FIELD MAP EXPERIENCE

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Contents

A NOTE IN INTRODUCTION

- This is not a review of the field.
I relate my own, limited, experience.

- There has been much more done, by many colleagues, with possibly different tools, in that very field of the use and manipulation of field maps in periodic structures, and related computer tool developments, as we know :

Japan R/D

PAMELA

EMMA

...

1 Motivations

- The presumable interest of the studies presented here stems from three hypotheses :

(i) "Realistic 6-D transmission simulations in FFAGs
can only be based on ray-tracing methods. "

The idea is not new however : it motivated active ray-tracing code developments, (on the first computers !) for that very purpose of FFAG design and experiental studies, in the MURA times, 1950s.

In addition to what,

(ii) "the closest distance we can get to real life
is by representing fields by means of field maps, "

- and in particular not just by analytical models of $\vec{B}(x, y, z)$, however sophisticated they be - although... see later : KEK 150 MeV FFAG and RACCAM simulation experience in that matter -

- neither by sticking to first or higher order types of mapping methods.

and, third

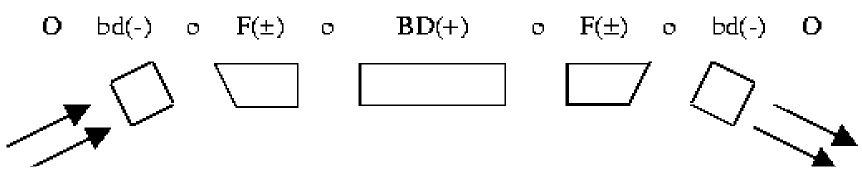
(iii) Stepwise ray-tracing computer codes (that's what we need to handle field maps !) are nowadays, or can be made, as efficient as mapping-type codes in delivering machine parameters. They are unbeatable in large-excursion dynamics.

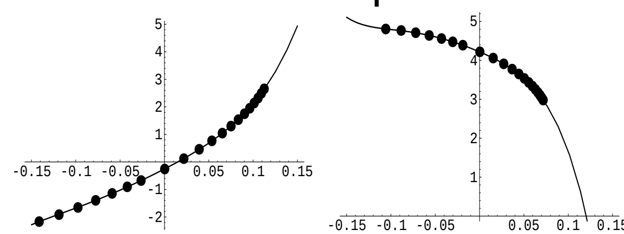
Let's discuss further some of the considerations that these hypotheses are based on.

- FFAG simulations deal with very special bending magnets, like
  - variable gap, sector, spiral dipoles,
  - with strong radial- and axial-dependence of fields
    $B_{zi}(r,\theta) = B_{z0,i}\,\mathcal{F}_i(r,\theta)\,\mathcal{R}_i(r)$

    as for instance in

    - Scaling FFAG, NC magnets,
      $\mathcal{R}_i(r) = \left(\frac{r}{R_{0,i}}\right)^{K_i}$
    - SC magnets,
      $\mathcal{R}_i(r) = b_{0_i} + b_{1_i}\frac{r-R_{0,i}}{R_{0,i}} + b_{2_i}\left(\frac{r-R_{0,i}}{R_{0,i}}\right)^2 + \ldots$
    - various classes of NS-FFAG dipoles, where those dependences are taylored so to reduce tune variation,
    - the isochronous pumplet cell with its highly non-linear radial field dependence.





    - etc.

- Proximity of the magnets in the multiplet arrangements, or compactness of the cell as for instance in the EMMA doublet, may strongly impact on the shaping of both $\mathcal{F}_i(r,\theta)$ and $\mathcal{R}_i(r)$.

• Gap variation, such as $g \approx g_0(R_0/r)^K$ in scaling magnets, and its effect on the shaping of the radial dependence of the field, have to be accounted for.

• Overlapping of fringe fields as well, since the flutter has a crucial role on the vertical focusing and on the correlated evolution of axial and radial tunes.

• Longitudinal dynamics (adding RF in the lattice) complicates things further, it raises such issues as

- determining precisely the geometry of the reference trajectory, at all momenta,
- accurate computation of time of flight and frequency law,
- finding the "accelerated orbit" (e.g., the layout of PSI cyclotron spiral sector magnet accounts for that [W. Joho, priv. comm.]),
- synchronous phase stabilty in relation with the above.

“Symplecticity” ?

• Resorting to simplectization methods is not necessary,
regardless of the fact that it can be dangerous, for instance in large excursion tracking, e.g. dynamiv aperture.

• There are ways of controlling the accuracy of numerical integration, as

(i) a good integrator and a good numerical usage of it,

(ii) the order of the integrator, e.g.,

- the order of the $\vec{R}$, $\vec{u}$, $\vec{S}, B\rho$, time, Taylor series expansions in Zgoubi,

- the value of “n” in RKn,

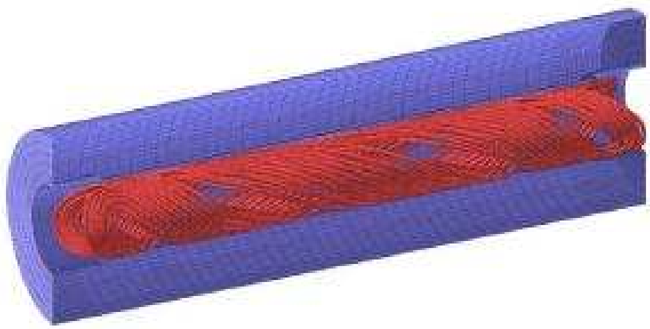
(ii) the integration step size.

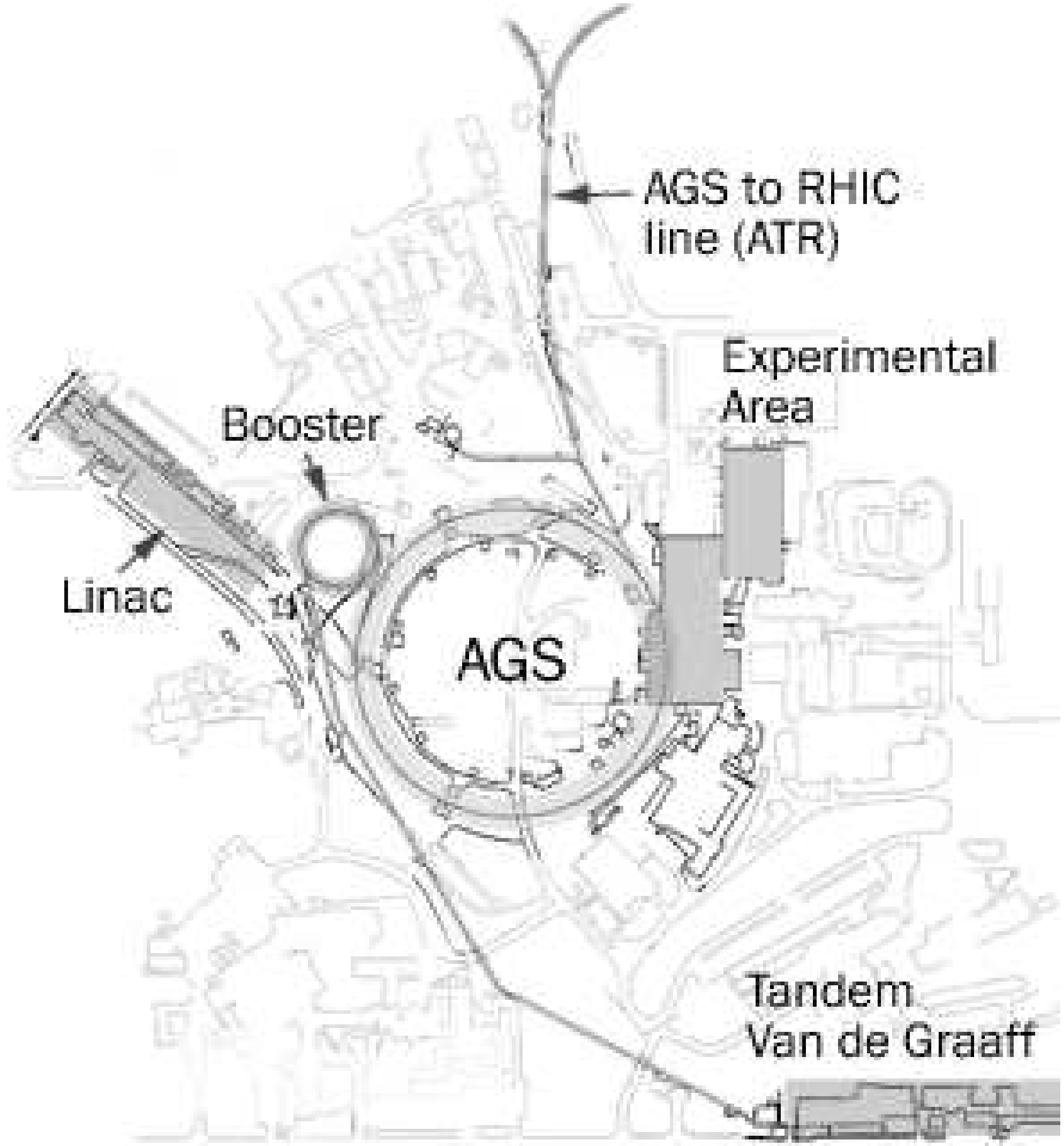
2 Two examples of the use of field maps - to warm up !

A recent one : the AGS - challenging, too !

- Particle and spin dynamics in the AGS are now routinely computed using 240+2 field maps

- 240 main magnets. Field maps are measured field maps
- 2 helical snakes, one SC, one warm, 3-D OPERA maps





- Typical parameters in the AGS :

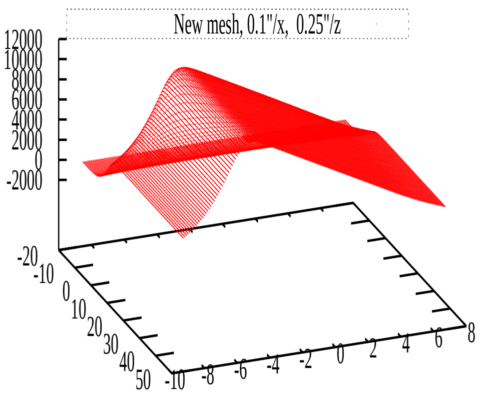
circumference	(m)	807.0909
bare tunes Qx, Qy		8.721, 8.785

- The 240 main dipoles form three families, A, B, C, and their three mirror-symmetric. B, 79 inches, is just a shorter version of the A dipole, 94 inches. Thus 2 measured field maps suffice to describe the AGS lattice.

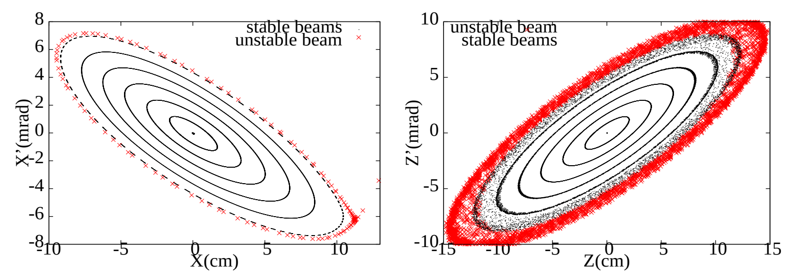
  12 measurement currents covering the AGS energy range are available.

  The mesh size is 0.1 inch transverse, 0.25 inch longitudinal.

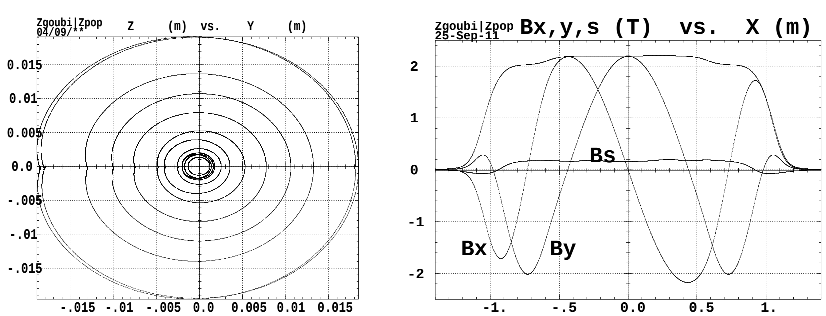
- Typical ray-tracing results.



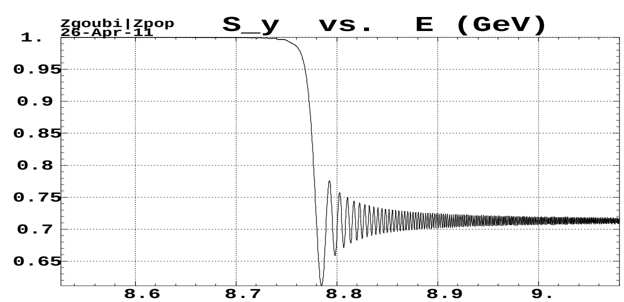
Measured field map.



1000-turn stability limits, horizontal (left) and vertical (right).

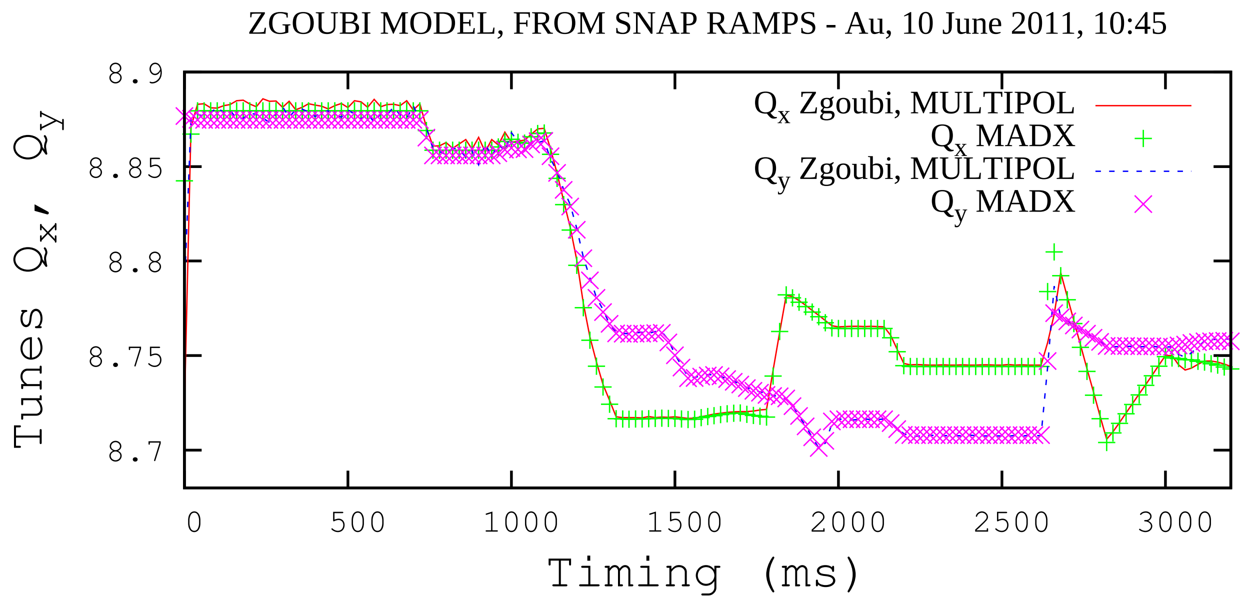


Orbits and field across a snake.



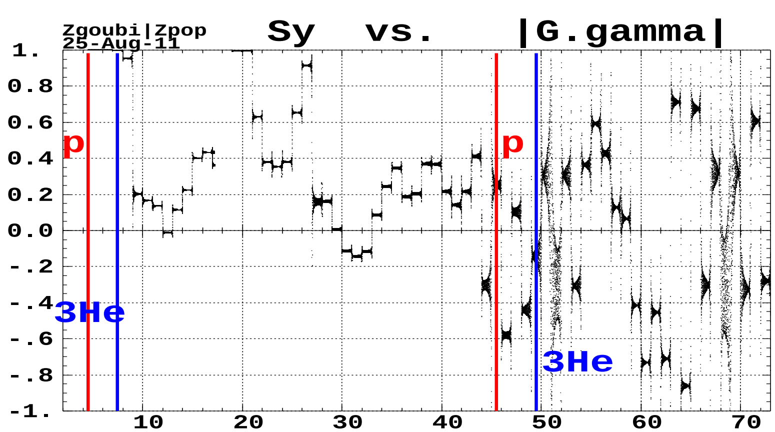
S_y, acceleration across $G\gamma = Q_y$ spin resonance, ≈3000 turns.

- A tune scan of the AGS over the all 3 second cycle,
using main magnet field maps,
together with real-life AGS optical settings as read from archived data files.

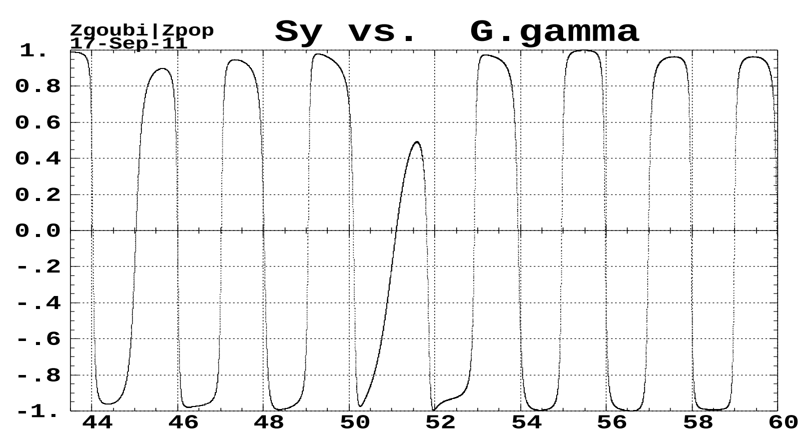


- Spin tracking across AGS cycle (analytical model of the main magnet + 3D OPERA maps of the helical snakes)

Imperfection resonances



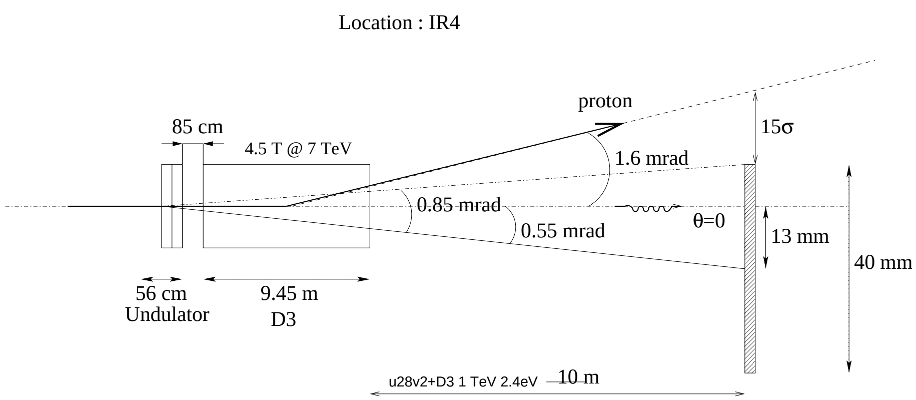
Snakes on !

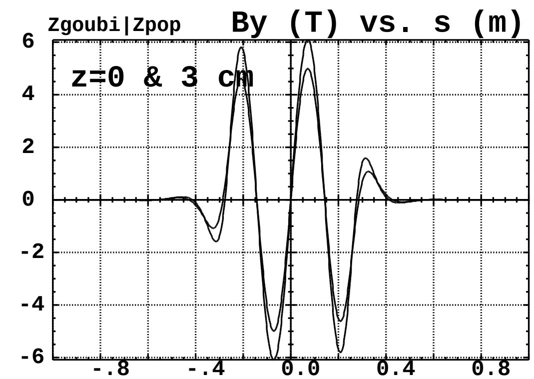


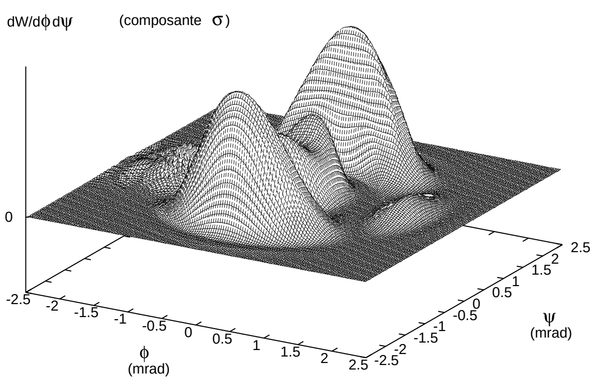
A curiosity, second example : LHC

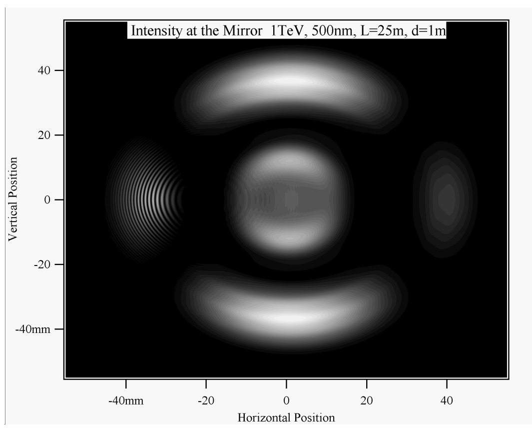
This won't be as convincing an argument as the previous one... a 1m/27000m one !
however it has the virtue of resulting from the same ideas about ray-tracing methods. It was also one of the first experiences of a field map in a periodic structure, in Zgoubi.

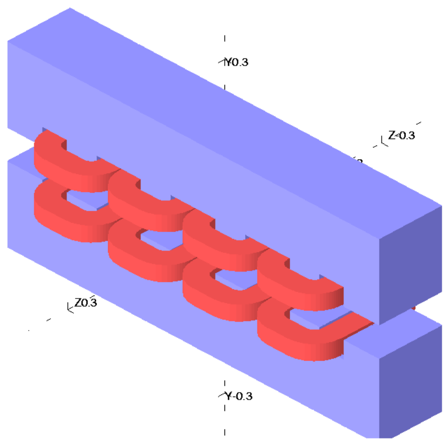
- The story is the following : the LHC undulator-SR diagnostic installation has been designed in the early 2000s using Zgoubi.











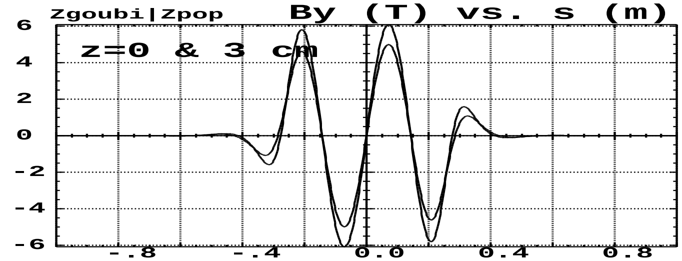
Radiation diagram at λ = 500 nm,
from 1 TeV proton in D3+U280 [Yellow Report CERN-2004-007].

You said, “strong field undulator in LHC” ?

- In 2004 the question was raised of the impact of the undulator on LHC parameters and BD.

Main ingredients in the problem :

- field is strong, reaches 5 Tesla, introduces non-linearities
- field integral $\int B_y(s)\,ds \approx$zero
- beam is shifted
- the undulator is short, 80 cm



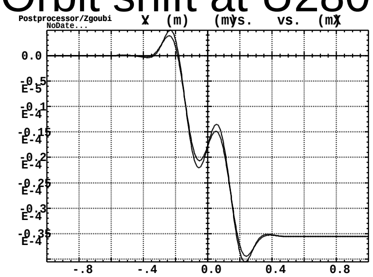
- Undulator field map had been calculated using OPERA 3D [M. Sassowsky]

Its emprical mid-plane behavior was devised for use in Zgoubi, as well :

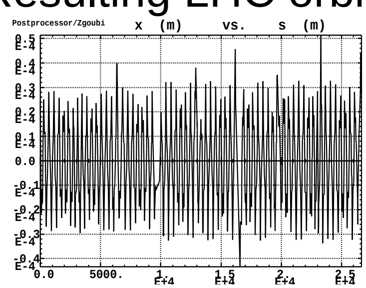
$$B_y(x,y,s)|_{y=0} = \frac{B_u}{1+(s/0.28901)^8}\sin\frac{-2\pi s}{\lambda_u},\quad B_u = 5.096\,\mathrm{T},\quad \text{peak field, and}\quad \lambda_u = 0.28538\,\mathrm{m},\quad \text{period.}$$

- Regular assessments followed, as we do in FFAGS !

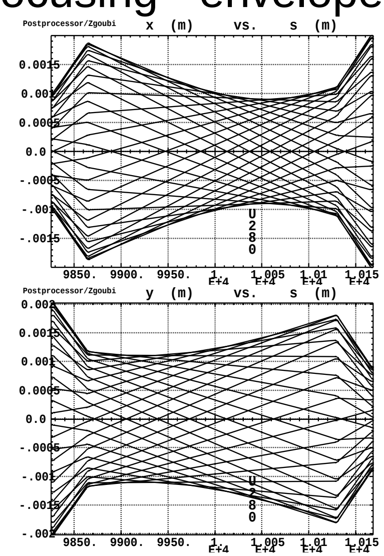
Orbit shift at U280 :



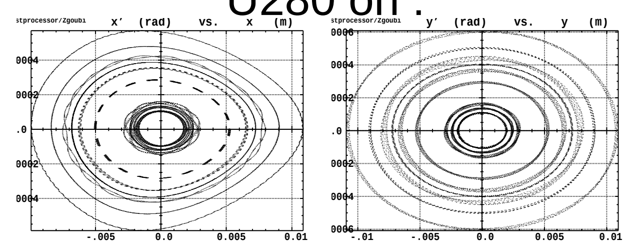
Resulting LHC orbit :



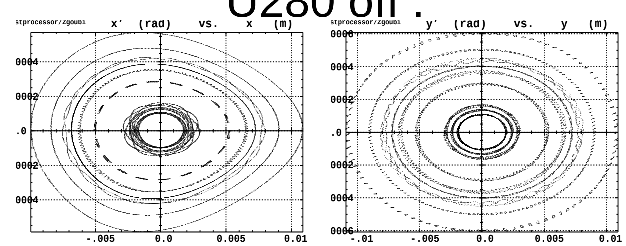
Focusing - envelopes :



Max. stable amplitudes, H, V, U280 on :



Max. stable amplitudes, H, V, U280 off :



3 Need to be able to “FIT”

The possibility of completing fitting trials is very useful in FFAG lattice and magnet design and optimization.

This is an indispensable tool for usual issues, as
- optimizing orbit, focusing, further including “accelerated orbit” in presence of RF,
- error corrections, resonance compensation studies, beam transmission efficiency, ...

In matter of fit constraints :
We want, and we can get from the ray-tracing method, much more than the classical constraints found in matrix- and other mapping-type codes.

- Trajectories, orbit :
  - coordinates, orbit excursion, ...
  - orbit closure, i.e., final coordinates == initial coordinates, be it spin orbit, particle orbit, ...
- Focusing :
  - tunes, chromaticites, anharmonicities
  - first and higher order transport coefficients
  - beam matrix coefficients (waist, divergence)
  - non-paraxial waists and envelopes
- Beam transmission
  - phase-space or physical collimation - allowing for e.g. decay (muons)

and so on, table next slide.

• An example of real-life application of these ideas : Cf. table of constraints, Zgoubi users' Guide

Type of constraint	Parameters defining the constraints								Object definition (recommended)
	IC	I	J	Constraint	Parameter(s) #	values			
σ-matrix	0	1 - 6	1 - 6	σ_{IJ} $(\sigma_{11}=\beta_Y, \sigma_{12}=\sigma_{21}=\alpha_Y$, etc.)					*OBJET/KOBJ=5,6*
Periodic parameters	0.N	1 - 6	1 - 6	σ_{IJ} $(\sigma_{11}=\cos\mu_Y+\alpha_Y\sin\mu_Y$, etc.)					*OBJET/KOBJ=5.0N*
		7	any	Y-tune = $\mu_Y/2\pi$					
(N=1-9 for *MATRIX*		8	any	Z-tune = $\mu_Z/2\pi$					
block 1-9))		9	any	$\cos(\mu_Y)$					
		10	any	$\cos(\mu_Z)$					
First order transport coeffs.	1	$1-6$	$1-6$	Transport coeff. R_{IJ}					*OBJET/KOBJ=5*
		7	i	$i\neq 8$: YY-determinant ; i=8 : YZ-det.					
		8	j	$j\neq 7$: ZZ-determinant ; j=7 : ZY-det.					
Second order transport coeffs.	2	$1-6$	$11-66$	Transport coeff. $T_{I,j,k}$ $(j=[J/10], k=J-10[J/10])$					*OBJET/KOBJ=6*
Trajectory coordinates	3	$1-IMAX$	$1-7$	$F(J,I)$					*[MC]OBJET*
		-1	$1-7$	$< F(J,i) >_{i=1,IMAX}$					
		-2	$1-7$	$Sup(\|F(J,i)\|)_{i=1,IMAX}$					
		-3	$1-7$	$Dist\|F(J,I)\|_{i=I1,I2,dI}$	3	I1	I2	dI	
	3.1	$1-IMAX$	$1-7$	$\|F(J,I)-FO(J,I)\|$					
	3.2	$1-IMAX$	$1-7$	$\|F(J,I)+FO(J,I)\|$					
	3.3	$1-IMAX$	$1-7$	min. (1) or max. (2) value of $F(J,I)$	1	1-2			
	3.4	$1-IMAX$	$1-7$	$\|F(J,I)-F(J,K)\|$ $(K=1-IMAX)$	1	K			
Ellipse parameters	4	$1-6$	$1-6$	σ_{IJ} $(\sigma_{11}=\beta_Y, \sigma_{12}=\sigma_{21}=\alpha_Y$, etc.)					*OBJET/KOBJ=8 ; MCOBJET/KOBJ=3*
Number of particles	5	-1	any	$N_{survived}/IMAX$					*OBJET*
		$1-3$	any	$N_{in\ \epsilon_{Y,Z,X}}/N_{survived}$	1	ϵ/π			*MCOBJET*
		$4-6$	any	$N_{in\ best\ \epsilon_{Y,Z,X,rms}}/N_{survived}$					*MCOBJET*
Spin	10	$1-IMAX$	$1-4$	$S_{X,Y,Z}(I)$, $\|\vec{S}(I)\|$					*[MC]OBJET*
	10.1	$1-IMAX$	$1-3$	$\|S_{X,Y,Z}(I)-SO_{X,Y,Z}(I)\|$					*+SPNTRK*

• The usage of field maps is to a large extent compatible with these requisites of flexibility.

This has been done already : cf. design of the spiral dipole in RACCAM, and in the KURRI 1 GeV spiral FFAG. Lattice and magnet design and optimization were handled as a one and same task, using Ray-Tracing+OPERA field maps.

• Various "tricks" bring flexibilty in field map manipulations :

- Normalization factor to the field amplitude.

- Developped for EMMA :
  * Independent field maps can be linearly combined
  * The coefficients in the linear combination can ve varied in the `'FIT'` procedure, as well as the transverse position of the maps.
  * Field maps can be slided (like EMMA quads)
  * In a general manner accounting for positioning errors is straightforward,

- Running field map can be interpolated from a set of field maps at different currents.

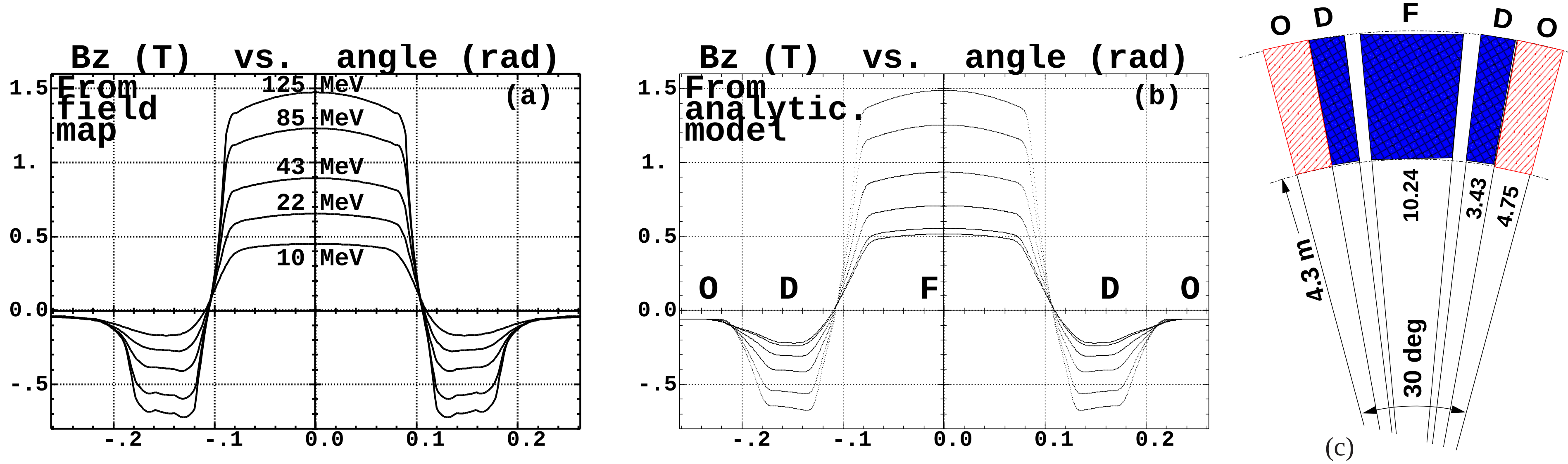
- The field at particle location, as interpolated in the field map, can be modified by a perturbation factor.
  For instance in the case of the AGS we apply
  $(1+dK_1/K_1)$ for tweaking tunes, for instance so to have equal model's and measured tunes,
  $(1+dK_2/K_2)$ for tweaking chromaticities.

4 More examples of the use of field maps

4.1 KEK 150 MeV FFAG. 6-D tracking in field maps

• A lot has been done at KEK using the OPERA field maps. I did a few things on my side, we'll go over that, here.

• A **"FFAG"** procedure. Which withstands the comparison with field from field maps.



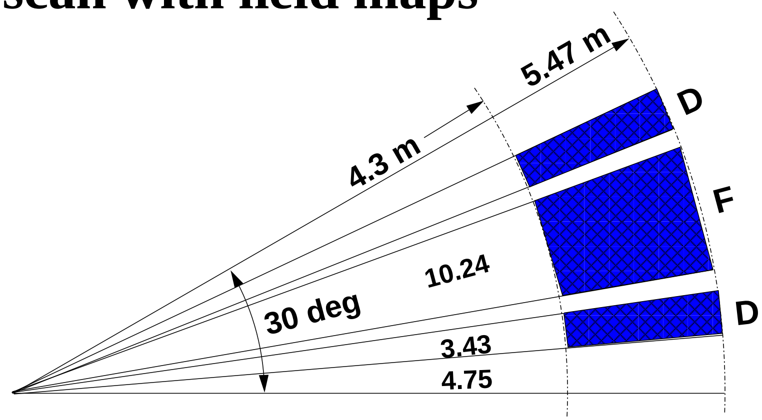
(a) TOSCA 3-D map representative of the 150 MeV FFAG, and,

(b) Field from the "3+2"-dipole geometrical model.

(c) Geometry of the "3+2"-dipole design, including two additional dipole regions (hatched) that simulate 700 G field extent over the two end drifts.

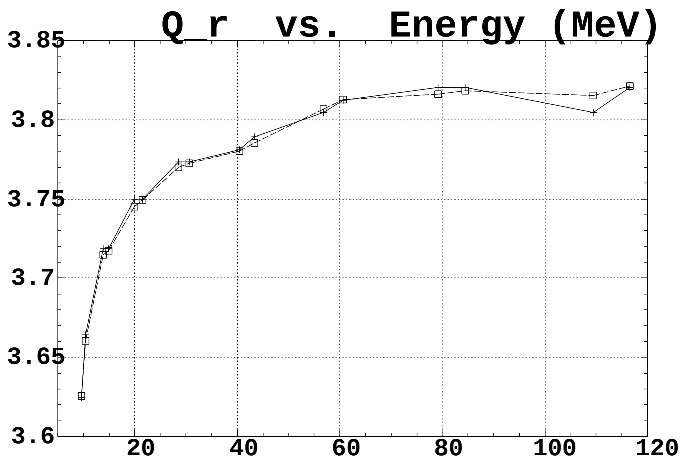
Note : the positioning of the dipoles with resp. to one another is arbitrary, so that method can be used to insert a magnetic element *inside* the triplet, for instance a septum [Kurasho-san].

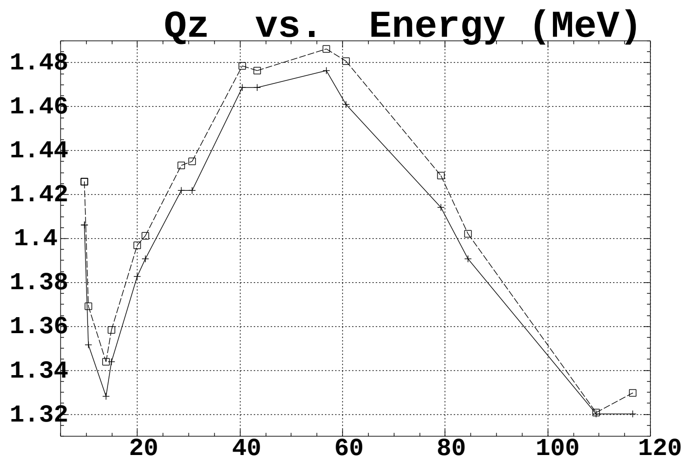
Tune scan with field maps



[Details in : CERN NuFact Note 140 (2004)]

Geometry of the DFD sector triplet and
30 degrees sector cell.

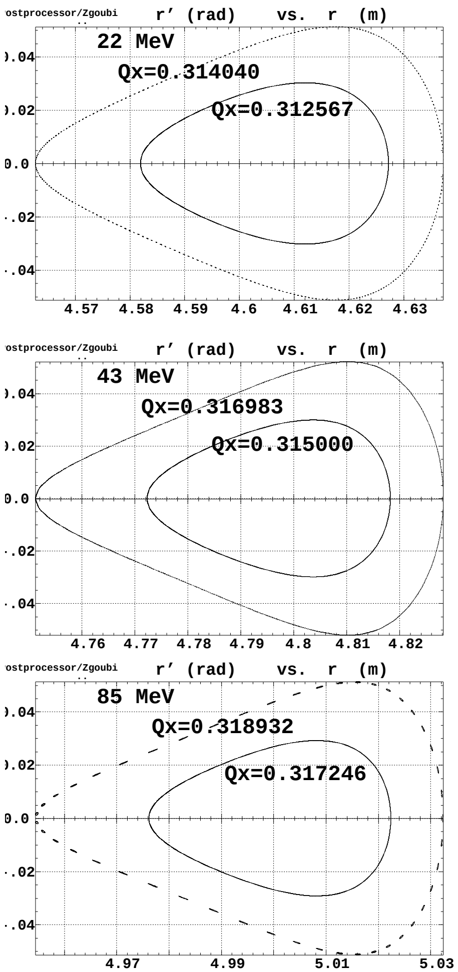




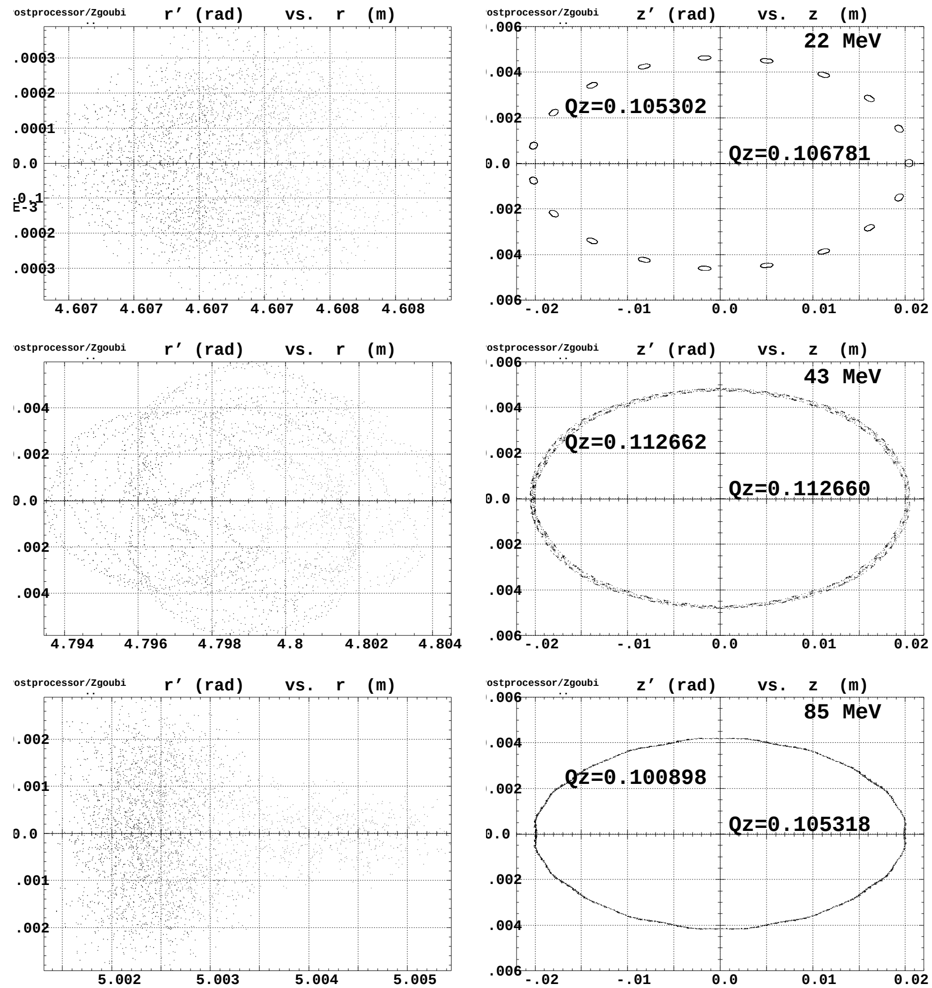
Radial tune (left plot) and axial tune (right) as a function of energy,
as obtained using RK4 integration (solid lines/crosses - M. Aiba),
or using Zgoubi (dashed line/squares).

4-D tracking, "long-term"

The results below make clear that the symplecticity is very good (precision to order Δs^6 here). Note that, the mesh size needs be very small, $\approx$mm, and the integration step size must follow.

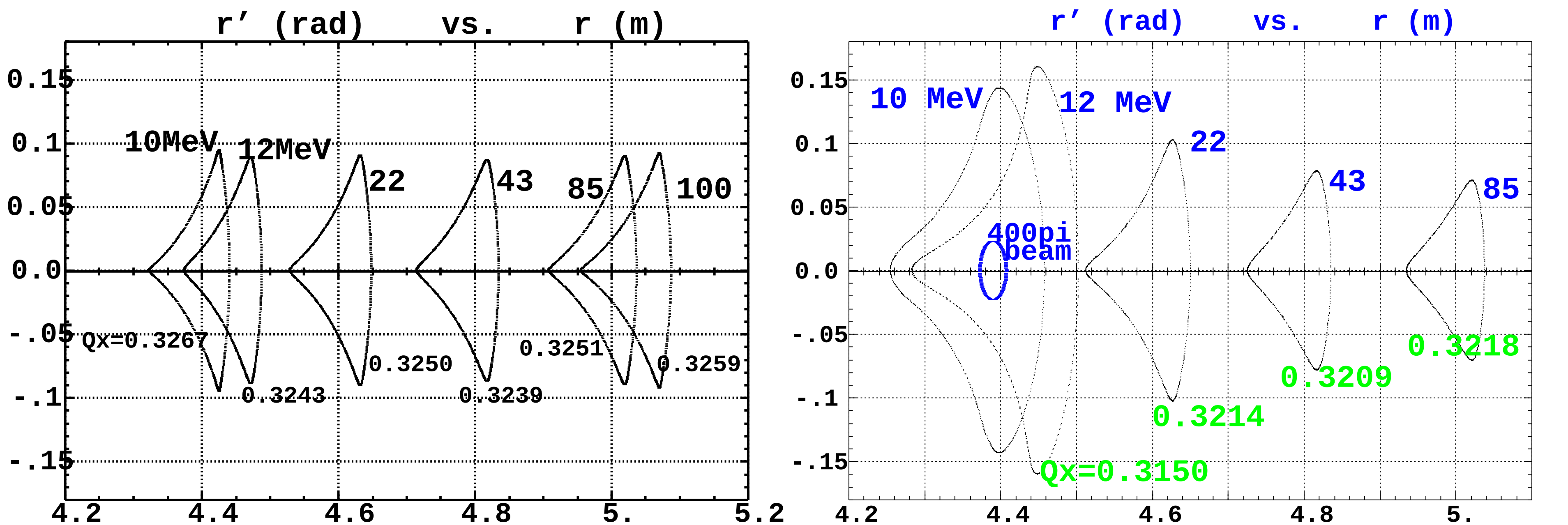


Horizontal motion, near stability limit. The inner motion is 3500 pass in a cell the outer one is 4700.



Right column : vertical phase-space for $z_0 = 2$ cm with $r_0 = r_{closed\ orbit}$. Left column : corresponding horizontal motion. 3200 periods.

Large excursion motion : dymanmic aperture



150MeV FFAG : horizontal phase space, the limits of stable motion, for 5 energies.

For comparison : tracking with geometrical model (left), or using TOSCA map (right).

6-D transport using field maps

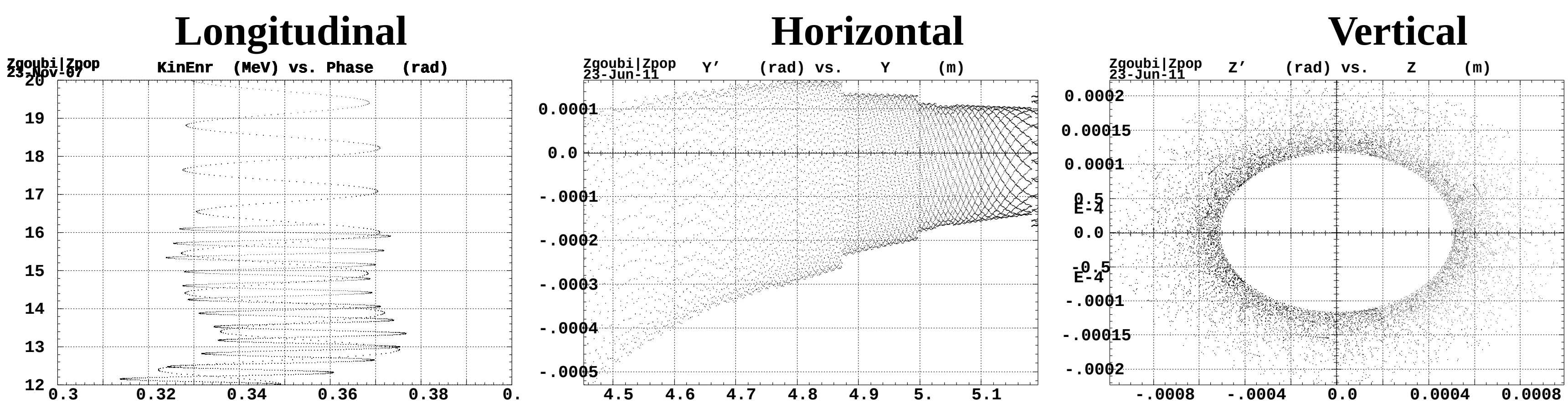
- Here, the frequency law is generated using

$$\tau = 2\pi r/\beta c = \tau_0 \left(\frac{p}{p_0}\right)^{\frac{-k}{k+1}} \frac{E+m}{(E_0+m)}$$

and saved in a dedicated file.

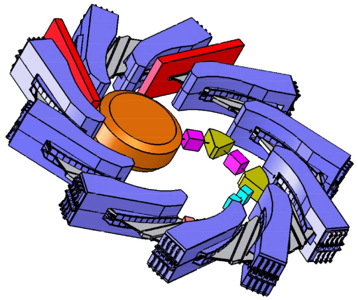
An alternate option is to generate it from series of closed orbits spanning the full energy bite.

- Acceleration to 100 MeV, using that method :

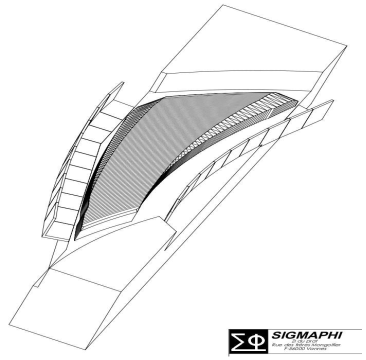


Note, PAMELA has been accelerated using that procedure, as well [ref. : T. Yokoi].

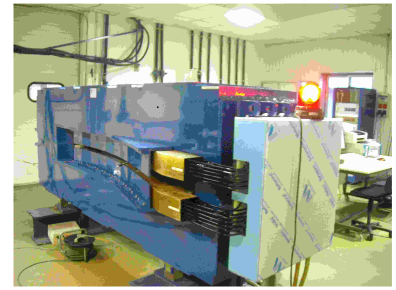
4.2 RACCAM [Ref. : PAC 09]



RACCAM protontherapy ring (by Jaroslaw, Thomas, Joris...)

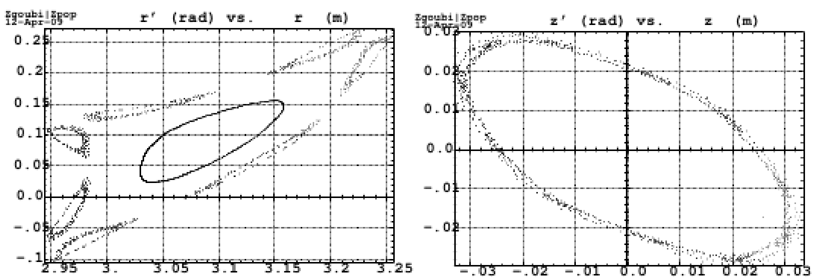


A sketch of the spiral dipole.

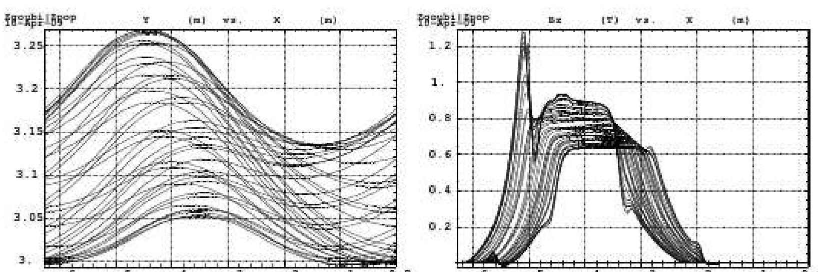


RACCAM dipole at SIGMAPHI during mag. measurements.

- Tracking in the measured 3-D field maps

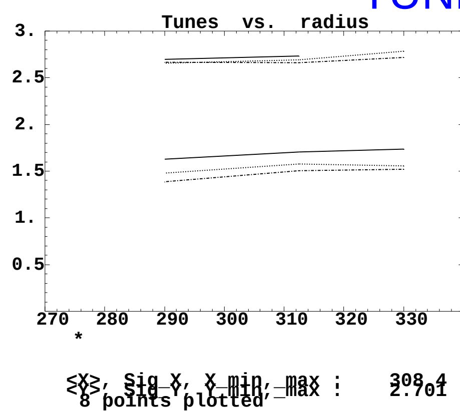


H and V stability limits, 1000-turn.

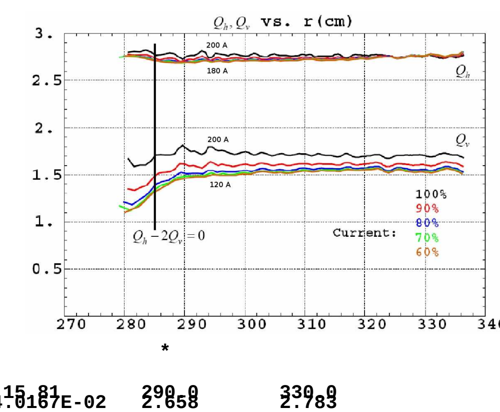


Multiturn trajectory & field, on H limit.

TUNES



From measured field maps.



From design stage, TOSCA field maps.

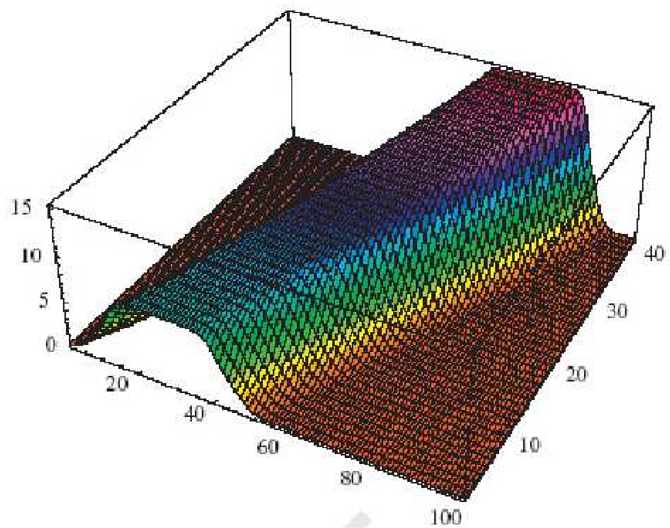
'FFAGSPI'

• Just like for the radial DFD triplet, a procedure has been developped to simulate a spiral FFAG dipole [Ref. : NIM A 589].

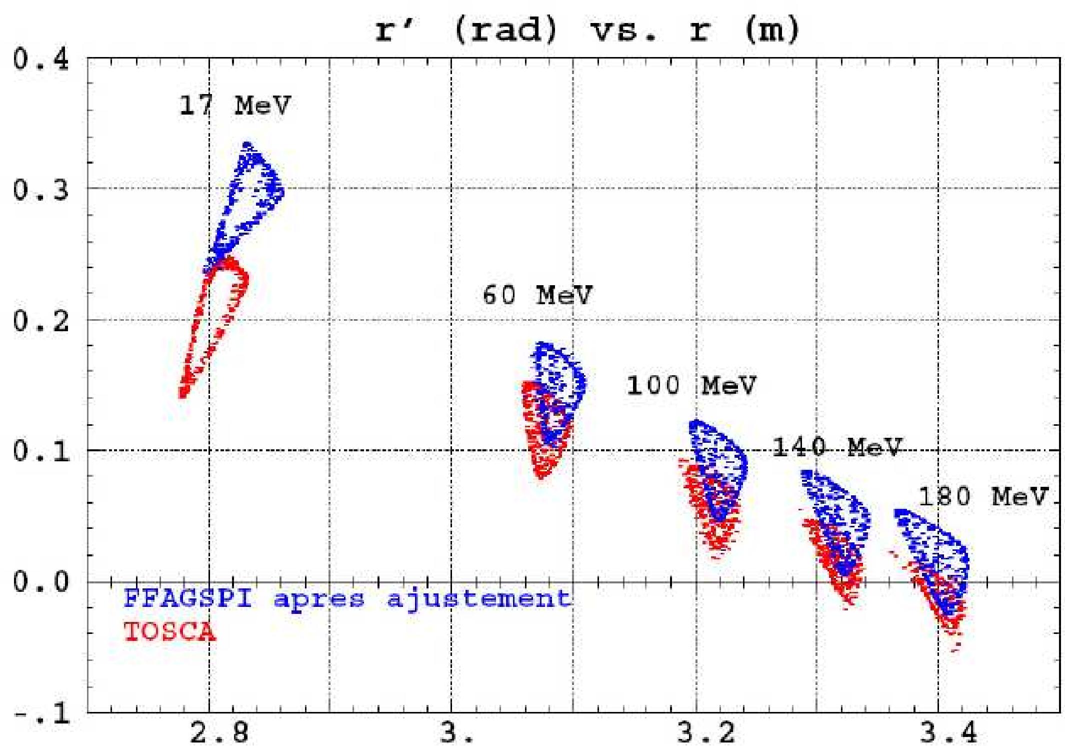
• 'FFAGSPI' yields, as 'FFAG', very close agreement with OPERA field map as to large excursion beam dynamics, once
- the r-dependence of the fringe field extent on the one hand,
- the r-dependence of B_y,

have been adjusted properly.

Mid-plane field
from 'FFAGSPI'

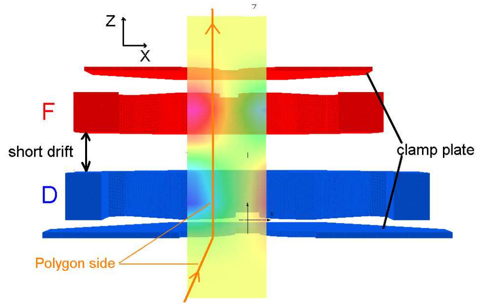


Dynamic aperture
Blue : 'FFAGSPI'
Red : OPERA field map

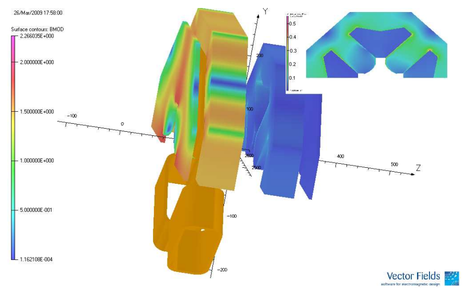


4.3 EMMA [Ref. : PAC 09]

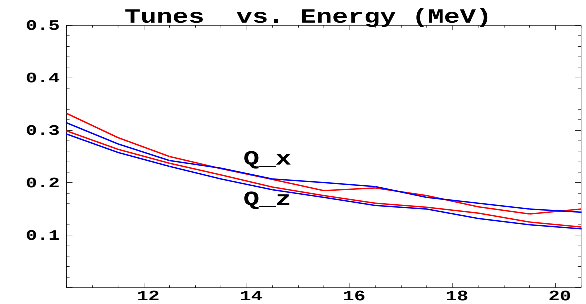
- Key point in the method was to convince ourselves that linear superimposition is legitimate, in that case. It seems it is, and yields very nice beam dynamics results - right column below.



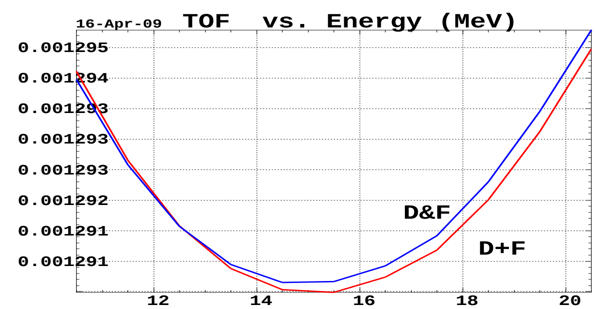
EMMA cell.



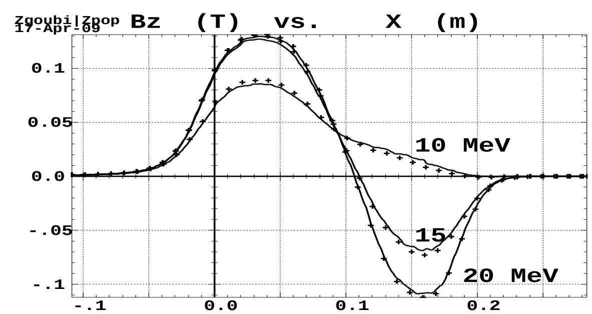
Field seen by the F yoke when only D is "on"



Tunes versus energy, case "D&F" (blue, thick lines) and case "D+F" (red, thin lines).



Time of flight parabola, "D&F" (blue, thick line), "D+F" (red, thin line).



Field along closed orbits, case "D&F" (solid line) and case "D+F" (crosses). .

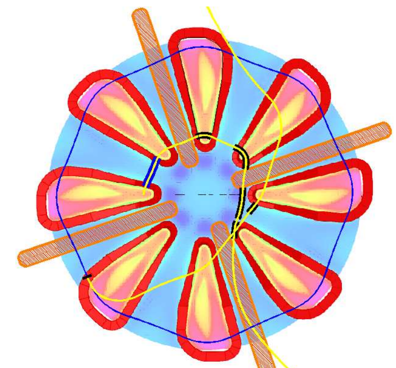
- Superimposing field maps :

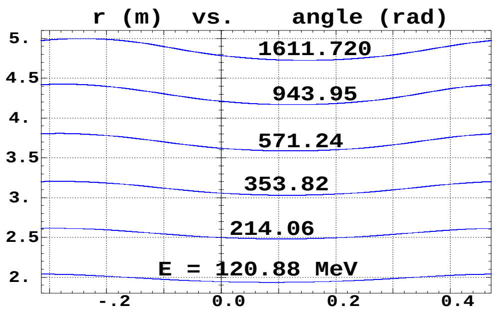
Zgoubi.dat file :

```
'OBJET'
[...]                Defines a set of closed orbits
'EMMA'
0  0
-1E-3 1. 1. 1.     Global normalization of map data
emma cell field map                          Comment
197 81 1  0          Number of nodes in Y,X,Z. Mode
1.  1.  0.                a_F,  a_D,  distance d_FD
Dax265.Fon.cart.table          Name of F quad map
Dax265.Don.cart.table          Name of D quad map
0 0 0 0
2                             Interpolation method
.1                        Integration step size, cm
2 0 0 0                     Field map positionning
'CHANGREF'
0. 0. -8.57142857152            Cell positionning
'END'
```

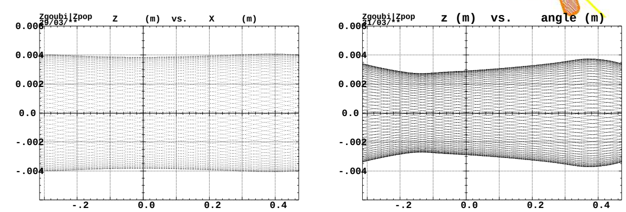
4.4 ADS cyclotron [Ref. : IPAC 12]

- "DAEDALUS", molecular H_2^+, 800 MeV/u, up to 8 MW, 6 Tesla peak field.

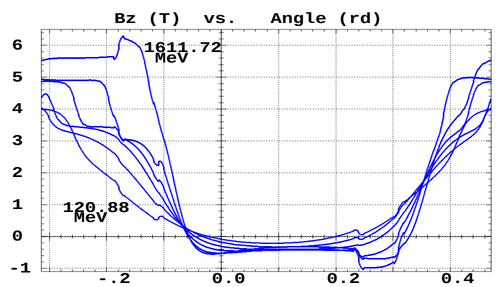




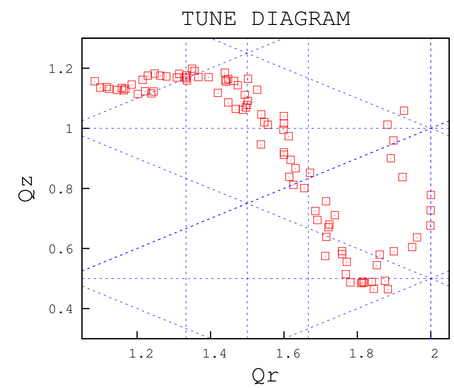
Closed orbits across one of the eight cells of the cyclotron, at six different energies.



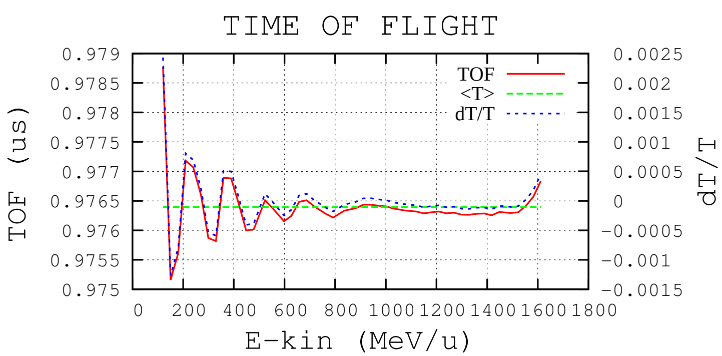
Beam envelopes, typical.



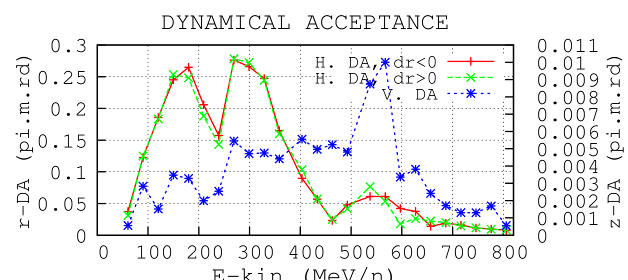
Field on closed orbits across a cell, featuring "shimming" bumps.



Beam path in tune diagram.



Time of flight.



Dynamical acceptance, 500-turn.

5 Conclusions

• Stepwise ray-tracing based methods and the use of field maps in lattice and magnet design and optimization have gone a long way,

- since the MURA experience in the 1950s,
- and, as far as Zgoubi is concerned, since the first steps (!) in the late 1980s when periodic optics concepts and spin tracking were introduced in that former spectrometer code - that was in view of assessing polarization transmission in SATURNE in presence of a partial solenoidal snake.

• The FFAG experience in Zgoubi has confirmed, beyond earlier SATURNE, LHC, now RHIC, two crucial steps towards the efficient use of field maps in machine design, namely

- the feasibility of using stepwise ray-tracing methods and codes in the same way that matrix based techniques are,
- the fact that the ray-tracing method can be made to speak the same language as the matrix-type method in matter design parameters, and of I/Os in a general manner,

• whereas, in addition,

- **using field maps** brings the possibility of making the design and the optimization of the magnet and of the lattice cell a same and single problem, which ray-tracing methods allow handling it as such,
- it is so far the best path to high accuracy, large excursion,
- as shown in these slides, **the use of field maps can be made very flexible, if handled in appropriate manner, and will preserve all the qualities and advantages of the ray-tracing method, including "fitting" techniques, while bringing even further precision.**

THANK YOU FOR YOUR ATTENTION