Fast Integer Resonance Crossing in a Scaling FFAG

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Purpose

To verify experimentally

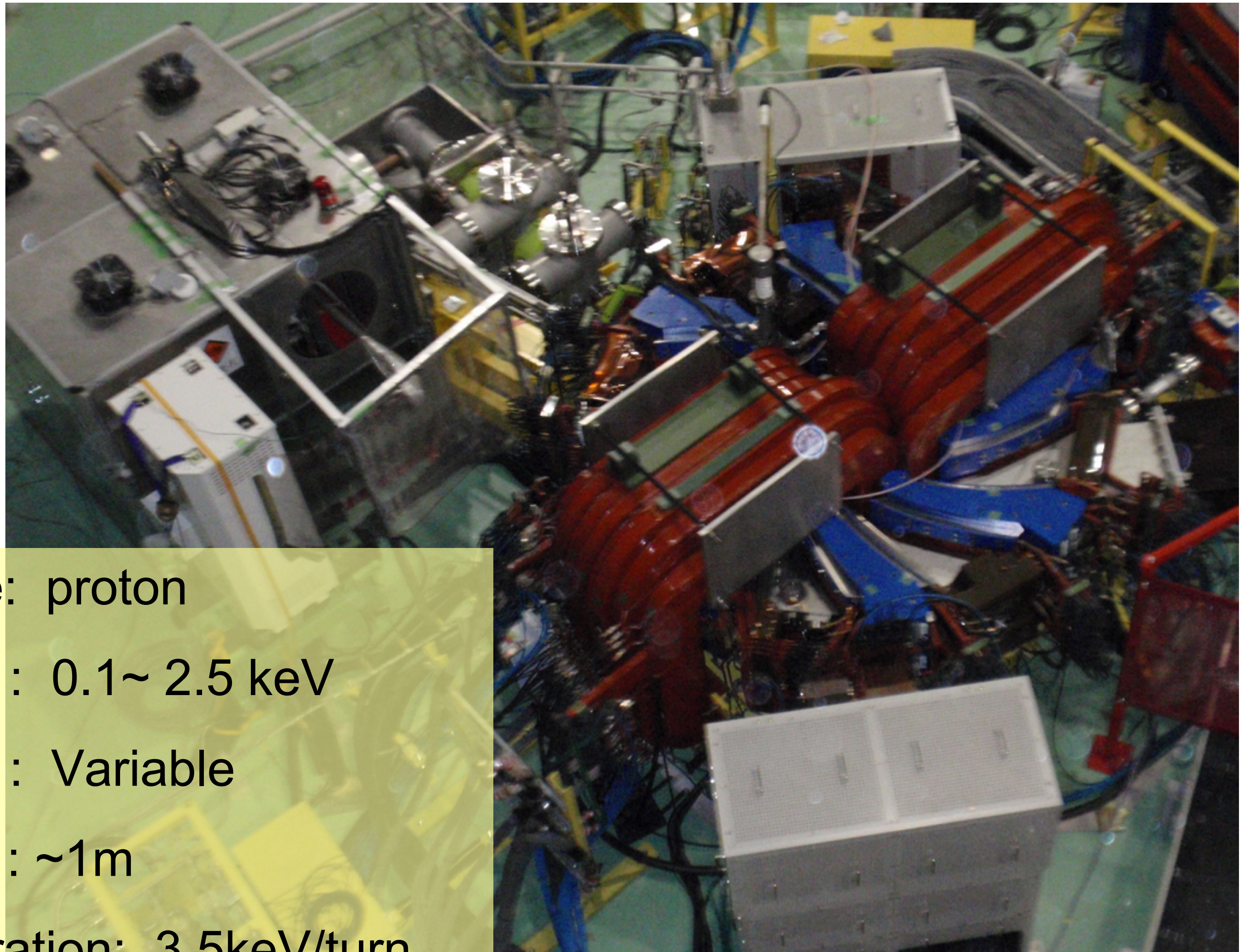
Integer resonance can be crossed,
when the crossing speed is high enough.

R. Baartman, May 2004, ?

$$\Delta A = \pi \frac{b_{n,1}}{\sqrt{Q_\tau}} = \frac{\pi}{\sqrt{Q_\tau}} \frac{\overline{R}}{\overline{B}} \frac{B_n}{Q}$$

This appears in early cyclotron theory because the $Q = 1$ resonance was used to extract the beam. See for example [Al Garren et al, Nucl. Instr. Meth. **18,19**

Injector FFAG in KURRI



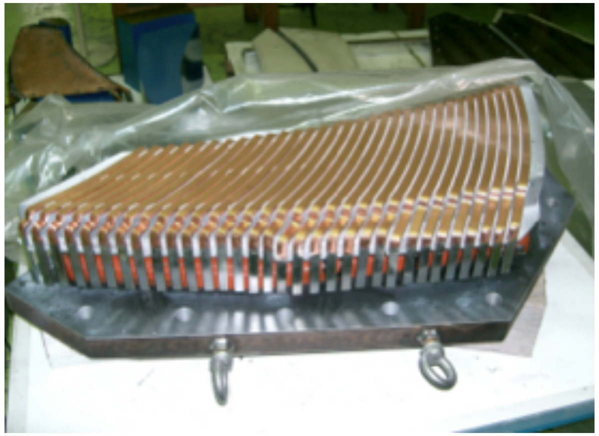
Particle: proton

Energy : 0.1~ 2.5 keV

K : Variable

Radius : ~1m

Acceleration: 3.5keV/turn

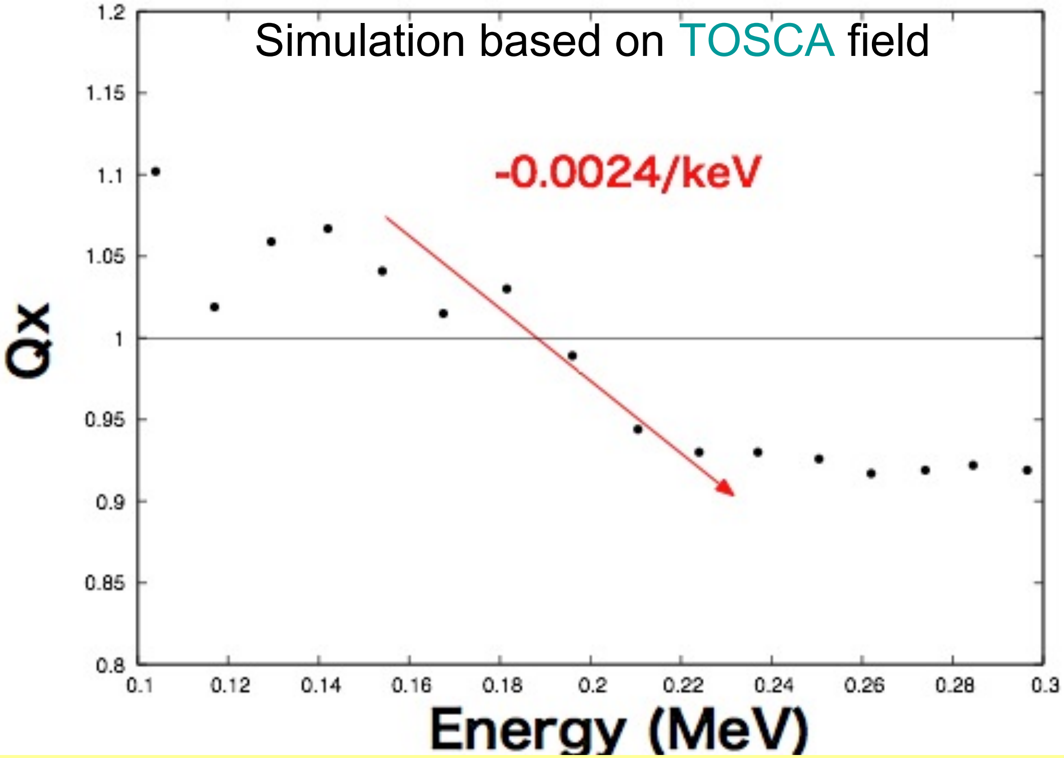


- Variable k, by means of 32 trim-coils
  --> Hori. tune is controllable
  (depending on energy !)
  --> easy to demonstrate resonance crossing

- Induction acceleration
  --> No longitudinal focus,
  --> no energy oscillations,
  which affects horizontal betatron oscillations

Tune Variation

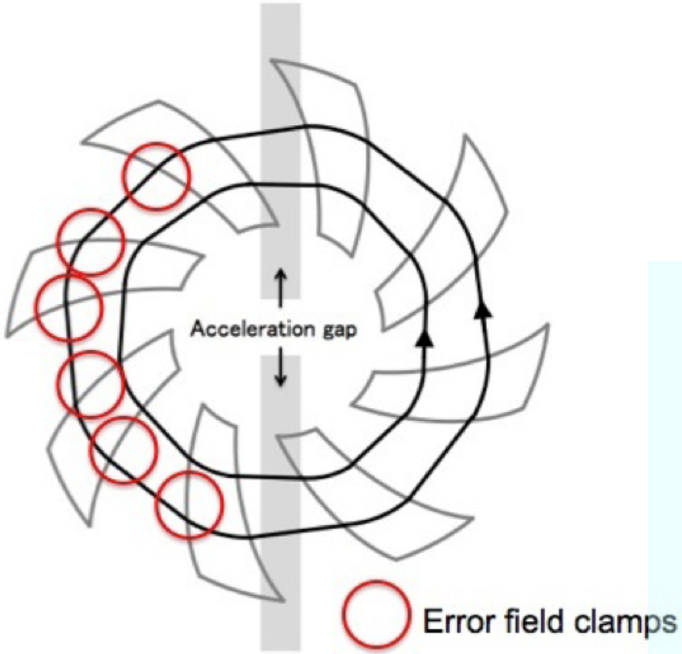
Without exciting trim-coils,
Qx~~1, but depending on E

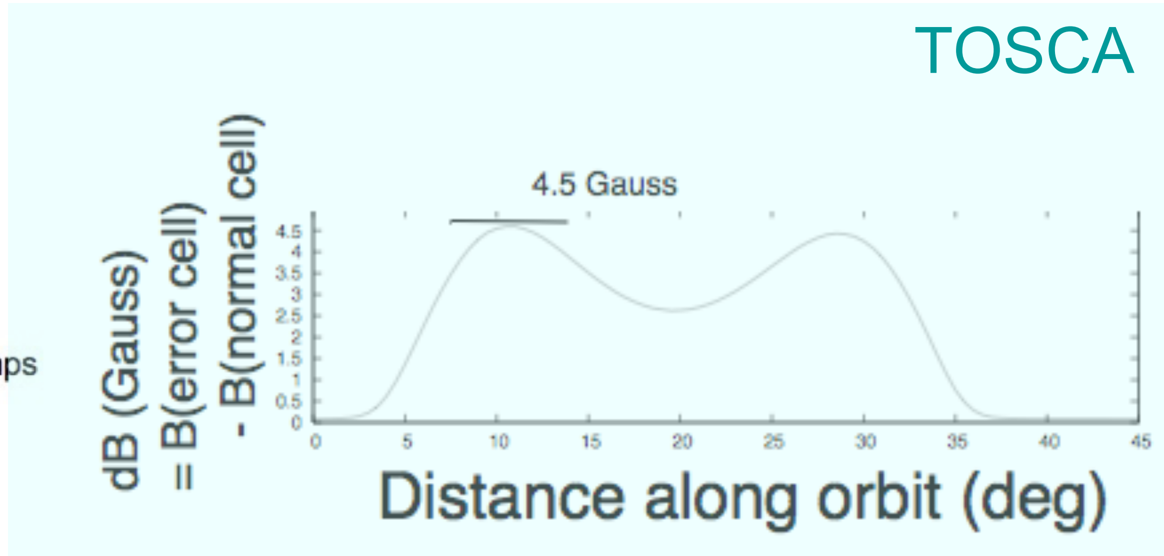


(Crossing speed)
= 0.0024/keV * (Accel. Voltage) < 0.0084/turn

First Harmonic Force

was applied by 'Error field clamps' which has wider gap than the others.

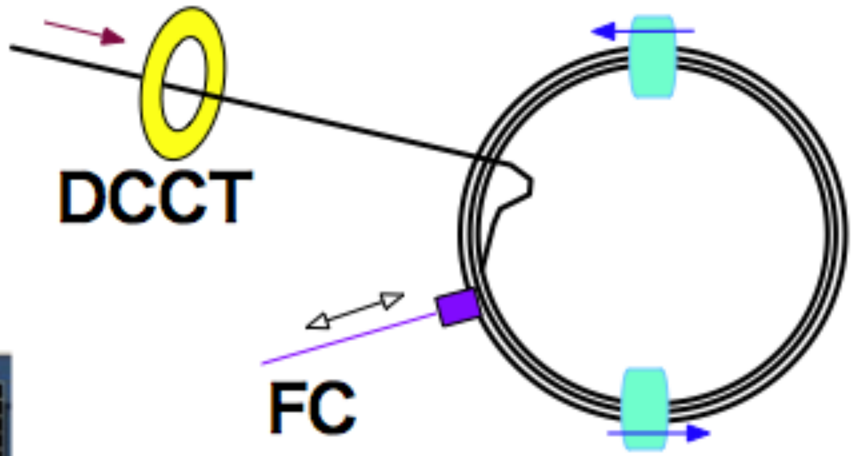




* Effects of accelerations at two gaps work in counter-phase

when Qx=~ 1.

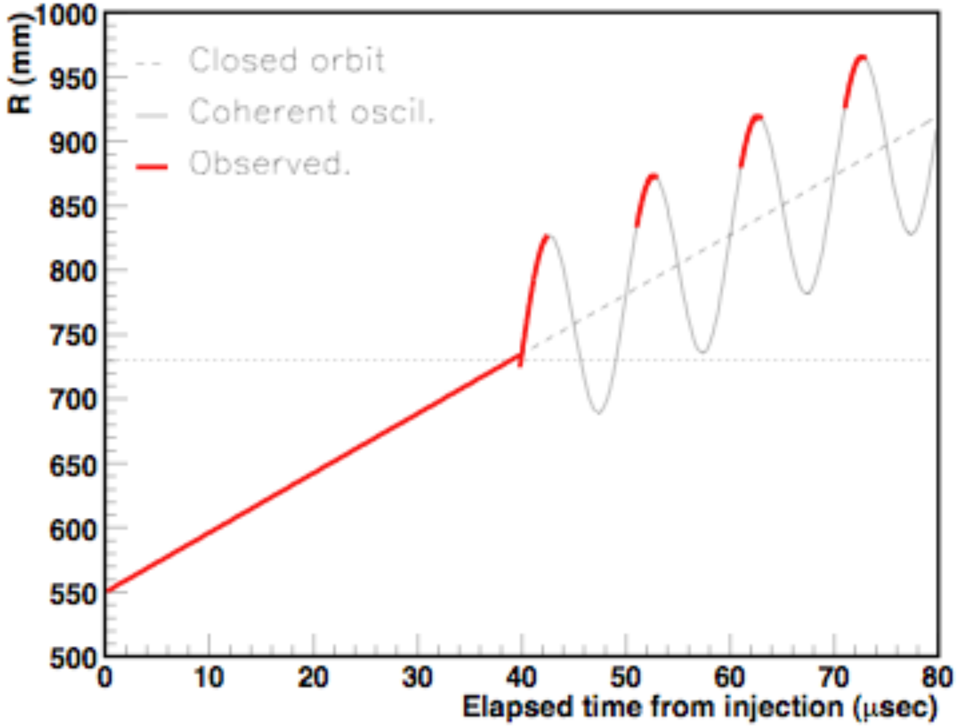
Observing Coherent Oscillations



Elapsed time was measured at different radius

then .....

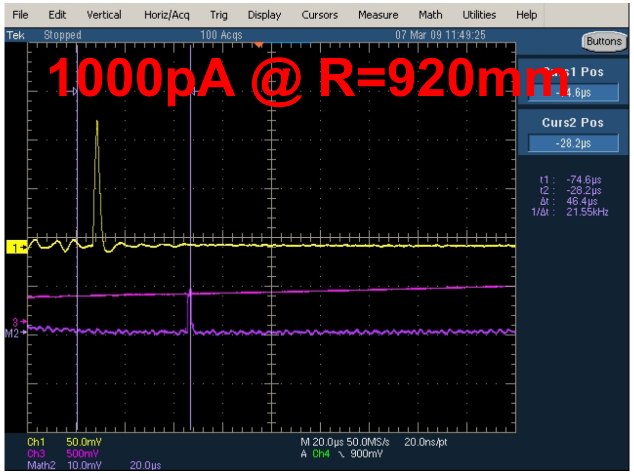
What is expected

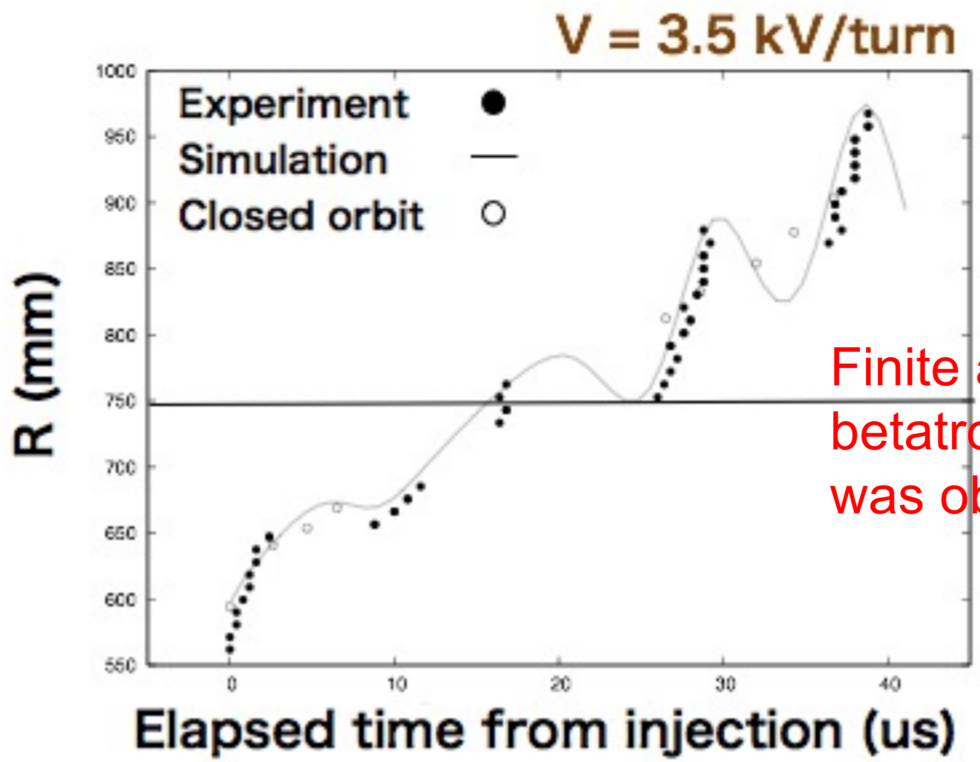


Coherent oscillations will be observed

Experimental Results

A part of beam survived after resonance crossing !



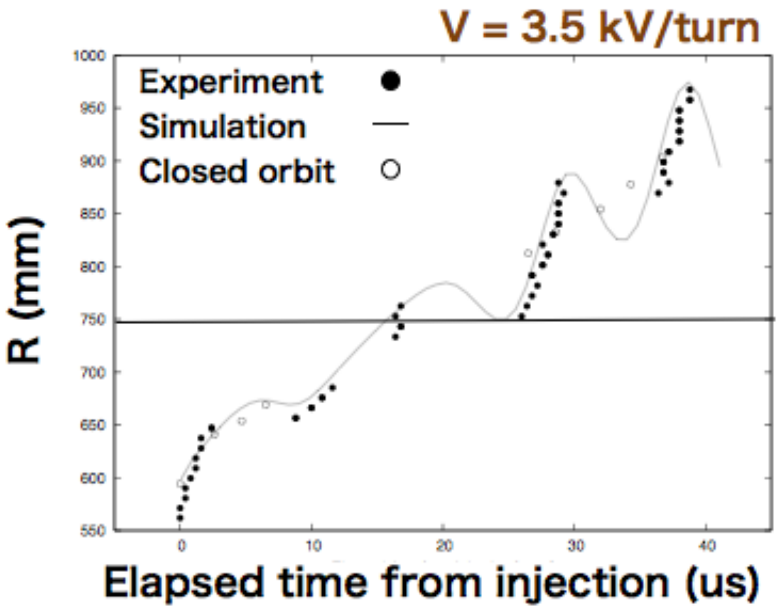


Finite amplitude of betatron oscillations was observed.

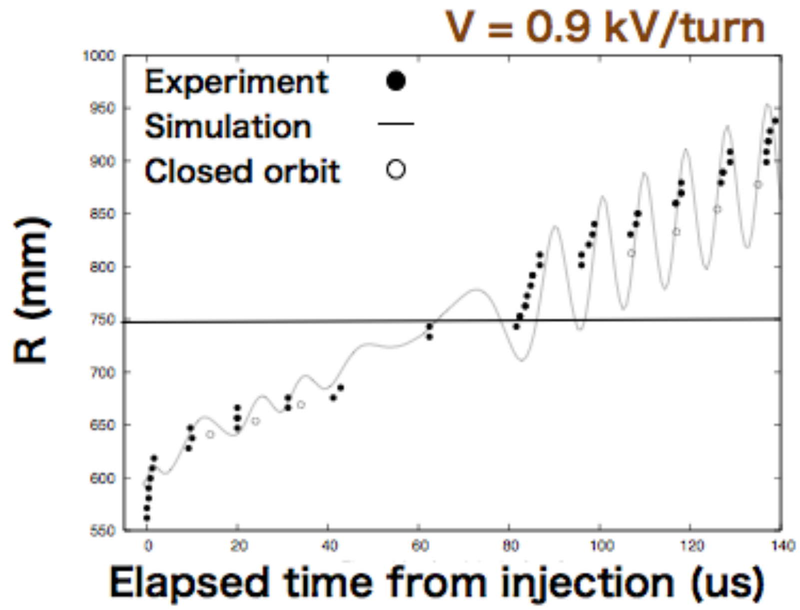
The curve shows simulation results with fitting Initial condition (amplitude and phase).

Dependence on Crossing speed

FAST



SLOW



No difference in the final amplitude ?

It's possible because …..

Final Amplitude depends on Initial Conditions

$$\frac{d^2x}{d\phi^2} + \nu(\phi)^2 x = f\sin(n\phi)$$

General Solution

$$x(\phi) = \underline{A(x_0, x_0'; \phi)} \; + \; \underline{S(\phi)}$$

Solutions of homogeneous eq. (f=0);
Oscillating in freq ν,
Little resonant blow up,
Initial amplitude, phase ..

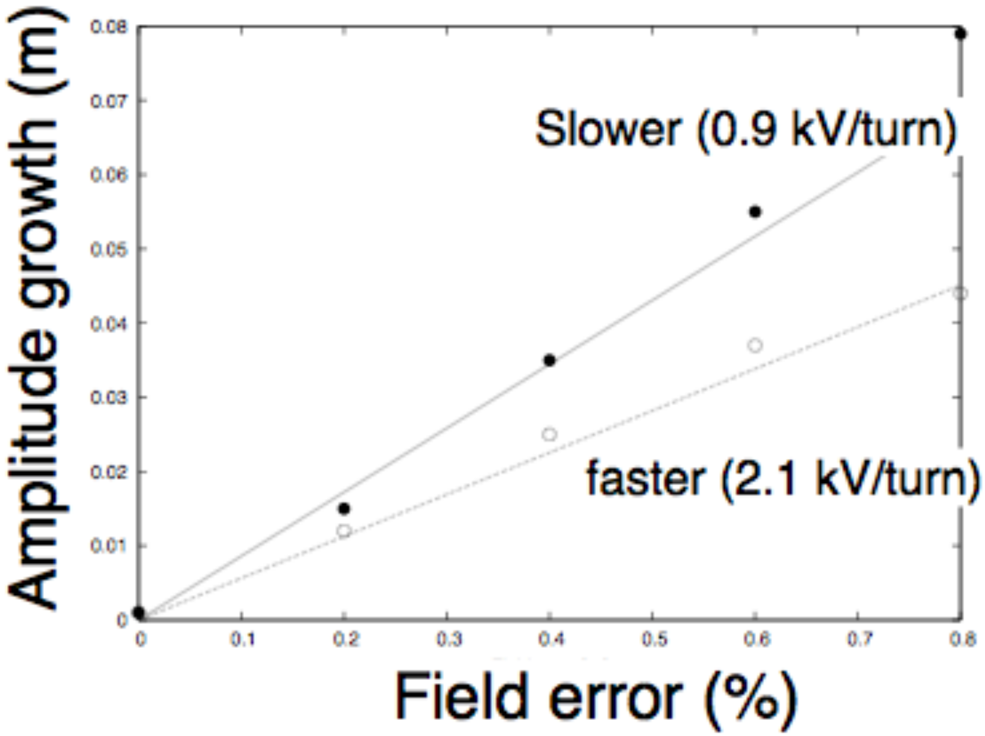
A solution of inhomogeneous eq.;
Oscillating in freq n,
Big resonant blow up

If this part is not negligible

Phase difference ?

. In phase --> Maximum amplitude growth |S|+|A|
. Counter phase --> Minimum growth |S|-|A|

Maximum Amplitude Growth (Simulation)



Worst cases of simulations with different initial phase

- simul. (single kick approx. of driving force)
- model   (field err) / sqrt(crossing speed)

Summary

- Fast crossing of Qx=1 resonance has been examined in Injector FFAG of KURRI.
- The beam survived after the crossing, because of the fast tune variation (and large horizontal acceptance).
- The measured oscillation was reproduced by Runge-Kutta simulations.
- Simulated amplitude growth was proportional to 1/sqrt(dQ/dt)