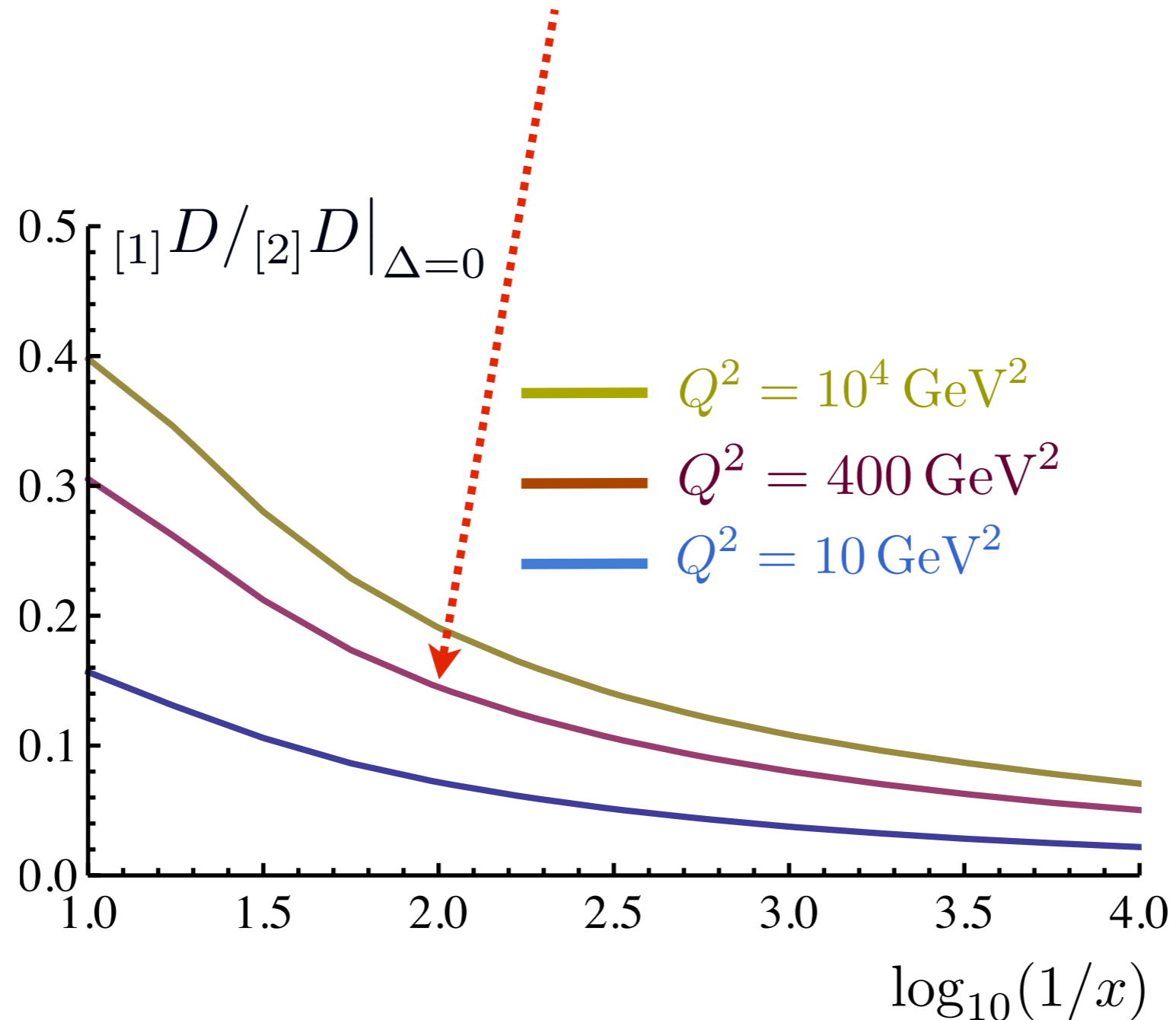
3-> 4 contribution vs. **4-> 4** is enhanced by a factor

$$2 \times \frac{7}{3} \simeq 5$$

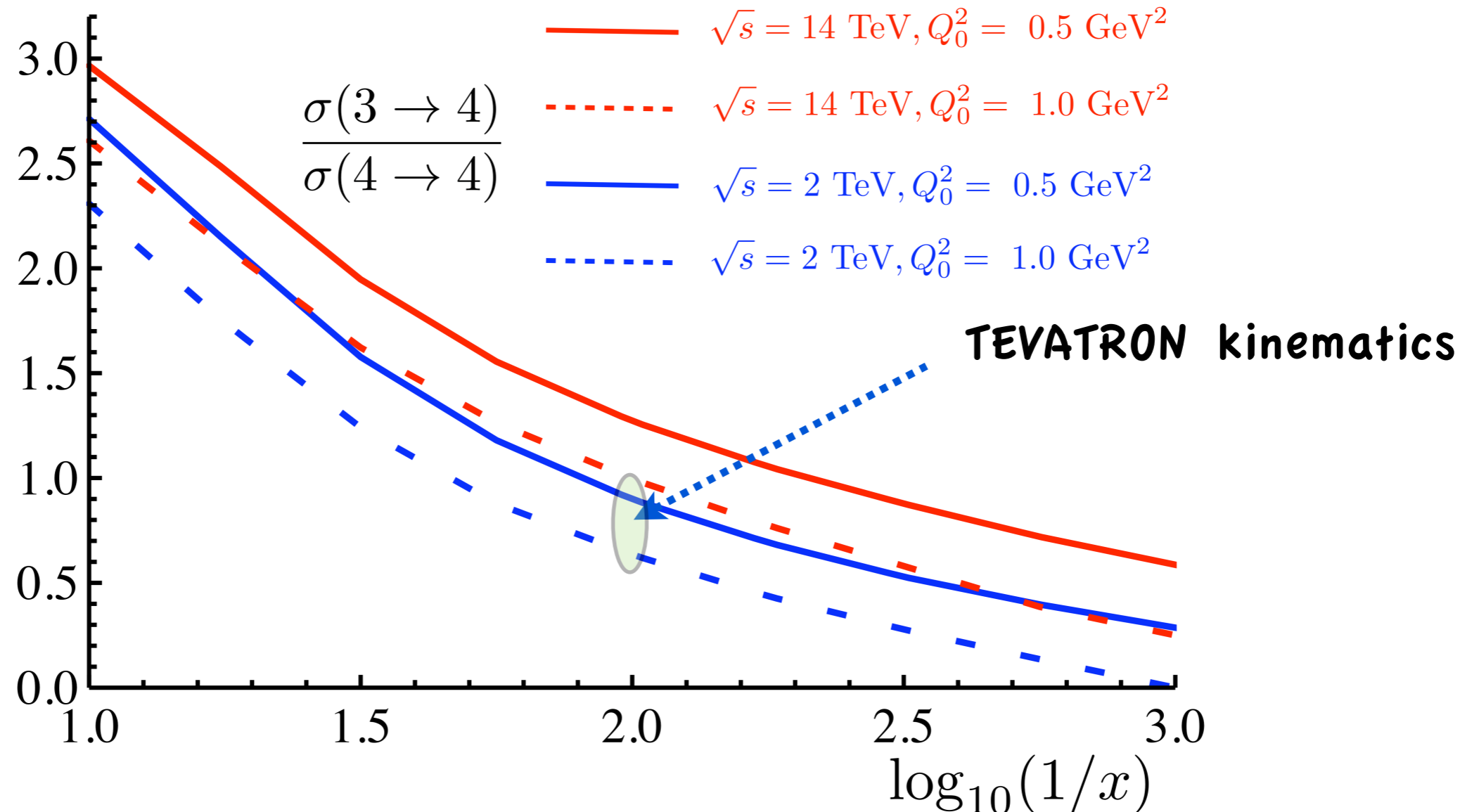


Results of quantitative analysis to appear in PRL (2012)

At TEVATRON energies, the $3 \rightarrow 4$ contribution amounts to about 15-20% of the $4 \rightarrow 4$:



the ratio of **3->4** to **4->4** contributions to the “total” Xsection



Uncertainty due to choice of the Q_0 parameter !

IMPORTANT ! *there is no contradiction.*

Q_0 *is not a formal parameter but a physical one:*

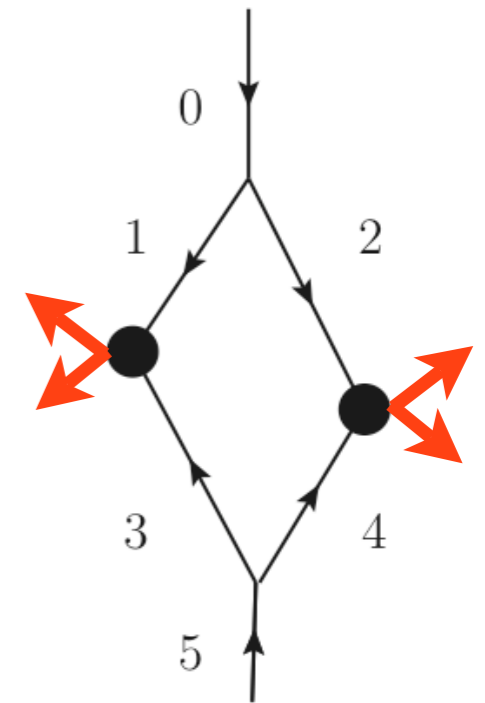
*The parton resolution scale at which $F_g^2(\Delta^2)$ has **no PT tail** $1/\Delta^2$!*

2 -> 4 processes

What if *both parton pairs* originate from PT splittings ?

No Δ —dependence whatsoever. The integral diverges...?..

This is **NOT** an amplitude of a *4-parton collision* but a one-loop correction to the *2-parton collision*



4-parton interaction is a “higher twist” effect

hard 2-parton scattering :

$$\frac{d\sigma^{(2\rightarrow 2)}}{d\hat{t}} \propto \frac{\alpha_s^2}{Q^4}$$

plus two additional jets :

$$\frac{d\sigma^{(2\rightarrow 4)}}{d\hat{t}_1 d\hat{t}_2} \propto \frac{\alpha_s^4}{Q^6}$$

4 jets from 4-parton scattering :

$$\frac{d\sigma^{(4\rightarrow 4)}}{d\hat{t}_1 d\hat{t}_2} \propto R^{-2} \cdot \left(\frac{\alpha_s^2}{Q^4}\right)^2 \propto \frac{\alpha_s^4}{R^2 Q^8}$$

extra $\frac{m_g^2}{Q^2}$

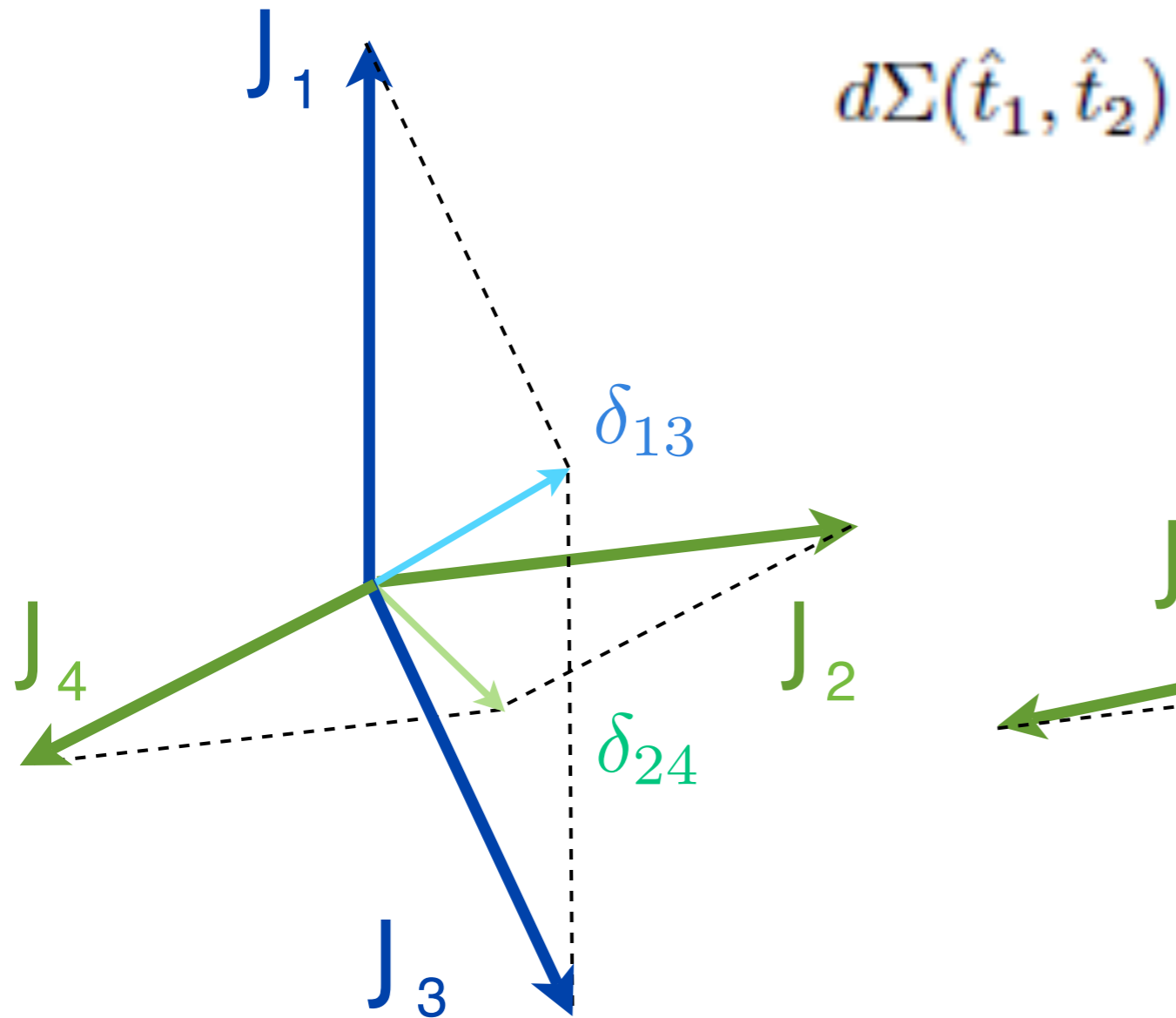
Always a *small contribution* to the *total 4-jet production cross section*

End of story?.. Not at all

What distinguishes “double hard collisions” is the *differential jet spectrum*

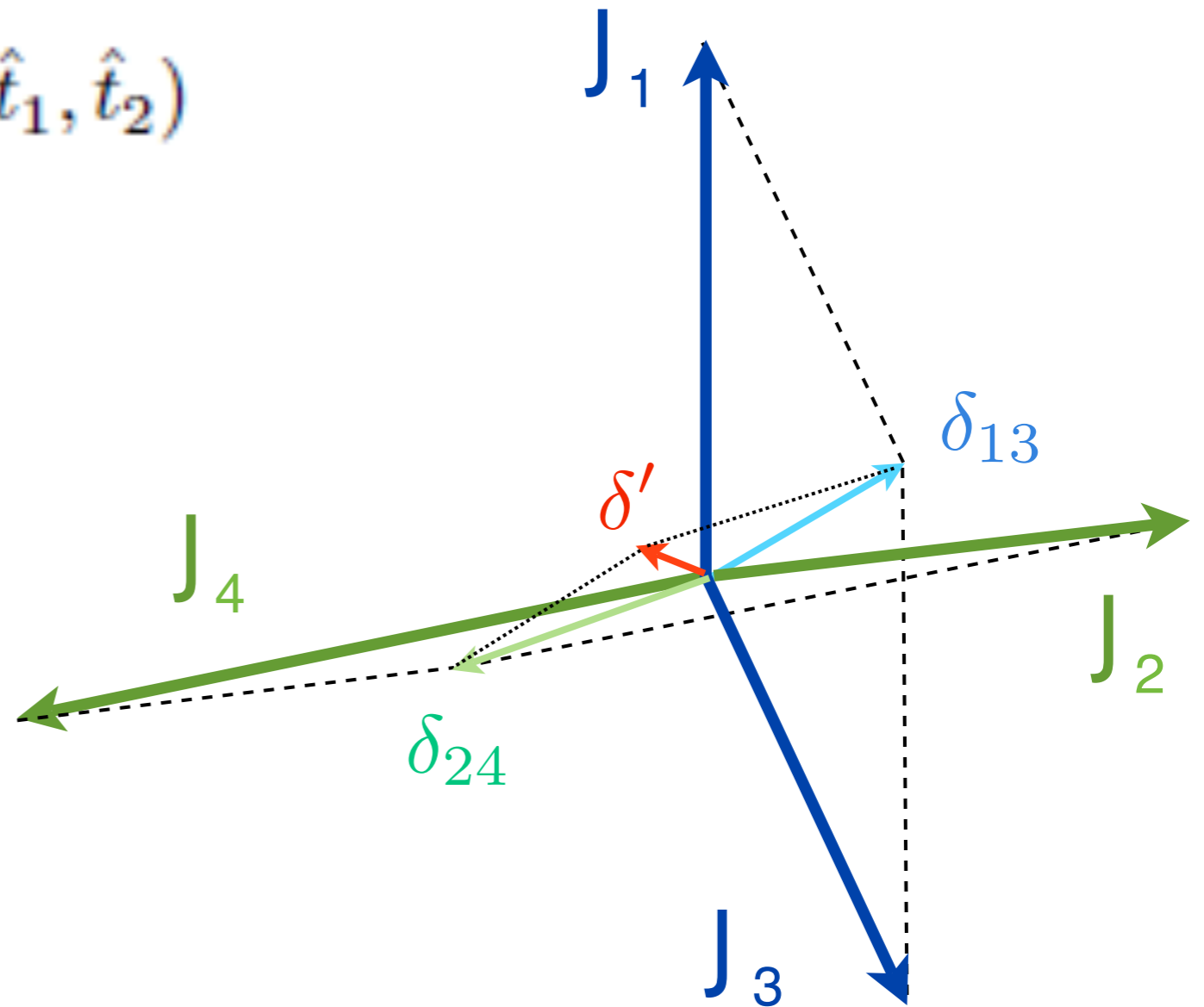
back-to-back kinematics

$$\delta_{13}^2, \delta_{24}^2 \ll J_{i\perp}^2$$



$$d\sigma^{(4\rightarrow 4)} \propto \frac{\alpha_s^2}{\delta_{13}^2 \delta_{24}^2} d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

$$\delta'^2 \ll \delta_{13}^2 \simeq \delta_{24}^2 \ll J_{i\perp}^2$$



$$d\sigma^{(3\rightarrow 4)} \propto \frac{\alpha_s^2}{\delta'^2 \delta^2} d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

Both these regimes are present
in differential imbalance
distributions due to
Multi-Parton Interactions

reminder :

Massive lepton pair production cross section

$$\frac{d\sigma}{dq^2 dq_{\perp}^2} = \frac{d\sigma_{\text{tot}}}{dq^2} \times \frac{\partial}{\partial q_{\perp}^2} \left\{ D_a^q(x_1, q_{\perp}^2) D_b^q(x_2, q_{\perp}^2) S_q^2(q^2, q_{\perp}^2) \right\}$$

Quark form factor :
$$S_q(Q^2, \kappa^2) = \exp \left\{ - \int_{\kappa^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_0^{1-k/Q} dz P_q^q(z) \right\}$$

Gluon form factor :
$$S_g(Q^2, \kappa^2) = \exp \left\{ - \int_{\kappa^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_0^{1-k/Q} dz [z P_g^g(z) + n_f P_g^q(z)] \right\}$$

Parton splitting probabilities

$$P_q^q(z) = C_F \frac{1+z^2}{1-z},$$

$$P_q^g(z) = P_q^q(1-z),$$

$$P_g^q(z) = T_R [z^2 + (1-z)^2],$$

$$P_g^g(z) = C_A \frac{1+z^4 + (1-z)^4}{z(1-z)}$$

Generalization of the DDT-formula for back-to-back 4-jet production spectrum

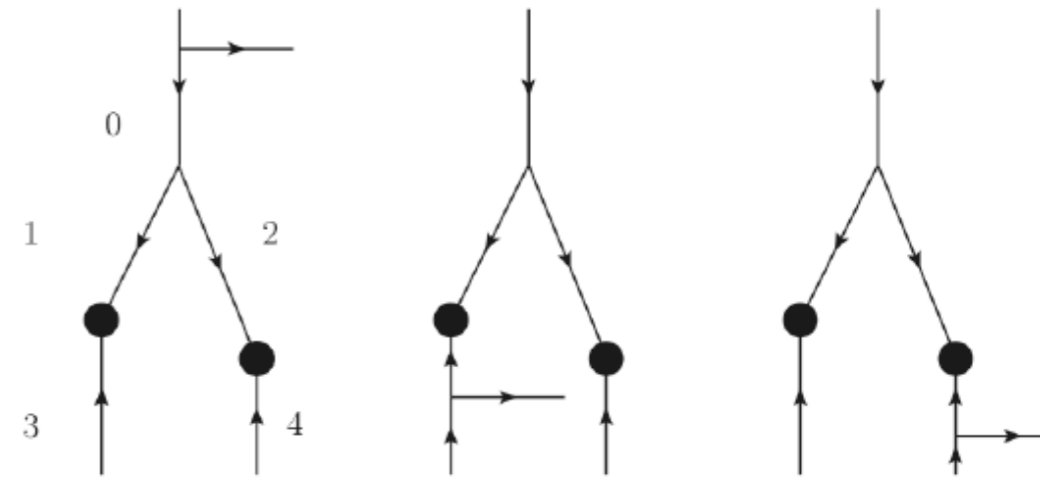
$$\pi^2 \frac{d\sigma^{(4 \rightarrow 4)}}{d^2\delta_{13} d^2\delta_{24}} = \frac{d\sigma_{\text{part}}}{d\hat{t}_1 d\hat{t}_2} \cdot \frac{\partial}{\partial\delta_{13}^2} \frac{\partial}{\partial\delta_{24}^2} \left\{ [2]D_a^{1,2}(x_1, x_2; \delta_{13}^2, \delta_{24}^2) \times [2]D_b^{3,4}(x_3, x_4; \delta_{13}^2, \delta_{24}^2) \right. \\ \left. \times S_1(Q^2, \delta_{13}^2) S_3(Q^2, \delta_{13}^2) \times S_2(Q^2, \delta_{24}^2) S_4(Q^2, \delta_{24}^2) \right\}$$

Not forgetting the Δ —integration and short-range correlations :

$$[2]D_a \times [2]D_b + [2]D_a \times [1]D_b + [1]D_a \times [2]D_b$$

Additional 3 -> 4 contribution :

$$\frac{\pi^2 d\sigma_2^{(3 \rightarrow 4)}}{d^2\delta_{13} d^2\delta_{24}} = \frac{d\sigma_{\text{part}}}{d\hat{t}_1 d\hat{t}_2} \cdot \frac{\alpha_s(\delta^2)}{2\pi \delta^2} \sum_c P_c^{1,2} \left(\frac{x_1}{x_1 + x_2} \right)$$



$$S_1(Q^2, \delta^2) S_2(Q^2, \delta^2) \frac{\partial}{\partial\delta'^2} \left\{ S_c(\delta^2, \delta'^2) \frac{G_a^c(x_1 + x_2; \delta'^2, Q_0^2)}{x_1 + x_2} S_3(Q^2, \delta'^2) S_4(Q^2, \delta'^2) \times [2]D_b^{3,4}(x_3, x_4; \delta'^2, \delta'^2) \right\}$$

Parton Entanglement in 3->4 interactions ?

Single Perturbative Splitting Diagrams in Double Parton Scattering

Jonathan R. Gaunt

[arXiv:1207.0480](https://arxiv.org/abs/1207.0480) [hep-ph]

● $_2\text{GPD} \neq \text{GPD} \times \text{GPD}$ a factorized model too naive

● Correct? Yes. Important? Not sure.

Things to sort out :

● to develop physical picture of entanglement

● to look for sources of potential suppression

"crosstalk interactions are suppressed by colour effects..." (wrong)

● Otherwise, an object too complex, too new... Ain't nice

- Multi-parton collisions contribute substantially to 4 jet production in the back-to-back kinematics
- 4→4 and 3→4 parton subprocesses are both **enhanced** in the back-to-back region, while “double perturbative parton splittings” generate effectively 2→4, which is **not**
- To describe multi-parton collisions one has to introduce and explore a new object
– **Generalized Double-Parton Distributions**

$$[2] D_h^{a,b}(x_1, x_2; q_1^2, q_2^2; \vec{\Delta})$$

→ the parameter $\vec{\Delta}$ encodes the information about the impact-parameter-space correlation between the two partons from one hadron

- experimentally observed **enhancement** of a 4-jet cross section indicates the presence of short range two-parton correlations in the nucleon parton wave function, as determined by the range of integral over $\vec{\Delta}$
- A model of partons uncorrelated at scale of about 0.5 GeV^2 , supplied with PT evolution (“3→4” processes), is capable of explaining Tevatron MPI findings
- I’d rather experimental studies employed QCD-motivated **jet finding algorithms** and concentrated on correlations in **transverse momenta** rather than **angles**

Where and how
to look for
4-parton interactions