Straight scaling FFAG experiment

J.-B. Lagrange, E. Yamakawa, T. Uesugi, Y. Ishi,
Y. Kuriyama, B. Qin, K. Okabe, Y. Mori, Y. Yoshino,
R. Nakano, Y. Niwa, T. Minamikawa, Y. Ito, M. Takabatake

FFAG12 - Nov. 2012

Outline

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- Theory

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- Experiment

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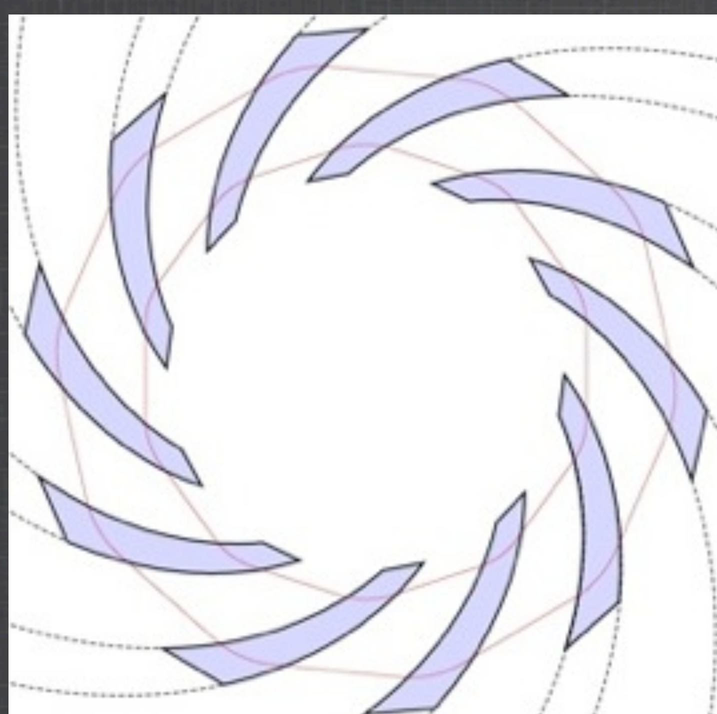
Circular case

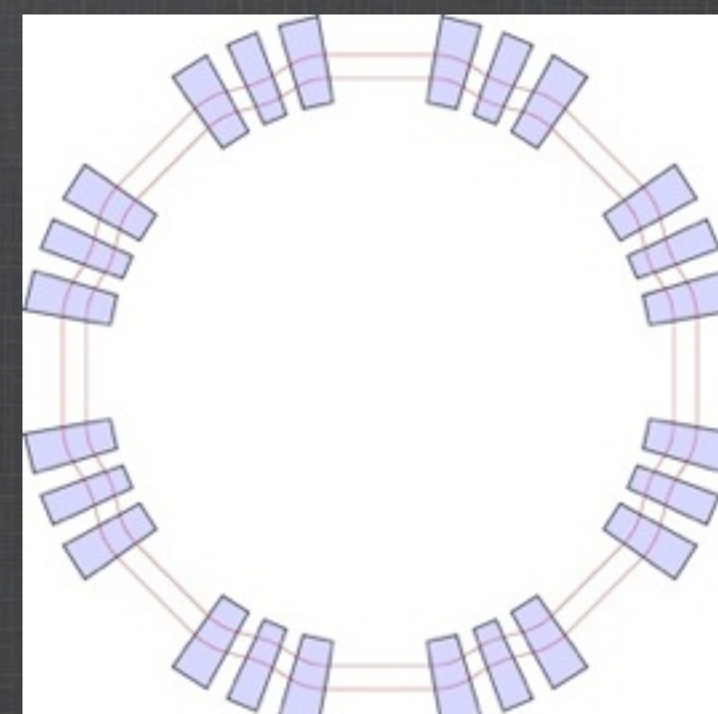
Invariance of the betatron oscillations ⟷ similarity of the closed orbits <u>and</u> invariance of the field index

Geometrical field index: $k = \frac{R}{\bar{B}} \frac{d\bar{B}}{dR}$

$$B(r,\theta) = B_0 \left(\frac{r}{r_0}\right)^k \cdot \mathcal{F}(\theta - \tan\zeta \ln \frac{r}{r_0})$$



Spiral sector: $\zeta = const.$



Radial sector: $\zeta = 0$

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Straight case

Linearized equations of motion for a momentum p:

$$\begin{cases} \dfrac{d^2x}{ds^2} + \dfrac{1-n}{\rho^2}x = 0, \\ \dfrac{d^2z}{ds^2} + \dfrac{n}{\rho^2}z = 0. \end{cases}$$

(x, s, z): curvilinear coordinates

n: field index

ρ : curvature radius

Independent of momentum p:

$$\begin{cases} \left(\dfrac{\partial \rho}{\partial p}\right)_s = 0, \\ \left(\dfrac{\partial n}{\partial p}\right)_s = 0. \end{cases}$$

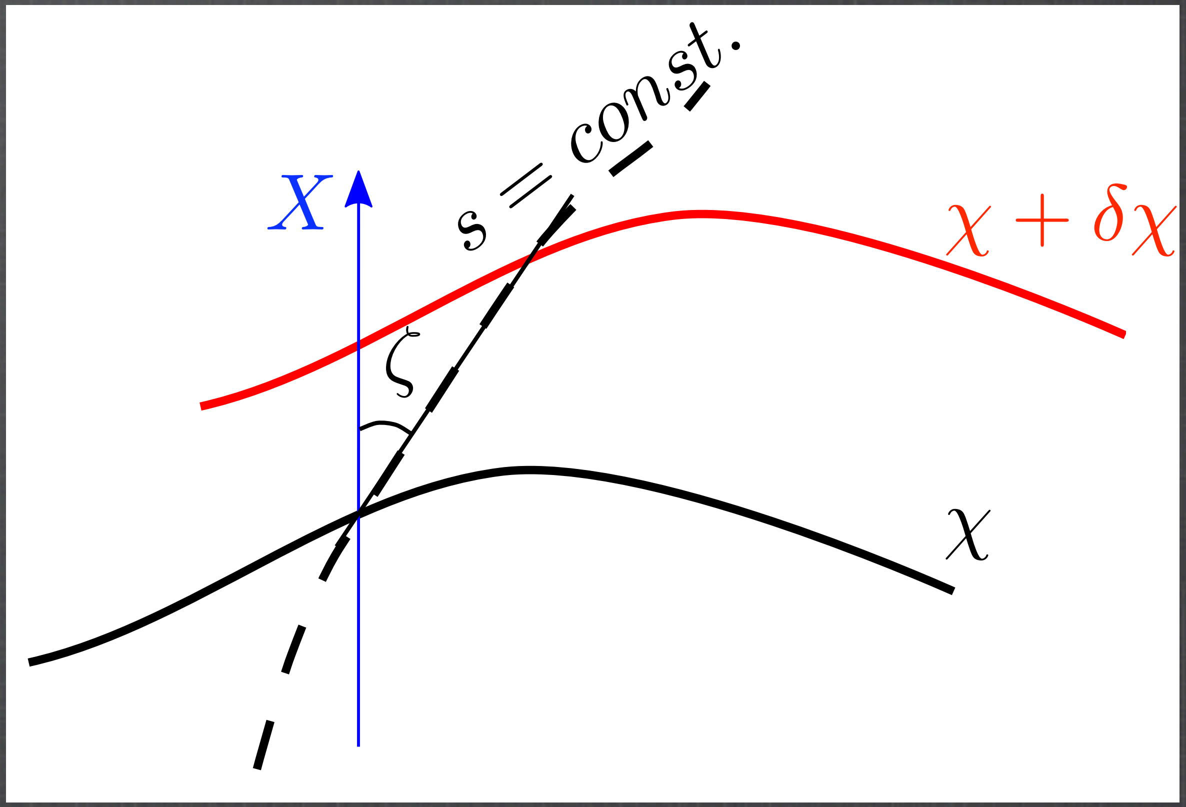
$\left(\frac{\partial \rho}{\partial p}\right)_s = 0$ ⇒ Similarity of the reference trajectories

$\left(\frac{\partial n}{\partial p}\right)_s = 0$ ⇒ Invariance of the focusing strength

Change of coordinates: introduction of average abscissa χ.

Straight case

Introduction of normalized field gradient: $m = \frac{1}{\bar{B}} \frac{d\bar{B}}{d\chi}$



Invariance of the focusing strength gives condition on *m*:

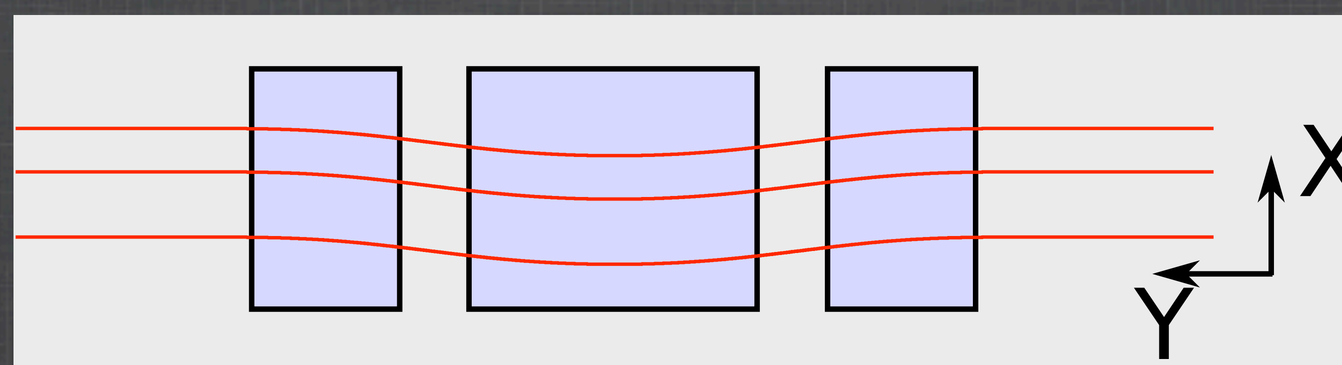
$m = m_1 + m_2 \tan\zeta(\chi)$

$$B(\chi, s) = B_0 e^{\left[m_1(\chi - \chi_0) + m_2 \int_{\chi_0}^{\chi} \tan\zeta(\chi) d\chi\right]} \mathcal{F}(s)$$

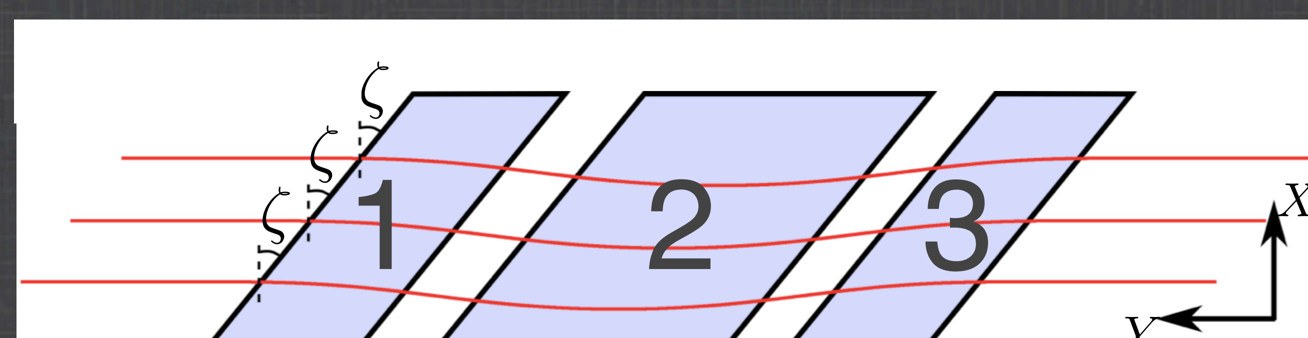
New case: straight case

$\zeta = const. \Longleftrightarrow m = const.$

$$B(X,Y) = B_0 e^{m(X-X_0)} \mathcal{F}\left(Y - (X - X_0)\tan\zeta\right)$$



Rectangular case: $\zeta = 0$



Tilted straight case: $\zeta = const.$

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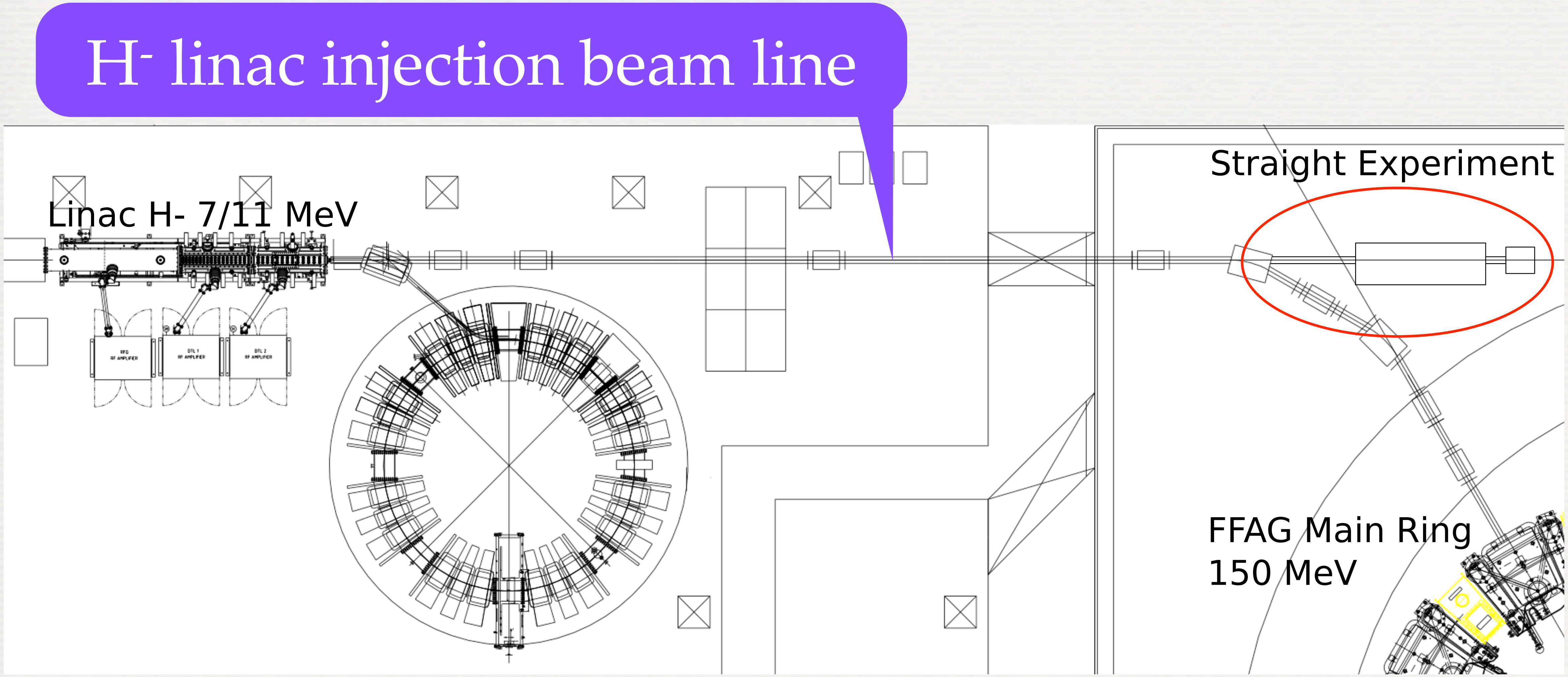
Goal of the experiment

The circular scaling law has been confirmed by experiment in the past.

The straight scaling law is new, and needs to be confirmed through experiment.

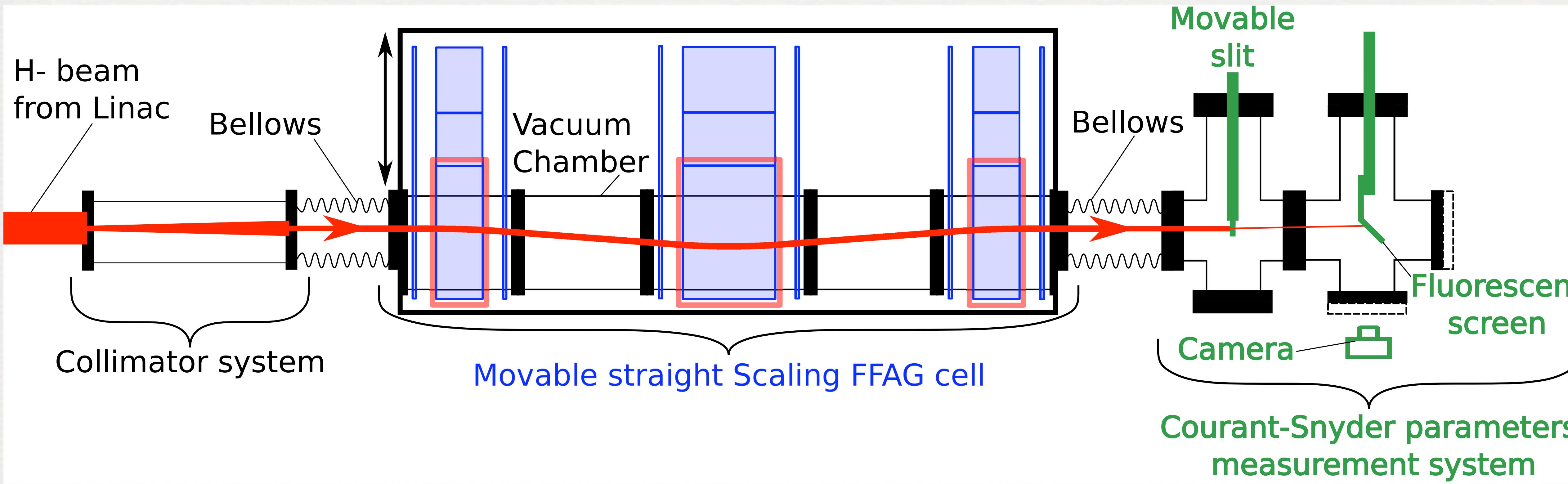
➡**Design and manufacturing of a straight scaling cell prototype, and measure of the horizontal phase advance for 2 different energies.**

Layout of the experiment

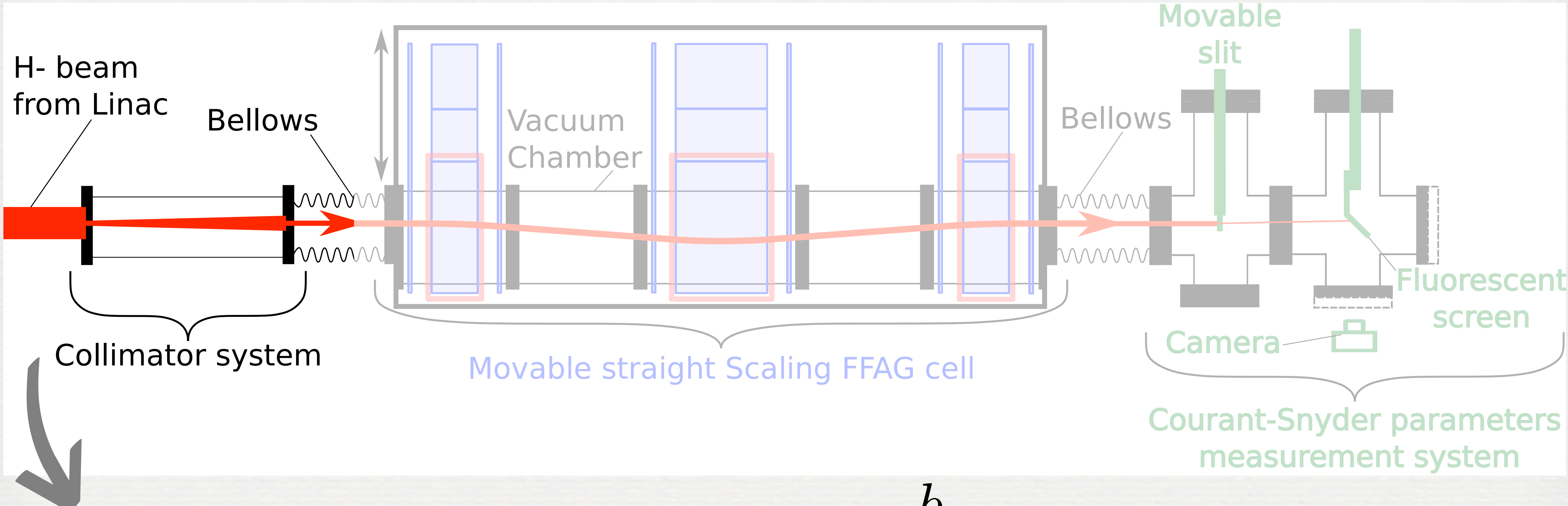


Use of 2 energies: 7 MeV and 11 MeV.

Layout of the experiment



Layout of the experiment



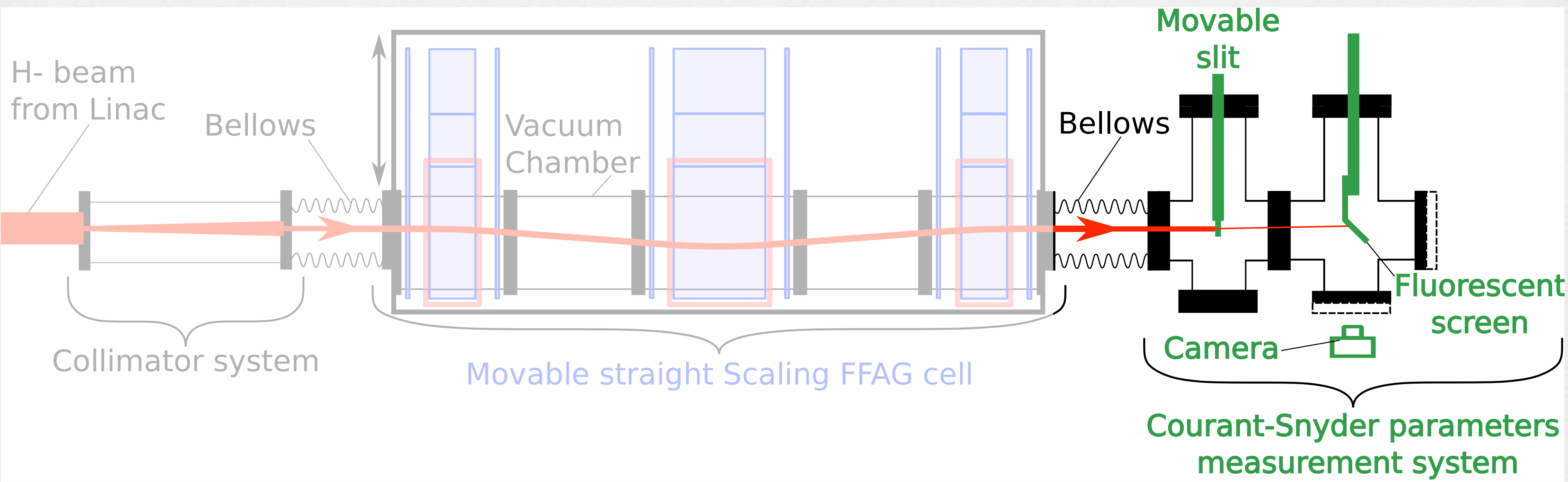
Set the entrance linear parameters (β_0 and α_0) and emittance (ε).

$$\beta_0 = \frac{b}{2} = 0.77\ m$$

$$\alpha_0 = 0$$

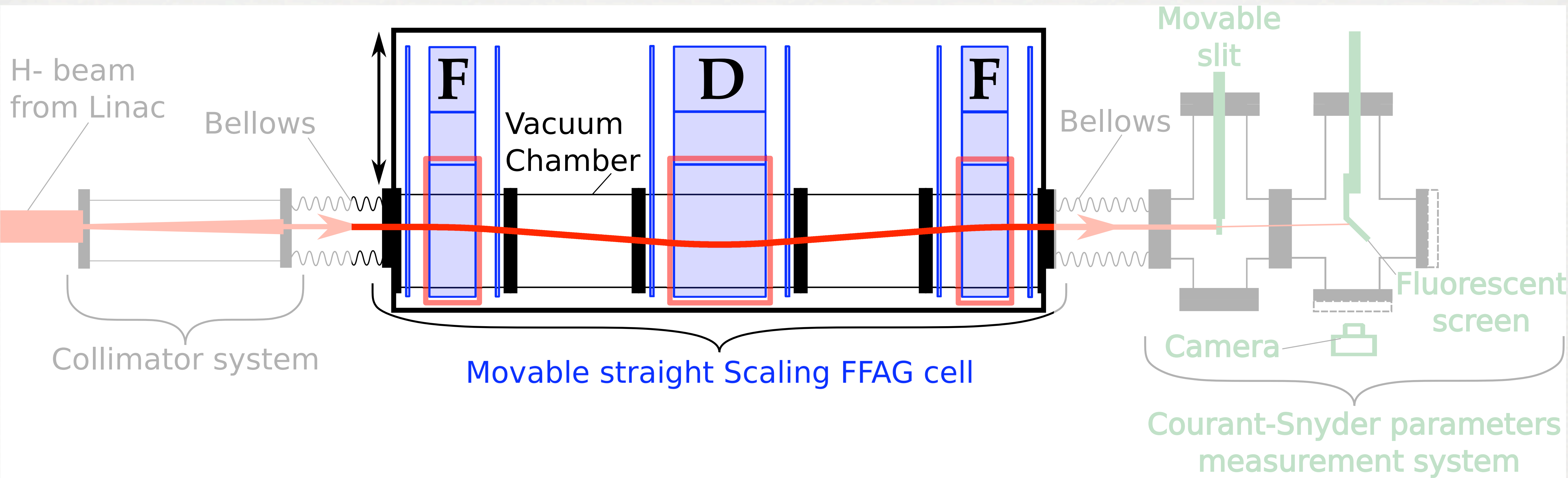
$$\epsilon_{rms} = \frac{a^2}{18b} = 0.14\ \pi\ mm.mrad$$

Layout of the experiment



Measure linear parameters (β_1 and α_1), position and angle of the beam.

Layout of the experiment



Straight Scaling FFAG cell design

- C-shape Magnets to have easy access to the pole.
- Cell able to move horizontally to match the different reference trajectories.
- Rectangular magnets.

Type	FDF
m-value	11 m^{-1}
Total length	4.68 m
Length of F magnet	15 cm
Length of D magnet	30 cm
Max. B Field (D magnet)	0.3 T
Max. B Field (F magnet)	0.2 T
Horizontal phase advance	87.7 deg.
Vertical phase advance	106.2 deg.

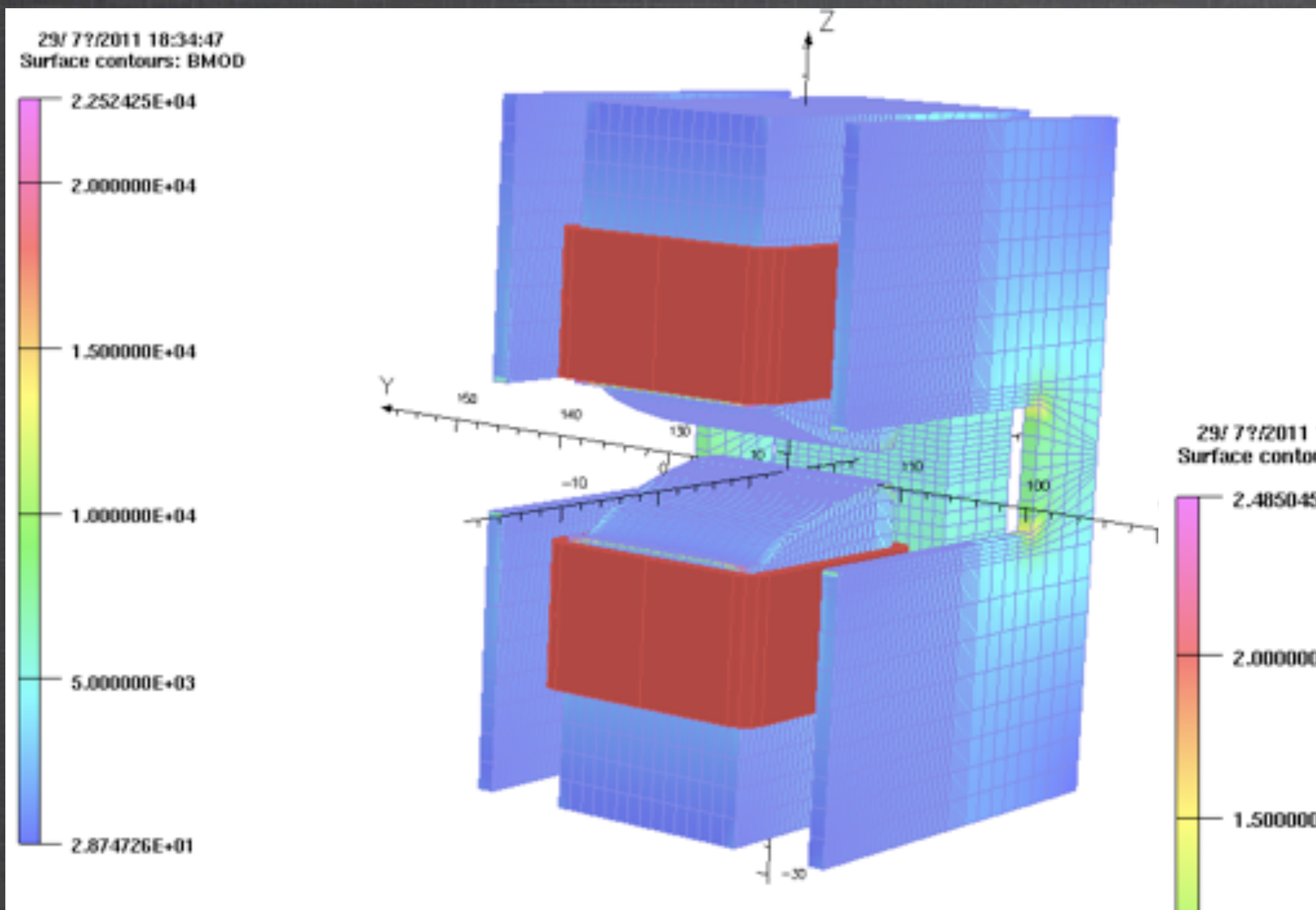
- Coils:
  - Max 3500 A.T/coil.
  - 18 turns x 4 layers = 72 turns of 5 mm x 2 mm cross section wire.

→ ~5 A/mm^2 → Indirect water cooling system.

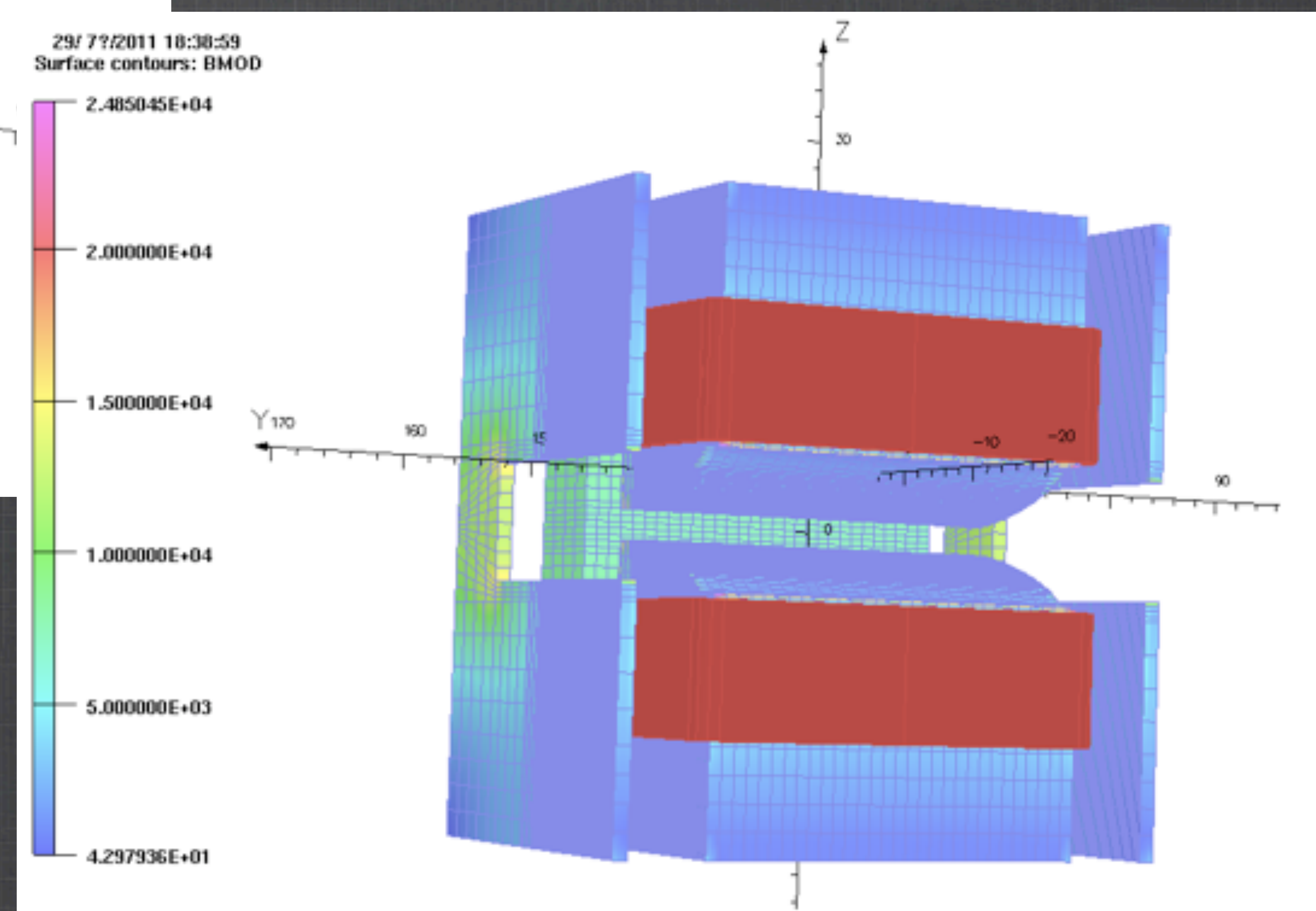
- Power supply per magnet (D): 100 A, 30 V.
- Whole system power consumption: ~1 kW.

Magnet design

Pole shape configured with POISSON, then TOSCA.

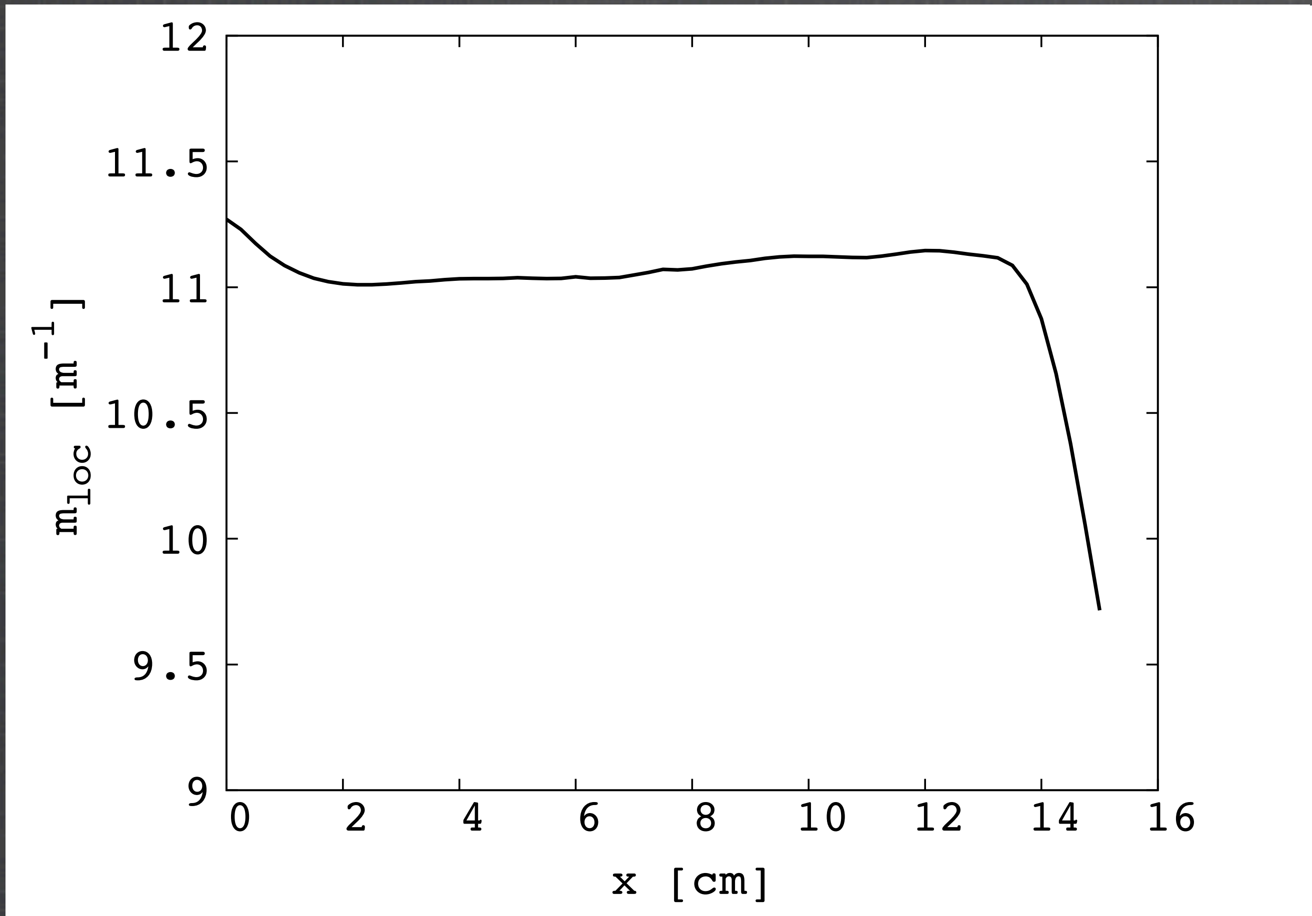


Magnetic field in
D magnet (30 cm long).
TOSCA model.

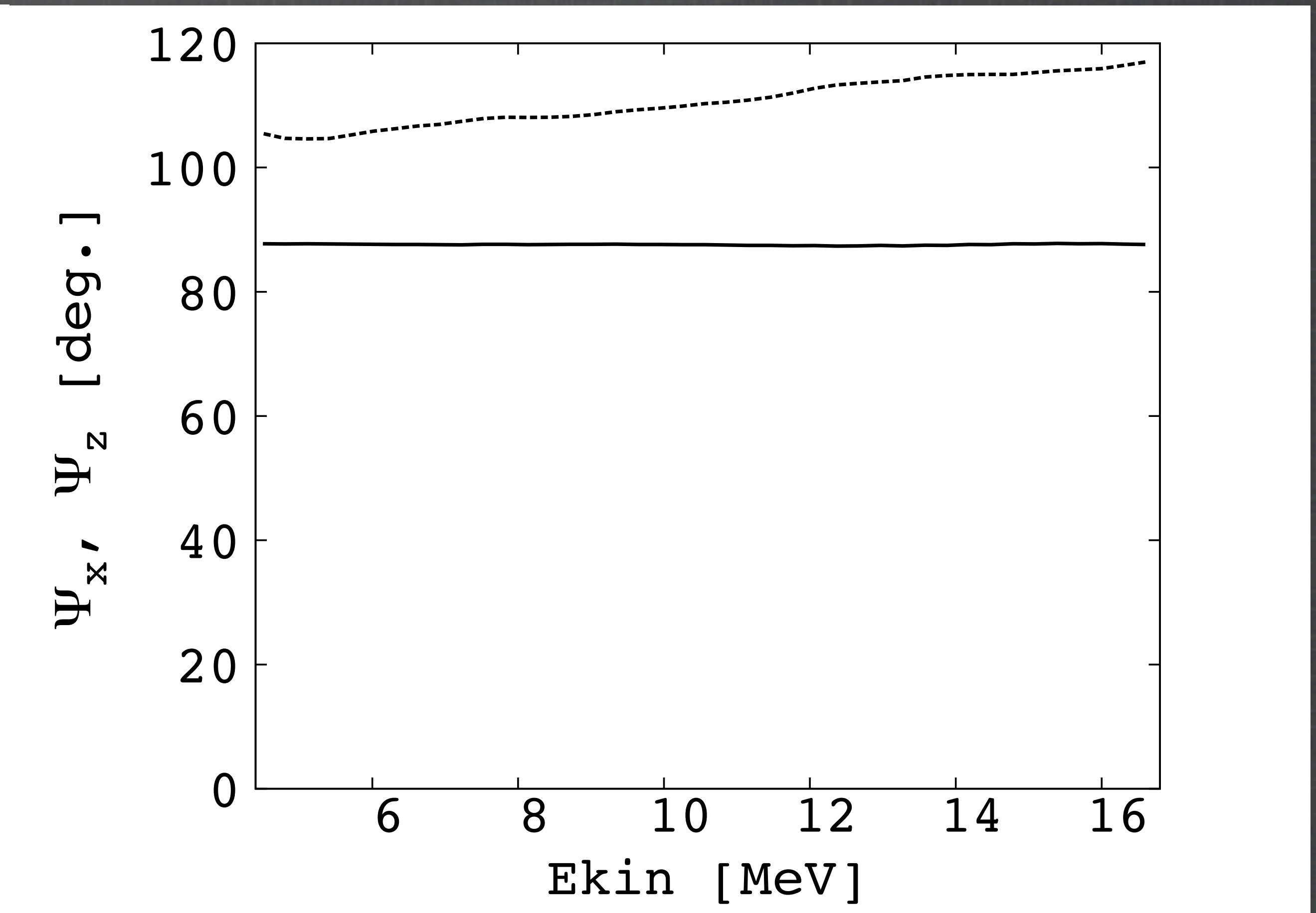


Magnetic field in
F magnet (15 cm long).
TOSCA model.

Tracking in TOSCA Field map



Local m value vs horizontal abscissa



Horizontal (plain) and vertical (dot) phase advances vs kinetic energy

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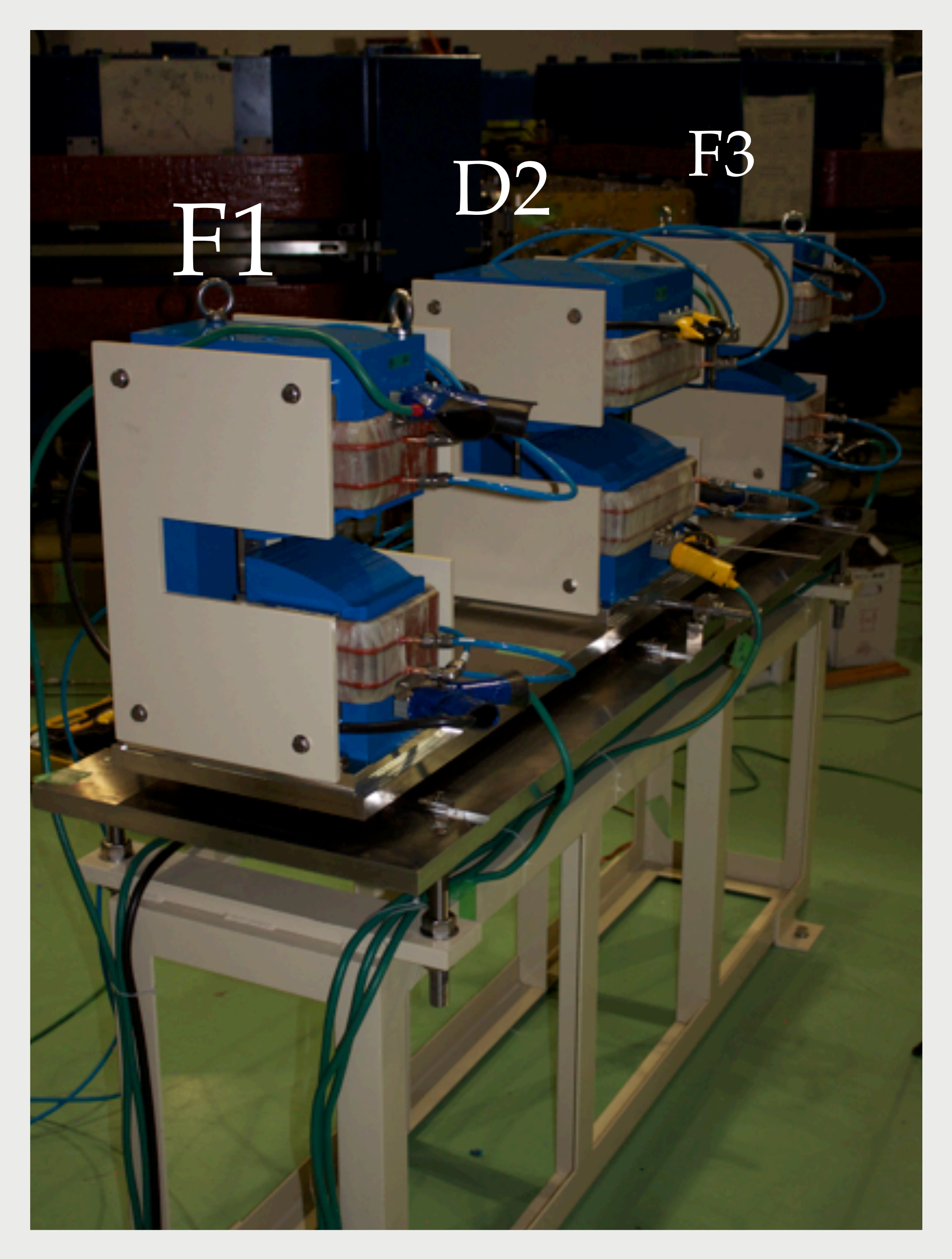
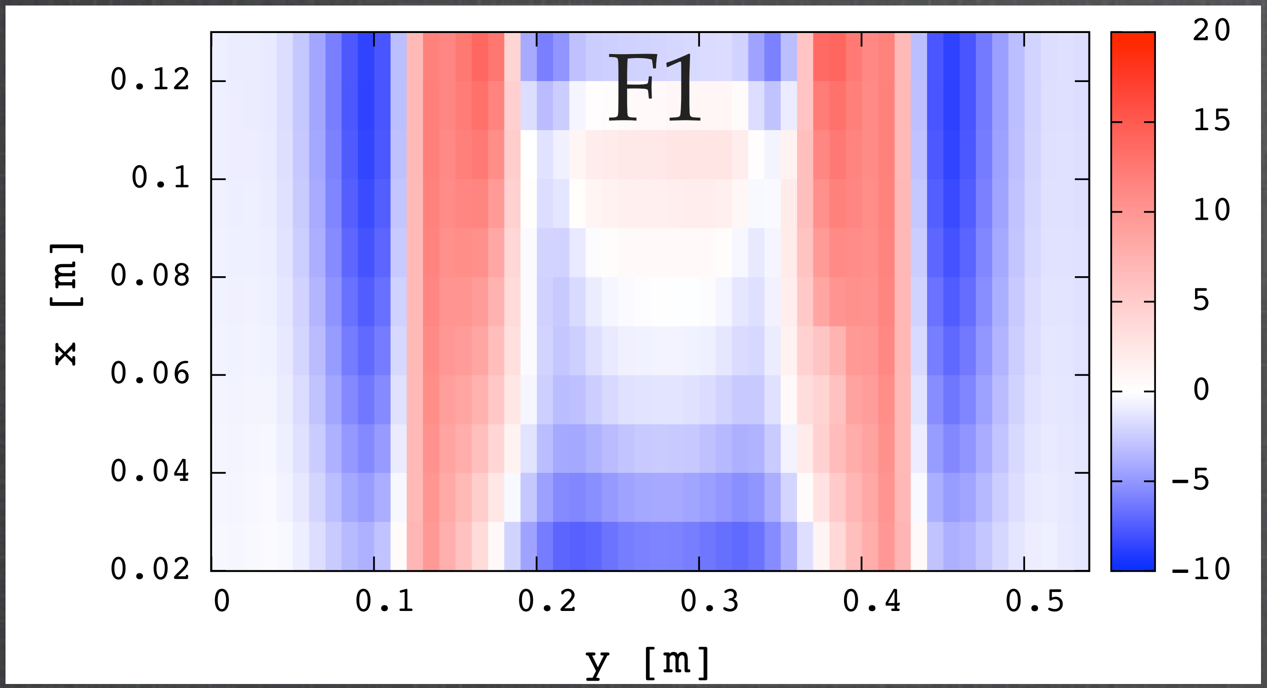
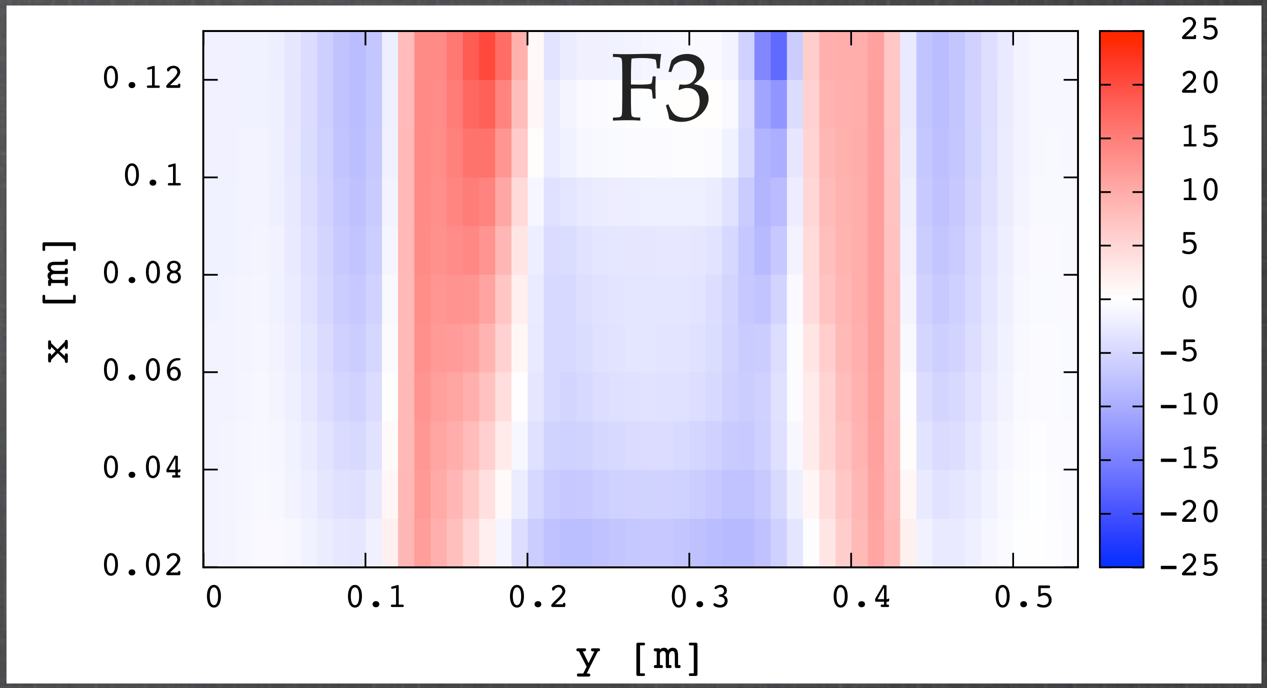
FIELD MEASUREMENT

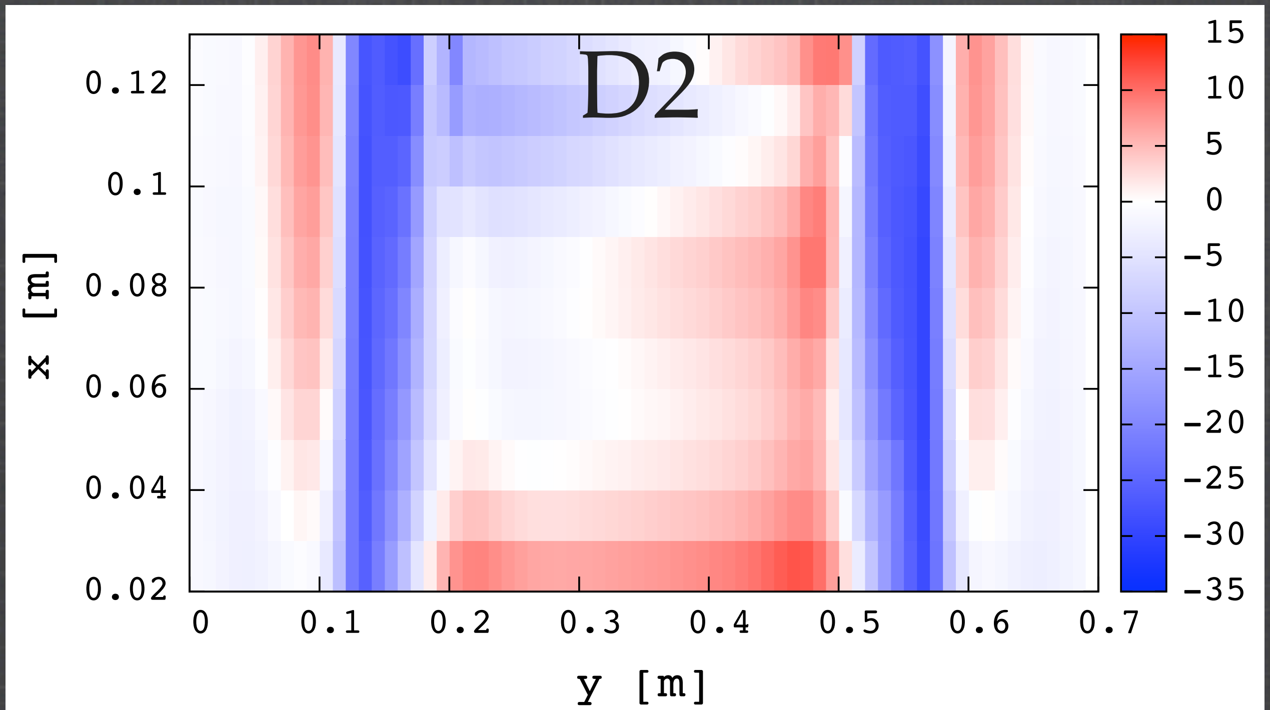
Measured field map created



Comparison TOSCA-Measure

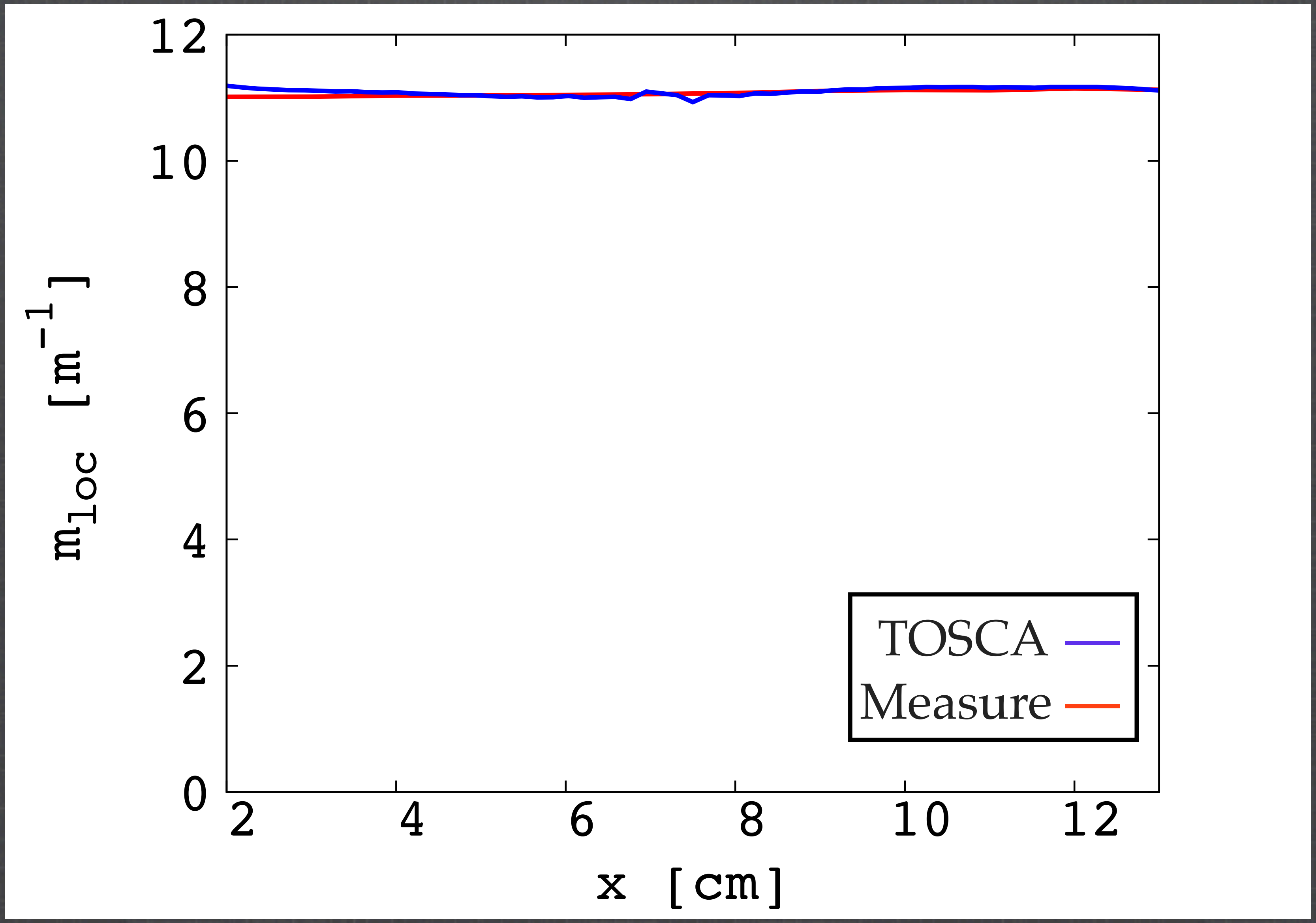






Difference of vertical field in the mid-plane in Gauss between TOSCA map and measured map

Comparison TOSCA-Measure



Local m-value vs horizontal abscissa

→ **<u>Good agreement</u>** (difference $< 1\%$)

Experimental measurement

$$\begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{\beta_1}{\beta_0}} \cos\psi & a_{12} \\ \frac{-\alpha_1 \cos\psi - \sin\psi}{\sqrt{\beta_1 \beta_0}} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \tan\psi = -\alpha_1 - \frac{\beta_1 x'_1}{x_1}$$

Exit parameters to measure: x_1, x'_1, β_1 and α_1.

For each energy, the beam is launched 3 times:

- on the reference trajectory,
- -10 mm off the reference trajectory,
- +10 mm off the reference trajectory.

$$\rightarrow \quad x_1 = \frac{x_{+10} - x_{-10}}{2} \qquad x'_1 = \frac{x'_{+10} - x'_{-10}}{2}$$

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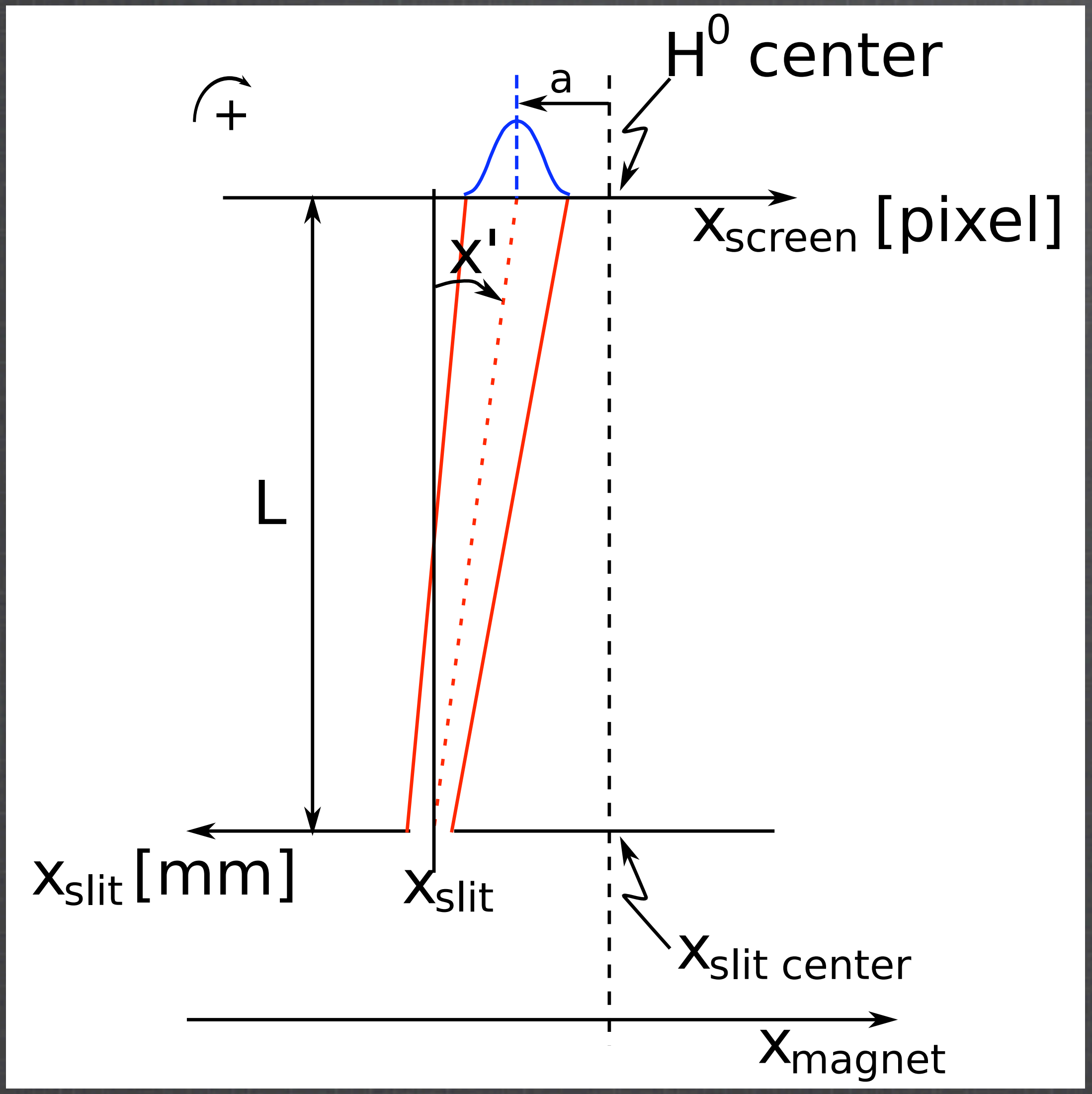
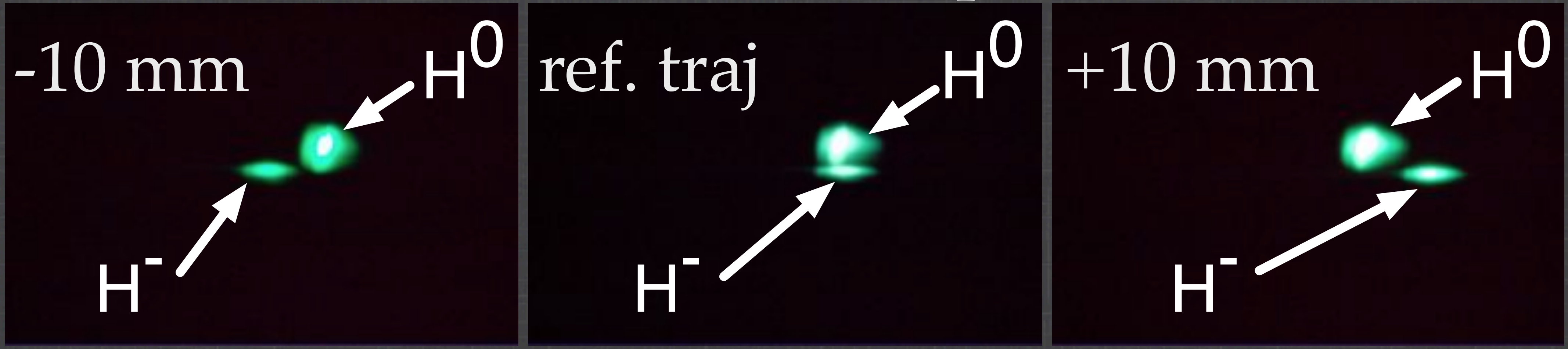
Experimental measurement



angle measurement scheme.

position and beta measurement
from pictures without slit.



Experimental measurement

α_1 measurement from

- the slope of the line x' vs. x_{slit}: $slope = -\left(\frac{\alpha}{\beta}\right)_{slit}$
- the beta value

→ Drift transfer matrix tracking to obtain α_1.

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Experimental results

$$\tan\psi = -\alpha_1 - \frac{\beta_1 x_1'}{x_1}$$

	$\bar{x_1}$ (mm)	$\bar{x_1'}$ (mrad)	$\bar{\beta_1}$ (m)	$\bar{\alpha_1}$	$\psi_{exp.}$ (deg)	ψ_{TOSCA} (deg)
11 MeV	**2.0**	**-2.4**	**17.7**	**-1.5**	**87.5** $\pm$ 3.3	**87.5**
7 MeV	**1.8**	**-2.1**	**11.7**	**-1.0**	**86.1** $\pm$ 9.6	**87.6**

ψ_{exp}(11 MeV)=87.5 deg

ψ_{exp}(7 MeV)=86.1 deg

Straight scaling law confirmed.

Summary

- Straight scaling FFAG theoretical magnetic field:

$$B(X,Y) = B_0 e^{m(X-X_0)} \mathcal{F}\left(Y - (X - X_0)\tan\zeta\right)$$

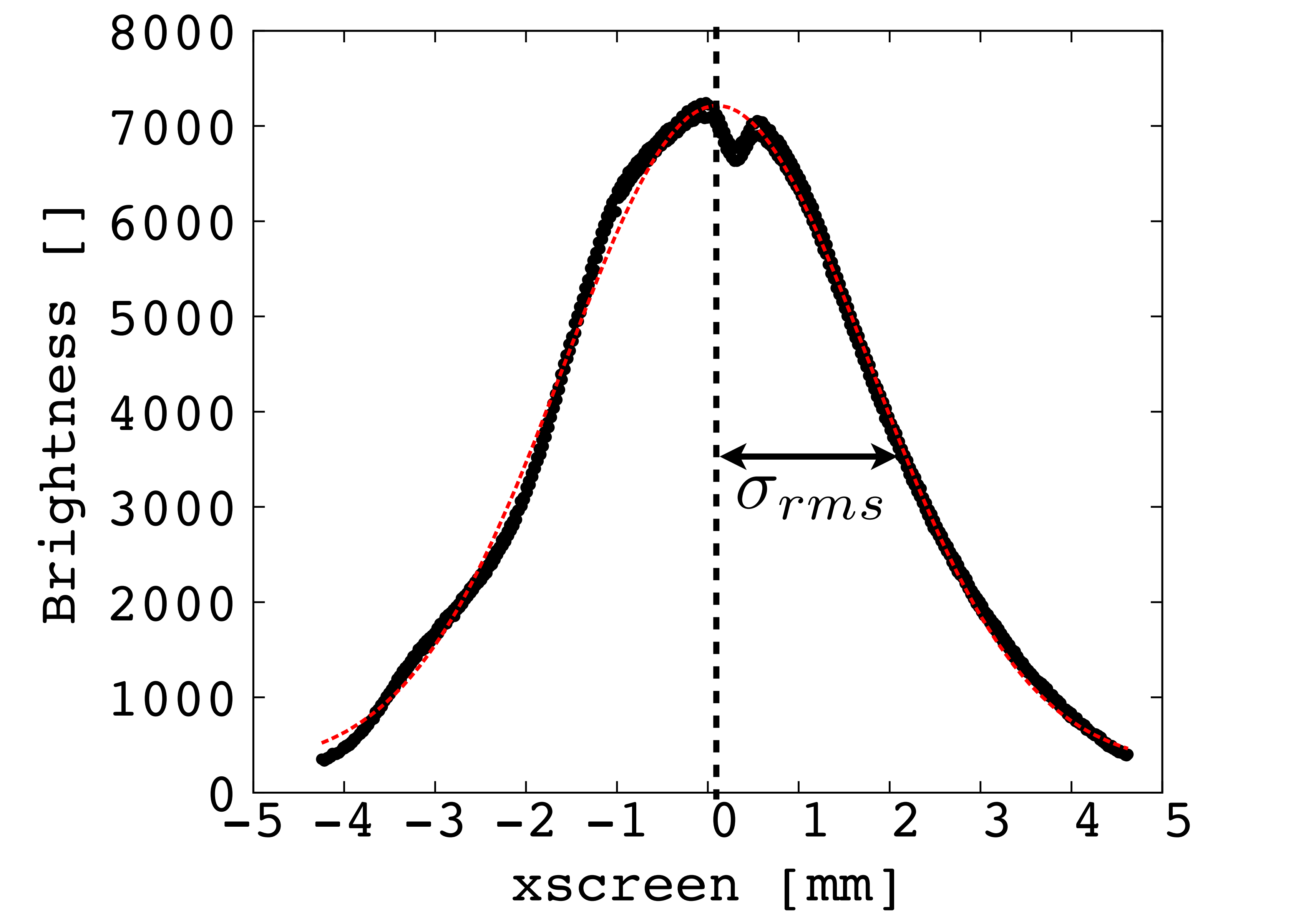
- To confirm the theory, an experiment has been conducted:
  - Design of a straight scaling FFAG cell,
  - Manufacturing of a prototype,
  - Measure of the magnetic field of the prototype,
  - Measure of the horizontal phase advances for 2 different energies

→ **STRAIGHT SCALING LAW CLARIFIED**

A special thank to all the people of the lab for their hard work!!

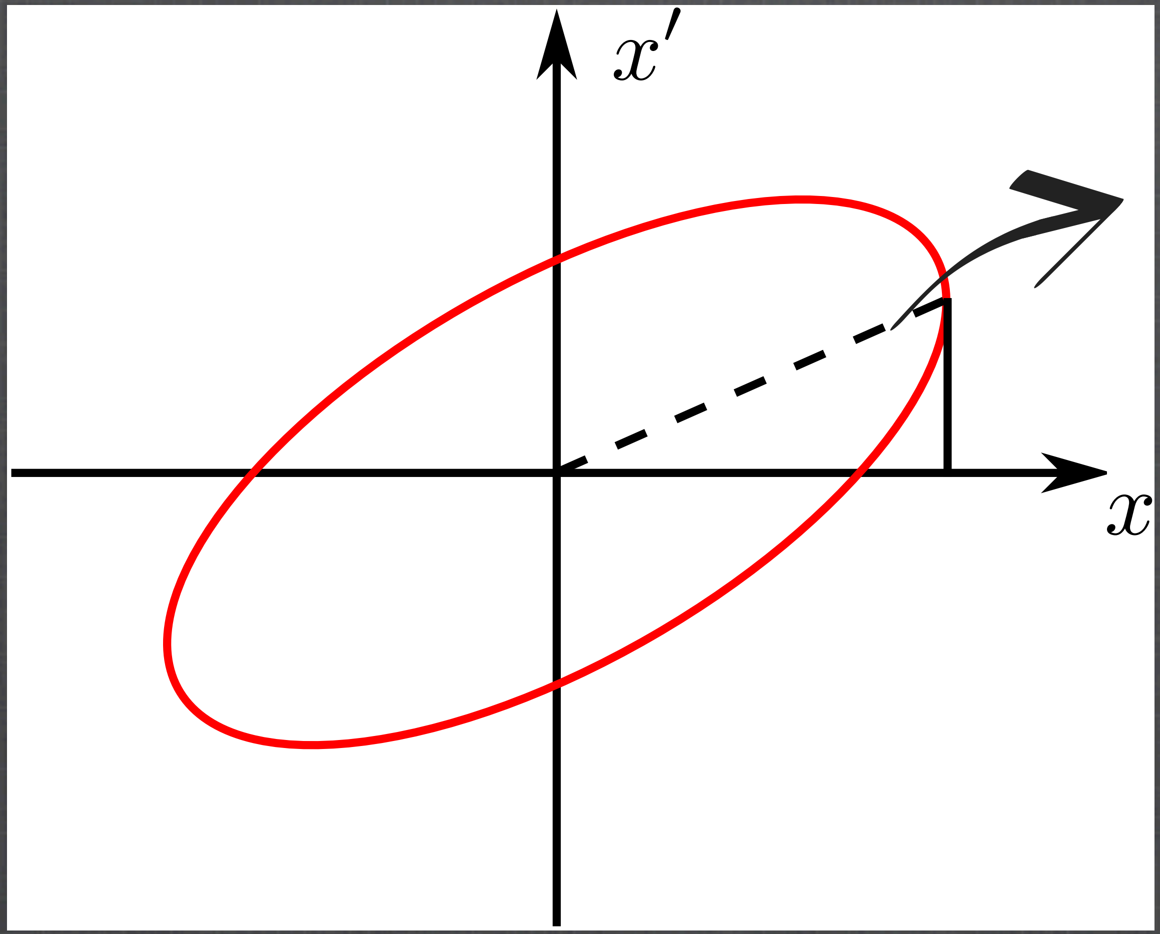
Thank you for your attention

β_1 measurement



$$\beta_1 = \frac{\sigma_{rms}^2}{\epsilon_{rms}}$$

α_1 measurement



$$slope = -\left(\frac{\alpha}{\beta}\right)_{slit}$$

$$\alpha_1 = \beta_{slit}\left(\frac{\alpha}{\beta}\right)_{slit} - L\left(\frac{1}{\beta_{slit}} + \beta_{slit}\left(\frac{\alpha}{\beta}\right)_{slit}^2\right)$$

$$\beta_{slit} = \frac{\beta_1 + \sqrt{\beta_1^2 - 4L^2\left(1 - 2L\left(\frac{\alpha}{\beta}\right)_{slit} + L^2\left(\frac{\alpha}{\beta}\right)_{slit}^2\right)}}{2\left(1 - 2L\left(\frac{\alpha}{\beta}\right)_{slit} + L^2\left(\frac{\alpha}{\beta}\right)_{slit}^2\right)}$$

Experimental results

	x_1 (mm)	x'_1 (mrad)	σ (mm)	$\left(\frac{\alpha}{\beta}\right)_{slit}$ (m^{-1})
Ref. traj. 11 MeV	0.1	-0.5	1.7	-0.09
+10 mm 11 MeV	1.4	-1.8	1.6	-0.08
-10 mm 11 MeV	-2.6	2.9	1.4	-0.11
Ref. traj. 7 MeV	0.6	0.5	1.1	-0.09
+10 mm 7 MeV	0.3	-1.9	1.1	-0.05
-10 mm 7 MeV	-3.4	2.2	1.8	-0.11

Error estimation

Dominated by the beta error due to the fluctuation of the beam size.

statistical and independent error from the variation of rms beam size σ_{rms}.

error for 11 MeV: 3.7%

error for 7 MeV: 11.1%