# Serpentine acceleration in zero-chromatic FFAG

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# Outline

- Motivations and introduction
- Serpentine acceleration scheme in scaling FFAG
- Experiments for demonstration of serpentine acceleration with electron scaling FFAG
- Application of using a serpentine acceleration scheme
- Summary and Conclusion

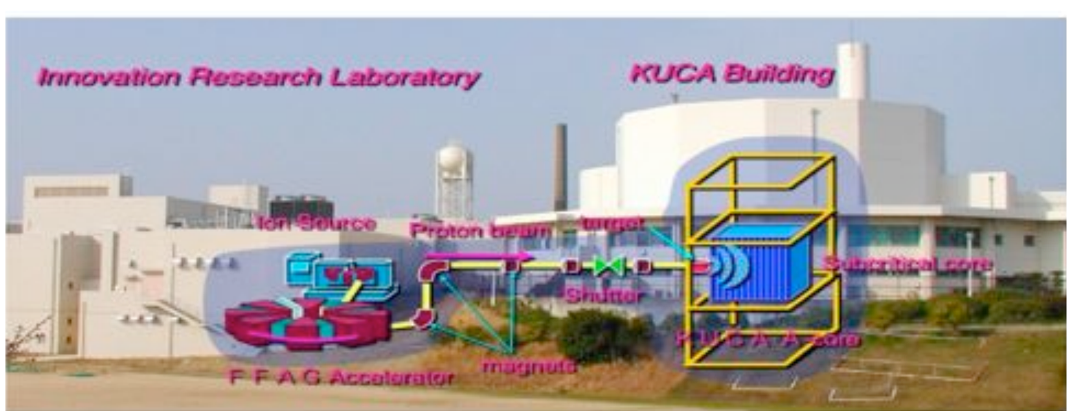
# Requirements for accelerators

- **Accelerators are used in many fields these days…for instance,**
  - Elementary Physics


**Muon accelerator & muon collider** are good candidates for the next generation of elementary physics experiments.

Rapid-acceleration scheme is required to accelerate short-lived particles within lifetimes.

- Accelerator Driven System (ADS)




**ADS : A system which keeps nuclear fission chain reaction induced by spallation neutrons obtained by irradiation of a target with high energy proton beams generated by accelerators.**

**High power proton beam is needed to generate enough neutrons**

# Necessary conditions for accelerators

- Requirements for accelerators

→

- ・Achievable of Rapid acceleration (~2μsec)
- ・Generating of High power beam (~10 MW)

- Accelerators are necessary satisfied with the following conditions:

| Requirements | Necessary conditions | Reason |
|---|---|---|
| Rapid acceleration | Static magnetic field | Variable magnetic field →Short-lived particles cannot be accelerated within the lifetime. |
| High-intensity beam | Fixed rf frequency acceleration | **Continuous wave (CW)** acceleration can be achieved |
| High-energy beam | Strong focusing | Large dynamic aperture is secured for secondary particle acceleration scheme. |

# Candidate of accelerators

- Candidate so far

| | Problems |
|---|---|
| **Linac** | Lots of cavities are needed |
| | Large space to put many rf cavities is required |

→ Expensive

- Circular accelerators

| | Strong focusing | Static magnetic field | Fixed rf frequency acceleration |
|---|---|---|---|
| Cyclotron | NO | YES | NO |
| Synchrotron | YES | NO | MAYBE<br>( $\beta \sim 1$ : Possible) |

- New candidate of accelerators

**Fixed Field Alternating Gradient (FFAG) Accelerators**

# FFAG accelerators

## Fixed Field Alternating Gradient (FFAG) Accelerators

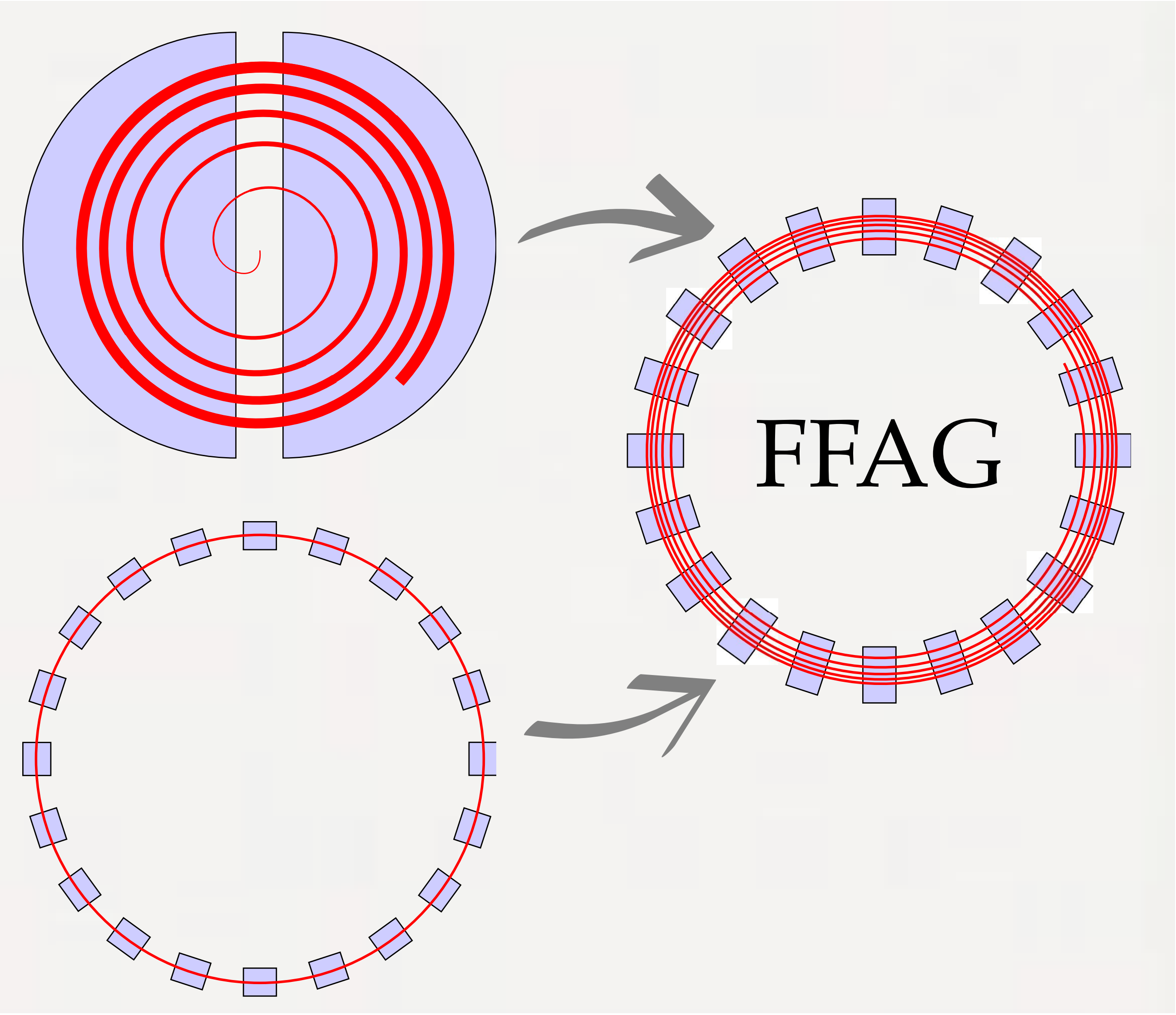
**Fixed field in time**
**(Like cyclotron)**

**+**

**Strong focusing**
**(Like synchrotron)**

**+**

**Fixed rf frequency acceleration**




**FFAG satisfies three necessary conditions to realize high-intensity and rapid-acceleration scheme.**

# Features of FFAGs

## Category of FFAGs

FFAGs
- scaling FFAG
- non-scaling FFAG

| | Features | |
|---|---|---|
| **scaling FFAG** | non-linear magnetic field ($B \propto r^k$) | Constant betatron tune |
| **non-scaling FFAG** | Linear magnetic filed | Betatron tune is not constant |

## Fixed rf frequency acceleration scheme in FFAGs

| Accelerators | Acceleration scheme | Energy |
|---|---|---|
| **scaling FFAG** | Stationary bucket acceleration | **Relativistic energy (β~1) beam is suitable.** |
| **non-scaling FFAG** | Serpentine acceleration | |

# Purpose of the study

- Beam acceleration in non-relativistic energy region

| | Energy | Requirements |
|---|---|---|
| Low energy muon acceleration | non-relativistic energy region | Rapid acceleration |
| Proton driver for ADS | | High-intensity proton beam |

→ **Fixed rf frequency acceleration scheme is required**
**Non-relativistic ($\beta<1$) energy region**

→ **Serpentine acceleration is applied in scaling FFAG**

The purpose of this study is to examine a serpentine acceleration scheme in scaling FFAG both theoretically and practically, allowing fixed rf frequency acceleration in non-relativistic energy region.

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  - Longitudinal phase space
  - Minimum rf voltage to make a serpentine channel
  - Total energy gain and phase acceptance in serpentine acceleration
  - Square rf voltage wave form in serpentine acceleration
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# Longitudinal hamiltonian for fixed rf frequency in scaling FFAG

## Phase equation

Phase difference ($\Delta\Phi$) per turn at the cavity.

$$\Delta\phi = 2\pi(f_{rf}\cdot T - h)$$

*T* : revolution period of no-synchronous particle

*h* : harmonic number

$f_{rf}$ : rf frequency

The relation between the equivalent radius (*R*) and momentum (*P*) is exactly given by

$$R = R_0\left(\frac{P}{P_0}\right)^{\frac{1}{k+1}}$$

$$\begin{aligned}\frac{T}{T_s} = 1 + \frac{\Delta\phi}{2\pi h} &= \left(\frac{R}{R_s}\right)\Big/\frac{P/E}{P_s/E_s} \\ &= \left(\frac{P}{P_s}\right)^{\alpha}\frac{P_s}{E_s}\left(\frac{E^2}{E^2-m^2}\right)^{\frac{1}{2}} \\ &= \left(\frac{E^2-m^2}{P_s^2}\right)^{\frac{\alpha}{2}}\frac{P_s}{E_s}\left(\frac{E^2}{E^2-m^2}\right)^{\frac{1}{2}} \\ &= (E_s^2-m^2)^{\frac{1-\alpha}{2}}\frac{E}{E_s}(E^2-m^2)^{\frac{\alpha-1}{2}}\end{aligned}$$

$\alpha$ : momentum compaction factor

$$\alpha = \frac{1}{k+1}$$

Solve the equation for $\Delta\Phi$

# Longitudinal hamiltonian for fixed rf frequency in scaling FFAG

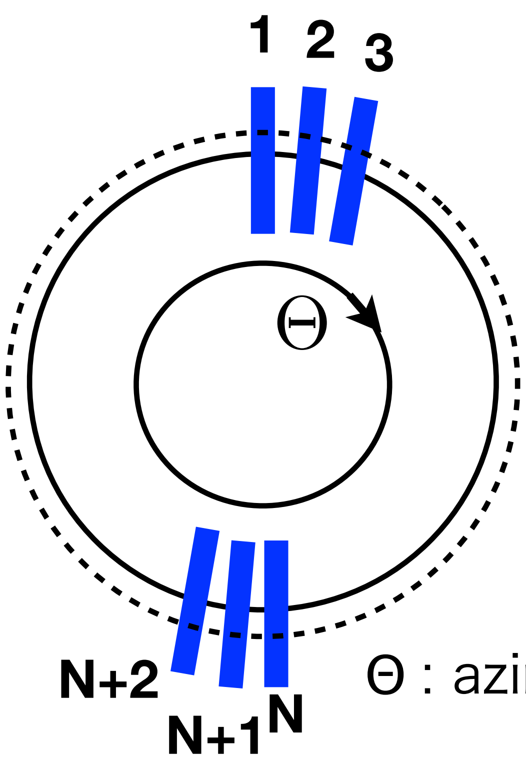
- Phase equation

$$\Delta\phi = 2\pi h\left[\frac{(E_s^2 - m^2)^{\frac{1-\alpha}{2}}}{E_s}E(E^2 - m^2)^{\frac{\alpha-1}{2}} - 1\right]$$

- Energy equation

Energy gain ($\Delta E$) per turn at the cavity

$$\Delta E = eV_{rf}\sin\phi$$




**We assume that there are many rf cavity in a ring**

➡ **Phase and energy differences are very small.**

$$\frac{\Delta\phi}{2\pi} \leftrightarrow \frac{d\phi}{d\Theta} \qquad \frac{\Delta E}{2\pi} \leftrightarrow \frac{dE}{d\Theta}$$

**Equation of motions**

➡

$$\frac{d\phi}{d\Theta} = h\left[\frac{(E_s^2 - m^2)^{\frac{1-\alpha}{2}}}{E_s}E(E^2 - m^2)^{\frac{\alpha-1}{2}} - 1\right]$$

$$\frac{dE}{d\Theta} = \frac{eV_{rf}}{2\pi}\sin\phi$$

# Longitudinal hamiltonian for fixed rf frequency in scaling FFAG

**Equation of motions**

$$\frac{d\phi}{d\Theta} = h\left[\frac{(E_s^2 - m^2)^{\frac{1-\alpha}{2}}}{E_s} E(E^2 - m^2)^{\frac{\alpha-1}{2}} - 1\right]$$

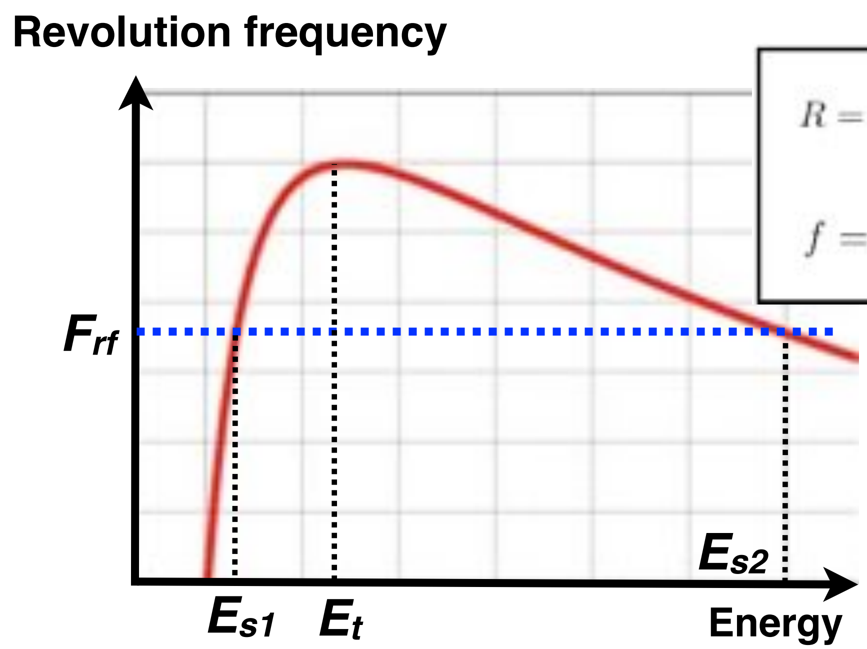
$$\frac{dE}{d\Theta} = \frac{eV_{rf}}{2\pi} \sin\phi$$

We choose the energy variable $E$ canonically conjugate to the coordinate variable $\phi$.

**Longitudinal hamiltonian**

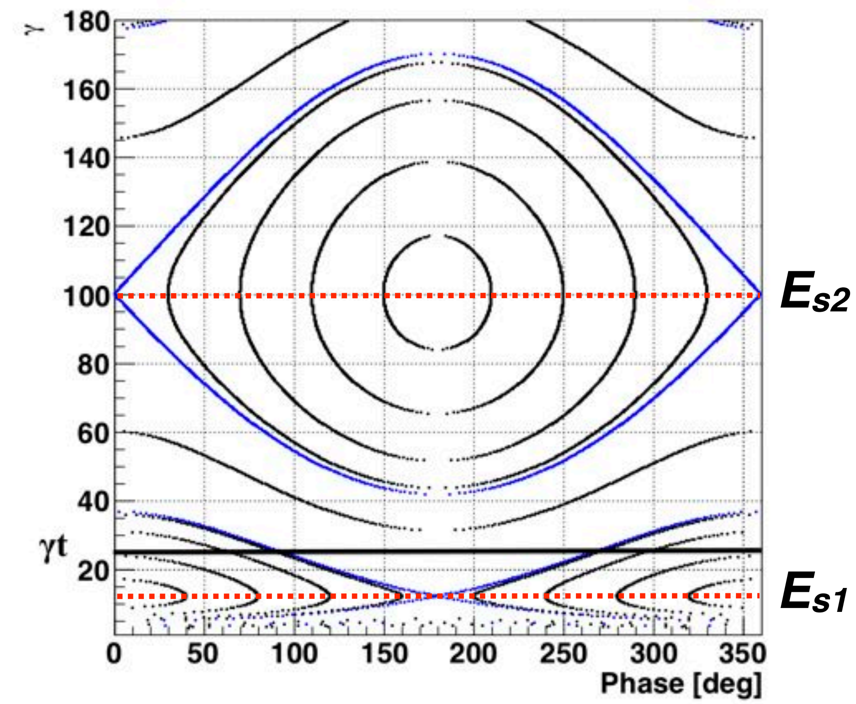
$$H(E,\phi;\Theta) = h\left[\frac{1}{\alpha+1}\frac{(E^2 - m^2)^{\frac{\alpha+1}{2}}}{E_s(E_s^2 - m^2)^{\frac{\alpha-1}{2}}} - E\right] + \frac{eV_0}{2\pi}\cos\phi.$$

# Longitudinal phase space

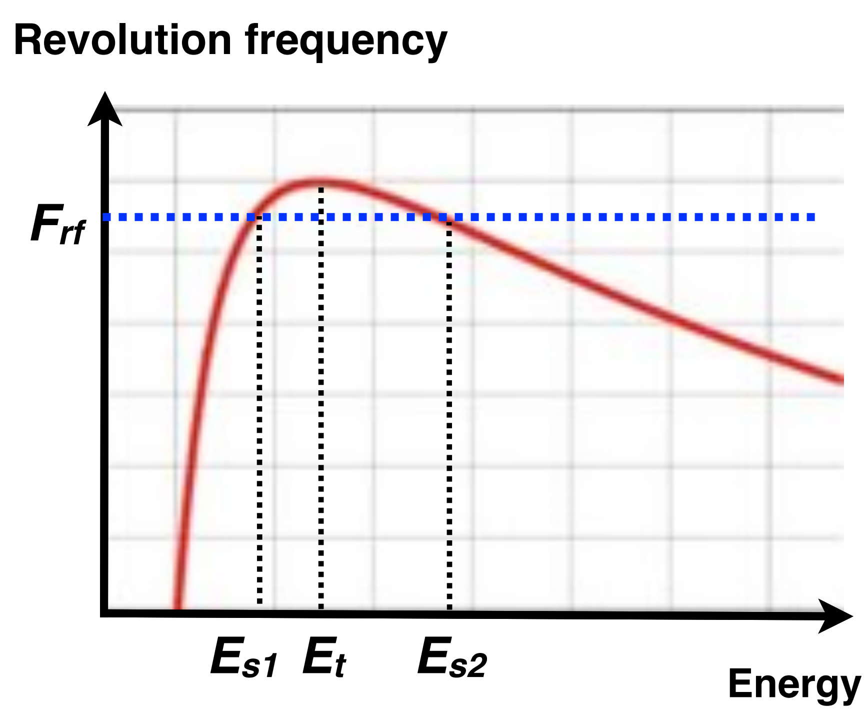


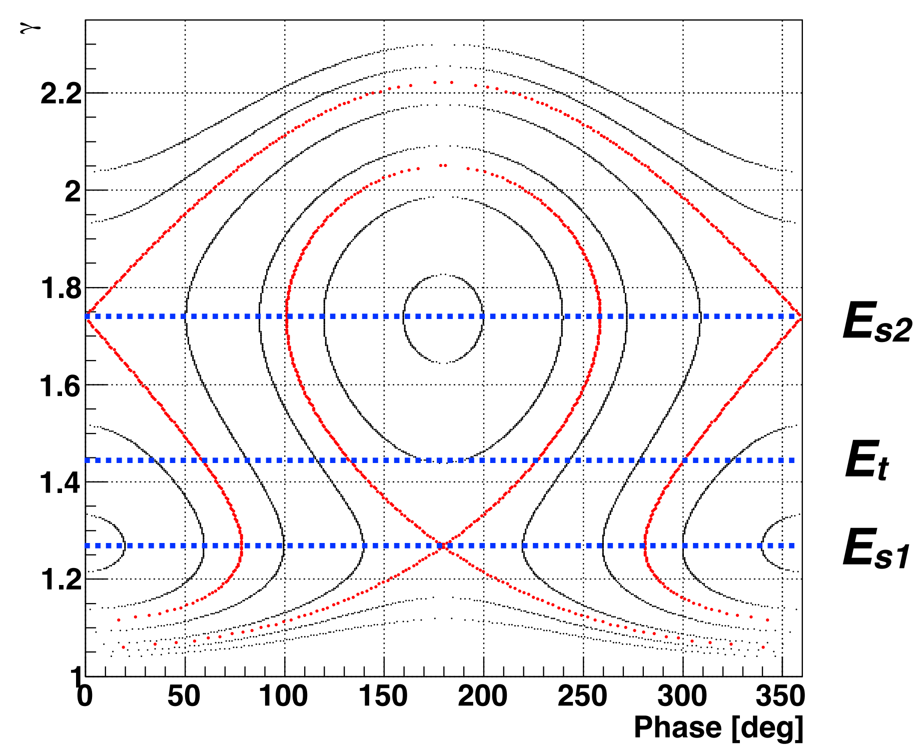

$$R = R_0 \left(\frac{P}{P_0}\right)^{\frac{1}{k+1}}$$

$$f = \frac{\beta c}{2\pi R}$$




Rf frequency is fixed near the transition energy




**Serpentine acceleration can be achieved in non-relativistic energy region with appropriate selection of transition energy.**

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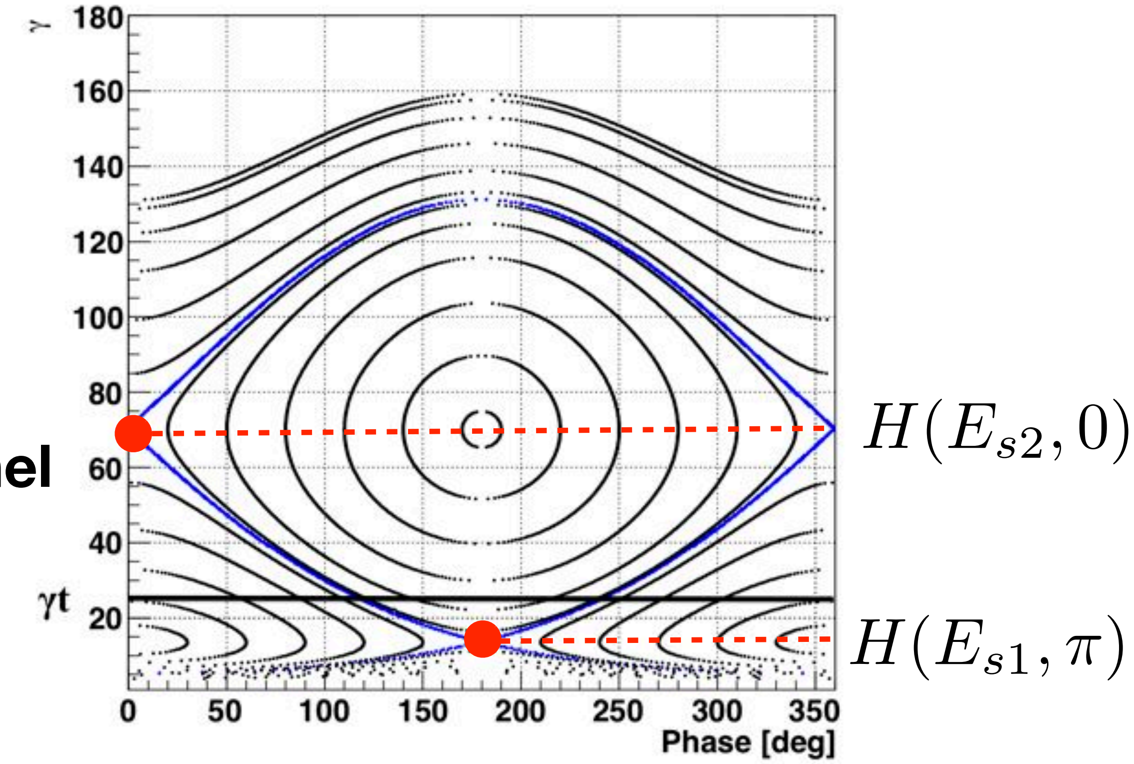
# Minimum rf voltage

- Limit condition of making serpentine channel

**Both stationary buckets are just connected.**

**Separatrix satisfies the equation :**

$$H(E_{s1}, \pi) = H(E_{s2}, 0)$$

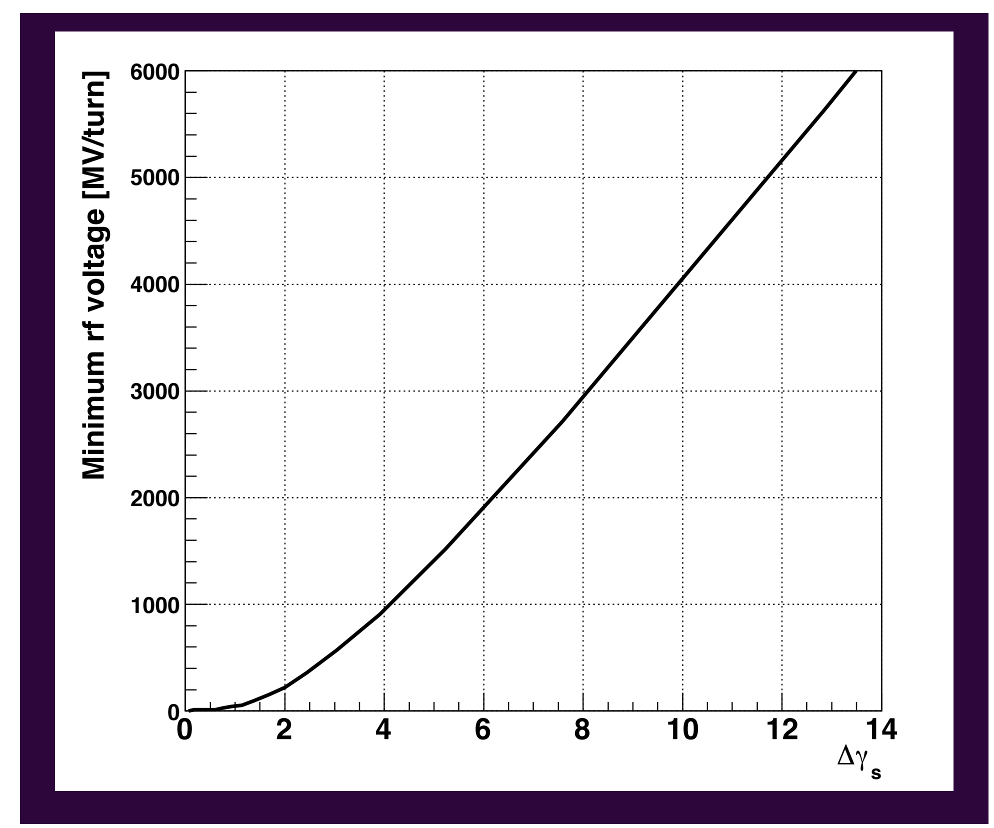



**Lowest limit of rf voltage to make a serpentine channel**

$$V_{min} = \pi h \left[ \frac{1}{\alpha + 1} \left( \frac{(E_{s1}^2 - m^2)}{E_{s1}} - \frac{(E_{s2}^2 - m^2)}{E_{s2}} \right) + (E_{s2} - E_{s1}) \right]$$

**Relation between minimum rf voltage and energy difference of stationary energies $\Delta\gamma_s$**

**( k-value & h are fixed )**

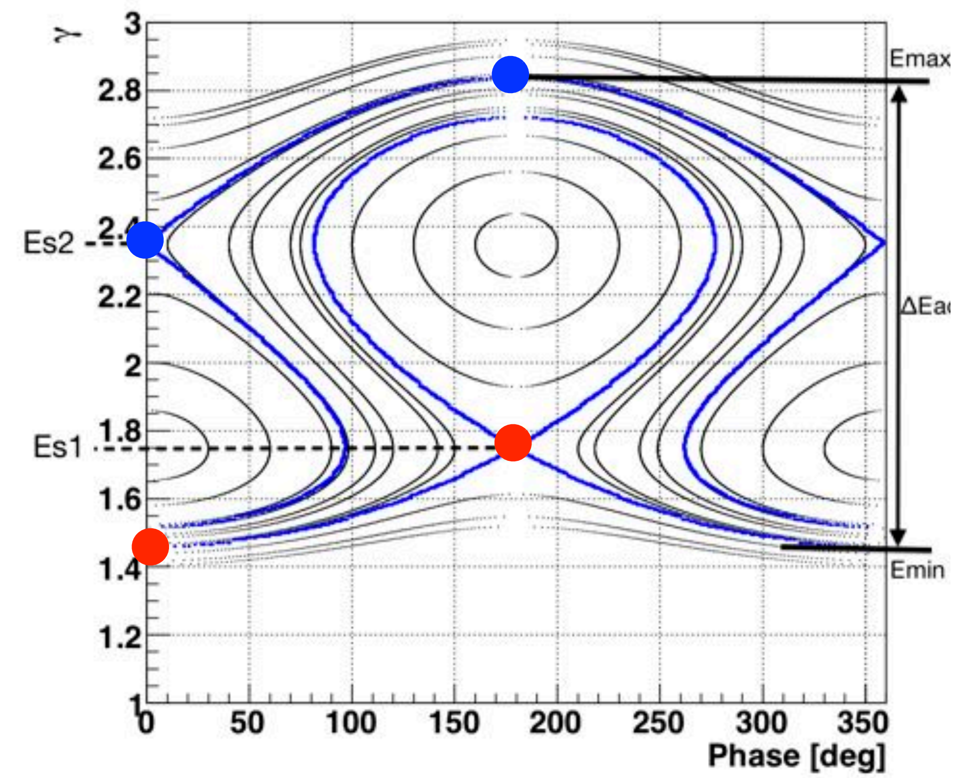



**Minimum rf voltage becomes big with difference of stationary energies ($\Delta\gamma_s$).**

# Maximum energy gain of serpentine acceleration

- Maximum energy gain

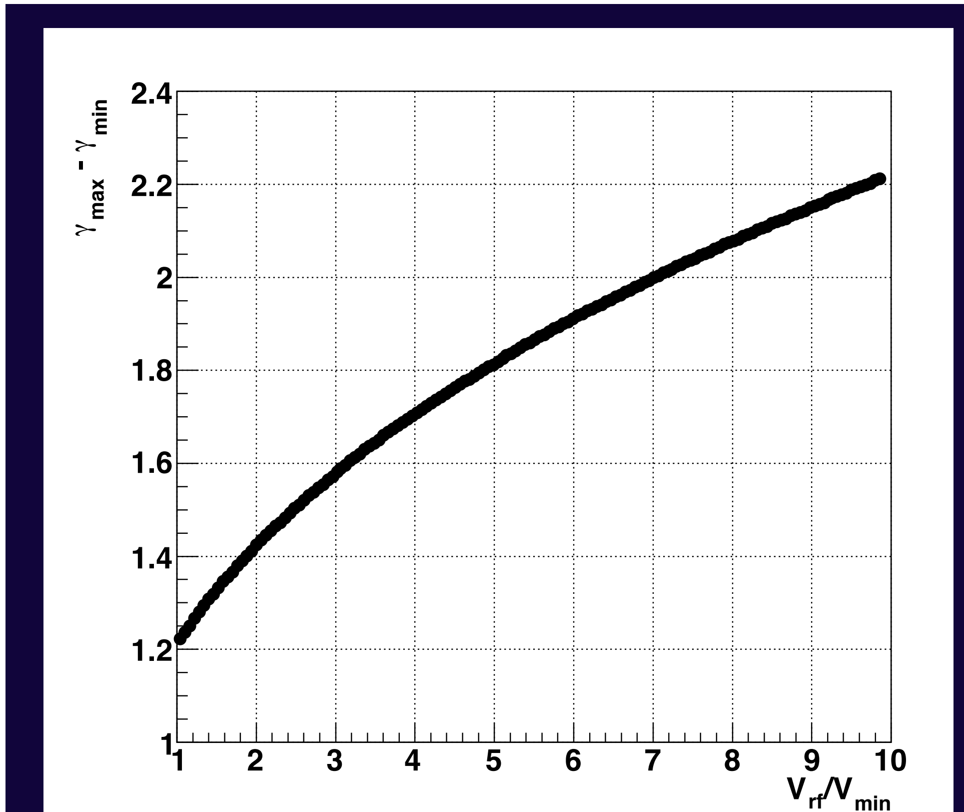
**Maximum energy range is determined by minimum and maximum energy on serpentine channel.**




**Separatrices for Minimum energy $E_{min}$ and Maximum energy $E_{max}$ satisfy these equations:**

$$\begin{cases} H(E_{max}, \pi) = H(E_{s2}, 0), \\ H(E_{min}, 0) = H(E_{s1}, \pi). \end{cases}$$

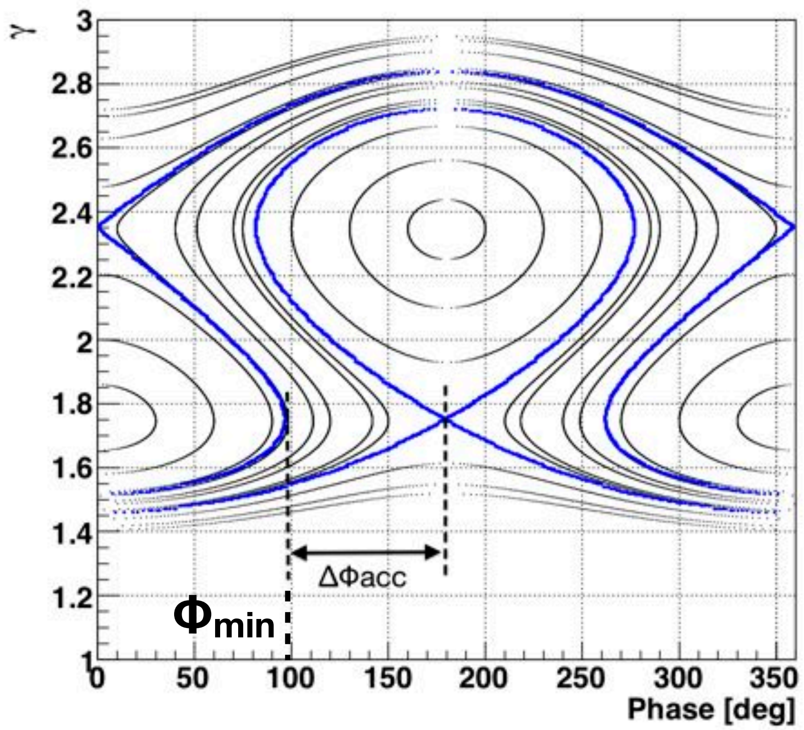
**Relation between maximum energy gain Δ*E* and rf voltage $V_{rf}$**
**(Stationary energies, harmonic number and k-value are fixed.)**




**Maximum energy gain becomes large with rf voltage.**

# Phase acceptance

- Phase acceptance of serpentine channel at stationary energy below transition

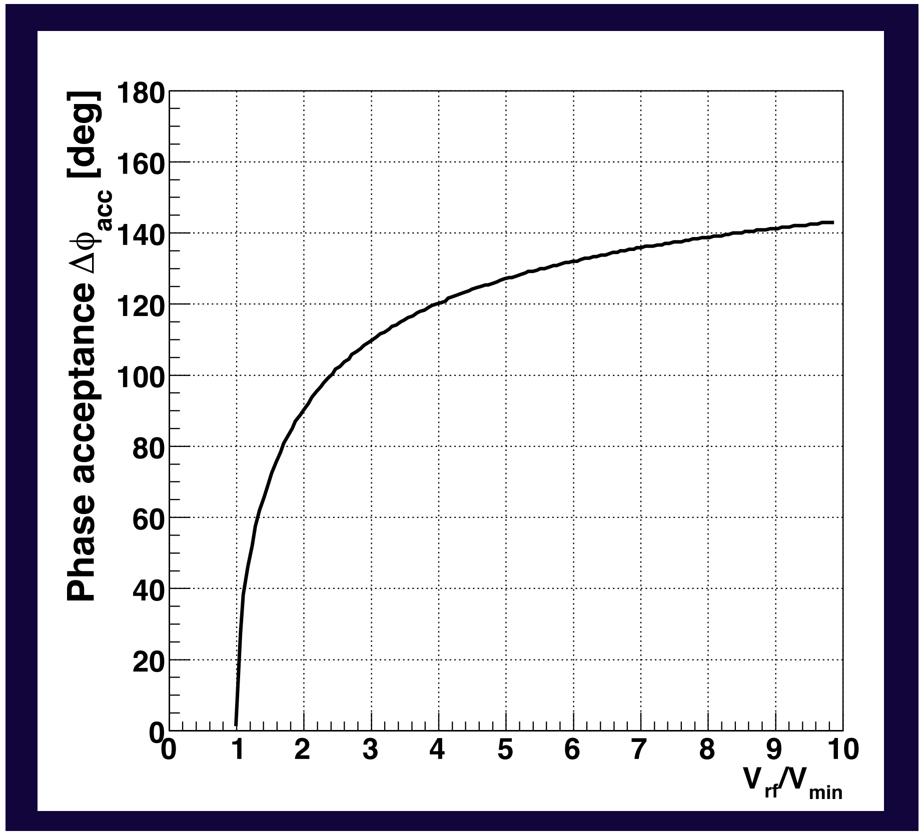



Separatrix for minimum phase ($\Phi_{min}$) of phase acceptance satisfies the equation:

$$H(E_{s1}, \phi_{min}) = H(E_{s2}, 0)$$

**Phase acceptance $\Delta\Phi_{acc}$**

$$\Delta\phi_{acc} = \pi - \phi_{min}$$
$$= \pi - \arccos\left[\frac{2\pi h}{eV_{rf}}\left(\frac{1}{\alpha+1}\left(\frac{(E_{s2}^2 - m^2)}{E_{s2}} - \frac{(E_{s1}^2 - m^2)}{E_{s1}}\right) + (E_{s1} - E_{s2})\right) + 1\right]$$




**Relation between phase acceptance ΔΦ and rf voltage $V_{rf}$**

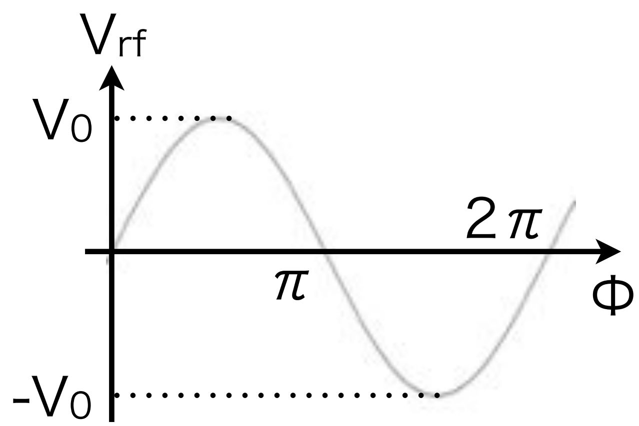
**( Stationary energies, harmonic number and k-value are fixed. )**

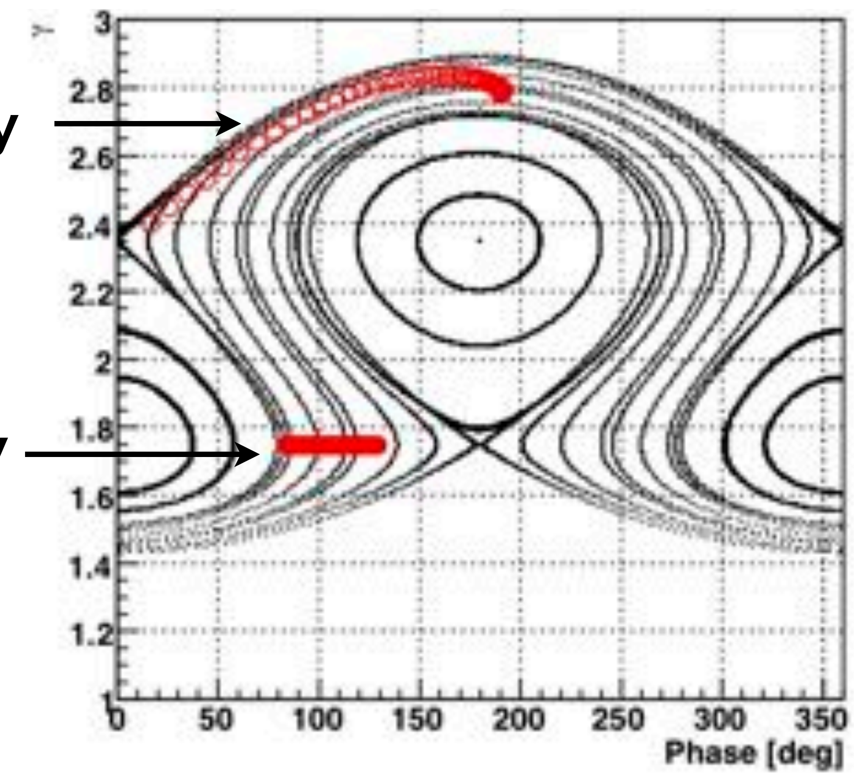
**Phase acceptance becomes large with rf voltage.**

# Square rf voltage wave form in serpentine acceleration

- Energy spread at extraction with sinusoidal rf voltage wave

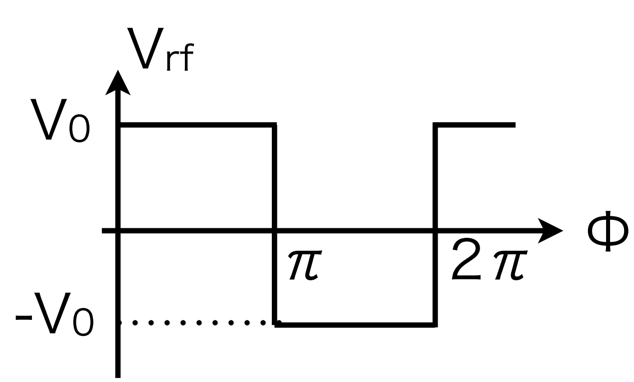
$$V_{rf} = eV_0 \sin\phi$$




Square rf voltage wave form is applied to obtain a mono-energetic extraction beam.

$$V_{rf} = \begin{cases} V_0 & : \quad 0 < \phi \leqq \pi, \\ -V_0 & : \pi < \phi \leqq 2\pi. \end{cases}$$

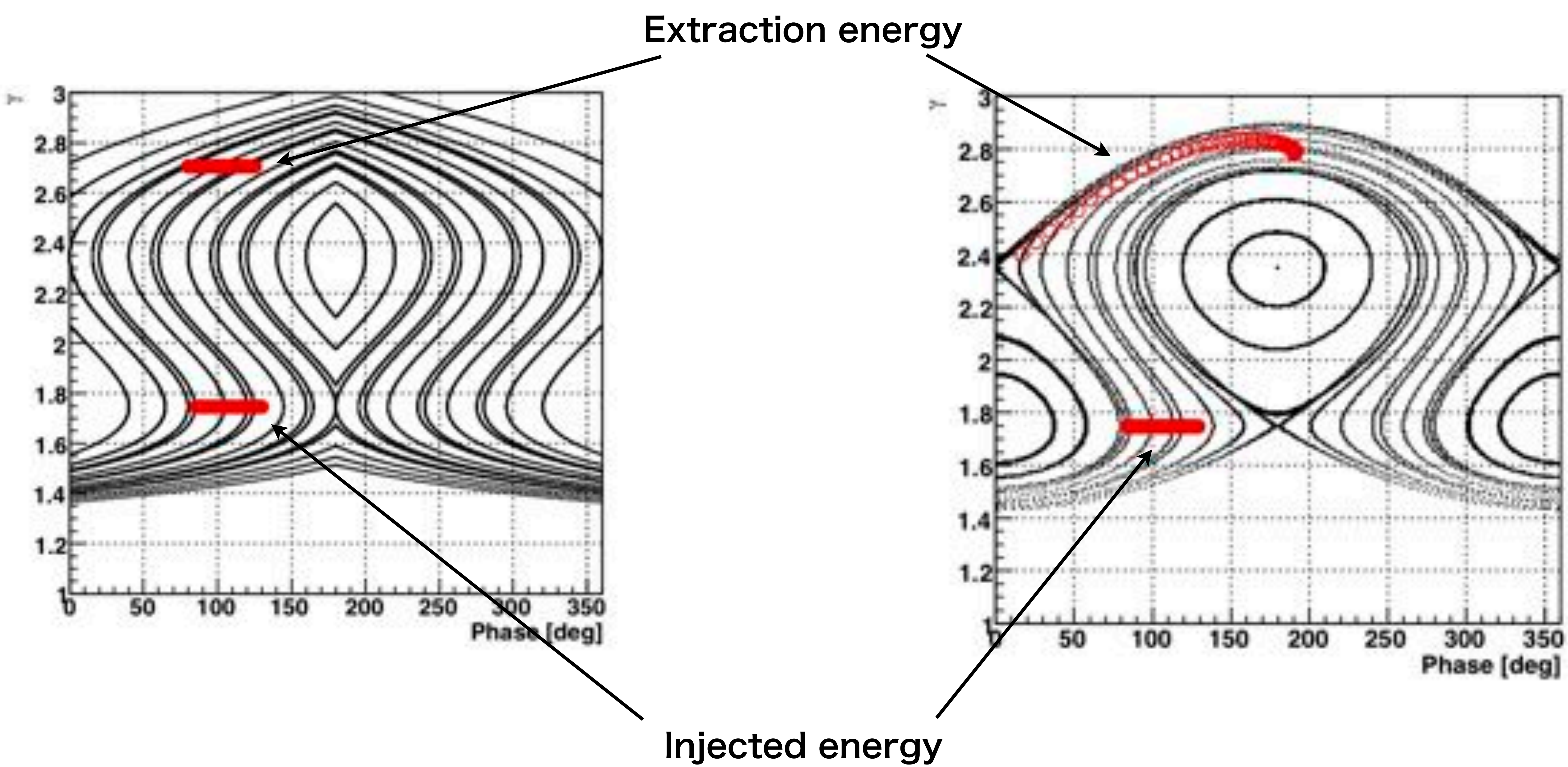



## Hamiltonian

$$H(E,\phi;\Theta) = \begin{cases} h\left[\frac{1}{1+\alpha}\frac{(E^2-m^2)^{\frac{1+\alpha}{2}}}{E_s(E_s^2-m^2)^{\frac{\alpha-1}{2}}} - E\right] - \frac{V_0}{2\pi}\phi & : 0 < \phi \leqq \pi, \\ h\left[\frac{1}{1+\alpha}\frac{(E^2-m^2)^{\frac{1+\alpha}{2}}}{E_s(E_s^2-m^2)^{\frac{\alpha-1}{2}}} - E\right] + \frac{V_0}{2\pi}\phi - V_0 & : \pi < \phi \leqq 2\pi, \end{cases}$$

# Square rf voltage wave form in serpentine acceleration

- Energy spread at extraction with sinusoidal and square rf voltage wave



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- Serpentine acceleration in scaling FFAG
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  - Purpose of experiments

1. Demonstration of serpentine acceleration in scaling FFAG

2. Phase acceptance study with rf voltage in serpentine acceleration

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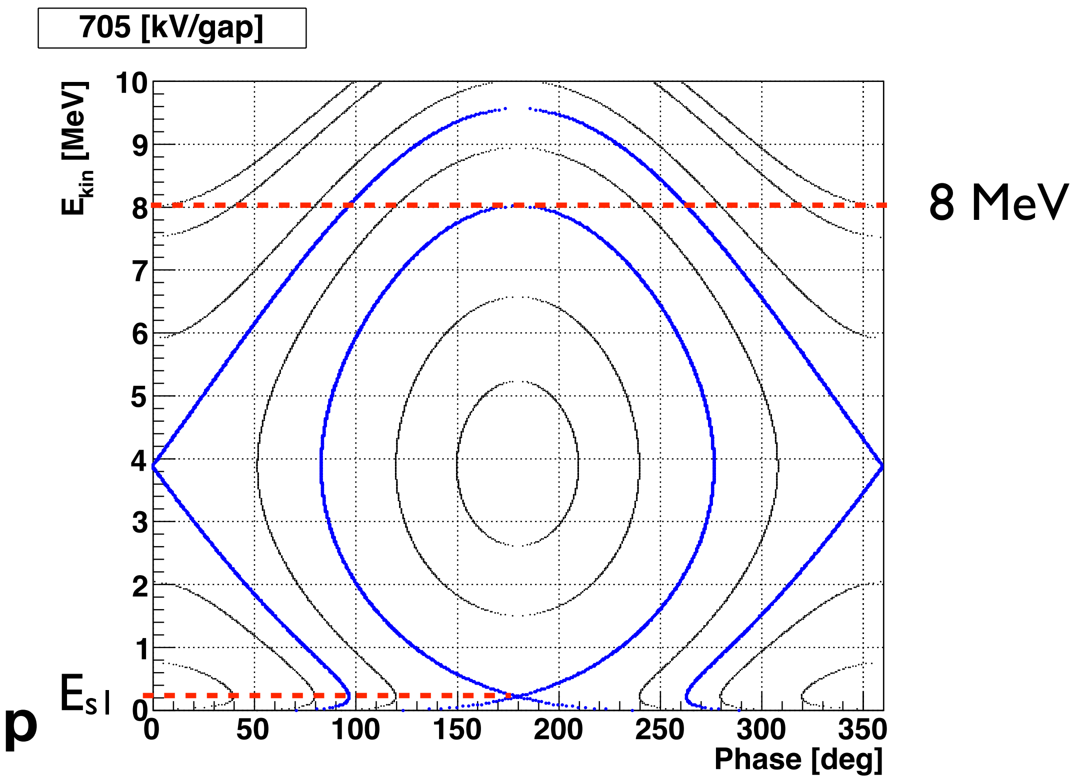
# Demonstration of serpentine acceleration

## Injected beam parameters

| | |
|---|---|
| Injected energy | 160 keV |
| Stationary energy ($E_{s1}$) | 205 keV |
| Injected beam phase | < 20% of rf period |
| Rf voltage | 705 kV/gap |




## Measurement conditions

- **Beam current in the ring is measured by Faraday cup**

  **Faraday cup is fixed at 8 MeV closed orbit**

- **Central phase of injected beam is changed with respect to the rf phase at the cavity.**

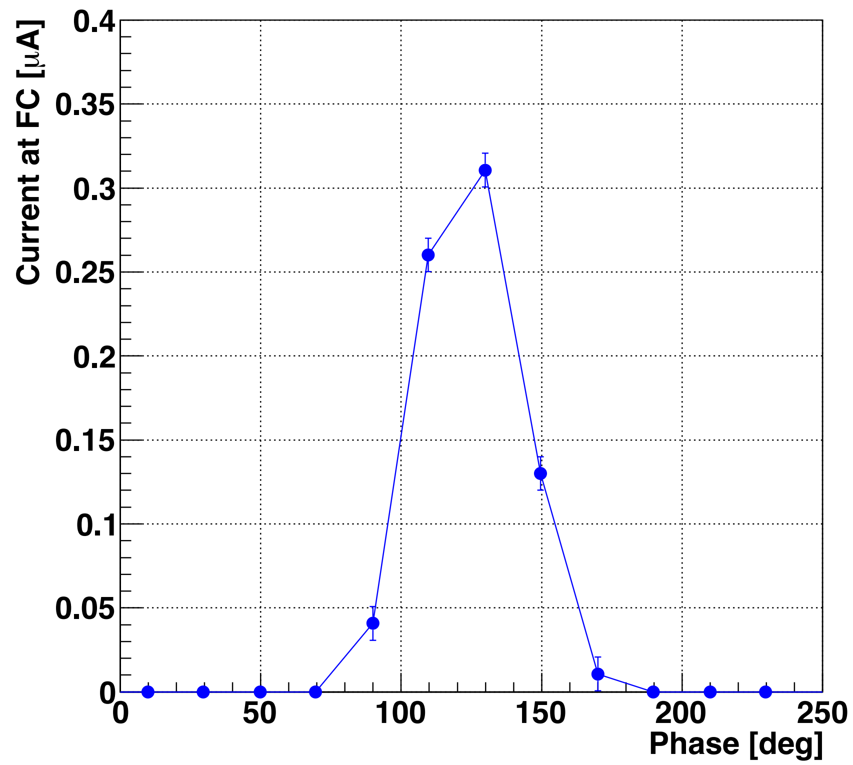
**Beam cannot be injected in the stationary bucket above transition.**

**Particles injected in the stationary bucket below transition cannot be detected by the Faraday cup.**

→ **Only particles accelerated in serpentine channel can be measured by the Faraday cup.**

# Serpentine加速法の実証実験

- **Results**

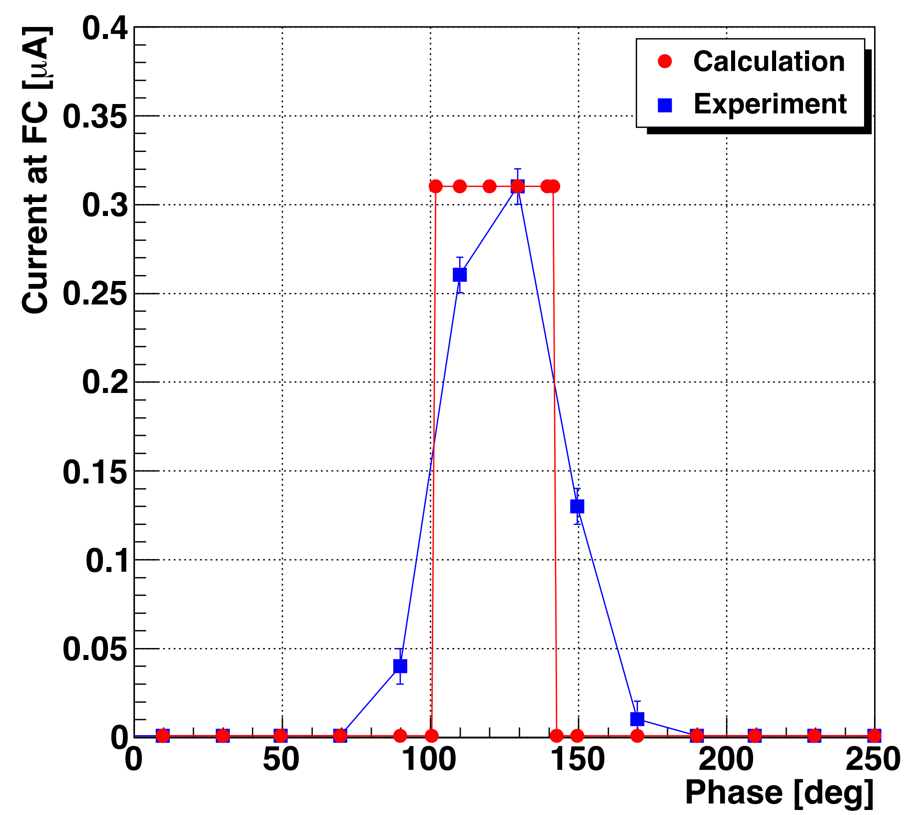



- **Simulation**

**Single particle longitudinal tracking has been done.**

When the particle reaches 8 MeV,
current is set to the maximum measured value.

**There is a discrepancy due to the phase range of injected beam.**



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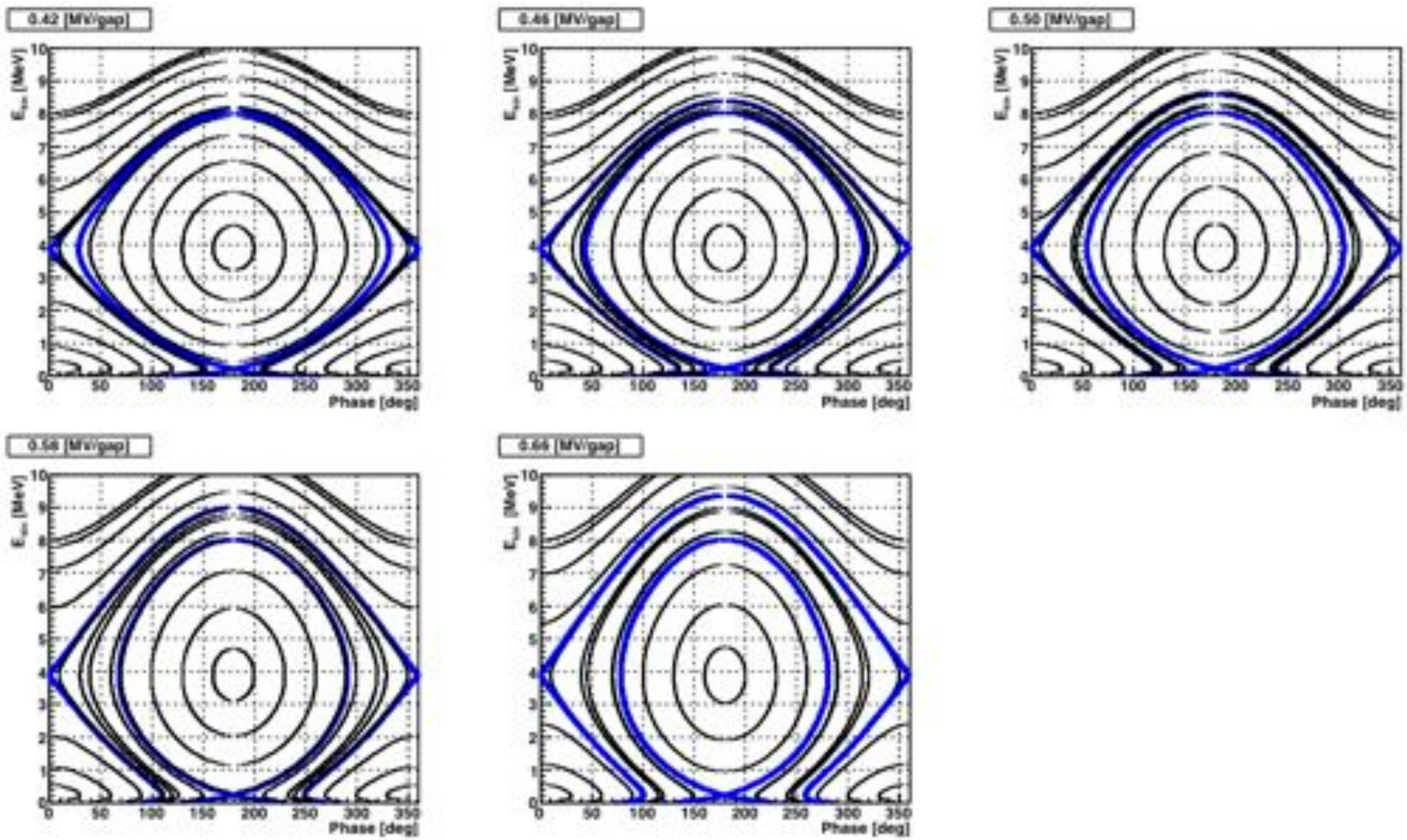
# Phase acceptance measurement

- **Phase acceptance becomes large with rf voltage**



# Phase acceptance measurement

- Injected beam parameters

| | |
|---|---|
| Injected energy | 200 keV |
| Stationary energy ($E_{s1}$) | 205 keV |
| Injected beam phase | < 20% of rf period |
| Rf voltage | 420 to 660 kV/gap |

- Measurement conditions

**- Beam current in the ring is measured by Faraday cup**

**Faraday cup is fixed at 8 MeV closed orbit**

**- Central phase of injected beam is changed with respect to the rf phase at the cavity.**

↓

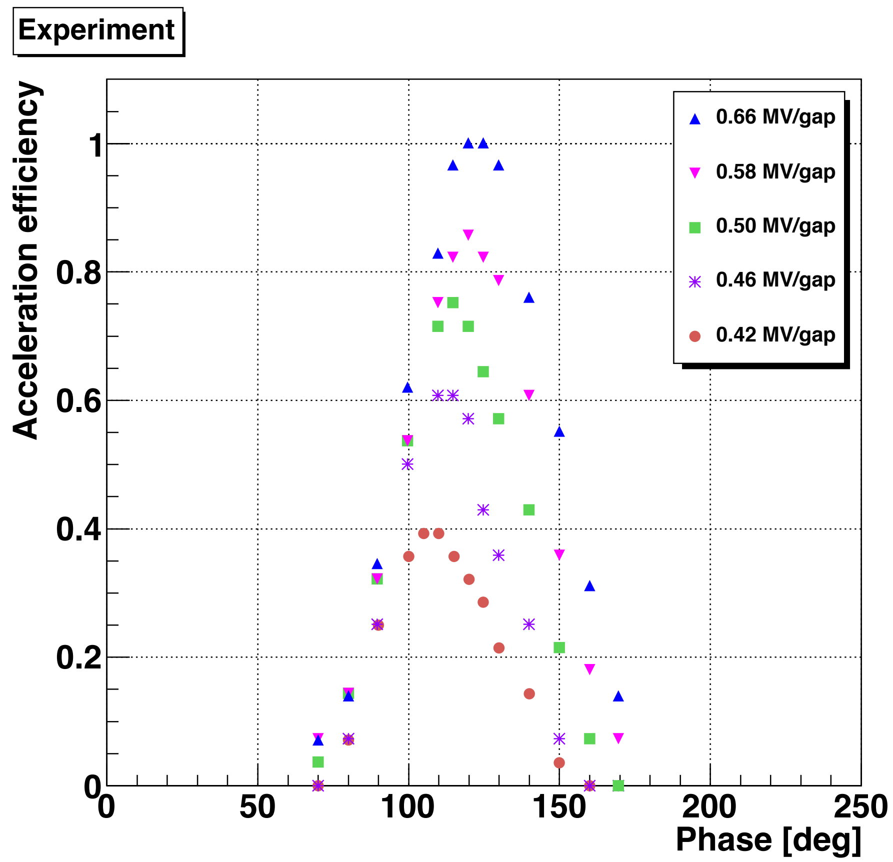
**Beam cannot be injected in the stationary bucket above transition.**

**Particles injected in the stationary bucket below transition cannot be detected by the Faraday cup.**

→ **Only particles accelerated in serpentine channel can be measured by the Faraday cup.**

# Phase acceptance measurement

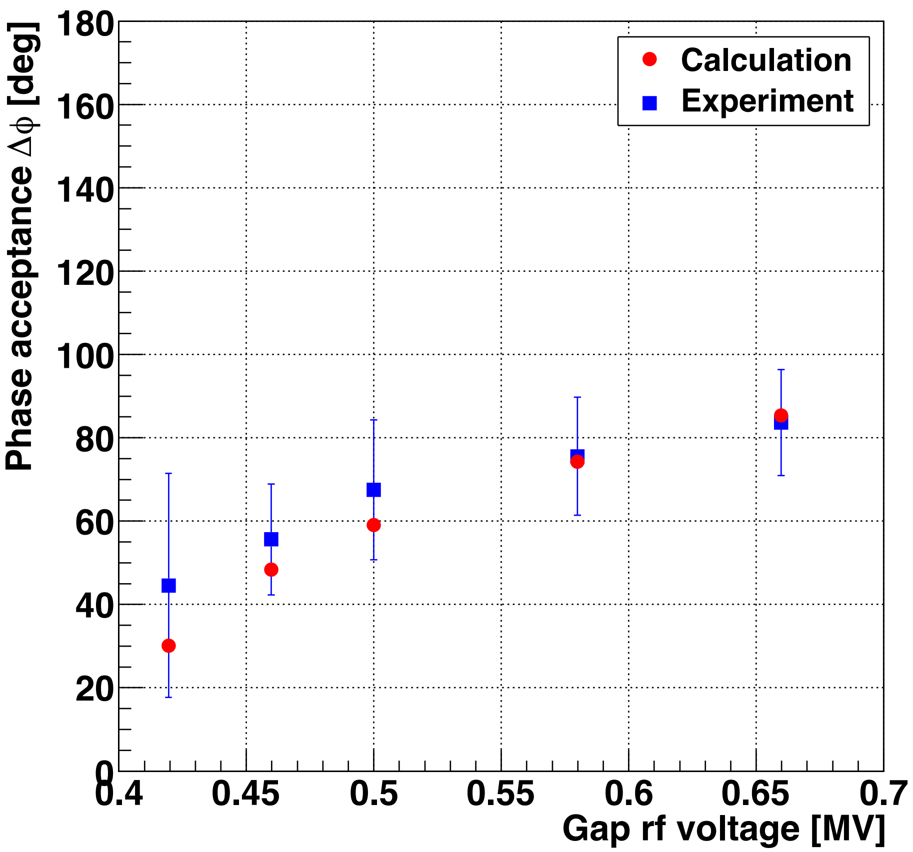
- **Results**




- **Simulations**

**Single particle longitudinal tracking has been done.**

**Tendency of measurement results agree with the simulations.**



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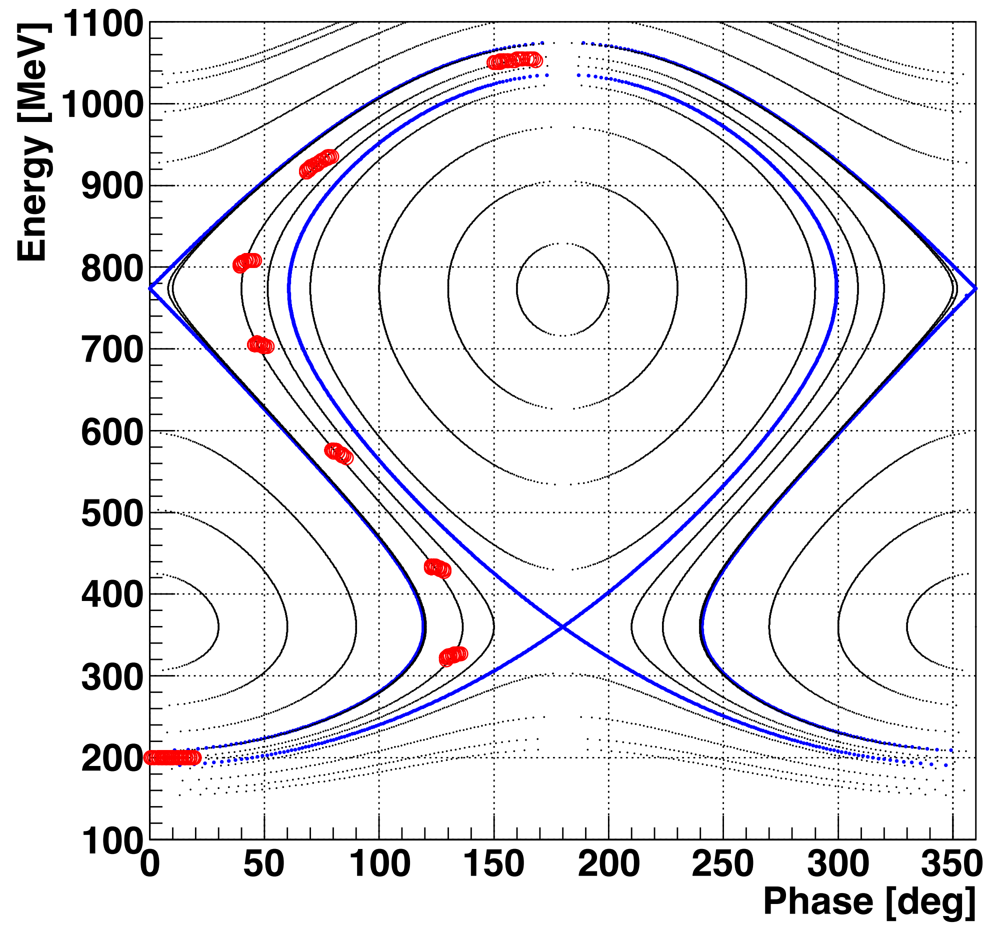
# Application of serpentine acceleration

- Proton driver for ADS

| | |
|---|---|
| $k$-value | 1.45 |
| Equivalent mean radius at 200 MeV [m] | 3 |
| Equivalent mean radius at 1 GeV [m] | 5.9 |
| Stationary kinetic energy below transition [MeV] | 360 |
| rf voltage [MV/turn] | 15 ($h$=1) |
| rf frequancy [MHz] | 9.6($h$=1) |



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Summary and Conclusion

# Summary and Conclusion

- A new type of fixed rf acceleration in scaling FFAG has been developed analytically.
- Proof of principle has been conducted experimentally with electron machine.
- Application of serpentine acceleration has been proposed with longitudinal design of proton driver for ADS
- This success also opens new possibilities for neutrino factory.