QCD evolution of the parton correlation functions

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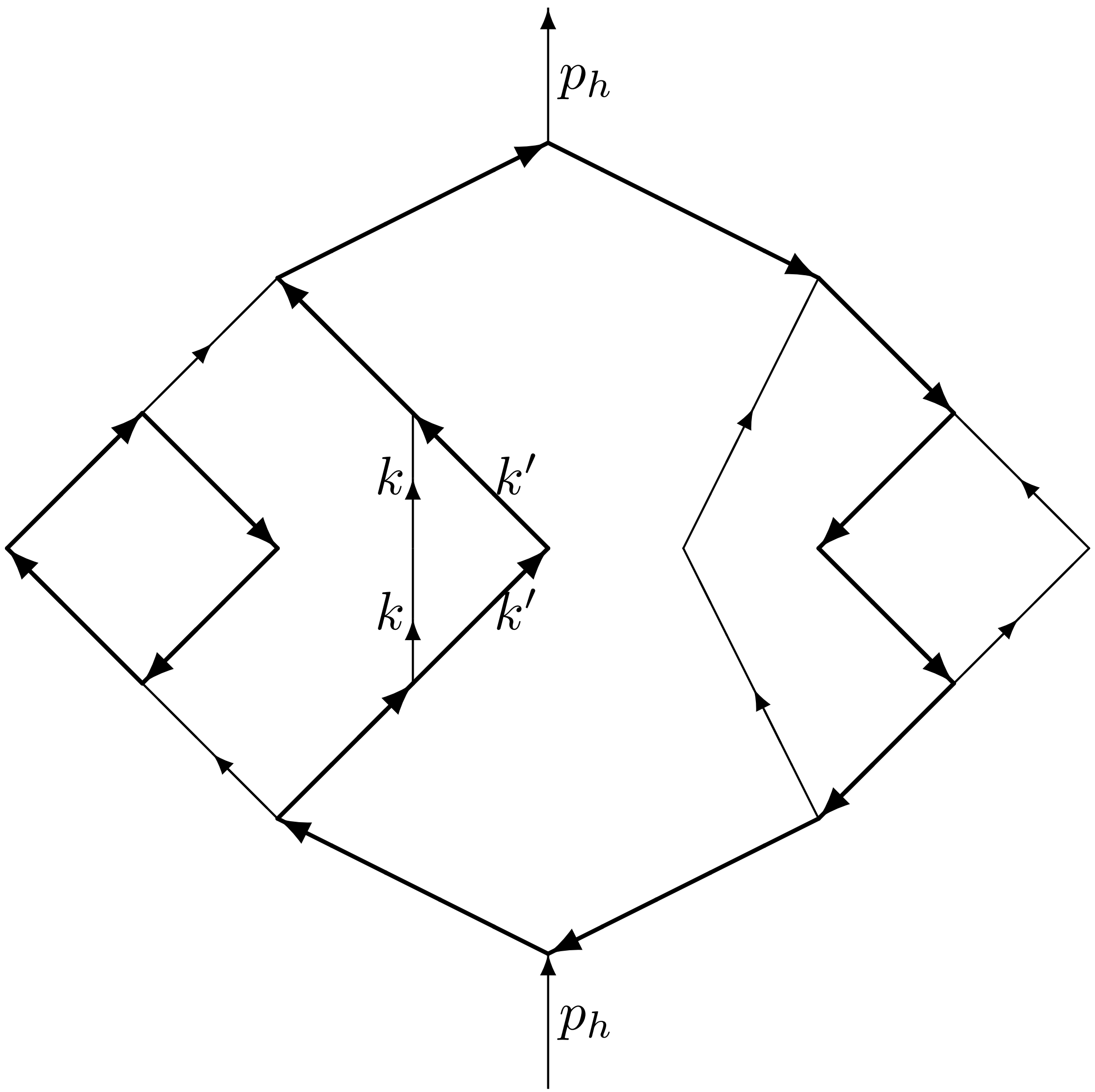
Content

Normalization condition for partonic wave function



1 Probabilistic DGLAP dynamics

Parton distributions

$$n_l(x) = \sum_n \int \prod_{i=1}^{n} \frac{d\beta_i d^2 k_i^{\perp}}{(2\pi)^2} \left|\Psi(\beta_1, k_1^{\perp} ... \beta_n, k_n^{\perp})\right|^2 \delta(1-\sum_{i=1}^{n} \beta_i)\, \delta^2(\sum_{i=1}^{n} k_i^{\perp}) \sum_{i\in l} \delta(\beta_i - x)$$

DGLAP equation

$$\frac{d}{d\xi}\, n_l(x) = -w_l\, n_l(x) + \sum_r \int_x^1 \frac{dy}{y}\, w_{r\to l}\left(\frac{x}{y}\right) n_r(y)\,,\ \ \xi = -\frac{2N_c}{\beta_0} \ln \frac{\alpha(Q^2)}{\alpha_\mu}$$

Decay probability

$$w_r = \sum_l \int_0^1 dx\, x\, w_{r\to l}(x)\,.$$

Twist-2 operators in the light cone gauge

$$O^{\psi}_{...} = \bar{\psi}\gamma_{.}\partial_{.}^{j-1}\psi\,.\ O^{G}_{...} - (\partial_{.} A^{\perp}_{\sigma})\partial_{.}^{j-2}(\partial_{.} A^{\perp}_{\sigma})\,,\ A_{.} = 0\,,\ C_{.} = q'_{\mu} C_{\mu}\,,\ q'^{\,2} = 0$$

2 Correlators of parton numbers

Parton number correlators (L. (1974))

$$< h|n_{l_1} n_{l_2} ... n_{l_k}|h >= Z_h \prod_{r=1}^{k} \delta_{l_r h} + \sum_{n=2}^{\infty} \sum_{i_1 ... i_n} n_{l_1} n_{l_2} ... n_{l_k} P_{i_1 ... i_n} ,$$

$$P_{i_1 ... i_n} - \int \prod_{i=1}^{n} \frac{d^2 k_{\perp i}}{(2\pi)^3} \frac{d\beta_i}{\beta_i} \theta(\beta_i) Z_g^{n_g} Z_q^{n_q} Z_{\bar{q}}^{n_{\bar{q}}} |\psi|^2 (2\pi)^3 \delta^2 (\sum_i k_{\perp i}) \delta(1 - \sum_i \beta_i)$$

Evolution equations (L. (1974))

$$\frac{d}{d\xi} < h| \prod_{r=1}^{k} n_{l_r} |h >= - \sum_s w_s < h|n_s \prod_{r=1}^{k} n_{l_r} |h > + \sum_s \sum_{i_1, i_2} w_{s \to (i_1, i_2)} N_{s, i_1, i_2}^{(n+1)} ,$$

$$N_{s, i_1, i_2}^{(n+1)} =< h|n_s \prod_{r=1}^{k} (n_{l_r} - \delta_{l_r s} + \delta_{l_r i_1} + \delta_{l_r i_2}) |h > , \; < h| \prod_{r=1}^{k} n_{l_r} |h >_{|\xi=0} = \prod_{r=1}^{k} \delta_{l_r h}$$

3 Particular solutions (B'KL (1984))

Multiplicative evolution in gluedynamics

$$< h|T^{(k)}(n)|h >= k!\, e^{k\,w\,\xi}\ .\ T^{(k)}(n) = n(n+1)...(n+k-1)$$

Probability to find a hadron in the n-gluon state

$$P_n = \frac{1}{\bar{n}} \left(\frac{\bar{n}-1}{\bar{n}}\right)^{n-1}\ ,\ \sum_{n=1}^{\infty} T^{(k)}(n)\, P_n = k!\, \bar{n}^k\ ,\ \bar{n} = e^{w\,\xi}$$

Generating functional for multi-parton distributions

$$I(\phi_t) = \sum_{n_t} \int \prod_t \prod_{l=1}^{n_t} \phi_t(\beta_l^t)\, d\beta_l^t\, f_{n_t}(\beta_r^t)\ ,\ t = g_\pm, q_\pm, \bar{q}_\pm$$

A solution for the N=4 super-symmetric QCD

$$I(\phi) = \int_{-i\infty}^{i\infty} \frac{d\,p}{2\pi i}\, e^p\, \frac{w\, \int_0^\infty\, e^{-xp}\, \frac{d\,x}{x}\, \phi(x,\xi)}{2\,(1-\exp(-w\xi))\int_0^\infty\, e^{-xp}\, \frac{dx}{x}\, \phi(x,\xi) + w\, \exp(-w\xi)}$$

4 Quasi-partonic operators (B'F'KL)

Parton propagators in the light-cone gauge $A_\mu q'_\mu = A_, = 0$

$$d_{\mu\nu} = \sum_{\lambda=\pm 1} e_\mu^{\lambda *}(k') e_\nu^\lambda(k') + k^2 \frac{q'_\mu q'_\nu}{(kq')^2}\,,\ \hat{k} = \sum_{\lambda=\pm 1/2} u^\lambda \bar{u}^\lambda + \frac{k^2}{2kq'}\,\hat{q}'\,,\ k' = k - \frac{k^2}{2kq'}\,q'$$

Building blocks for quasi-partonic operators

$$G^\perp_{.\mu} = \partial_. A^\perp_\mu\,,\ D_. = \partial_.\,,\ \bar{\psi}\gamma_.\,,\ \gamma_.\,,\ \gamma_.\psi$$

Invariance to the field shift

$$A_\mu \to A_\mu + q'_\mu \phi\,,\ \bar{\psi} \to \bar{\psi} + \bar{\chi}\gamma_.\,,\ \psi \to \psi + \gamma_.\chi$$

Matrix elements of QPO and correlation functions

$$< h'|O^{\{r\}}|h > = \sum_{\lambda_1 \ldots \lambda_l} \int d\beta_1 \ldots d\beta_l\, O^{\{r\}}_{\lambda_1 \ldots \lambda_l} \prod_{t=1}^{l} \beta_t^{n_t}\, N^{\{r\}}_{\lambda_1 \ldots \lambda_l}(\beta_1, \ldots \beta_l)$$

5 Examples of QPO

Twist and number of fields for QPO

$$t = d - j = k\,.$$

Twist-2 operators

$$((-i\partial_.)^{n_1}\bar{\psi})\gamma_.(i\partial_.)^{n_2}\psi\,,\ ((-i\partial_.)^{n_1}\bar{\psi})\gamma_5\gamma_.(i\partial_.)^{n_2}\psi\,,\ ((-i\partial_.)^{n_1}\bar{\psi})\gamma_.\gamma_\rho^\perp(i\partial_.)^{n_2}\psi\,,$$

$$\left(-i\partial_. A_\rho^\perp\right)(i\partial_.)^{n+1}A_\rho^\perp\,,\ \epsilon_{\rho_1\rho_2}^{\perp\perp}\left(-i\partial_. A_{\rho_1}^\perp\right)(i\partial_.)^{n+1}A_{\rho_2}^\perp\,,\ S_{\rho_1\rho_2}\left(-i\partial_. A_{\rho_1}^\perp\right)(i\partial_.)^{n+1}A_{\rho_2}^\perp$$

Twist-3 operators

$$\left((-i\partial_.)^{n_1}\bar{\psi}\right)\gamma_.\gamma_5(i\partial_. A_\sigma^\perp)(i\partial_.)^{n_2}\psi\,,\ f_{abc}\epsilon_{\rho_1\rho_2}^{\perp\perp}\left((-i\partial_.)^{n_1}A_{\rho_1}^{a\perp}\right)(i\partial_. A_\sigma^{b\perp})(i\partial_.)^{n_2}A_{\rho_2}^{c\perp}$$

Twist-4 operators and ${}_2D(x_1, x_2, Q_1^2, Q_2^2, \vec{\Delta})$

$$\left((-i\partial_.)^{n_1}\bar{\psi}\right)\gamma_.\left((i\partial_.)^{n_2}\psi\right)\left((-i\partial_.)^{n_3}\bar{\psi}\right)\gamma_.(i\partial_.)^{n_4}\psi\,,$$

$$\left((-i\partial_.)^{n_1}A_\rho^\perp\right)\left((i\partial_.)^{n_2}A_\rho^\perp\right)\left((-i\partial_.)^{n_3}A_\sigma^\perp\right)(i\partial_.)^{n_4}A_\sigma^\perp\,,$$

6 Renormalization of QPO

DGLAP evolution of correlation functions for QPO

$$\frac{\partial N^{\{r\}}(\beta_1,...\beta_l)}{\partial\tilde{\xi}} = \sum_{i<k}\sum_{r_{i'}r_{k'}}\int d\beta_{i'}\,d\beta_{k'}\,\Phi^{r_i r_k}_{r_{i'}r_{k'}}(\beta_i,\beta_k|\beta_{i'},\beta_{k'})\,N^{\{r'\}}(\beta_1,...\beta_{i'}...\beta_{k'}...)$$

Gegenbauer polynomials as eigenfunctions of $\Phi^{r_i r_k}_{r_{i'}r_{k'}}$

$$R^{\bar{q}q}_{n+1}(\beta_1,\beta_2) = \sum_{k=0}^{n}(-1)^k\,\frac{n!\,(n+2)!\,\beta_1^k\,\beta_2^{n-k}}{k!\,(k+1)!\,(n-k)!\,(n-k+1)!}\,,$$

$$R^{gg}_{n+2}(\beta_1,\beta_2) = \sum_{k=0}^{n}(-1)^{k+1}\,\frac{n!\,(n+4)!\,\beta_1^k\,\beta_2^{n-k}}{k!\,(k+2)!\,(n-k)!\,(n-k+2)!}\,,$$

$$R^{qg}_{n+\frac{3}{2}}(\beta_1,\beta_2) = \sum_{k=0}^{n}(-1)^k\,\frac{n!\,(n+3)!\,\beta_1^k\,\beta_2^{n-k}}{k!\,(k+1)!\,(n-k)!\,(n-k+2)!}\,,$$

$$n = j - s_1 - s_2$$

7 Evolution equations for moments

Moments of correlation functions for QPO

$$N^{r_1 \dots r_l}(n_1, ..., n_l) = \int \prod_{t=1}^{l} \left(\frac{d\beta_t}{(n_t+1)!} \, \beta_t^{n_t} \right) N^{\{r\}}(\beta_1, ..., \beta_l)$$

Evolution equations in momenta representation

$$\frac{\partial}{\partial \tilde{\xi}} N^{r_1 \dots r_l}(n_1, ... n_l) = \sum_{i<k} \sum_{r_{i'}, r_{k'}} \sum_{n'_i, n'_k} \left[\Phi^{r_i r_k}_{r_{i'} r_{k'}} \right]^{n_i n_k}_{n_{i'} n_{k'}} N^{r_1 \dots r_l}(n_1, ..., n_{i'} ... n_{k'} ... n_l)\,,$$

$$n'_i + n'_k - n_i - n_k = -\Delta S_{ik} = s_i + s_r - s_{i'} - s_{k'}$$

Pair kernels

$$\left[\Phi^{r_1 r_2}_{r_{1'} r_{2'}} \right]^{n_1 n_2}_{n_{1'} n_{2'}} = \frac{a_{r_{1'}}(n_{1'})\, a_{r_{2'}}(n_{2'})}{a_{r_1}(n_1)\, a_{r_2}(n_2)} \sum_j V^{r_1 r_2\, j}_{n_1 n_2} \, V^{r_{1'} r_{2'}\, j}_{n_{1'} n_{2'}} \, \Lambda^{r_1 r_2}_{r_{1'} r_{2'}}(j) \left(\frac{j+2}{j-1} \right)^{\frac{1}{2}\Delta S_{12}}\,,$$

$$a_q(n) = \sqrt{n+1}\,,\ a_g = \sqrt{(n+1)/(n+2)}\,,\ V^{r_1 r_2\, j}_{n_1 n_2} = (-1)^{n_1 + 2s_1 - 1}\, C^{j,\Delta S}_{n/2, n/2; n/2+S, \Delta n}$$

8 Discussion

1. Parton number correlators.
2. Evolution of parton number correlators.
3. Quasipartonic operators.
4. Parton correlation functions.
5. Evolution of parton correlation function.
6. Relation between ${}_2GPD$, parton number correlators and QPO.
7. Twist 4 non-QPO operators (Braun, Manashov, Rohrwild).