# CORRECTION OF MULTIPOLAR FIELD ERRORS IN INSERTION REGIONS FOR THE PHASE 1 LHC UPGRADE AND DYNAMIC APERTURE

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*Abstract*

The Phase 1 upgrade of the LHC interaction regions aims at increasing the machine luminosity by reducing the beam size at the interaction point. This requires an in-depth review of the full insertion region layout and a large set of options have been proposed with conceptually different designs. This paper reports on a general approach for the compensation of the non-linear field errors of the insertion region magnets by means of dedicated correctors. The goal is to use the same correction approach for all the different layouts. The correction algorithm is based on the computation of the high orders of the polynomial transfer map using MAD-X and Polymorphic Tracking Code, while the actual performance of the method is estimated by computing the dynamic aperture of the layouts under study.


## INTRODUCTION

The design of the interaction region (IR) of a circular collider is one the most critical issues for the machine performance. Many constraints should be satisfied at the same time and the parameter space to be studied is huge (see Refs. [1, 2] and references therein for an overview of the problem). The strong focusing required to increase the luminosity generates large values of the beta-function at the triplet quadrupoles. This in turns enhance the harmful effects of the magnets field quality on the beam dynamics. It is therefore, customary to foresee a system of non-linear corrector magnets to perform a quasi-local compensation of the non-linear aberrations. This is the case of the nominal LHC ring, for which corrector magnets are located in the $Q_1$, $Q_2$, and $Q_3$ quadrupoles, the latter including non-linear corrector elements.

The strategy for determining the strength of correctors was presented in Ref. [3] and is based on the compensation of those first-order resonance driving terms that were verified to be dangerous for the nominal LHC machine. In general, the proposed approach is based on a number of assumptions that are in general valid for the nominal LHC machine, but not necessarily true for the proposed upgrade scenarios [4, 5], such as perfect antisymmetry of the IR optics between the two beams circulating in opposite directions. Indeed, some LHC upgrade options may not respect the antisymmetry of the IR optics between the two beams and the set of dangerous resonances might not be the same as for the nominal LHC or even be different among the LHC upgrade options. Furthermore, it might be advisable to use a method that should take into account all possible sources of non-linearities within the IR, such as the field quality of the separation dipoles and also collective beam effects like the long-range beam-beam interactions.

For these reasons a more general correction algorithm should be envisaged, thus allowing a direct and straightforward application to any of the upgrade options or, more generally, to any section of an accelerator. The propose method is based in the analysis of the non-linear transfer map a given section of a particle accelerator. The essential details about the non-linear effects of the elements comprised in the section of the machine under consideration are retained in the polynomial transfer map. For this reason the one-turn transfer map was proposed as an early indicator of single-particle instability with a reasonable correlation with the dynamic aperture [6–8].

In the next sections the proposed method is described and some applications to Phase 1 LHC upgrade layouts given.

## MATHEMATICAL BACKGROUND

The transfer map between two locations of a beam line is expressed in the form

$$\vec{x}_f = \sum_{jklmn} \vec{X}_{jklmn}\, x_0^j\, p_{x0}^k\, y_0^l\, p_{y0}^m\, \delta_0^n, \qquad (1)$$

where $\vec{x}_f$ represents the vector of final coordinates $(x_f, p_{xf}, y_f, p_{yf}, \delta_f)$, the initial coordinates being represented with the zero subindex, and $\vec{X}_{jklmn}$ is the vector containing the map coefficients for the four phase-space coordinates and the momentum deviation $\delta$, considered as a parameter. The MAD-X [9] program together with the Polymorphic Tracking Code (PTC) [10] provide the computation of the quantities $\vec{X}_{jklmn}$ up to any desired order.

To assess how much two maps, $X$ and $X'$ deviate from each other, the following quantity is defined:

$$\chi^2 = \sum_{jklmn} ||\vec{X}_{jklmn} - \vec{X}'_{jklmn}|| \qquad (2)$$

where $||\cdot||$ stands for the quadratic norm of the vector. To disentangle the contribution of the various orders to the global quantity $\chi^2$, the partial sum $\chi^2_q$ over the map coefficients of order $q$ is defined, namely

$$\chi^2_q = \sum_{j+k+l+m+n=q} ||\vec{X}_{jklmn} - \vec{X}'_{jklmn}|| \qquad (3)$$

so that

$$\chi^2 = \sum_q \chi^2_q. \qquad (4)$$

In principle, this definition could be used to introduce a weighting of the various orders, using a well-defined amplitude in phase space. This option is not considered in the applications described in this paper.

Furthermore, $\chi_q^2$ is split into a chromatic $\chi_{q,c}^2$ and achromatic $\chi_{q,a}^2$ contribution, corresponding to

$$\chi_{q,a}^2 = \sum_{j+k+l+m=q} ||\vec{X}_{jklm0} - \vec{X}'_{jklm0}||. \tag{5}$$

It is immediate to verify that $\chi_q^2 = \chi_{q,c}^2 + \chi_{q,a}^2$.

# CORRECTION OF MULTIPOLAR ERRORS

## *Algorithm*

The basic assumption is that the multipolar field errors of the IR magnets are available as the results of magnetic measurements. The ideal IR map $X$ without errors is computed using MAD-X and PTC to the desired order and stored for later computations. Including the magnetic errors to the IR elements perturbs the ideal map. To cancel or compensate this perturbation, distributed multipolar correctors need to be located in the IR. The map including both the errors and the effect of the correctors will be indicated with $X'$. The corrector strength is determined by simply minimising $\chi_q^2$ for these two maps. For efficiency, the minimisation is accomplished order-by-order (see, e.g., Ref. [11] for a description of the dependence of the various orders of the non-linear transfer map on the non-linear multipoles). In such an approach the sextupolar correctors are used to act on $\chi_2^2$, the octupolar ones on $\chi_3^2$, and so on.

The code MAPCLASS [12] already used in [13] has been extended to compute $\chi_q^2$ from MAD-X output. The correction is achieved by the numerical minimisation of $\chi_q^2$ using any of the existing algorithms in MAD-X for this purpose.

## *Performance evaluation*

The evaluation of the performance of the method previously described is carried out using two of the three layouts proposed for the upgrade of the LHC insertions (see, e.g., Refs. [2, 4, 5, 14] for the details on the various configurations under consideration).

The field quality of the low-beta triplets is considered to follow the assumption reported in Ref. [15]. This implies that the various multiple components $b_n, a_n$ given by

$$B_y + i\,B_x = 10^{-4}\,B_2 \sum_{n=2}^{\infty} (b_n + i\,a_n) \left(\frac{x + i\,y}{R_{ref}}\right)^{n-1}, \tag{6}$$

where $B_x, B_y$ represents the transverse components of the magnetic field, and $R_{ref}$ the reference radius, scale down linearly with the reference radius, taken at a given fraction of the magnet aperture $\phi$, according to [15]

$$\sigma(b_n, a_n; \alpha\,\phi, \alpha\,R_{ref}) = \frac{1}{\alpha}\sigma(b_n, a_n; \phi,\, R_{ref}). \tag{7}$$

As a natural consequence, large-bore quadrupoles will feature a better field quality than smaller aperture ones. The multipolar components used for the simulations discussed in this paper are listed in Table 1.

An example of the order-by-order correction is shown in Fig. 1 for the so-called low $\beta_{\max}$ configuration [2, 5]. A total of sixty realisations of the LHC lattice are used in the computations. It is worthwhile stressing that even though the random errors are Gaussian-distributed with zero mean and sigma given by the values in Table 1 re-scaled to the appropriate value of the magnet aperture, the limited statistics used to draw the values for a single realisation (corresponding to 16 magnets each divided into ?? slices) implies that in reality non-zero systematic errors are included in the simulations.

Table 1: Random part of the relative magnetic errors of the low-beta quadrupoles at 17 mm radius [16]. The components $b_n$ and $a_n$ stand for normal and skew multipolar errors, respectively.

| Order | $b_n$ $[10^{-4}]$ | $a_n$ $[10^{-4}]$ |
|---|---|---|
| 2 | 0.349431 | 0.477730 |
| 3 | 0.100570 | 0.309803 |
| 4 | 0.067294 | 0.062218 |
| 5 | 0.135565 | 0.057960 |
| 6 | 0.012633 | 0.016546 |
| 7 | 0.003812 | 0.014816 |
| 8 | 0.006825 | 0.003813 |
| 9 | 0.008446 | 0.003973 |

One corrector per IR side and per type (normal or skew component) are used, for a total of ?? correctors. Different locations of the non-linear correctors can be used for the minimisation of $\chi_q^2$. The configuration having the lowest $\chi_q^2$ after correction is selected for additional studies (see next section). The difference between a non-optimised positioning and the best possible one is illustrated in Fig. 2. There, the results of the proposed correction scheme in the case of a symmetric configuration (see Refs. [2, 4, 14]) are shown. The configuration corresponding to the grey dots achieves slightly better corrections over the ensemble of realisations and therefore is selected for further studies.

# DYNAMIC APERTURE COMPUTATION

## *Assessment of the non-linear correction algorithm*

The main goal of the error compensation is to increase the domain in phase space where the motion is quasi-linear, thus improving the single-particle stability. It is customary to quantify the stability of single-particle motion using the concept of dynamic aperture (DA). The DA is defined as the minimum initial transverse amplitude becoming unstable beyond a given number $N$ of turns. The standard

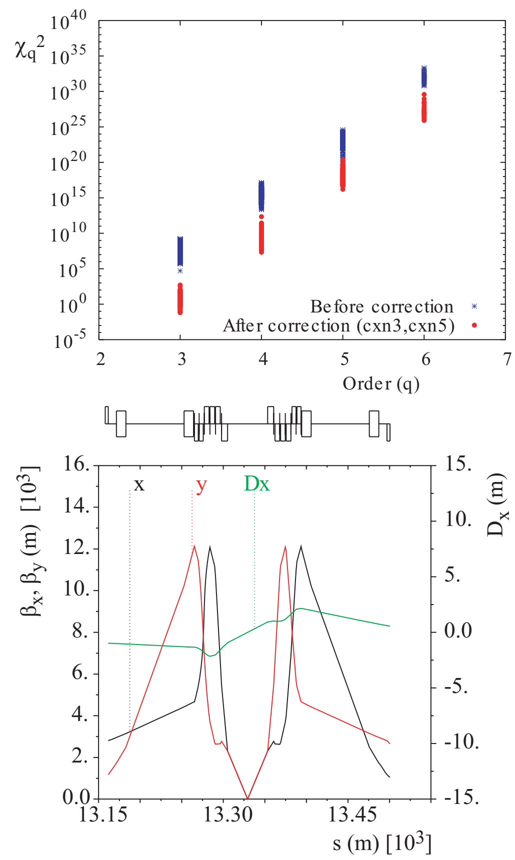


Figure 1: Evaluation of the various orders of $\chi_q^2$ (upper plot) before (blue markers) and after (red markers) correction. Sixty realisations of the random magnetic errors are used. The layout is the low $\beta_{\max}$, whose optics is also reported (lower plot).

protocol used to compute the DA for the LHC machine is based on $N = 10^5$ and a sampling of the transverse phase space $(x, y)$ via a polar grid of initial conditions of type $(\rho\cos\theta, 0, \rho\sin\theta, 0)$ with $\theta \in [0, \pi/2]$. In practise, five values for $\theta$ are used. The scan in $\rho$ is such that a $2\,\sigma$ interval is covered with $30$ initial conditions. The momentum offset is set to $3/4$ of the bucket height.

As far as the magnetic field errors used in the numerical simulations are concerned, the as-built configuration of the LHC is used. The information concerning the measured errors, as well as the actual slot allocation of the various magnets is taken into account in the numerical simulations. The errors on the results of the magnetic measurements are included in the numerical simulations by adding random errors to the various realisations of the LHC ring. On the other hand, the field quality of the low-beta triplets from Table 1 and the scaling law from Ref. [15] are used. It is worth mentioning that the layouts under studies are not finalised, yet. In particular, the details for the implementation of the separation dipoles D1 and D2 are not fixed. As a consequence, no estimate concerning their field quality was taken into account in the modelling of the LHC ring.

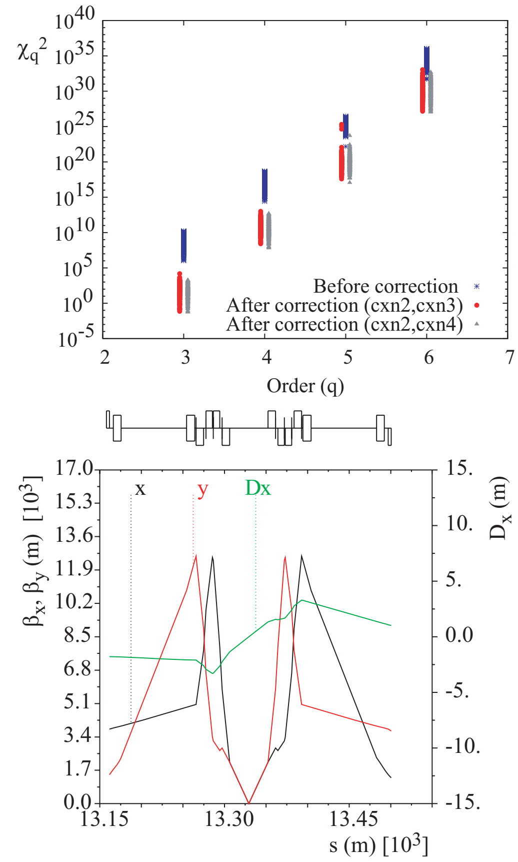


Figure 2: Evaluation of the various orders of $\chi_q^2$ (upper plot) before (blue markers) and after (grey markers) correction. The red markers represent a non-optimised (in terms of correctors location) compensation scheme. Sixty realisations of the random magnetic errors are used. The layout is the symmetric one, whose optics is also reported (lower plot).

As for the evaluation of the correction schemes, sixty realisations of the random multipolar errors in the triplets are used and the value of DA represents the minimum over the realisations. The accuracy of the numerical computation of the minimum DA is considered to be at the level of $\pm 0.5\sigma$.

In Fig. 3 the DA for the two LHC upgrade options, low $\beta_{max}$ and symmetric, as a function of phase space angle is plotted with and without non-linear corrections schemes.

The correction algorithm proved to be particularly successful in the case of the symmetric layout. Indeed, for this configuration about $2.5\ \sigma$ are recovered thanks to the correction of the non-linear $b_3$ and $b_6$ errors.

The compensation in the case of the low $\beta_{max}$ layout is less dramatic, allowing to recover $2.5\ \sigma$ for small angles, only. It is also important to stress that the baseline DA is not the same for the two layouts, as the low $\beta_{max}$ is already well above $14.5\ \sigma$ without any correction. Furthermore, not only the optics is different for the options, but also the triplets' aperture. The first implies a differ-

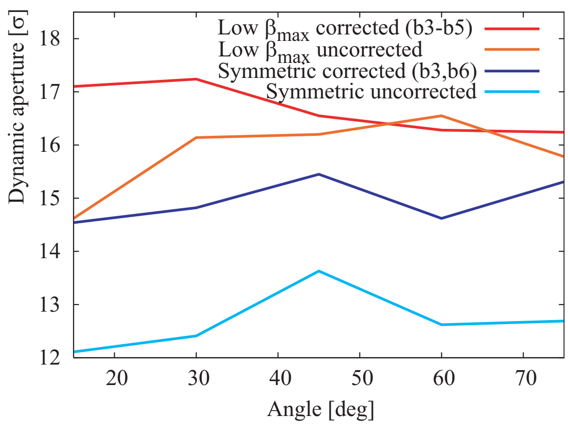


Figure 3: Comparison of the dynamic aperture for the so-called LHC upgrade layouts low $\beta_{max}$ and symmetric with and without correction of the non-linear magnetic errors in the low-beta quadrupoles.

ent enhancement of the harmful effects of the triplets field quality, while the latter has a direct impact on the actual field quality because of the scaling law [15]. It is clear that the DA for the low $\beta_{max}$ is already well beyond the targets used for the design of the nominal LHC even without non-linear correctors. The situation for the symmetric option is slightly worse and a correction scheme might be envisaged.

### *Digression: Dynamic aperture vs. low-beta triplet aperture*

A third layout proposed as a candidate for the LHC IR upgrade is the so-called compact [2, 5]. It features very large aperture triplet quadrupoles ($150$ mm diameter for $Q_1$ and $220$ mm for $Q_2$ and $Q_3$). Thanks to the proposed scaling law, the field quality is excellent and the results DA is beyond $16\ \sigma$ and hence does not require any correction scheme.

Nevertheless, a detailed study of the dependence of the dynamic aperture on the magnets aperture is carried out. The overall LHC model is the same as the one described in the previous sections, the main difference being the scan over the aperture of $Q_1$ and simultaneously over the apertures of $Q_2$ and $Q_3$. The optics is assumed to be constant, which implies that the configurations corresponding to larger magnets apertures than the nominal ones cannot be realised in practise.

The results are shown in Fig. 4. The minimum, average, and maximum (over the realisations) DA are shown for the two type of scans. The horizontal lines represent the asymptotic value of the DA and are obtained by using a huge (and unrealistic) value for the triplets aperture.

The dependence on the aperture of $Q_1$ is rather mild, because of the not too high value of the beta-function, and there exists a rather wide range of apertures for which the DA is almost constant. In particular for $\phi > 110$ mm the asymptotic value of the DA is reached. A constant drop of

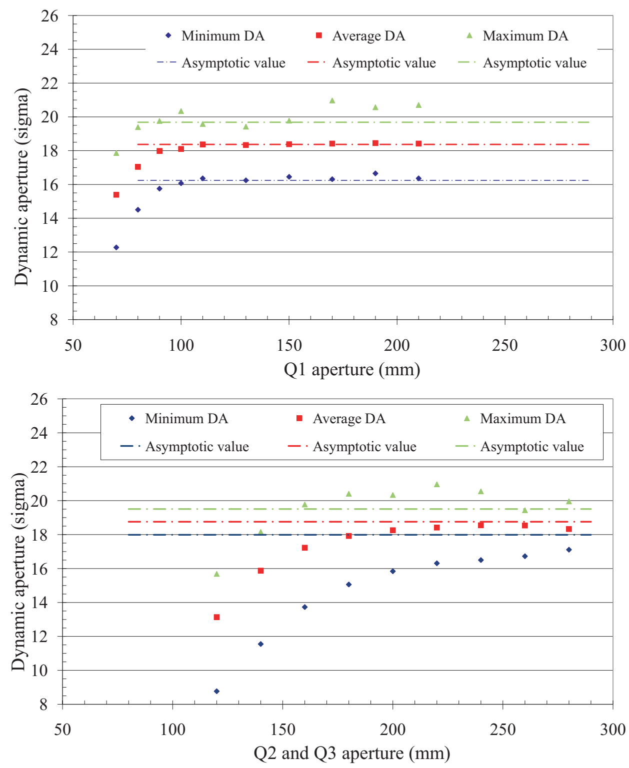


Figure 4: DA as a function of the low-beta quadrupoles aperture. The scan over the aperture of $Q_1$ is shown in the upper plot (nominal aperture $150$ mm), while $Q_2$ and $Q_3$ are considered in the lower plot (nominal aperture $220$ mm). The layout is the so-called compact one.

DA is observed for $\phi < 100$ mm and, in general, the three curves behave the same.

The dependence of DA on the $Q_2$ and $Q_3$ aperture is somewhat different. The asymptotic value is hardly reached for apertures larger than $250$ mm and the DA drop with aperture is monotonic and smooth. The spread between the asymptotic values for minimum, average, and maximum DA is smaller than for the case of the scan over the aperture of $Q_1$.

As an example, the behaviour of the DA as a function of aperture is fit with two functions (exponential and power law) and the results are shown in Fig. 5. The difference between the asymptotic and the actual DA value is plotted as a function of the $Q_2$ and $Q_3$ aperture. The agreement between the fit functions and the simulation results is excellent, even though, for the time being no theoretical argument explains these results.

## CONCLUSIONS

A general algorithm for the correction of multipolar errors in a given section of a circular accelerator has been developed. It is based on the computation and comparison of map coefficients obtained from standard accelerator codes such as MAD-X and PTC. The algorithm aims at minimis-

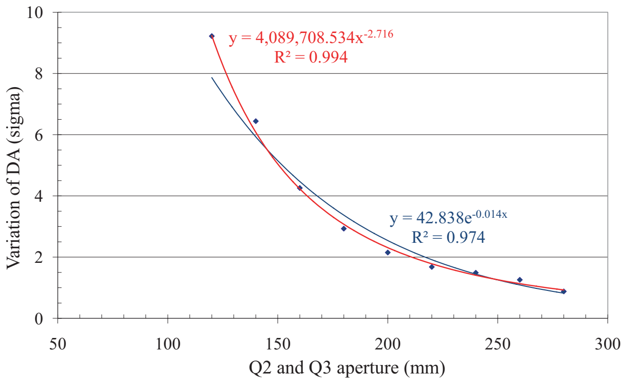


Figure 5: Behaviour of the minimum DA as a function of $Q_2$ and $Q_3$ aperture. Two types of fit functions are also shown.

ing the difference between a target transfer map and the actual one. Both order-by-order and global optimisation strategies are possible. Of course, the algorithm can be used also to optimise the location of the corrector elements. In its present form the non-linear magnetic field errors are the only source of non-linearities included in the transfer map. Nevertheless, other sources of non-linear effects in the transfer map could also be included in the correction algorithm, such as beam-beam kicks from long-range encounters. The efficiency of such an approach should be tested in practise with dedicated studies.

The correction algorithm was successfully tested on two layouts for the proposed IR upgrade of the LHC machine. The quality of the correction was also verified by means of numerical simulations aimed at computing the dynamic aperture. In the two cases under consideration a sizable increase of the dynamic aperture due to the correction scheme is observed.

In the numerical simulations used to evaluate the dynamic aperture a new scaling law for the magnetic field errors as a function of the low-beta quadrupoles aperture was used. The impact of such an assumption on the value of the dynamic aperture was assessed in details with a series of dedicated studies, where the triplets aperture is scanned. Smooth dependency of the dynamic aperture with respect to the magnets aperture is found, and exponential or power laws are fitted to the numerical data with very good agreement. These results could be used as an additional criterion for the definition of the required aperture of triplet quadrupoles. Indeed, one could derive the minimum aperture for which the dynamic aperture does not require any correction. Such a condition should then be taken into account together with the ones related to the needed beam aperture and energy deposition issues.

## ACKNOWLEDGMENTS

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