



# Advanced Scaling FFAG egg-shape design study for PRISM

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# Outline

- ① “Egg-shape” design parameters
- ② Linear fringe field fall-offs
- ③ Edge functions field fall-offs
- ④ Conclusion

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# Egg-shape design

Small Bending cell FDF triplet

k-value

3.82

total bending angle

39.15 deg.

Average radius

5 m

Phase advances:

Horizontal  $\mu_x$

90 deg.

Vertical  $\mu_z$

60 deg.

Dispersion

1 m

Large Bending cell FDF triplet

k-value

28.9503

total bending angle

11.7 deg.

Average radius

30 m

Phase advances:

Horizontal  $\mu_x$

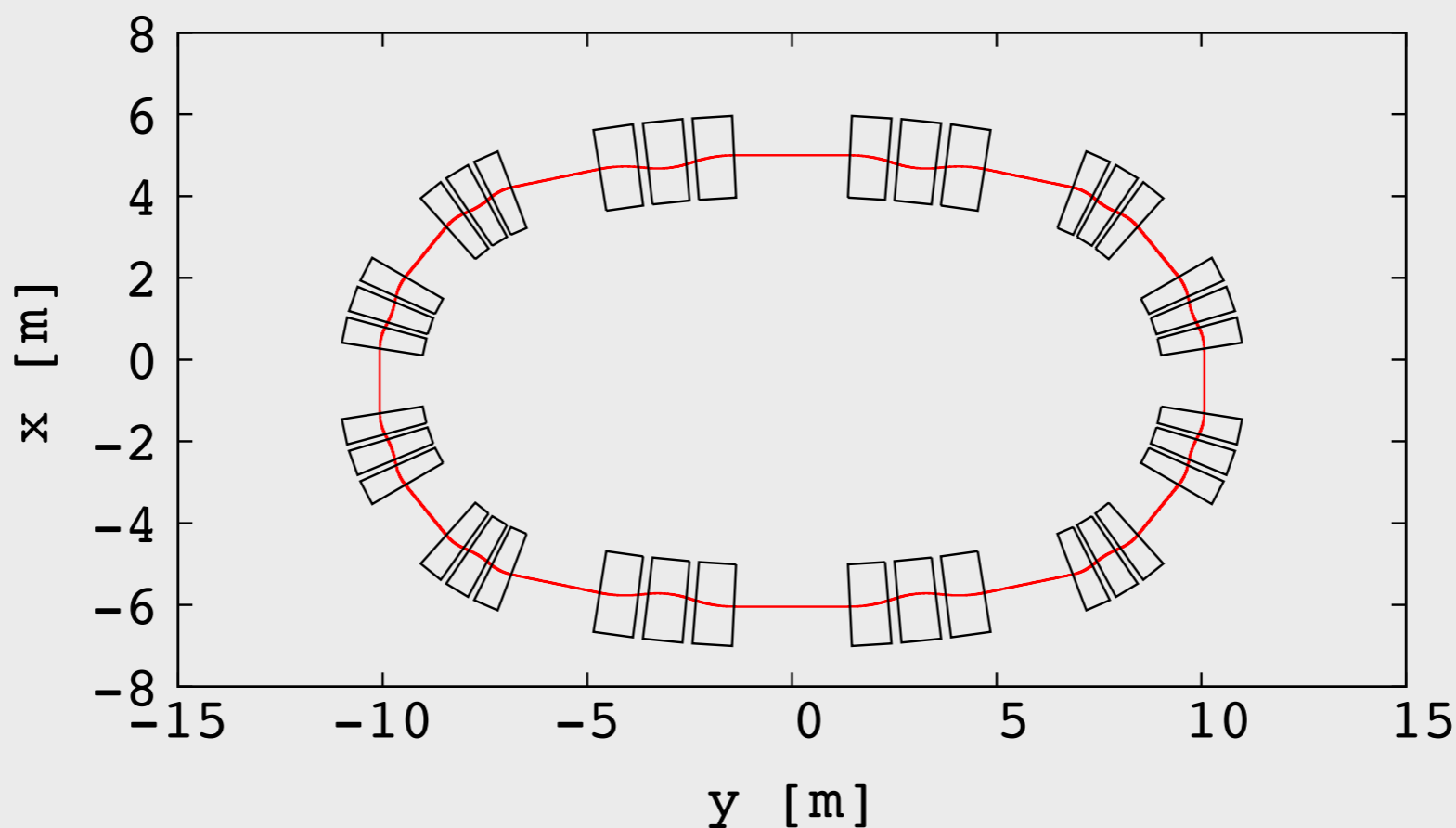
75 deg.

Vertical  $\mu_z$

81 deg.

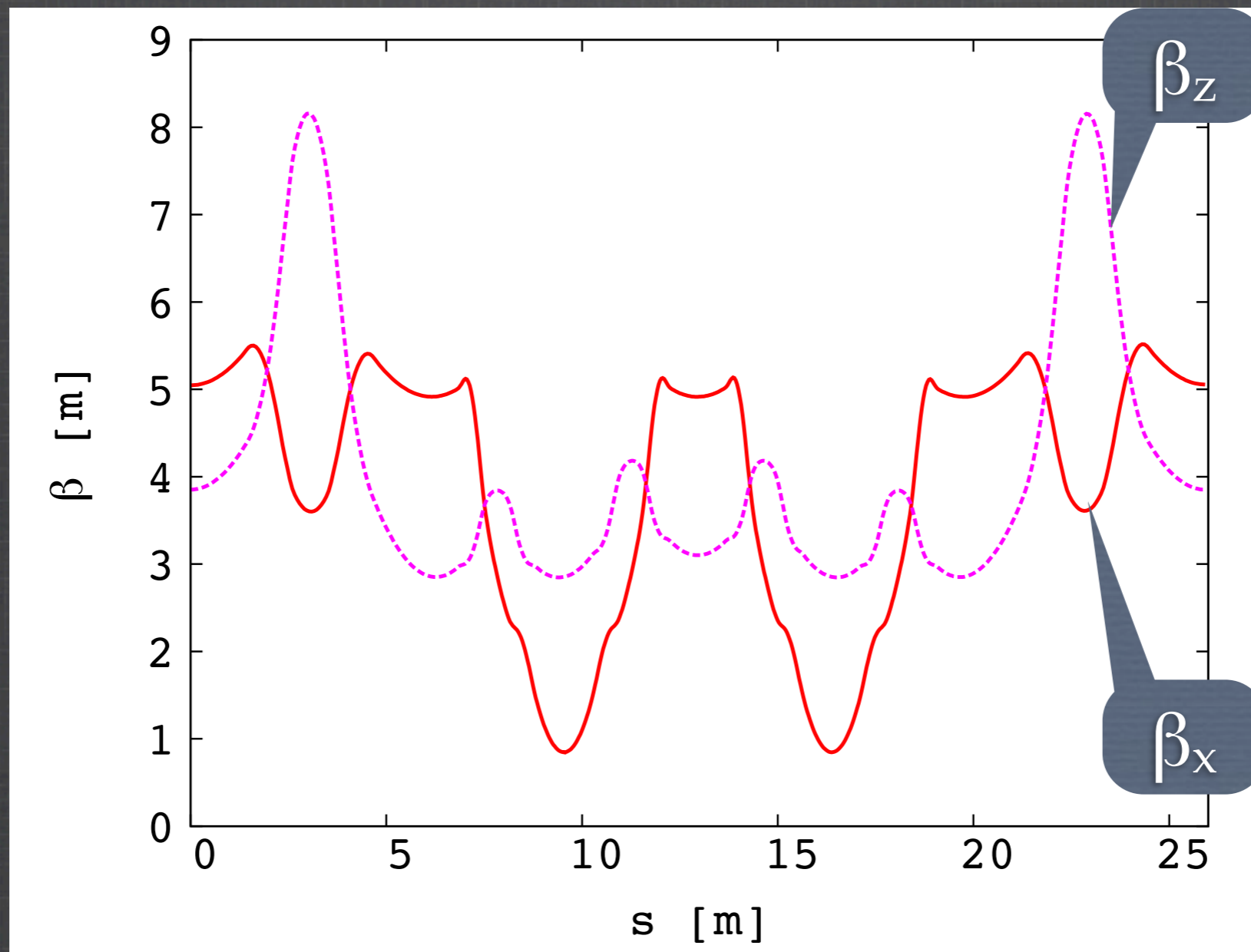
Dispersion

1 m

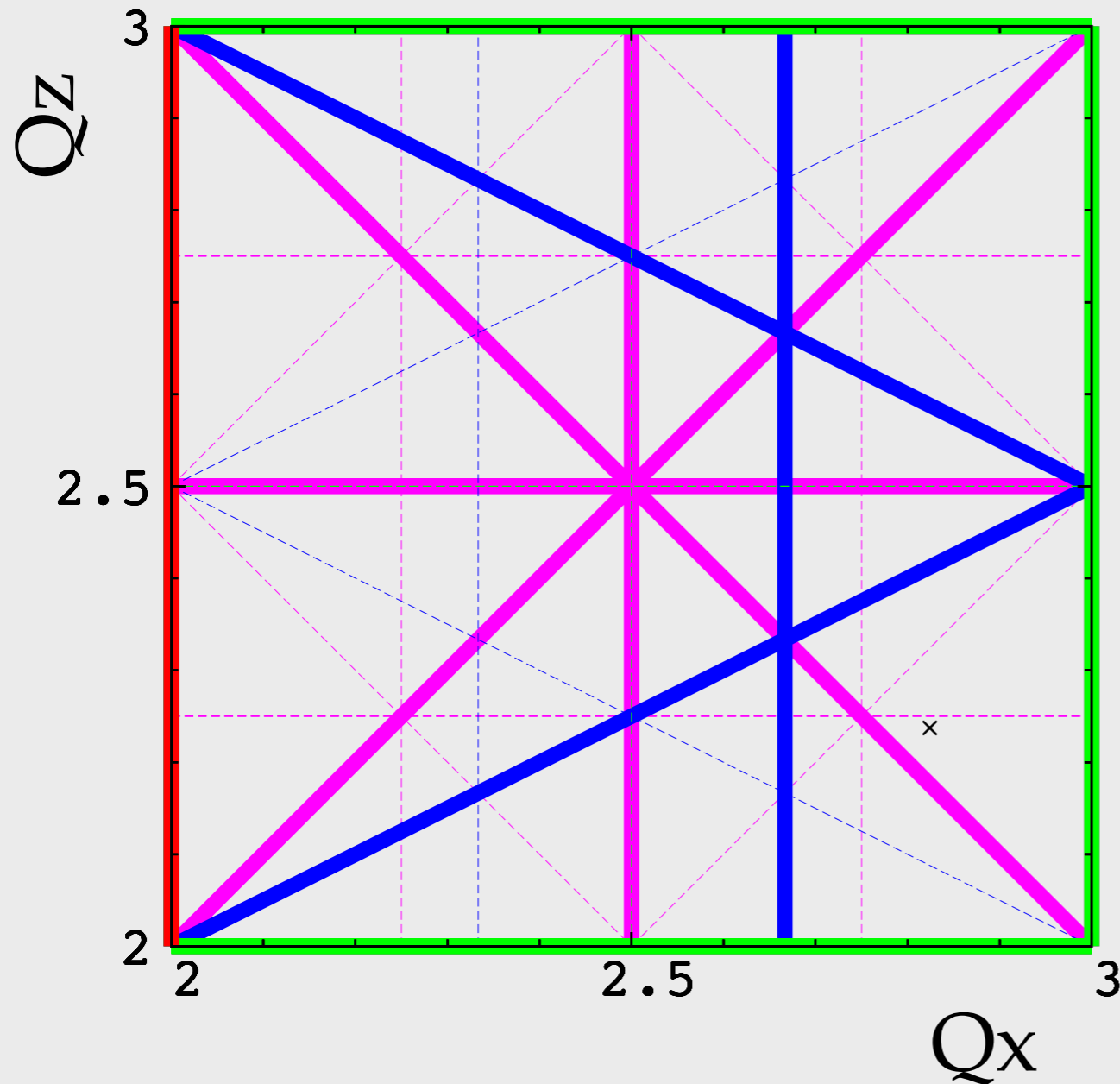


# Betafunctions

Horizontal (plain red) and vertical (dotted purple) betafunctions of half the ring in egg-shape PRISM



# Tune point



Working point of the ring in the tune diagram. Integer (red), quadrupole (green), sextupole (blue) and octopole (purple) normal resonances are plotted.

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# Linear fringe field fall-offs

- To simulate fringe fields, linear fall-offs are used first.

$$B_z(r, \theta) = B_0 \mathcal{F}(\theta) \left( \frac{r}{r_0} \right)^k$$

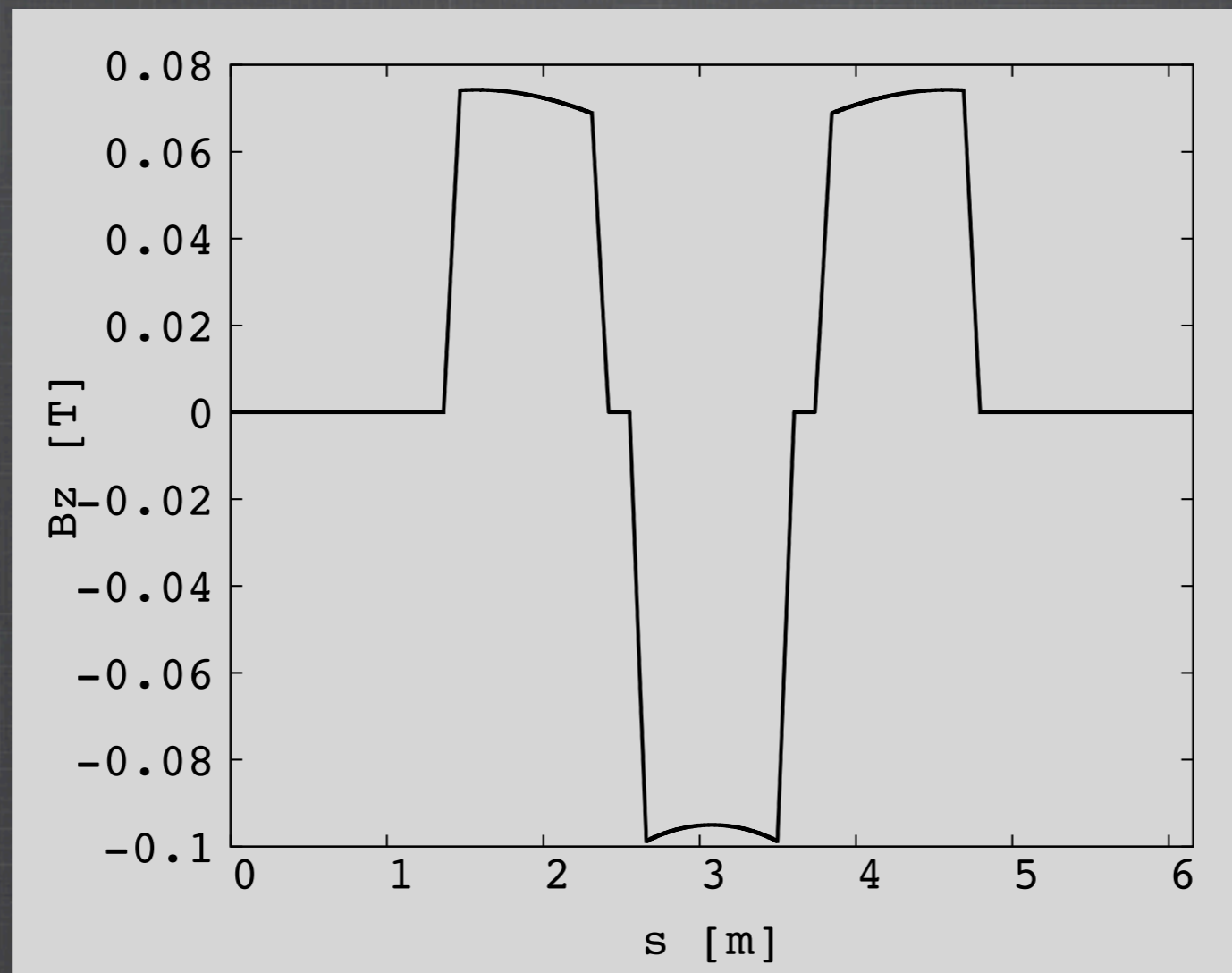
$$\left\{ \begin{array}{l} \mathcal{F} = \frac{\theta - \Theta_{EFB_{en}} + \Theta_{FE_{en}}}{2\Theta_{FE_{en}}}, \text{ for } \Theta_{EFB_{en}} - \Theta_{FE_{en}} \leq \theta < \Theta_{EFB_{en}} + \Theta_{FE_{en}} \\ \mathcal{F} = 1, \text{ for } \Theta_{EFB_{en}} + \Theta_{FE_{en}} \leq \theta < \Theta_{EFB_{ex}} - \Theta_{FE_{ex}} \\ \mathcal{F} = 1 - \frac{\theta - \Theta_{EFB_{ex}} + \Theta_{FE_{ex}}}{2\Theta_{FE_{ex}}}, \text{ for } \Theta_{EFB_{en}} + \Theta_{FE_{en}} \leq \theta < \Theta_{EFB_{ex}} - \Theta_{FE_{ex}} \\ \mathcal{F} = 0, \text{ for } \theta < \Theta_{EFB_{en}} - \Theta_{FE_{en}} \text{ or } \theta \geq \Theta_{EFB_{ex}} - \Theta_{FE_{ex}}, \end{array} \right.$$

- 1<sup>st</sup> order interpolation off the mid-plane.



# Linear fringe field fall-offs

- To simulate fringe fields, linear fall-offs are used first.

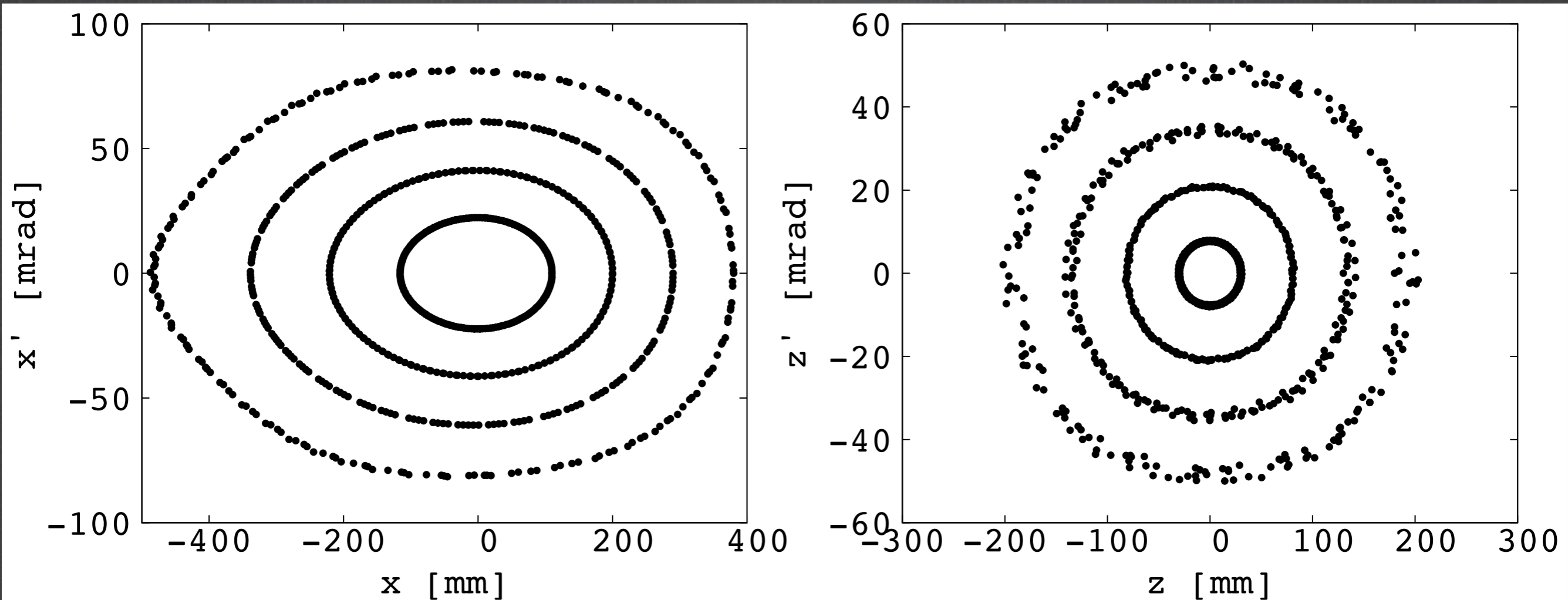


- 1<sup>st</sup> order interpolation off the mid-plane.

# Linear case: Dynamic aperture study

$\approx 40\,000 \pi \text{ mm.mrad}$

$\approx 8\,000 \pi \text{ mm.mrad}$



Horizontal (left) and vertical (right) maximum stable amplitude over 100 turns. Far collimators identify lost particles.

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# Enge fringe field fall-offs

- To check validity of dynamic aperture, enge fall-offs are then used.

$$B_z(r, \theta) = B_0 \mathcal{F}(\theta) \left( \frac{r}{r_0} \right)^k$$

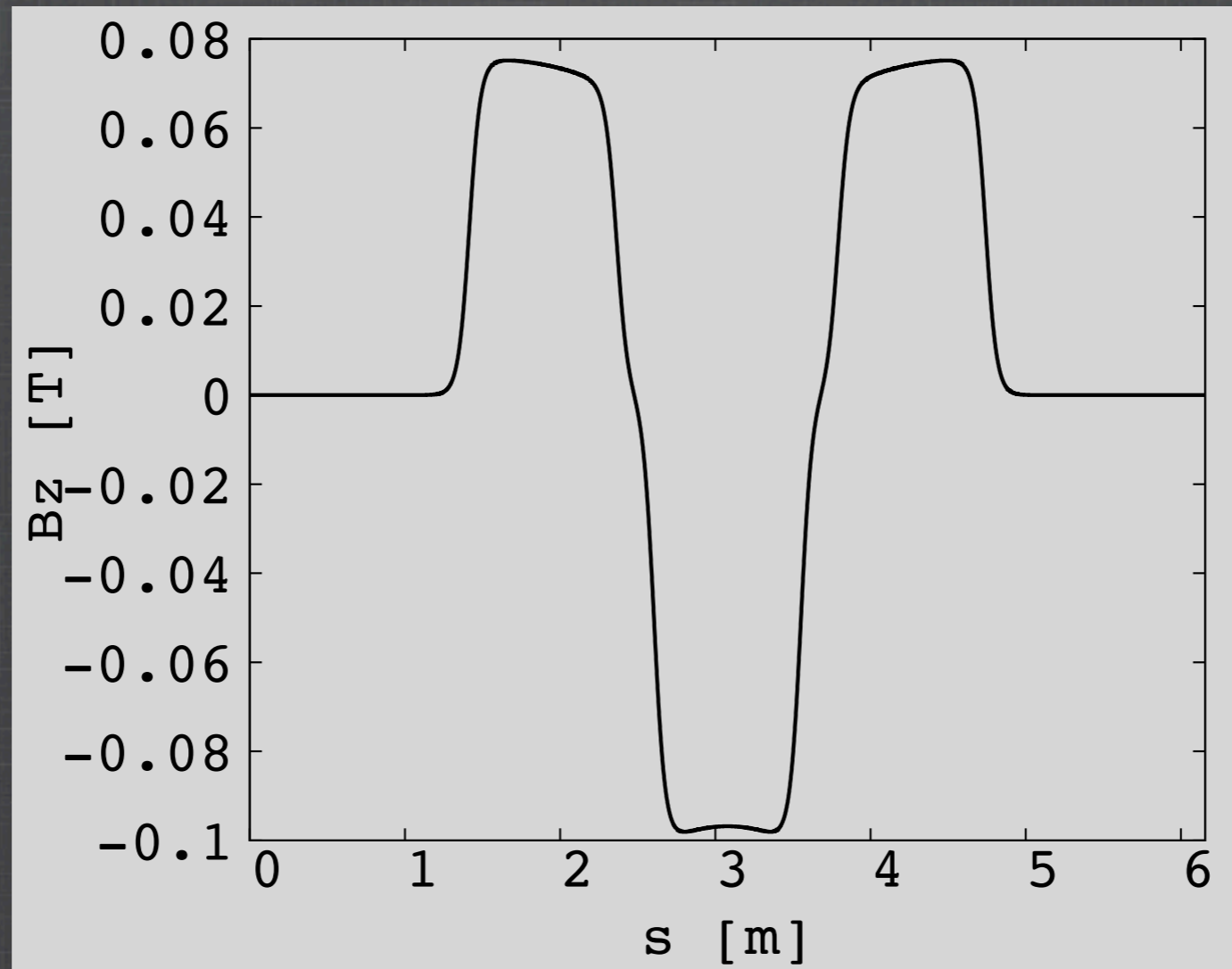
$$\mathcal{F}(\theta) = \mathcal{F}_{en}(\theta) \times \mathcal{F}_{ex}(\theta)$$

$$\text{with } \begin{cases} \mathcal{F}_{en}(\theta) = \frac{1}{1 + e^{C_1(\Theta_{EFB_{en}} - \theta)}}, \\ \mathcal{F}_{ex}(\theta) = \frac{1}{1 + e^{C_1(\theta - \Theta_{EFB_{ex}})}} \end{cases}$$

- 4<sup>th</sup> order interpolation off the mid-plane.

# Enge fringe field fall-offs

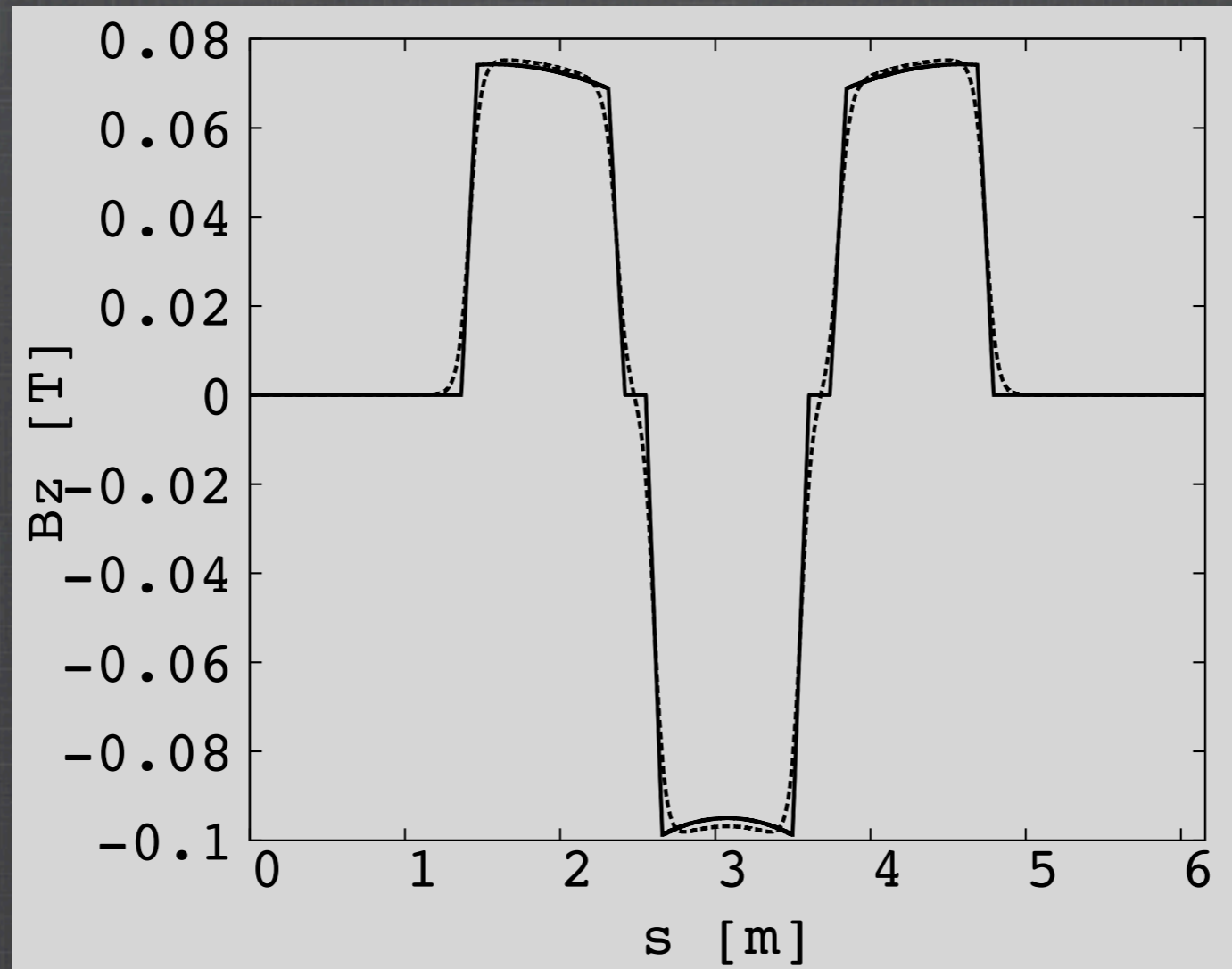
- To check validity of dynamic aperture, enge fall-offs are then used.



- 4<sup>th</sup> order interpolation off the mid-plane.

# Enge fringe field fall-offs

- To check validity of dynamic aperture, enge fall-offs are then used.

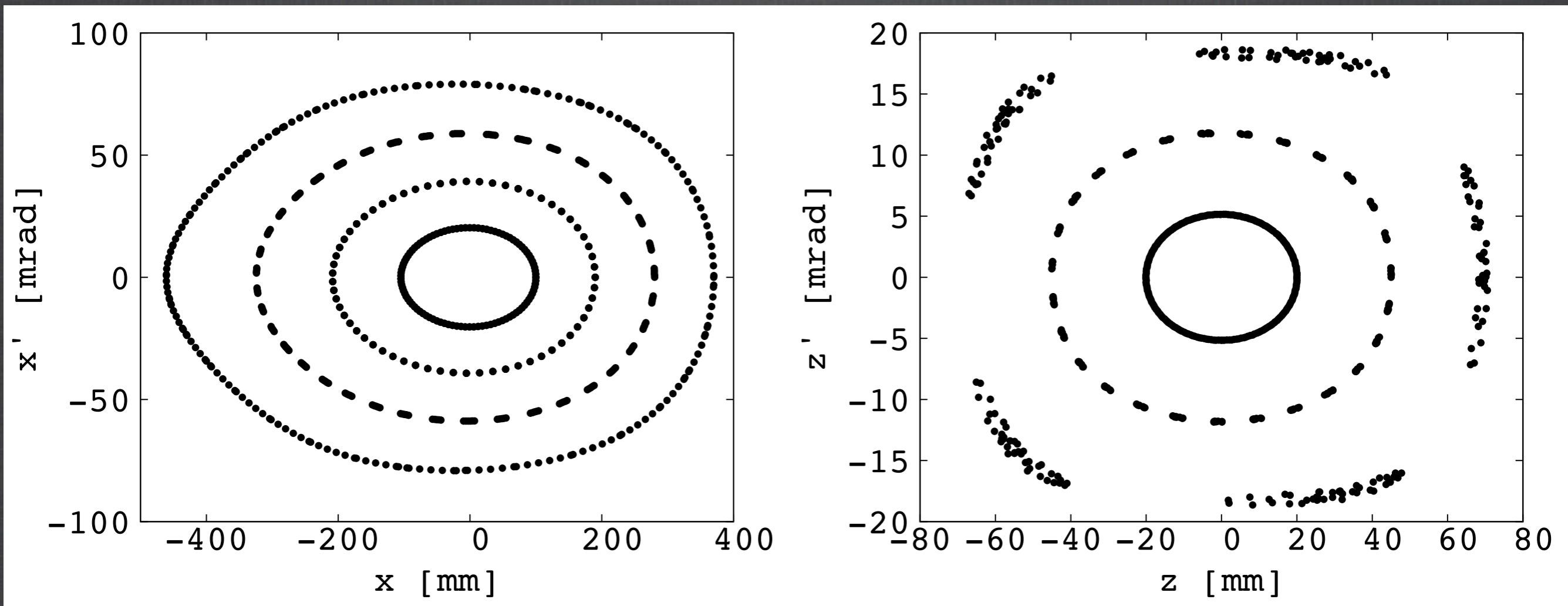


- 4<sup>th</sup> order interpolation off the mid-plane.

# Enge case: Dynamic aperture study

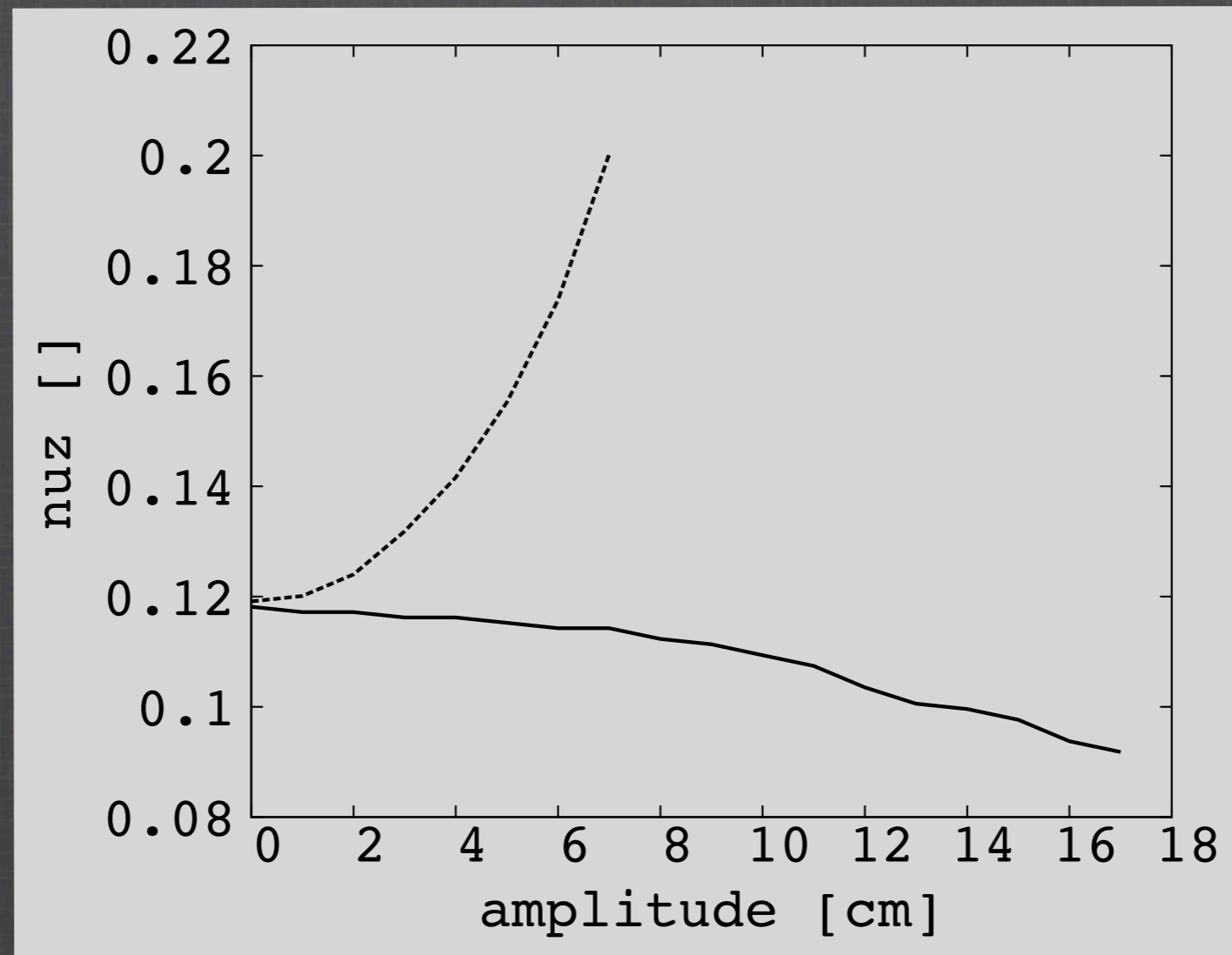
$\approx 40\,000 \pi \text{ mm.mrad}$

$\approx 1\,500 \pi \text{ mm.mrad!!!}$



Horizontal (left) and vertical (right) maximum stable amplitude over 100 turns. Far collimators identify lost particles.

# Amplitude dependance of the tune



Fractional part of the vertical tune for half of the ring in the linear case (plain) and in the Enge case (dotted).



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# Summary

- Linear and Enge fringe field fall offs have a good agreement for the horizontal dynamic aperture.
- Very different for the vertical dynamic aperture !
- Enge fringe field fall-offs are more realistic, and must then be used for any vertical dynamic aperture study.
- Egg-shape design has a vertical acceptance half of the requirements: back to the first stage of the design.

**Thank you for your attention**