Science & Technology Facilities Council

# FFAG without mid-plane symmetry

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# Introduction (1)

- Brooks (re-) invented a FFAG with vertical orbit excursion.
- Fateev and Yablokov, A ring cyclotron with a vertically increasing magnetic field, *Atomnaya Energiya* **8**, 552 (1960).
  - “VFFAG with reverse bend”

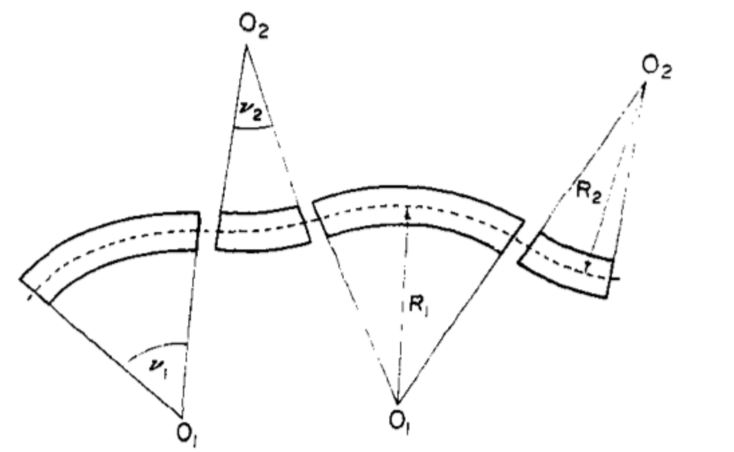



FIG. 1.—Arrangement of sectors and orbit in a ring cyclotron.

- Teichmann, Accelerators w sing field, *Atomnaya Energiya* **12,** 475 (1962).
  - “VFFAG with spiral focusing”

# Introduction (2)

- Teichmann, Accelerators with vertically increasing field, *Atomnaya Energiya* **12,** 475 (1962).
  - Orbit excursion in BOTH horizontal and vertical.

# Accelerators with vertically increasing field

*by J. Teichmann*

- Three design criteria: *complete isochronism*
  - the directing and focusing fields are constant with time. (fixed field)
  - the betatron oscillation frequency is independent of energy. (fixed tune)
  - the rotational frequency of the particles is also independent of energy. (fixed frequency)
- The all cannot be satisfied as long as a beam is confined on the medium plane (maybe true).

# Accelerators with vertically increasing field

*by J. Teichmann*

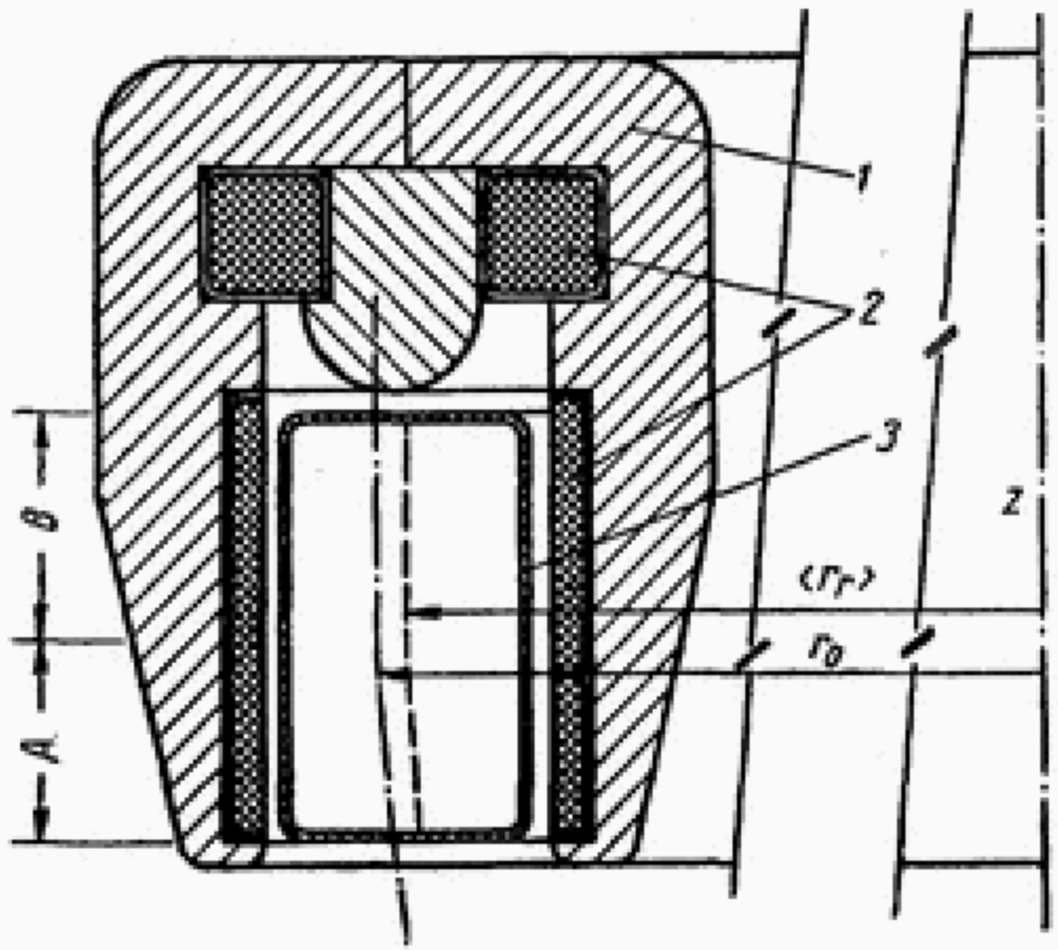



Fig. 1. Schematic section of accelerator with vertically increasing field: 1) ring magnet; 2) excitor windings for directing and focusing fields; 3) vacuum chamber; A) relativistic region; B) ultrarelativistic region.

- ultra-relativistic region
  - beam goes up vertically with constant circulating radius.
- relativistic region
  - small increase of circulating radius as well as vertical shift.

# Advantages

- Strong focusing
- CW operation

- I would like to add, "Turn separation in vertical direction at extraction."
- Then, this may solve the most critical problem of beam loss in a cyclotron.

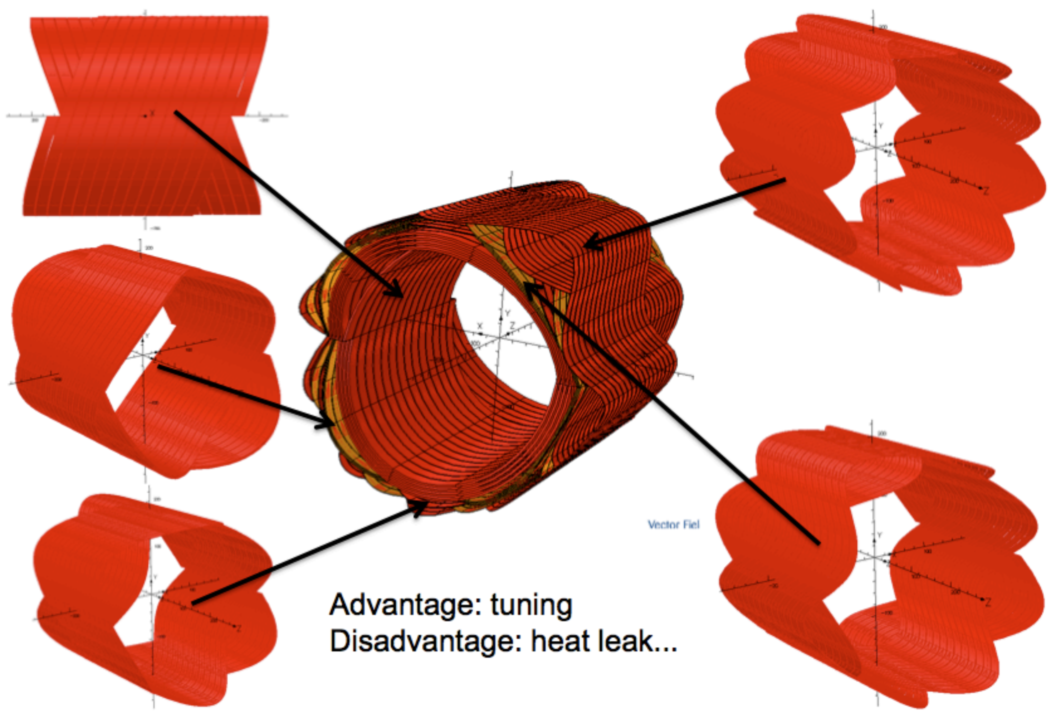
The ultimate cyclotron/FFAG.

# Question is how

- Details are not available, only a sketch.

- Magnetic field in general can be reconstructed by combination of multiple.
  - We did use the same trick for PAMELA.

# PAMELA lattice magnet




H. Witte

- Combination of rectangular multipole magnets.

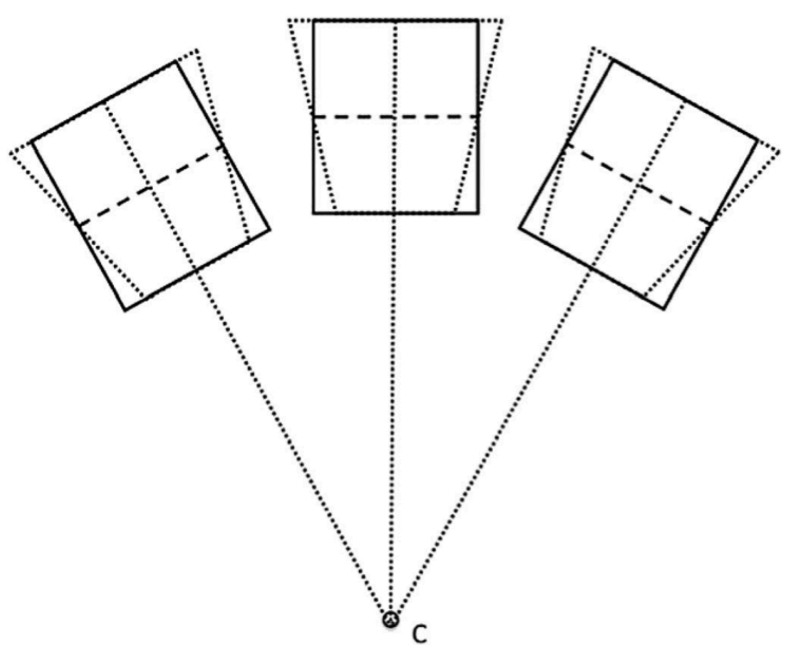



FIG. 5. Converting wedge-shaped magnets (dotted line) to rectangular magnets (solid line). The magnet center is unchanged and the three magnets face the machine center.

$$B_z = B_{z0}\left(\frac{r_0 + r}{r0}\right)^k = B_{z0}\left(1 + \sum_{n=1} \frac{1}{n!} \frac{k(k-1)\cdots(k-n+1)}{{r_0}^n} r^n\right). \quad (3)$$

- Relative strength can be a parameter.

Sheehy et al, PRST-AB 13 040101 (2010)

# Orbit in vertical direction

- In a synchrotron lattice, main source of the vertical dispersion is a skew field where horizontal dispersion is finite.

$$\frac{d^2 z}{ds^2} + K_z(s) z = \frac{1}{B\rho}\frac{\partial B_x}{\partial x} D_x \delta$$

- Vertical dispersion function is

$$D_z(s) = Q_z^2 \sqrt{\beta_z(s)} \sum_{k=-\infty}^{\infty} \frac{f_k \exp(ik\phi(s))}{Q_z^2 - k^2}$$

$$f_k = \frac{1}{2\pi B\rho}\int_0^{2\pi} \sqrt{\beta_z} D_x \frac{\partial B_x}{\partial x} \exp(-ik\phi(s)) ds$$

# Knobs we can use

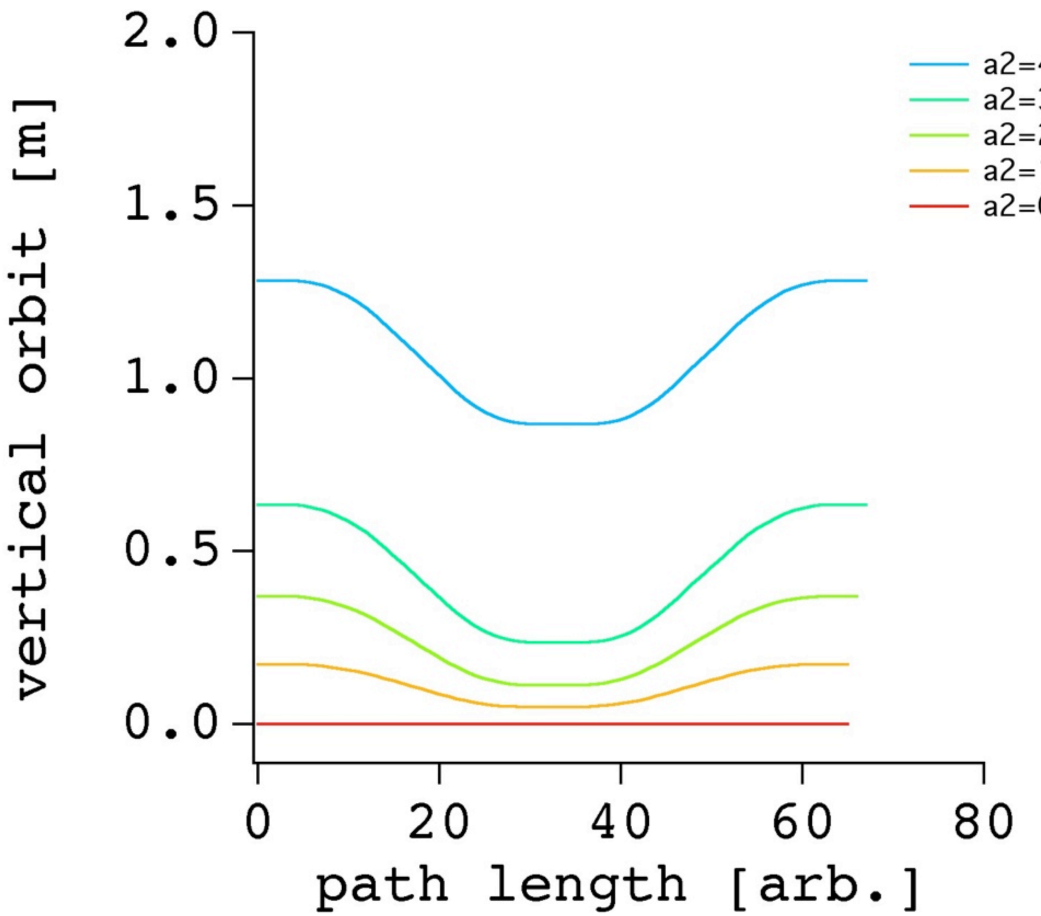
- One another knob we did not use at the time of PAMELA study is the *rotation* of the multipoles.

# Test lattice

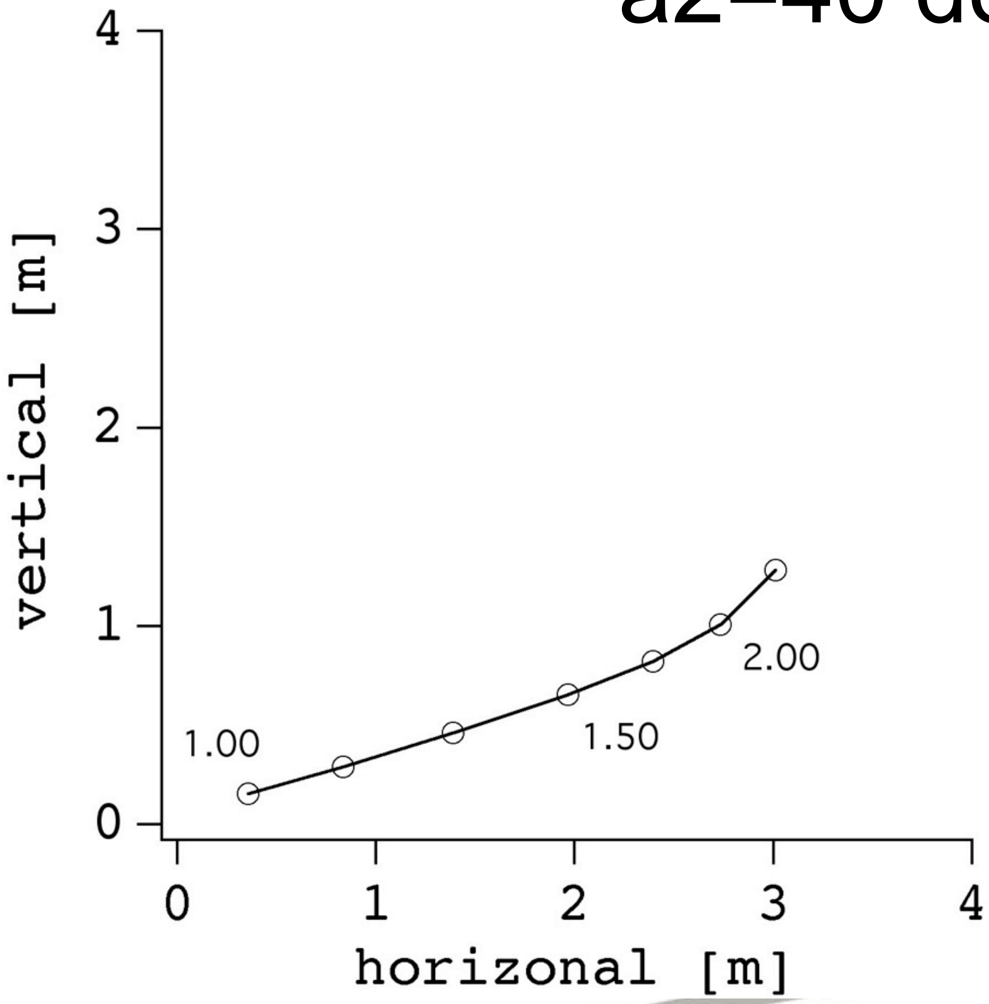
- Kinetic energy range: 500 MeV to 1.5 GeV (as an after-burner of a cyclotron).
- 24 FODO cell: large enough so that wedge shaped magnet can be replaced by rectangular shape magnet.
- Start from ordinary scaling FFAG design and introduce approximates as we did for PAMELA.

# Orbit excursion in both horizontal and vertical (1)

- Introduction of sextupole rotation (skew sextupole)
  - As rotated, vertical orbit excursion becomes larger.




a2=40 deg.



# Orbit excursion in both horizontal and vertical (2)

- First, start with normal multipoles which satisfied the scaling law.
- Optimisation of the rotation (skew) of multipoles (quad, sext, oct, deca, etc) is underway.

# Quick test using ADSR-FFAG

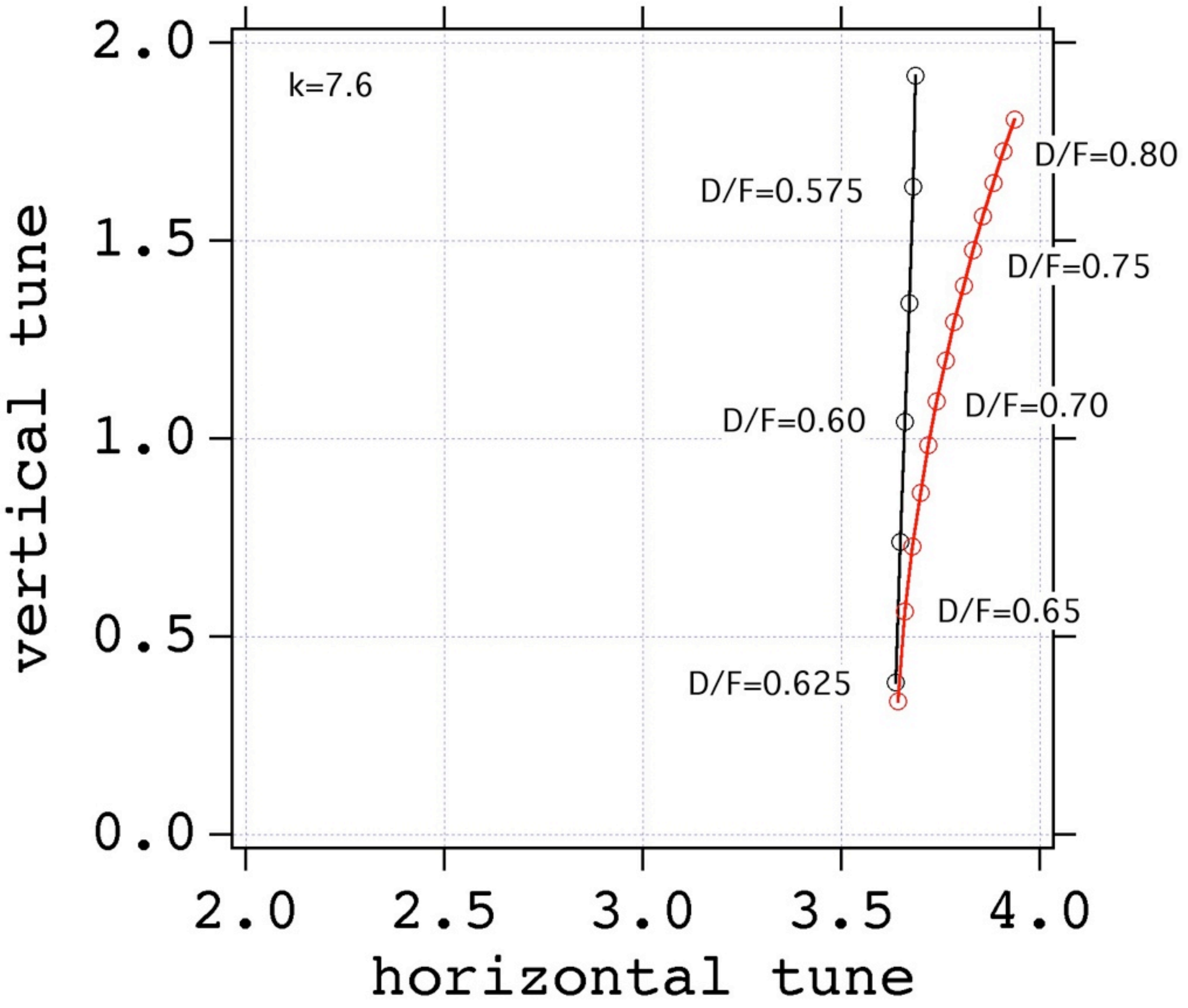
- A beam going vertically off-centred through sextupole magnets feels skew quadrupole field.
- What happens if a beam is injected at off mid-plane?

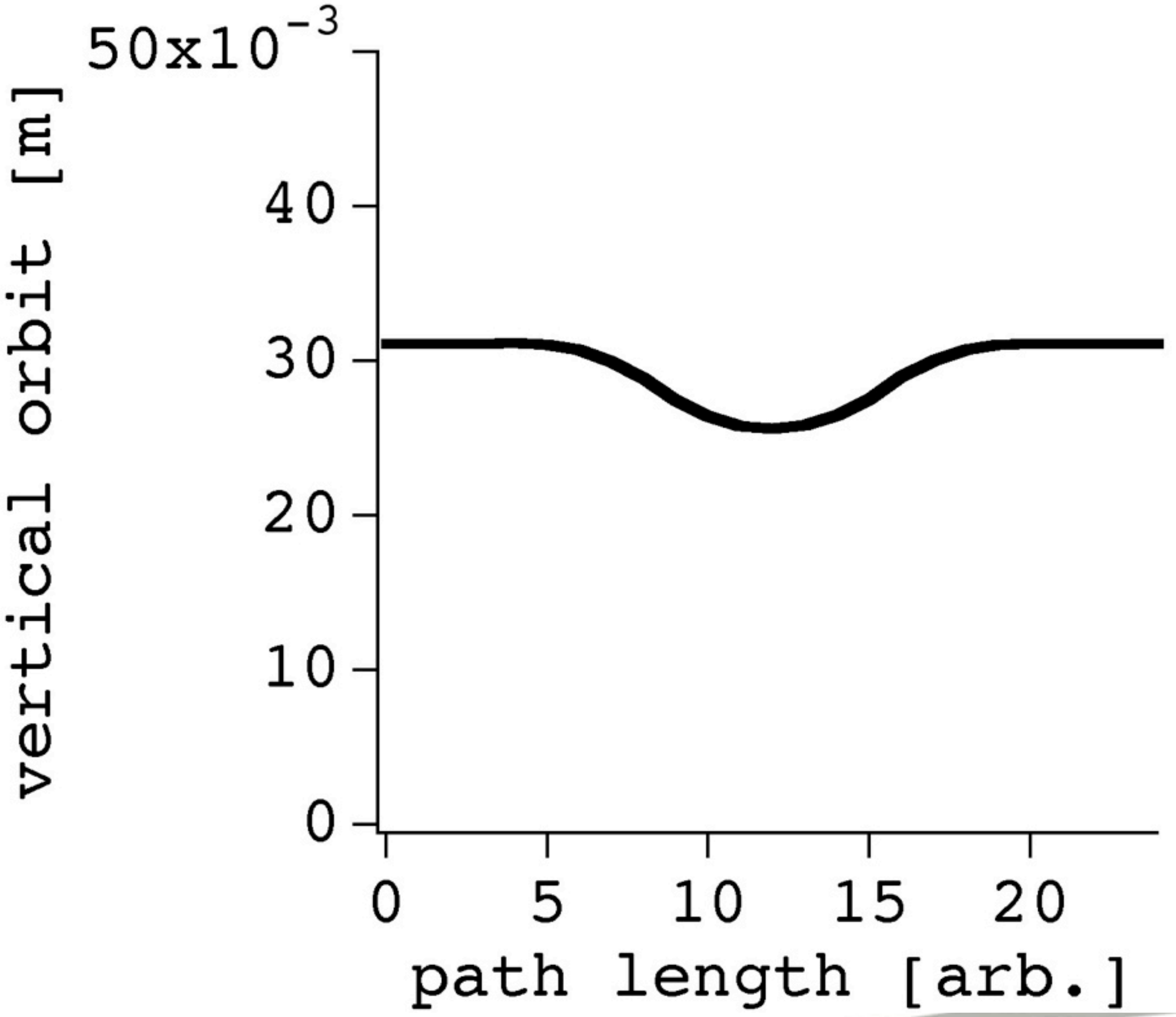
# Off mid-plane orbit

- An orbit going through off mid-plane exists (at least in simulation) with a certain F/D ratio.
- It is interesting (or maybe obvious) that one is unstable when both on and off mid-plane orbits exist.

# Stability

- Two region of F/D ratio.
- Displacement is independent of momentum.



# Summary

- FFAG with both horizontal and vertical orbit excursion can satisfy “complete isochronism” conditions (to be confirmed).
    - fixed field, fixed tune and fixed frequency.
- Field expansion with normal and skew multipoles is introduce to realise this FFAG.