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Modelling of ADSR-FFAG

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Back of the envelope calculation (0)
same as ERIT study

- Basic beam parameters

  T = 11 MeV (E = 949.272 MeV)

  P = 144.094 MeV/c

  γ = 1.011724

  β = 0.151794

  ε_{rms} = 8 π mm mrad (unnormalized, rms)

  $\varepsilon_{95\%}$ = 48 (=6 x 8) π mm mrad (unnormalized, 95%)

  dT/T = 0.1% (rms)

  (dp/p = 0.05% (rms))

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Back of the envelope calculation (1)

- Tune shift (for uniform beam) $$\Delta Q = -\frac{r_p n_t}{2\pi\varepsilon\beta^2\gamma^3}\frac{1}{B_f}$$

  $\varepsilon_{100\%}$ = 100 π mm mrad (unnormalized, 100%)

  B_f = 0.25

  $n_t = 6 \times 10^{11}$

  gives $\Delta Q = -0.25$

- Number of particles $$n_t = 2\pi R \cdot TN \cdot \frac{I}{e\beta c}$$

  $n_t = 6 \times 10^{11}$

  R = 4.5 m at injection

  I = 5 mA

  gives TN = 31 (or 18 micro s)

Back of the envelope calculation (2)

rf voltage is much lower than ERIT

- Bucket height

$$B_h = 2\sqrt{\frac{eV}{2\pi\beta^2 Eh|\eta|}}$$

V = 4 kV

E = 11 + 938 MeV

h = 1

\eta = \alpha_t – 1/gamma^2 =1/(7.6+1)-1/1.0117^2=-0.861

gives B_h (=dp/p) = 1.16×10^{-2}

- Synchrotron tune

$$Q_s = \sqrt{\frac{heV|\eta|}{2\pi\beta^2 E}}$$

gives Q_s = 5.01×10^{-3} (or 200 turns)

Back of the envelope calculation (3)

- Longitudinal space charge

$$F_{sc}/e = -\frac{eg_0}{4\pi\varepsilon_0\gamma^2}\frac{\partial\lambda}{\partial s}$$

$$g_0 = 1 + 2\log\frac{d}{a}, \lambda = -k_2 s^2 + k_0$$

k_0 = 7.317 x 10^{11}

k_2 = 1.935 x 10^{12}

Sqrt[k_0/k_2] = \lambda_rf/4 and n_t = 6 x 10^{11}

g_0 = 2

gives (F_{sc}/e)/s = 7.5 x 10^0

- rf voltage

$$F_{rf}/e = V\sin\left(2\pi s/\lambda_{rf}\right) = \frac{2\pi V s}{\lambda_{rf}}$$

V = 4 kV

\lambda_rf = 2πR/h = 28.274 m

gives (F_{rf}/e)/s = 8.8 x 10^2 (>> 7.5 x 10^0)

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Back of the envelope calculation (4)

- Energy loss by foil scattering

  dE = 0.760 kV per turn (Okabe at FFAG10)

  \phi_s = asin(0.760/4) = 11 degree (not negligible)

  Actual \phi_s shift (Uesugi at FFAG11)

- Overlapping of linac micro structure

$$dt = \eta \frac{dp}{p} t_{rev}$$

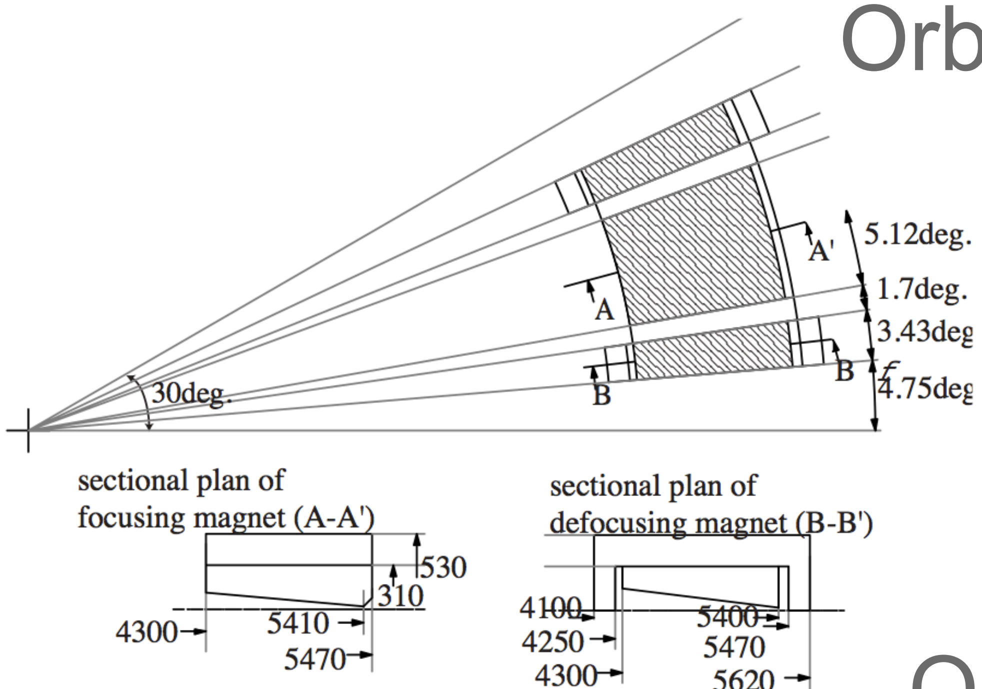
  f_Linac, rf = 425 MHz (Okabe at FFAG10)

  f_rev = 1.6 MHz (f_ADSR, rf = 1.6 MHz)

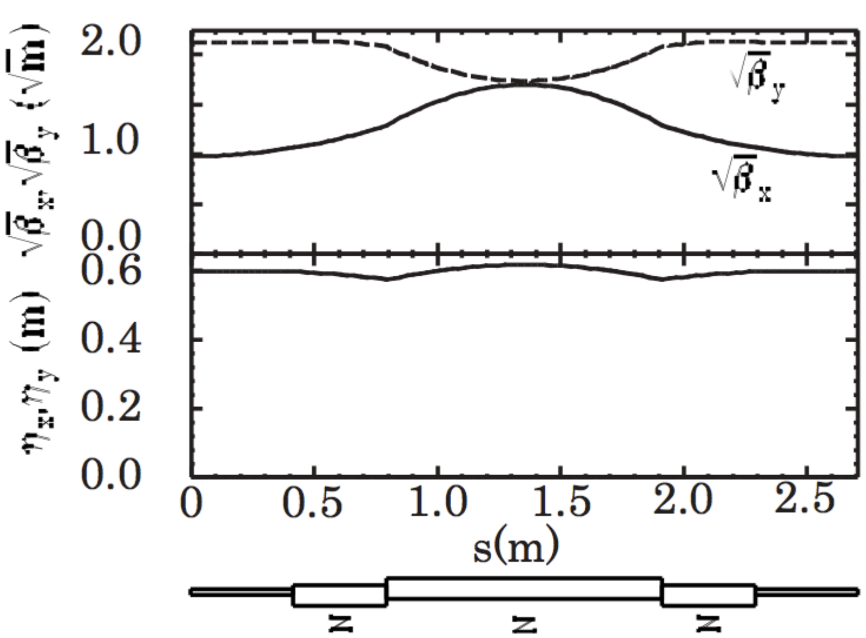
  gives f_Linac, rf/f_rev = 266

  n_debunch = (t_rf/t_rev)/(\eta x dp/p) = 1/(266 x 0.861 x 0.0005) = 9 turns

Modelling with Simpsons-FFAG (1)



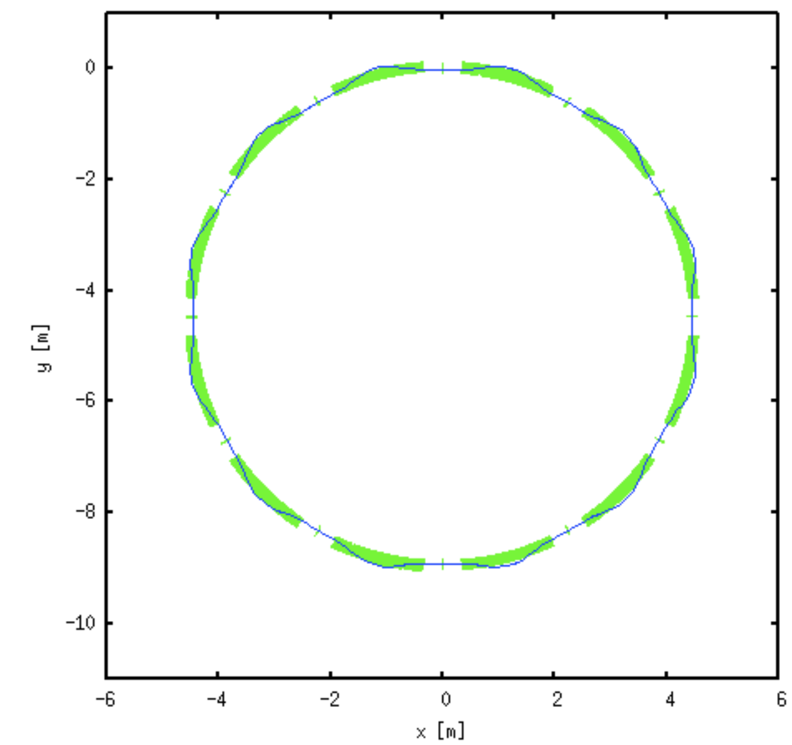
Aiba et al, EPAC2002



Orbit

T_{rev} = 636 ns or

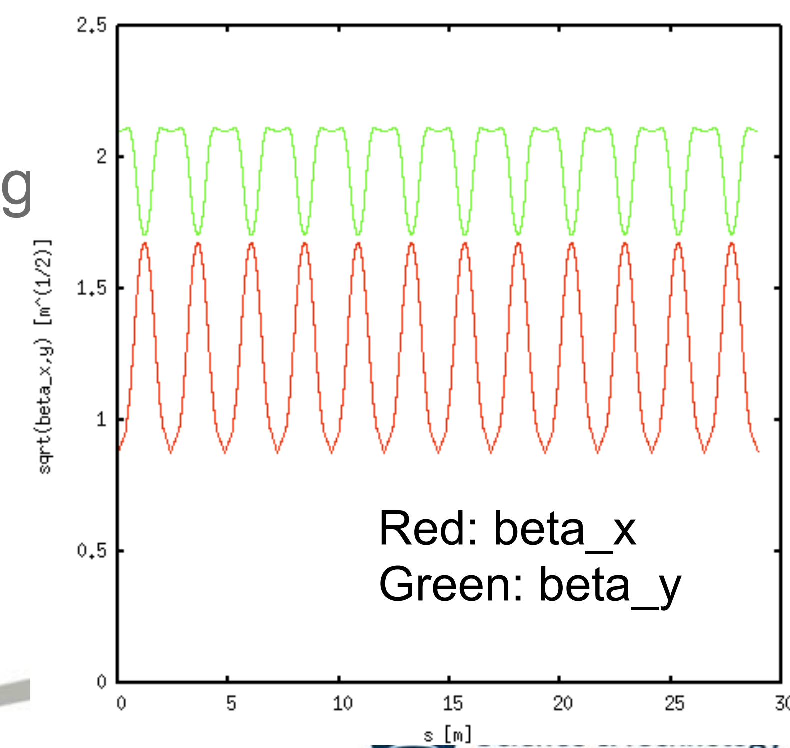
f_{rf} = 1.573 MHz (h = 1)



Optics

(Qx, Qy) = (3.720, 1.189)

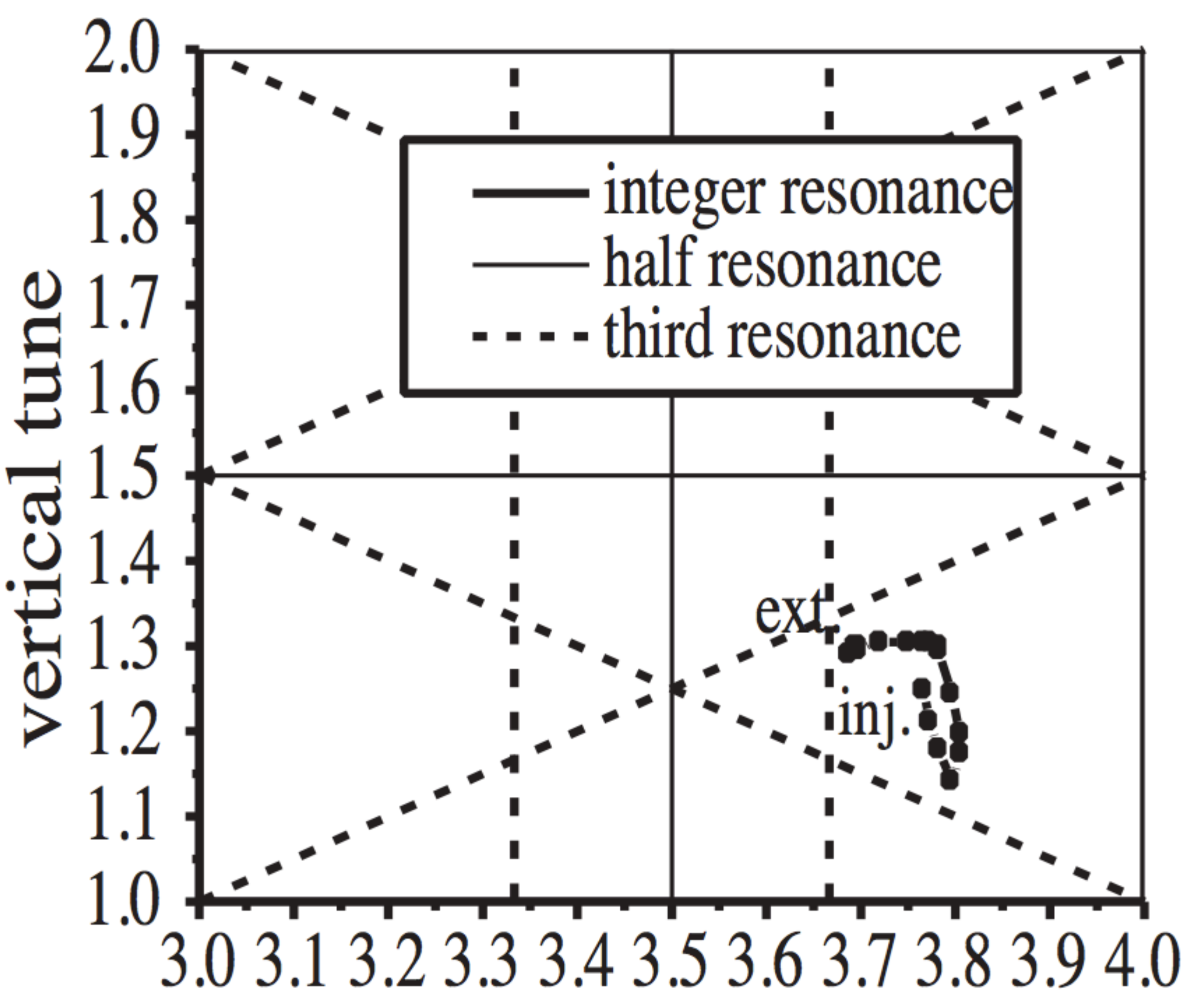
with Enge fringe (g=0.5 deg

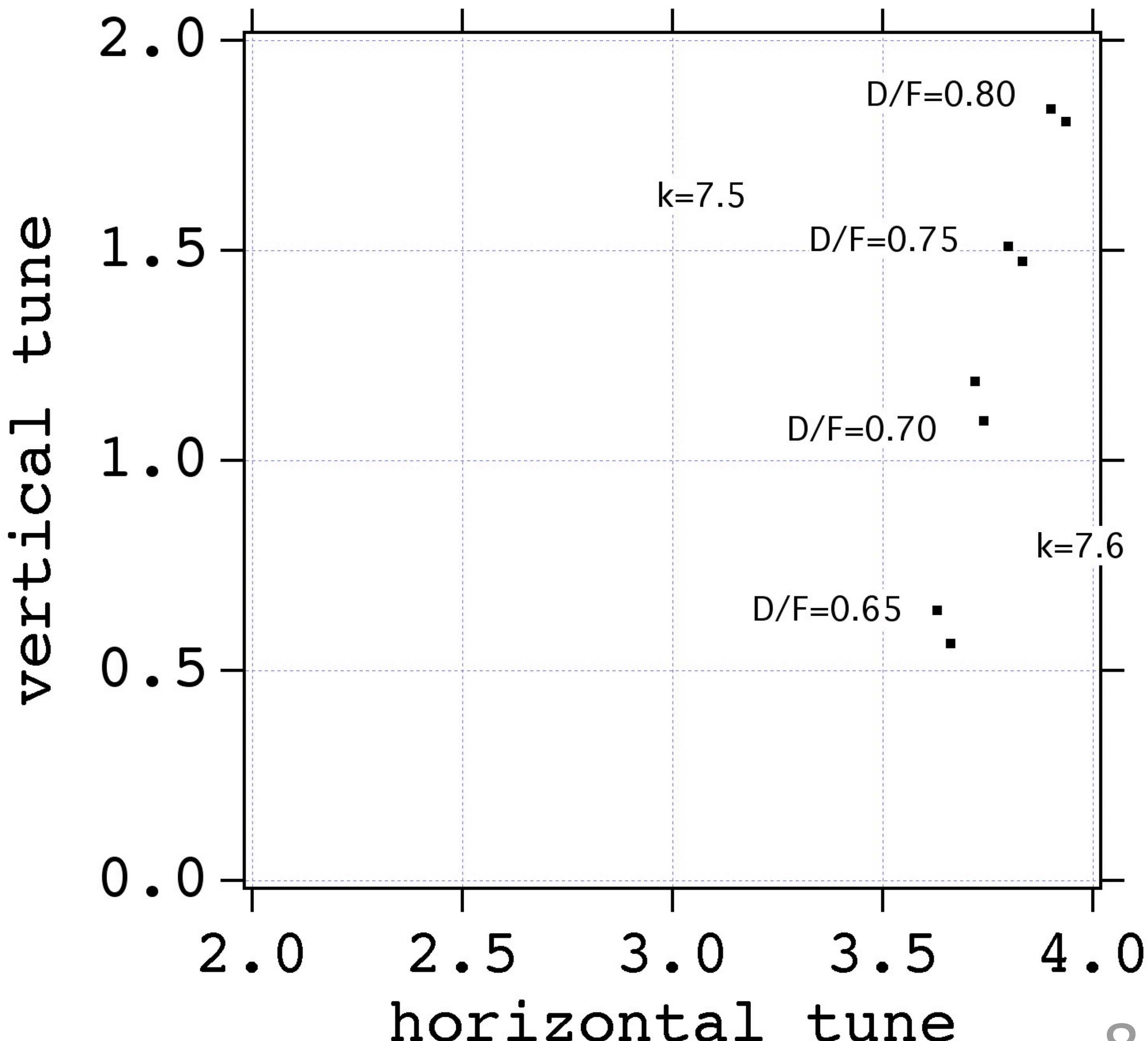


Modelling with Simpsons-FFAG (2)

- Tune vs (index k and D/F ratio) in the modelled lattice.

Aiba et al, EPAC2002

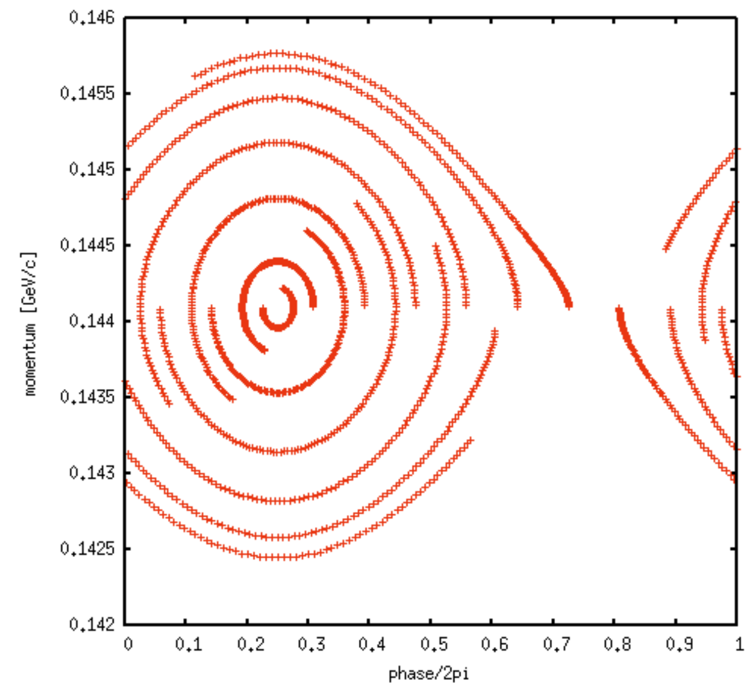




Modelling with Simpsons-FFAG (3)

- First, check dynamics in a stationary bucket.

Bucket height

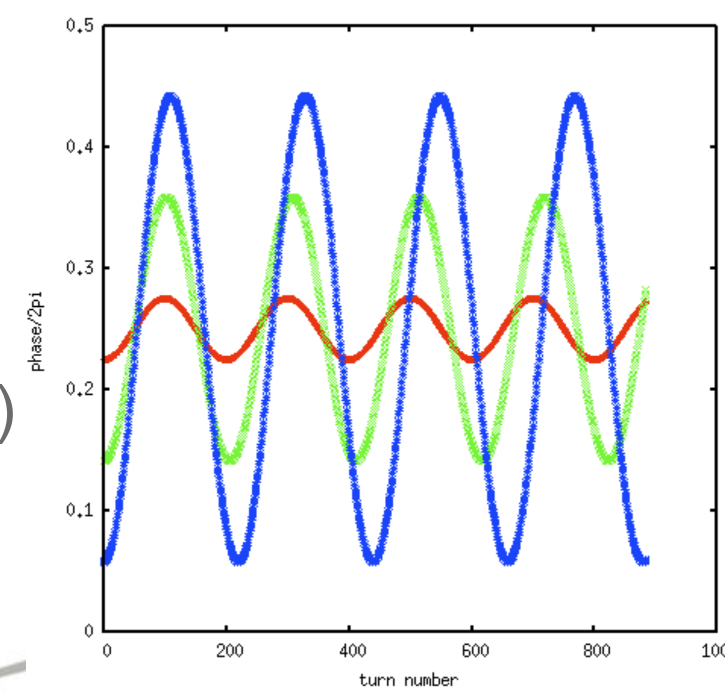


dp/p = (0.1458-0.1441)/0.1441

$= 1.18 \times 10^{-2}$

c.f. B_h (=dp/p) = 1.16×10^{-2}

Synchrotron tune



1 oscs/200 turns = 5×10^{-3}

c.f. Q_s = 5.01×10^{-3} (or 200 turns)

Modelling with Simpsons-FFAG (4)

- rf tables for acceleration
  1. Voltage vs time
  2. Frequency vs time

  (\phi_s is not an input parameter in reality)

r_i, p_i

$$f_{rev}(t) = \frac{\beta(t)c}{2\pi r(t)}$$

$$\frac{dE(t)}{dt} = f_{rev}(t)V(t)\sin(\phi_s(t))$$

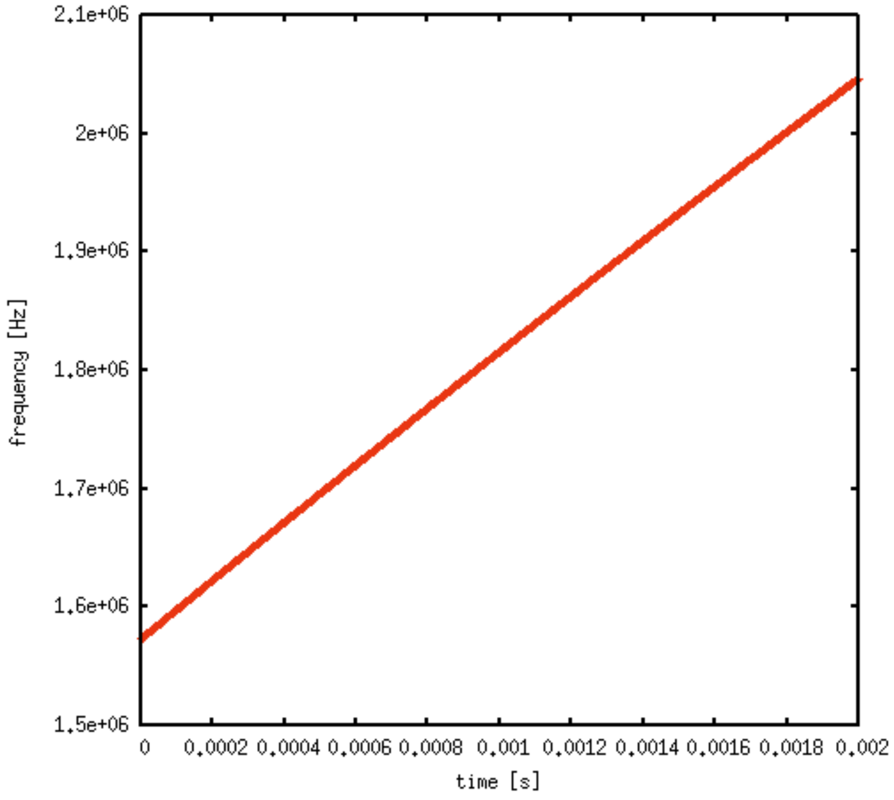
p_i -> p_i+1

$$\frac{r(t)}{r_0} = (\frac{p(t)}{p_0(t)})^{1/(k+1)}$$

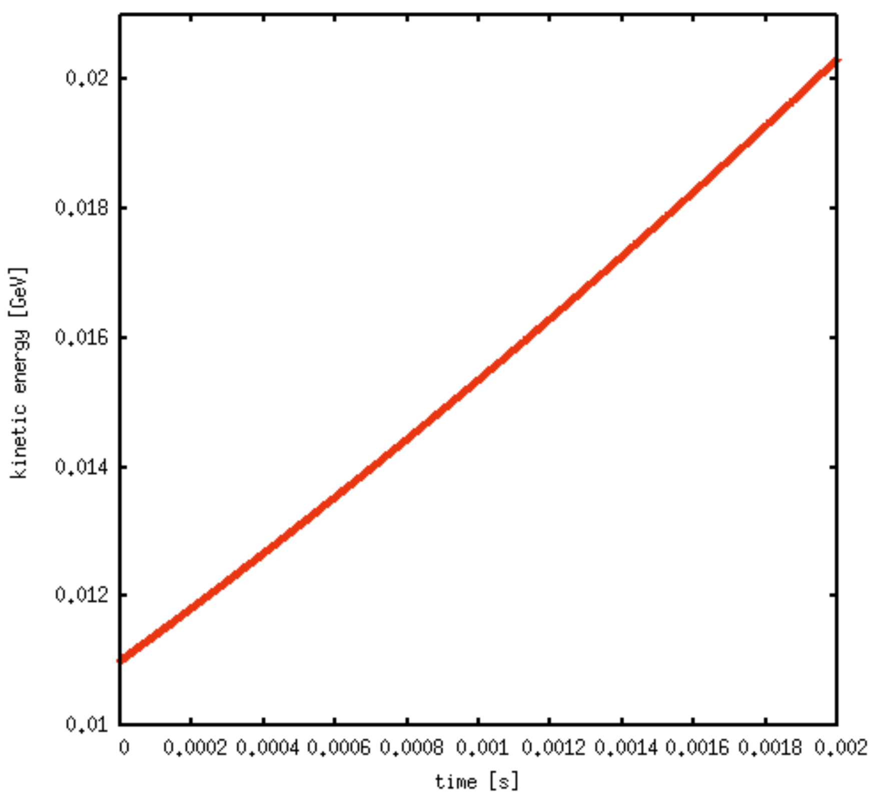
r_0 -> r_i+1

Modelling with Simpsons-FFAG (5)

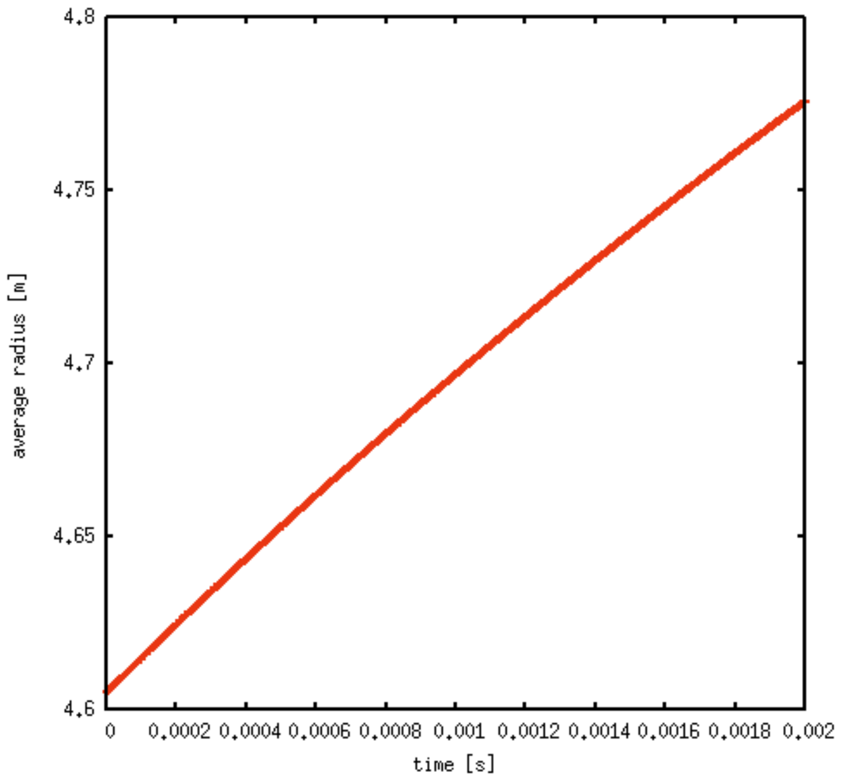
- When V=4 kV and \phi_s=40 deg (both are constant),



frequency



kinetic energy



average radius

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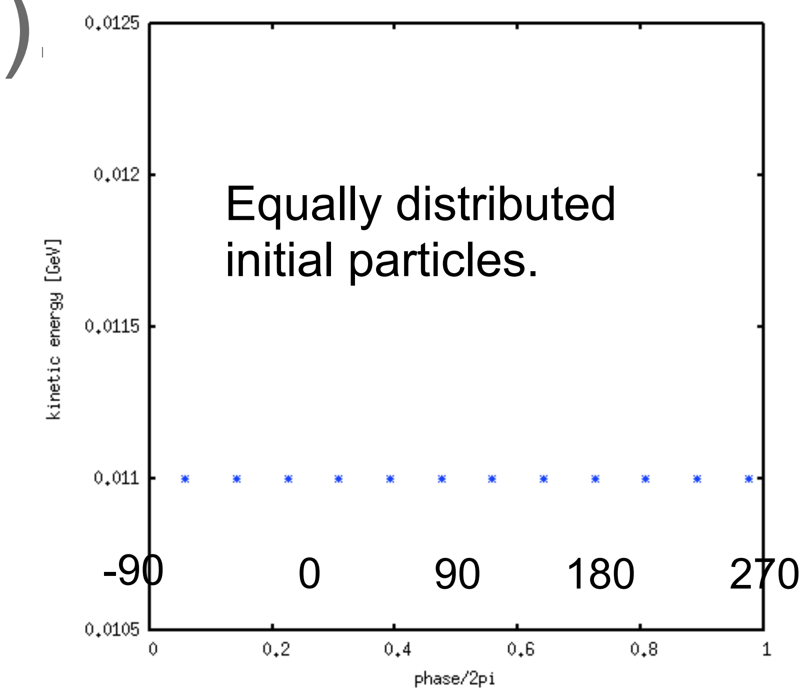
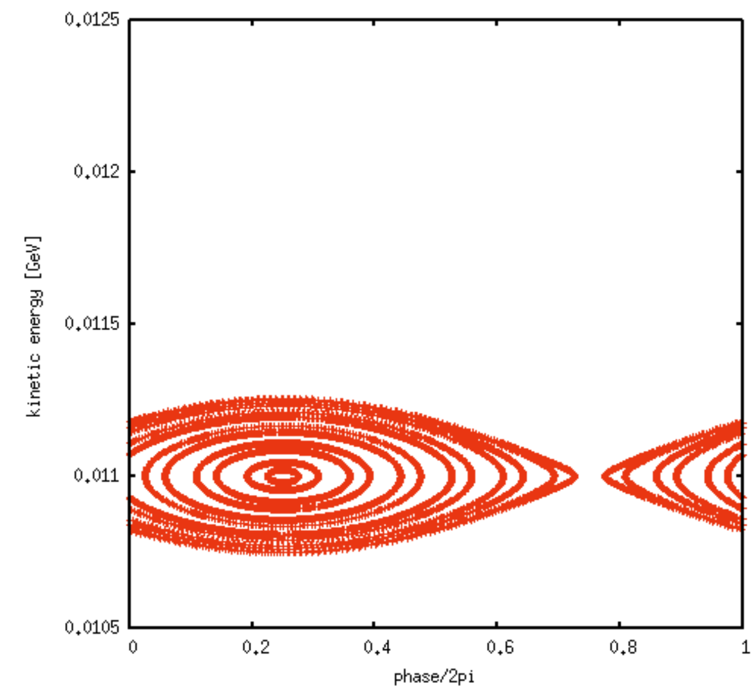
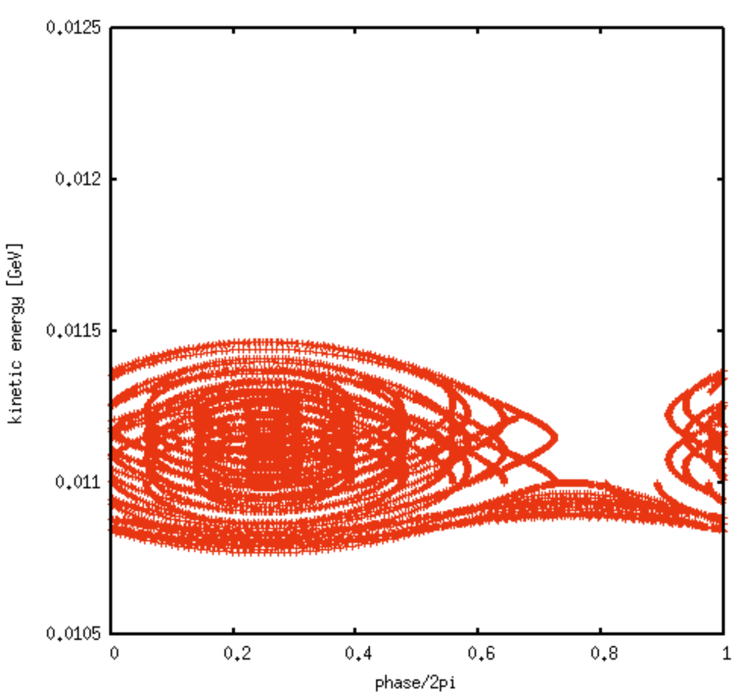
Modelling with Simpsons-FFAG (6)

- Particle tracking in 6D phase space (only show longitudinal)

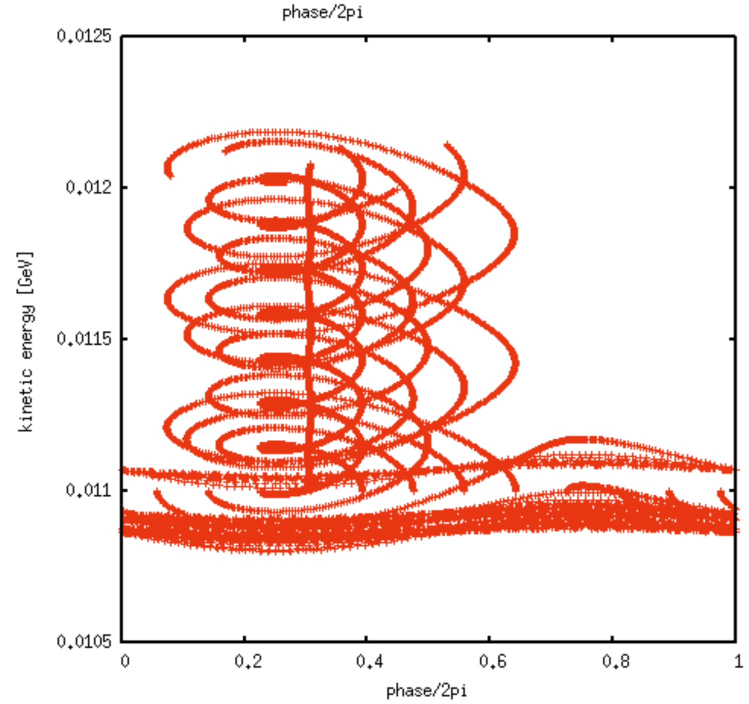




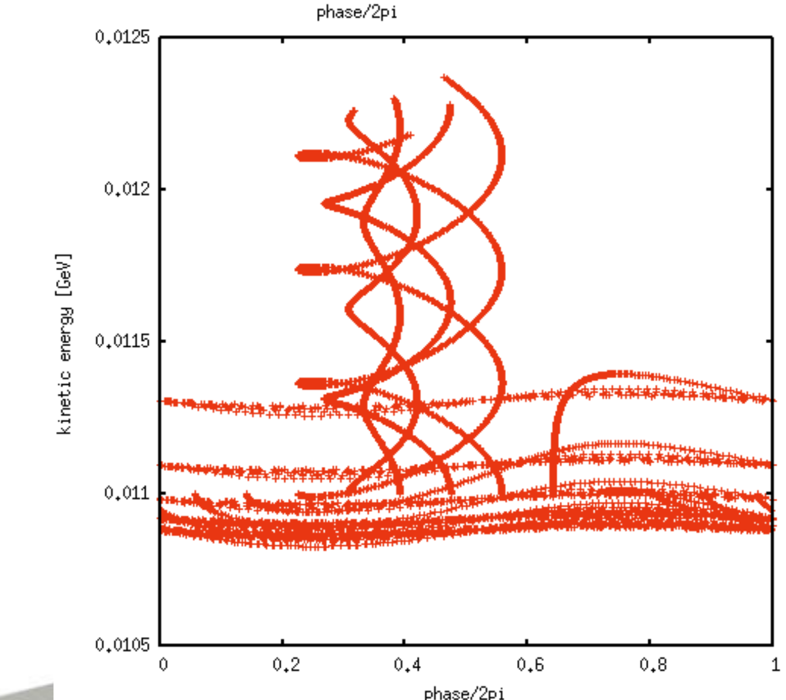
phi_s=0 deg. (x=0.250)



phi_s=5 deg. (x=0.264)



phi_s=20 deg. (x=0.306)

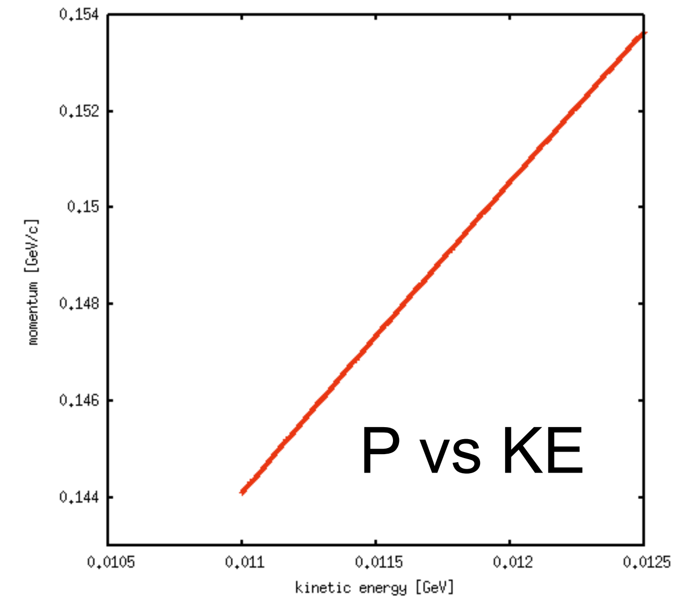


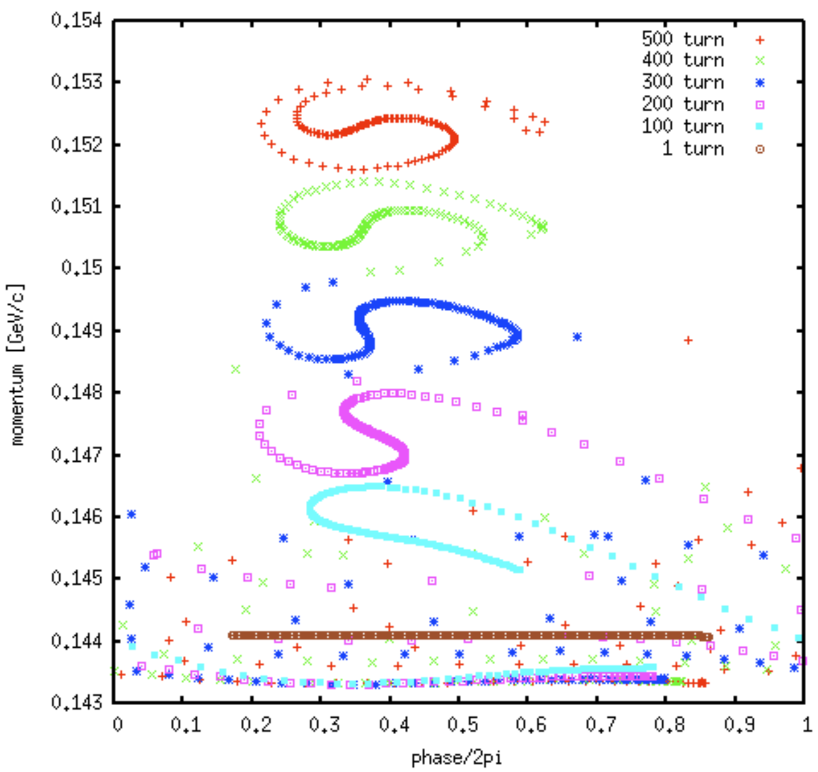
phi_s=40 deg. (x=0.361)

Modelling with Simpsons-FFAG (7)

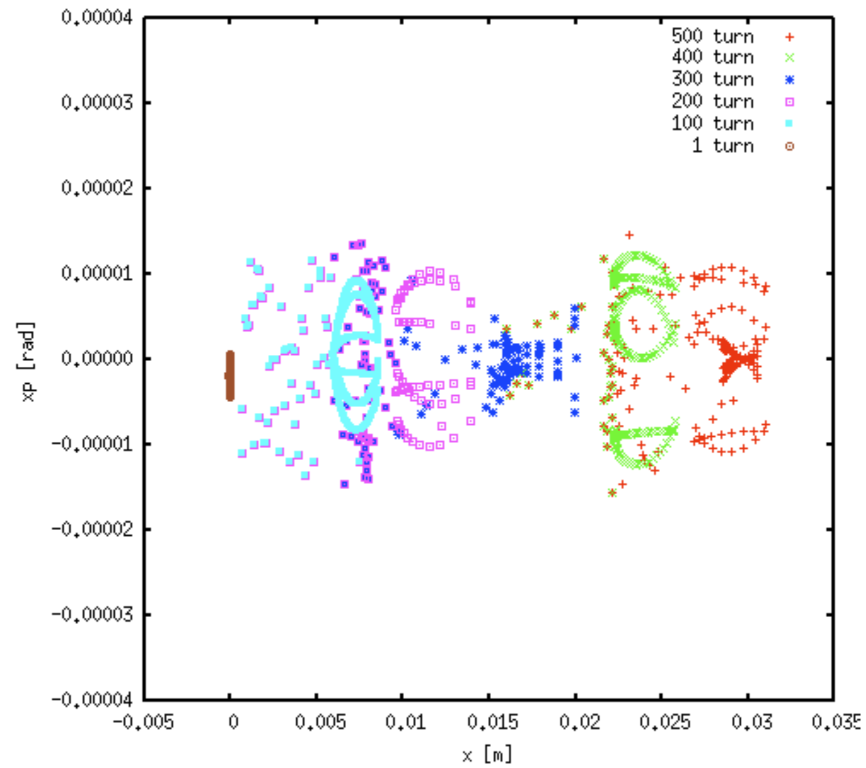
- Multi-particle tracking.

  Voltage=4 kV, phi_s=40 deg.

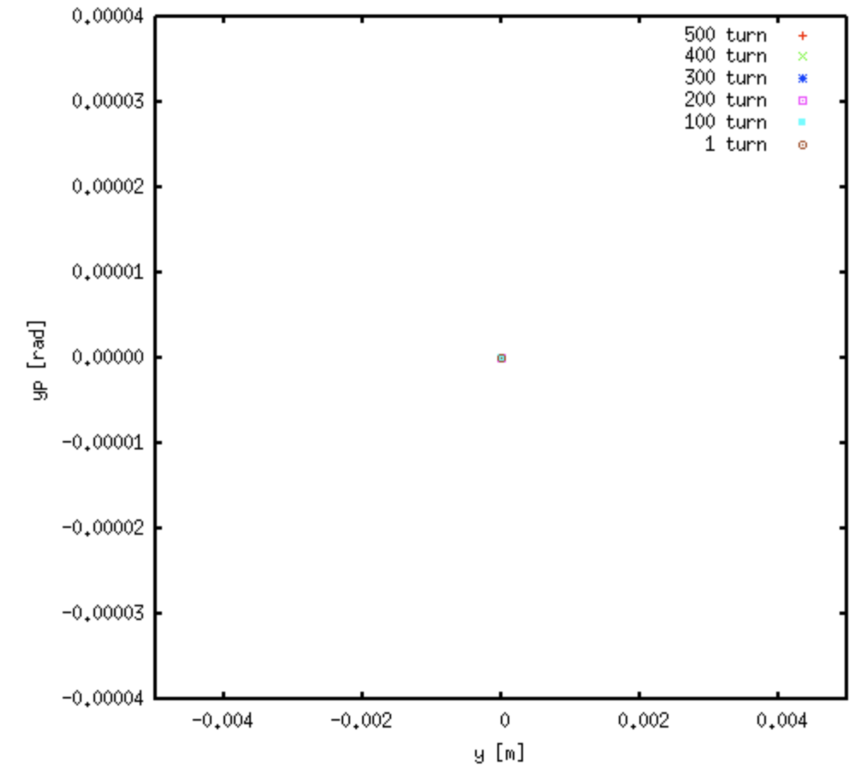




longitudinal



horizontal



vertical

Summary

- Single particle dynamics in 6D phase space with acceleration is modelled and compared with analytic formula.
- Next step is to include space charge and foil scattering.

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