Study of beam capture using a sawtooth RF field at Kyushu University

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- Purpose of this study
- Longitudinal simulation
- Power test of sawtooth
- Summary

Yushi Inaoka　(Kyushu Univ　M1)

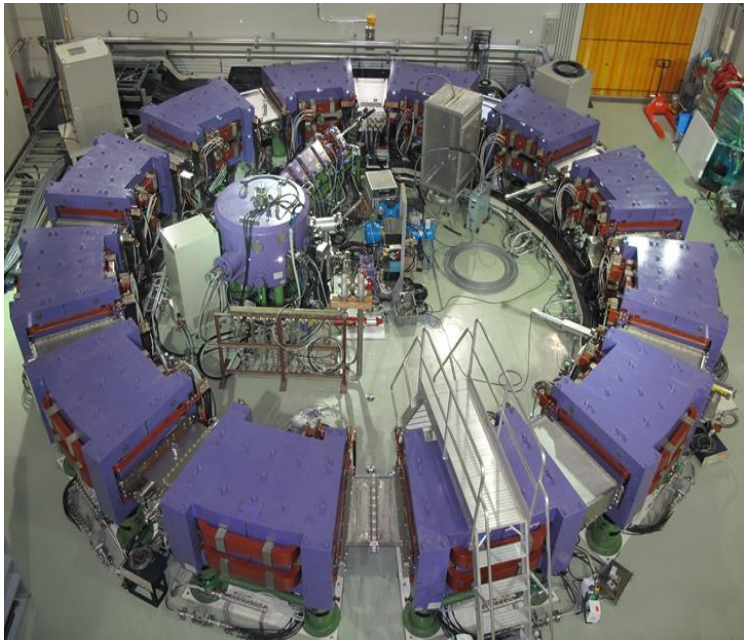
Introduction

- To increase beam intensity in 150 MeV FFAG,
  the operation of high repetition rate is required.
- It is necessary to make capture time short for high repetition rate.
- Adiabatic capture is NOT suitable for high repetition rate.

The capture with sawtooth voltage is faster than adiabatic capture to capture injected beam.

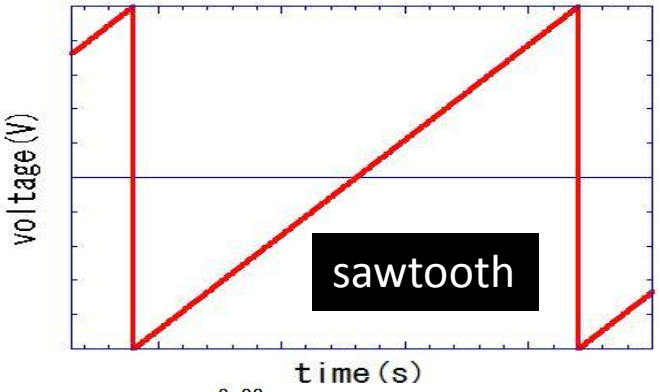
Purpose of this study

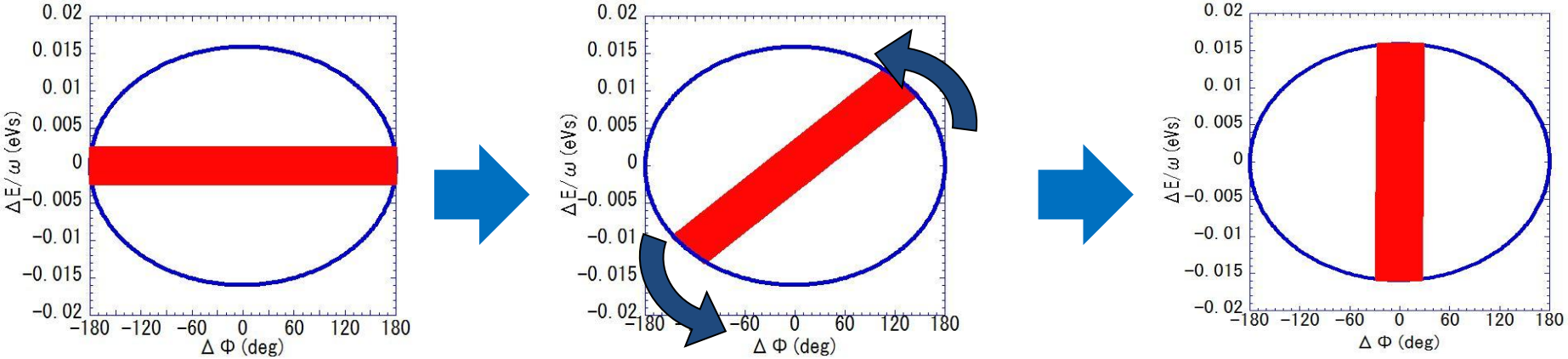
Quantitative evaluation of beam intensity with high repetition rate in case of capture with sawtooth voltage



Sawtooth capture

Sawtooth voltage is applied to the RF gap.





The angular speed of rotation is constant.

Advantage

It takes only 1/4 period of synchrotron oscillation.

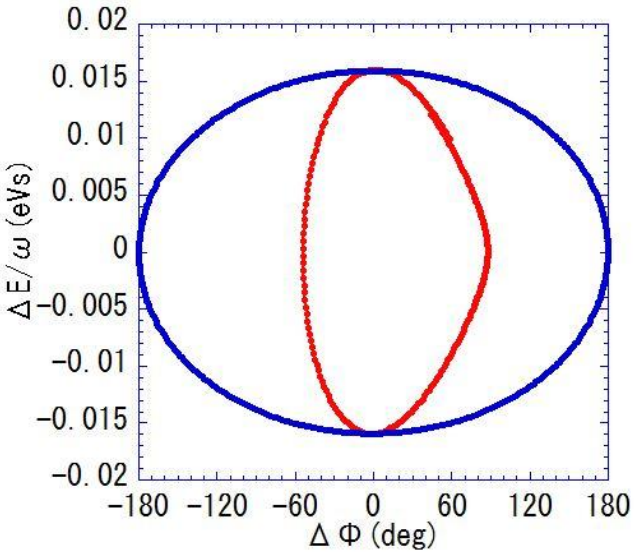
Problem

Difficulties of sawtooth capture

- Begin accelerating just after a 1/4 period of synchrotron oscillation.
- Accelerating bucket height is larger than stationary bucket height.

Solution

- Analytical solution of the period of synchrotron oscillation with sawtooth voltage.
- Match the stationary bucket height of sawtooth as accelerating bucket height



Longitudinal simulation

Purpose

Optimization of sawtooth capture for high repetition rate

Procedure

$$V_{gap} = \frac{V_c}{\pi}\phi$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\Delta \mathrm{E}}{\omega_{\mathrm{rf}}}\right) = \frac{\mathrm{e}V_c}{2\pi \mathrm{h}}\left[\frac{\phi + \Delta\phi}{\pi} - \frac{\phi}{\pi}\right]$$

Difference of energy

$$\frac{\mathrm{d}(\Delta\phi)}{\mathrm{dt}} = \frac{\omega_{\mathrm{rf}}^2 \eta_{\mathrm{s}}}{\beta_{\mathrm{s}}^2 \mathrm{E}_{\mathrm{s}}}\left(\frac{\Delta \mathrm{E}}{\omega_{\mathrm{rf}}}\right)$$

Difference of phase

ω_{rf} : revolution angular frequency

V_c : capture voltage

E_s : Total energy of synchronous particles

Parameter of sawtooth capture

$$\omega_{syn} = \sqrt{\frac{eV_c|\eta_s|\omega_{\mathrm{rf}}^2}{2\pi^2 h\beta_s^2 E_s}} \qquad T_{syn} = \sqrt{\frac{2\pi^2 h\beta_s^2 E_s}{eV_c|\eta_s|\omega_{\mathrm{rf}}^2}}$$

$$W_m = \sqrt{\frac{2eV_{ac}\beta_s^2 E_s}{\pi h|\eta_s|\omega_{\mathrm{rf}}^2}}\sqrt{-\cos\phi_s + \left(\frac{\pi}{2} - \phi_s\right)\sin\phi_s}$$

V_{ac} : acceleration voltage

ϕ_s : synchronous phase

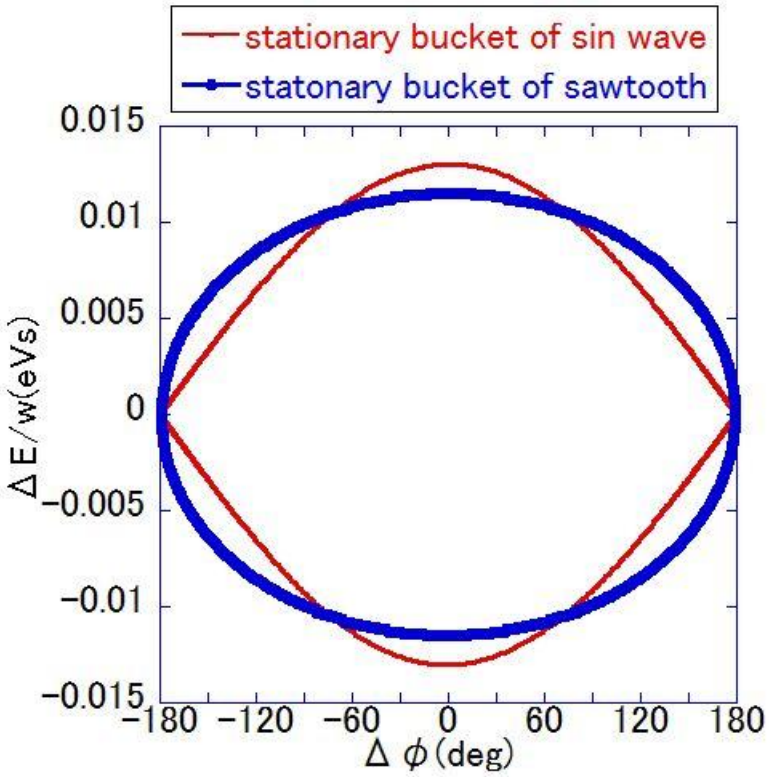
Capture voltage

$$V_c = \mathrm{W}_m^2 \times \frac{2h|\eta_s|\omega_{\mathrm{rf}}^2}{eE_s\beta_s^2}$$

Capture time

$$T_{cap} = T_{syn} \times \frac{1}{4}$$

V_c and T_{cap} only depends on ϕ_s since acceleration voltage .



Condition of Longitudinal simulation

Injector is cyclotron.

Particle	Proton
Energy	10 MeV
$\Delta p/p$	±0.23%

Condition of acceleration

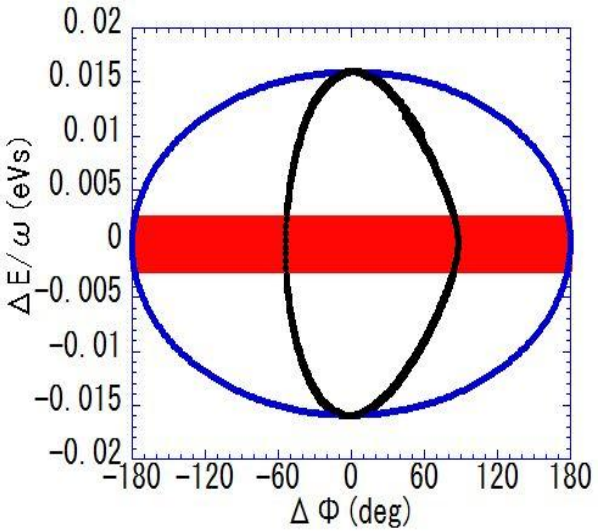
Gap voltage	8 kV_p (4kV_p/1 cavity × 2)
Final energy	125 MeV
Revolution Freq	1.5 – 4.2MHz

Definition

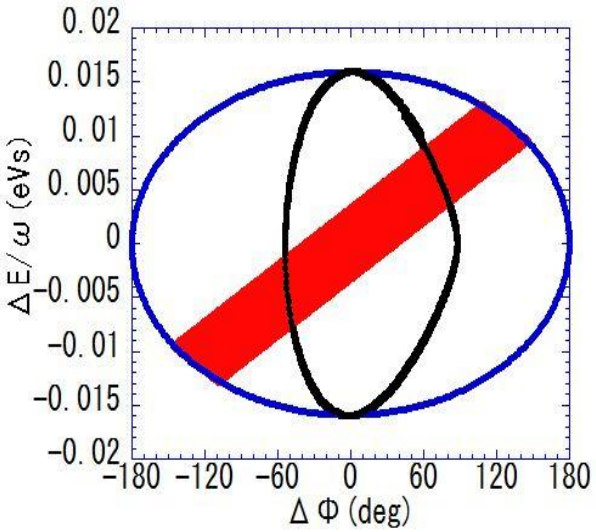
$$\text{Ratio of survived particles}(\%) = \frac{\text{Number of particles at extraction energy}}{\text{Number of injected particles}} \times 100$$

Result of the bunch rotation in the phase space

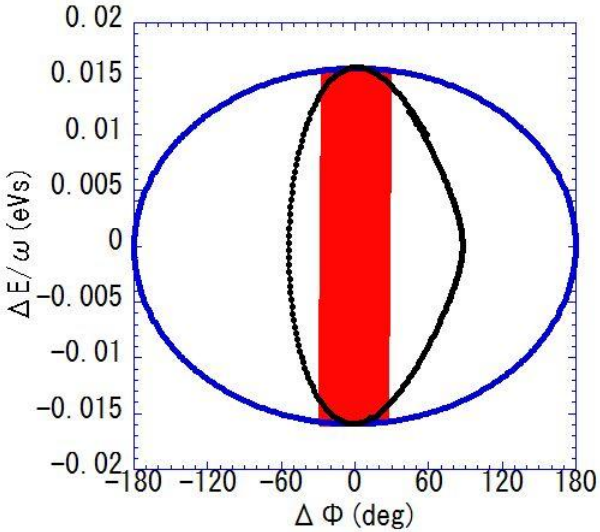
0(s)



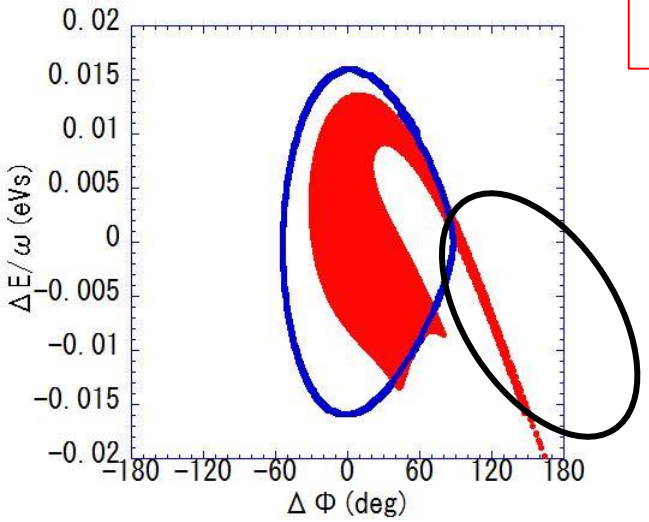
$^1/_8\,T_s$(s)



$^1/_4\,T_s$(s)



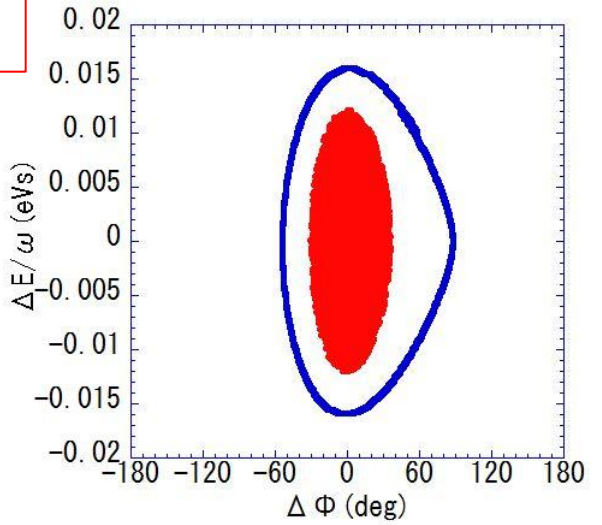
During acceleration



Synchronous phase=48deg
Ratio of survived particles=95%

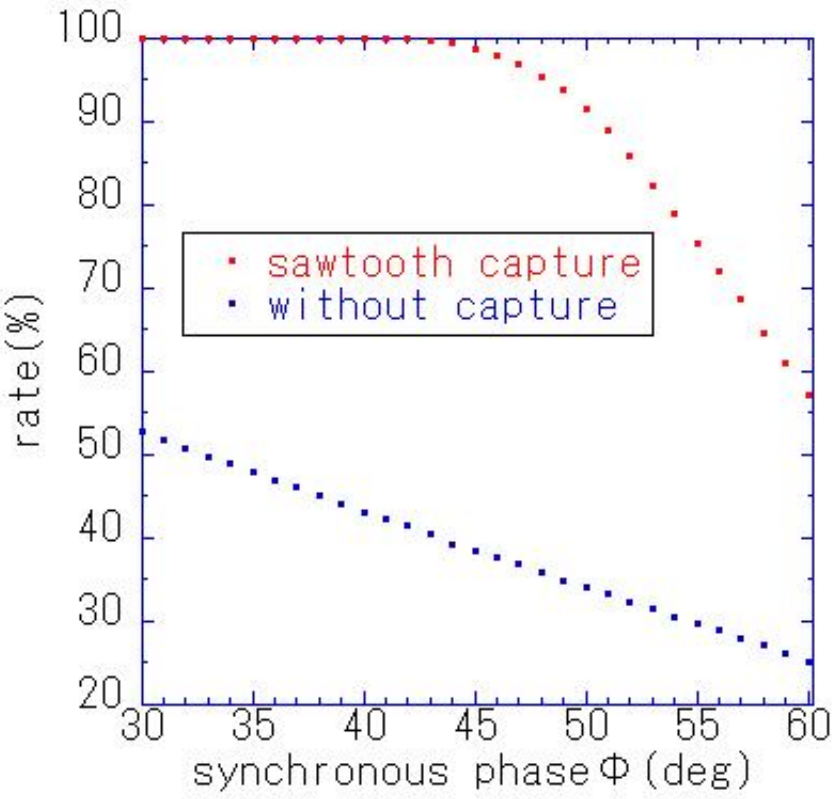
Beam outside accelerating bucket is lost.

Extraction



Result of simulation(ratio of survived particles)

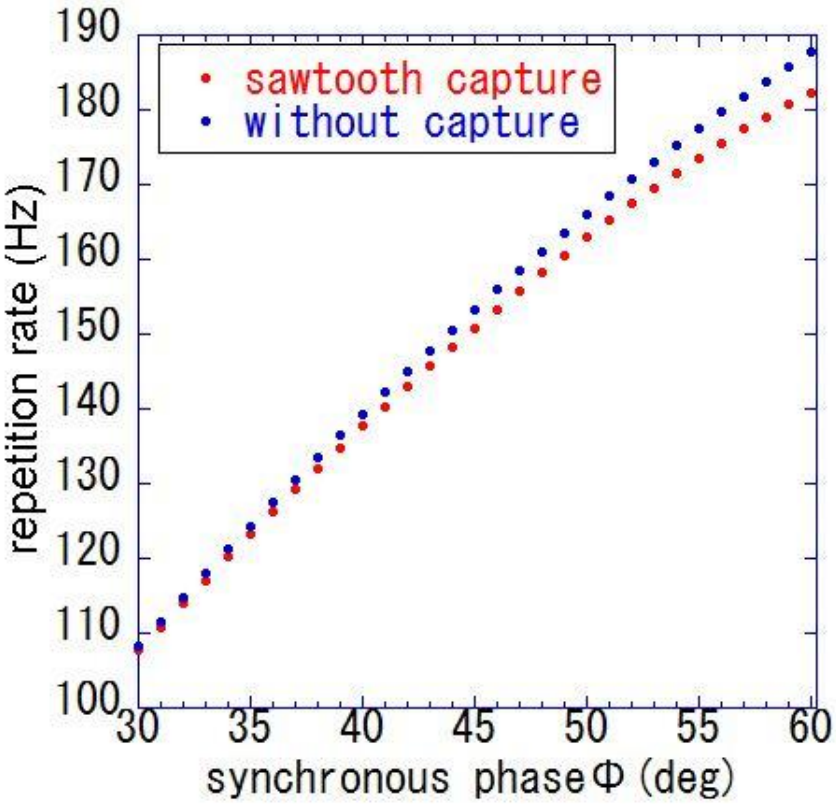
The synchronous phase dependent of the ratio of survived particles



- The ratio of survived particles suddenly decreases .

Result of simulation(repetition rate)

The synchronous phase dependent of the repetition rate



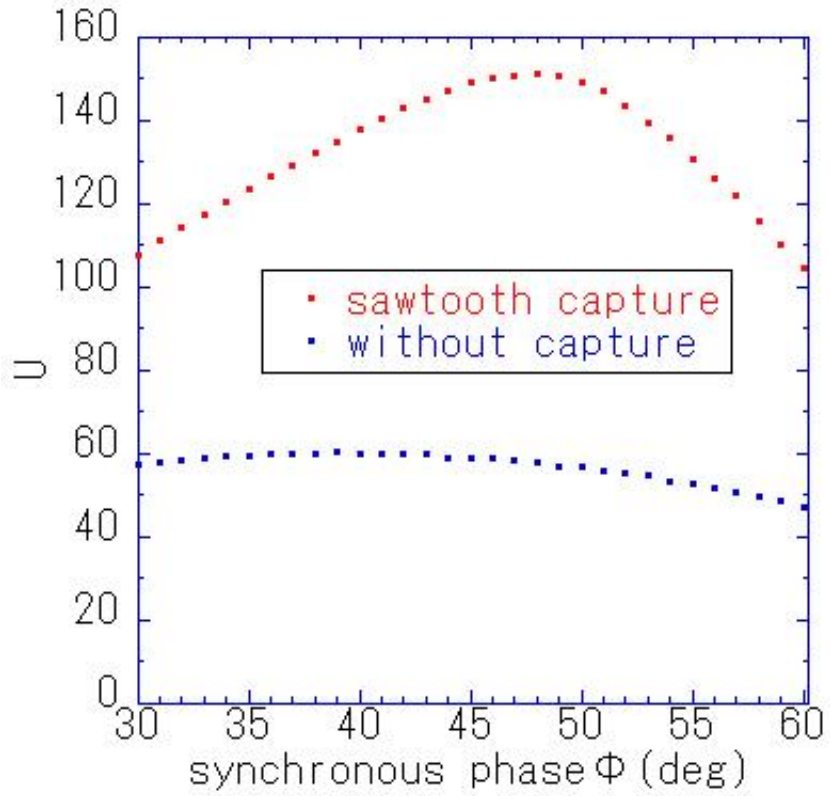
- Repetition rate is in proportion to synchronous phase.

Beam intensity parameter U

= Ratio of survived particle **×** repetition rate **(Hz)**

Result of simulation(U)

U= Ratio of survived particle × repetition rate **(Hz)**



Maximum value(sawtooth capture)

==========

U=151

The ratio of survived particles:95%

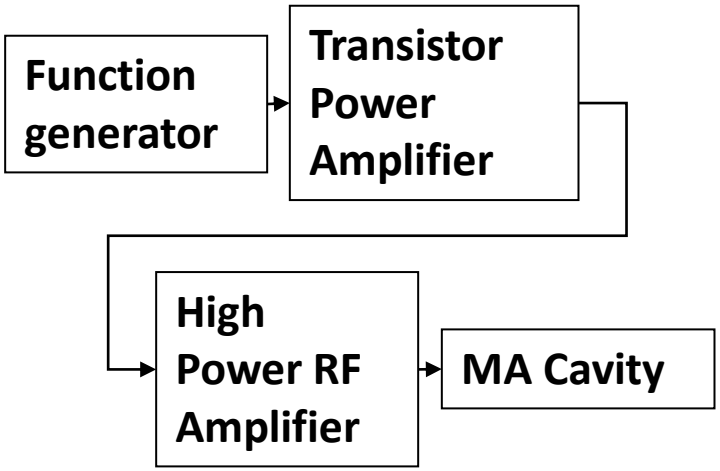
The repetition rate :158Hz

Synchronous phase ϕ:48 deg

Capture time :109 μs

- Sawtooth capture achieves U about 2.5 times higher than the case without capture.
- Capture time is 109 μs.

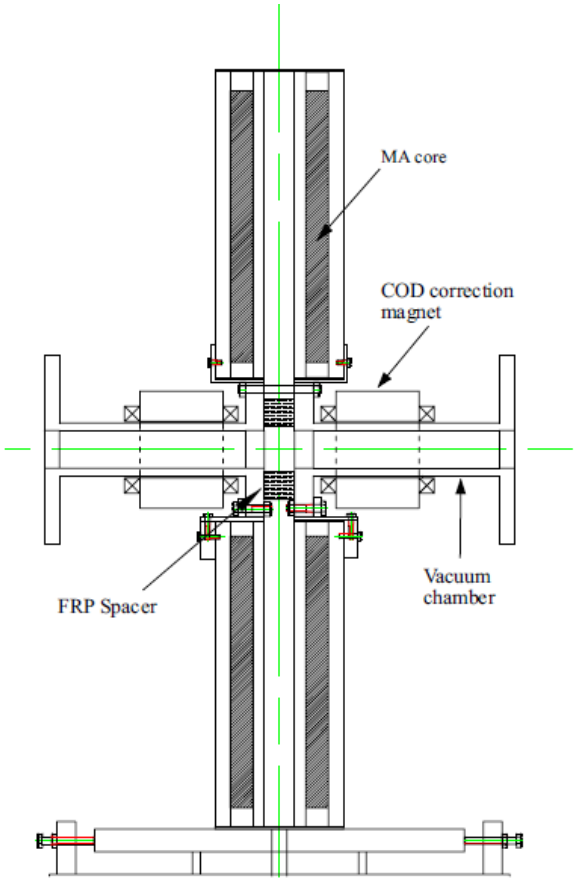
RF acceleration system



Designed parameters

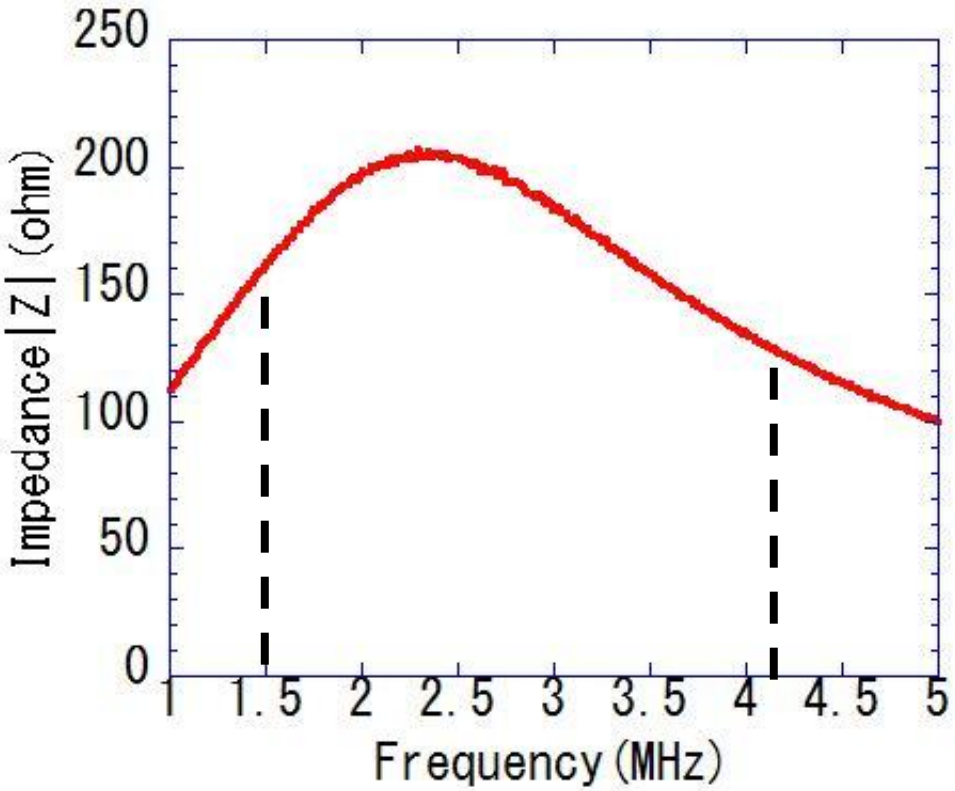
Gap voltage	4 kV_p
Frequency bandwidth	1.5～4.2 MHz
horizontal aperture	940 mm
Shunt impedance	200 Ω

RF cavity



Characteristic impedance

Measured impedance of the RF cavity



$$|Z| = \sqrt{R_s^2 + X_s^2}$$

Impedance Z

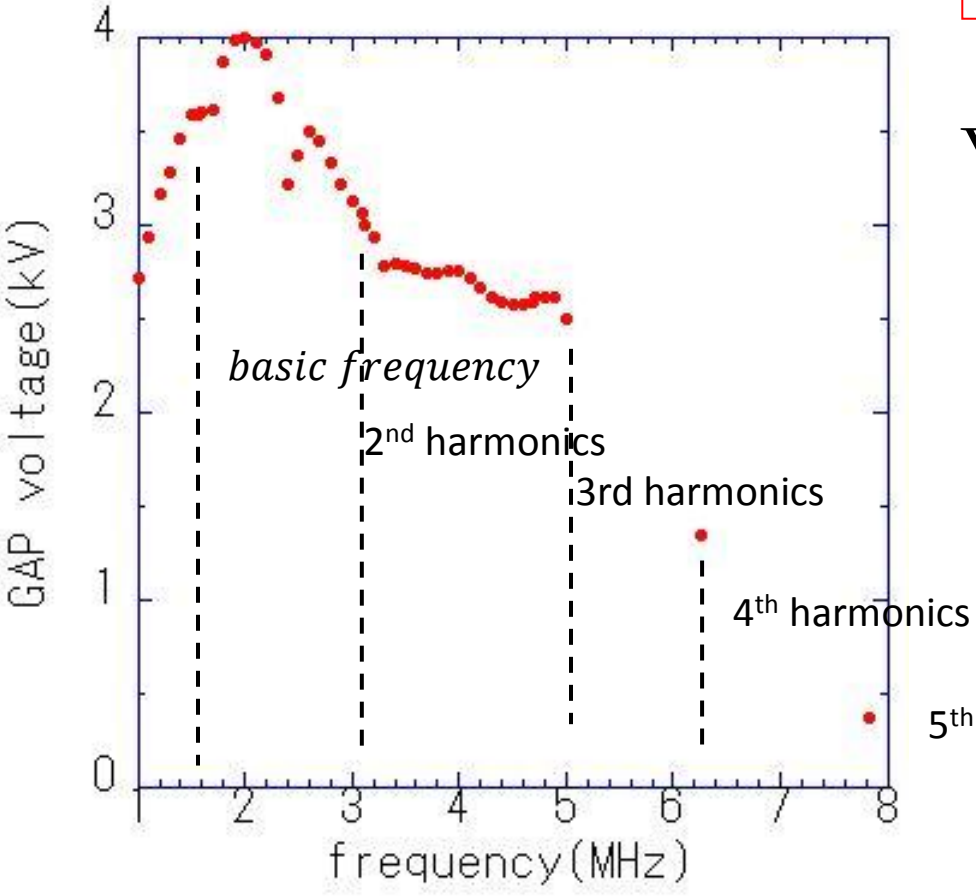
$$Z = R_s + jX_s$$

R_s: the real part of impedance

X_s: the imaginary part of impedance

- The impedance amplitude depends on frequency.

Power test



Sawtooth wave

$$V(t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\sin(2\pi n f_s t)$$

f_s: basic frequency

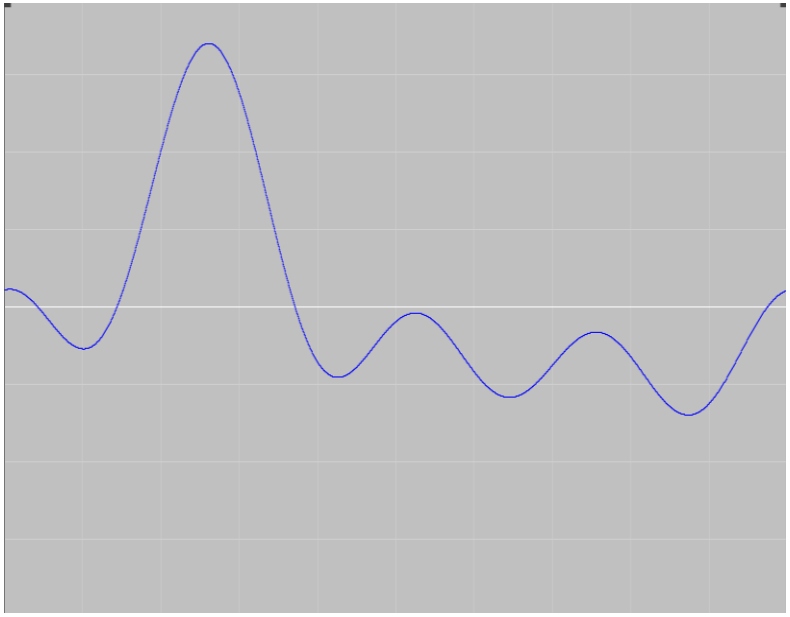
n=1 : 1.565 MHz
n=2 : 3.13 MHz
n=3 : 4.695 MHz
n=4 : 6.26 MHz
n=5 : 7.825 MHz

- gain of amplitude depends on frequency.
- Frequency bandwidths only cover up to 4th harmonics(n=4).

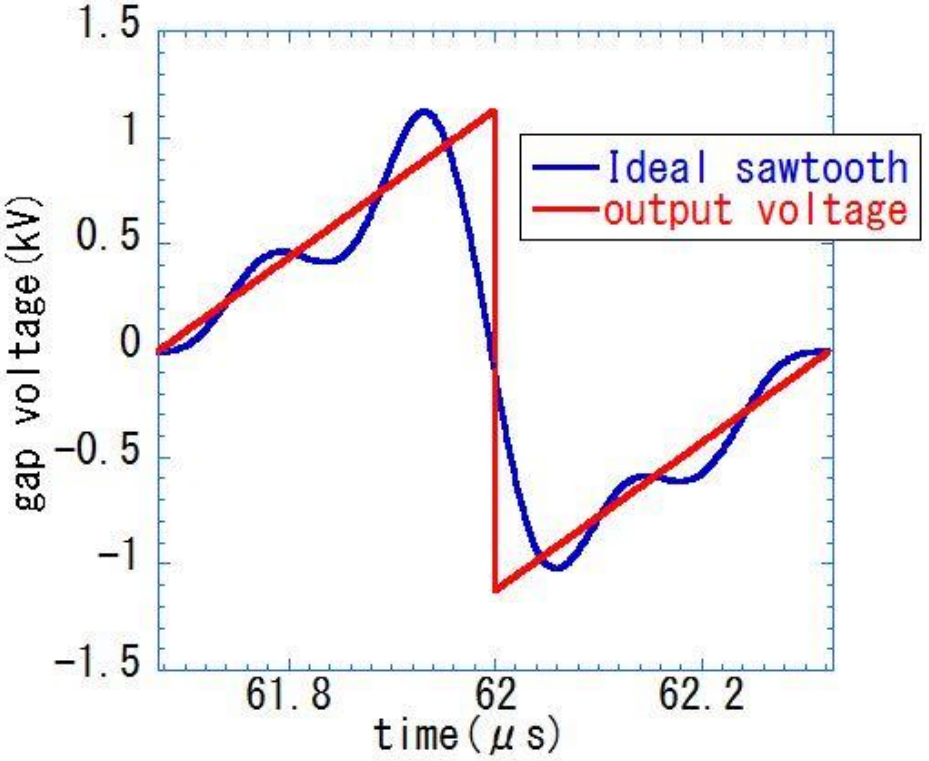
Power test

$$V_{in}(t) = \frac{2}{\pi}\sum_{n=1}^{4}\frac{(-1)^{n-1}}{n}A_n \sin(2\pi f_s t + B_n)$$

f_s: basic frequency A_n, B_n: correction term



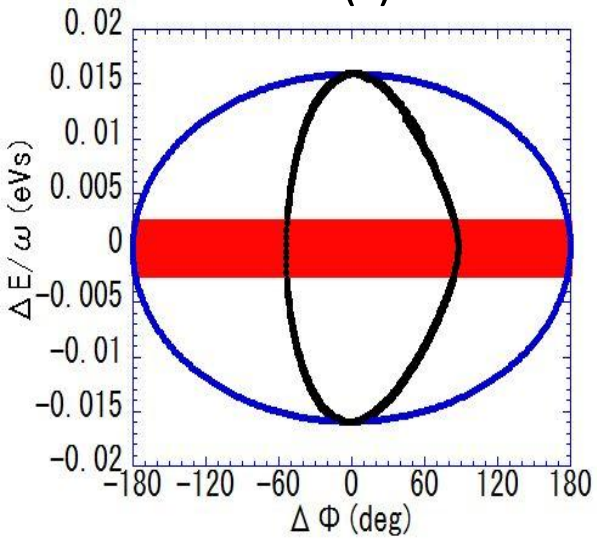
Input waveform



output waveform

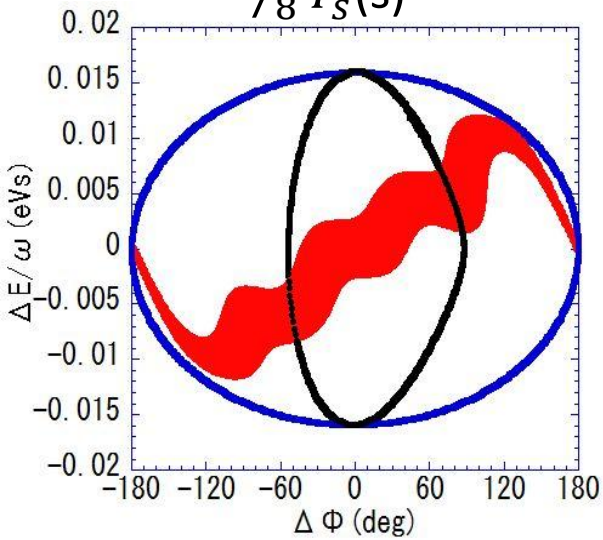
Result of the bunch rotation in the phase space

0(s)



Synchronous phase=48deg
Ratio of survived particles=91%

$^1/_8\ T_s$(s)



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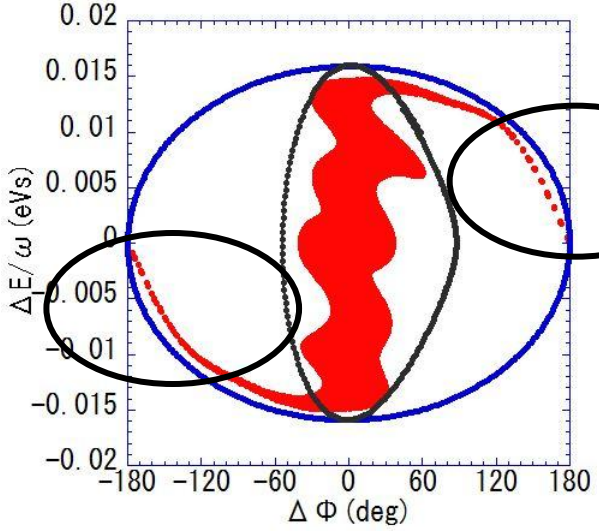
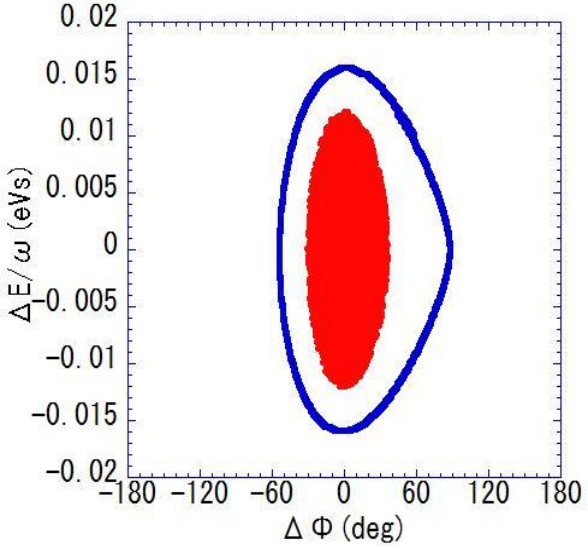


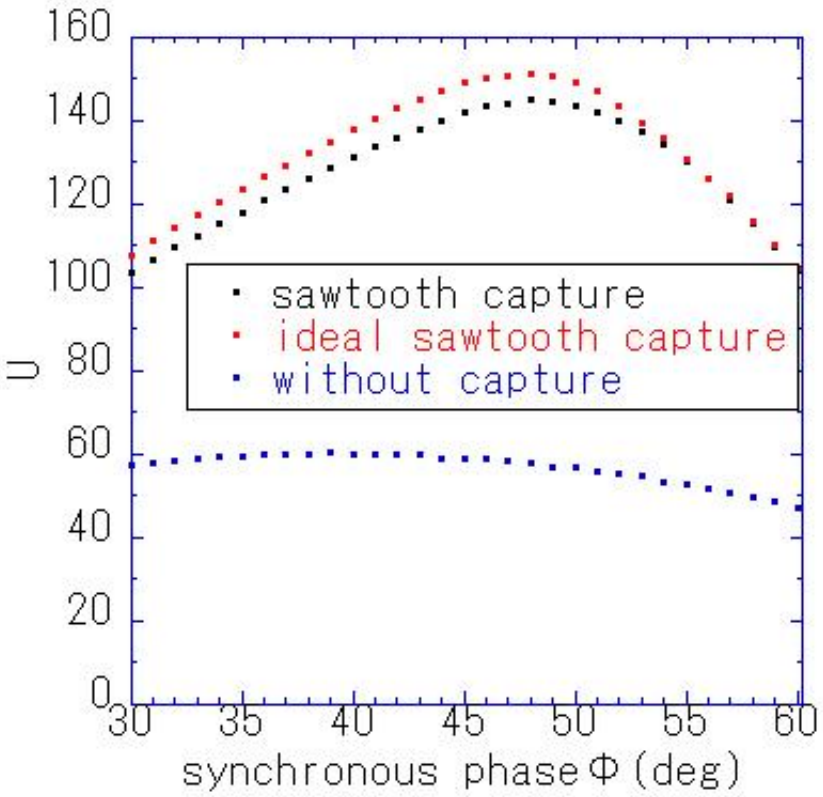
Beam outside accelerating bucket is lost.

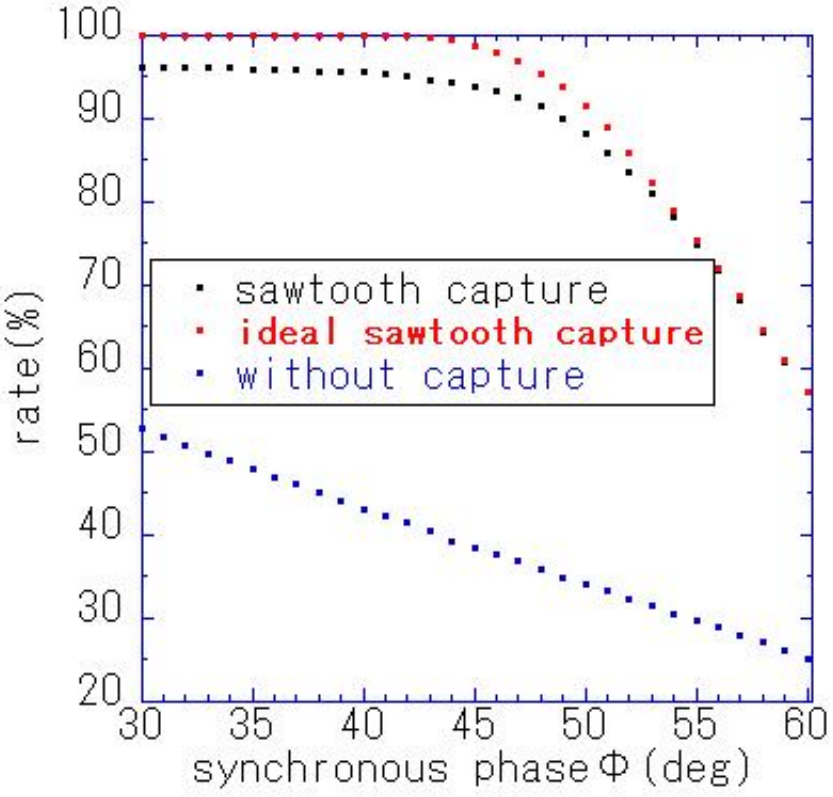
Extraction



Result of simulation

U= Ratio of survived particles × repetition rate **(Hz)**





$$\frac{U(\text{ideal sawtooth capture})}{U(\text{without capture})} \cong 2.5 \Rightarrow \frac{U(\text{sawtooth capture})}{U(\text{without capture})} \cong 2.4$$

Summary

- Sawtooth capture requires
  - correct timing of beginning acceleration.
  - Matching of bunch height as accelerating bucket height.
- Condition of sawtooth capture only depends on the synchronous phase.
- It's possible to output Sawtooth voltage(sum of 4^{th} harmonics).
- Sawtooth capture achieves the beam intensity about 2.4 times higher than without capture.

Capture time is only 109μs.

The Ratio of survived particles amount to 91%.

The repetition rate amount to 158 Hz.

Sawtooth capture is suitable for high repetition rate in 150 MeV FFAG .