Dynamic aperture studies in e+e- factories with crab waist

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Outline

- ❖ DA limitation in the crab waist colliders
  - Low emittance lattice
  - Strong crab sextupoles
  - Combined effect of the BB and magnetic nonlinearities
  - Field errors in the FF quadrupoles
- ❖ Simulation of the BB-effects with realistic lattice
- ❖ DA optimization
- ❖ Summary

Low emittance lattice (1)

Main source of DA limitation are strong chromatic sextupoles

Hamiltonian in harmonic representation:

$$
\begin{aligned}
H &= \nu_x J_x + \nu_y J_y \\
&+ (2J_x)^{3/2} \sum_n \left[3A_{1n}\cos(\varphi_x - n\theta) + A_{3n}\cos(3\varphi_x - n\theta)\right] \\
&- 3(2J_x)^{1/2}(2J_y) \sum_n \left[2B_{1n}\cos(\varphi_x - n\theta) + B_{+n}\cos(\varphi_+ - n\theta)\right]
\end{aligned}
$$

with 5 types of harmonics:

$$
A_{jn} = \frac{1}{48\pi}\sum_m \beta_{xm}^{3/2}(k_2 l)_m \cos(j\psi_x - \nu\theta + n\theta)_m
$$

$$
B_{1n} = \frac{1}{48\pi}\sum_m \beta_{xm}^{1/2}\beta_{ym}(k_2 l)_m \cos(\psi_x - \nu\theta + n\theta)_m
$$

$$
B_{\pm n} = \frac{1}{48\pi}\sum_m \beta_{xm}^{1/2}\beta_{ym}(k_2 l)_m \cos(\psi_\pm - \nu_\pm\theta + n\theta)_m
$$

Low emittance lattice (2)

For some lattice cells (at least DBA, FODO, TME) the following estimation of main (resonant) sextupole harmonics is possible:

$$A \cong -\frac{1}{12}\frac{\xi_x}{\sqrt{H_e}} \qquad \text{and} \qquad B \cong \frac{1}{12}\frac{\xi_y}{\sqrt{H_e}}$$

$$H_e = \gamma_x \eta^2 + 2\alpha_x \eta\eta' + \beta_x \eta'^2$$ = const outside the bending magnets

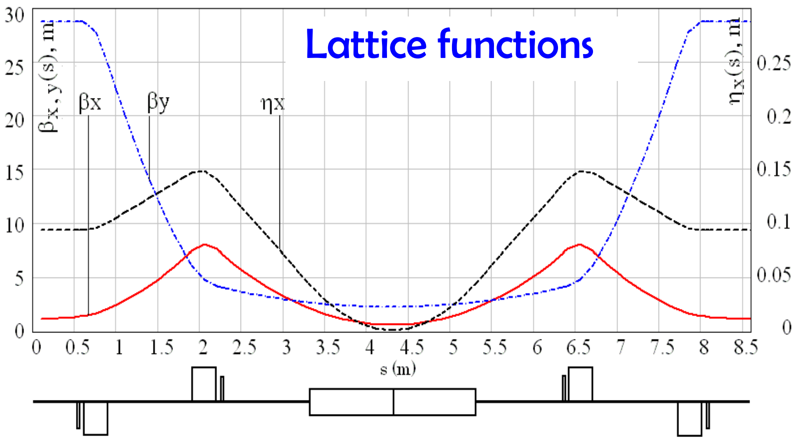
Estimation of DA from this harmonic representation on the basis of simple "fixed points" calculation gives

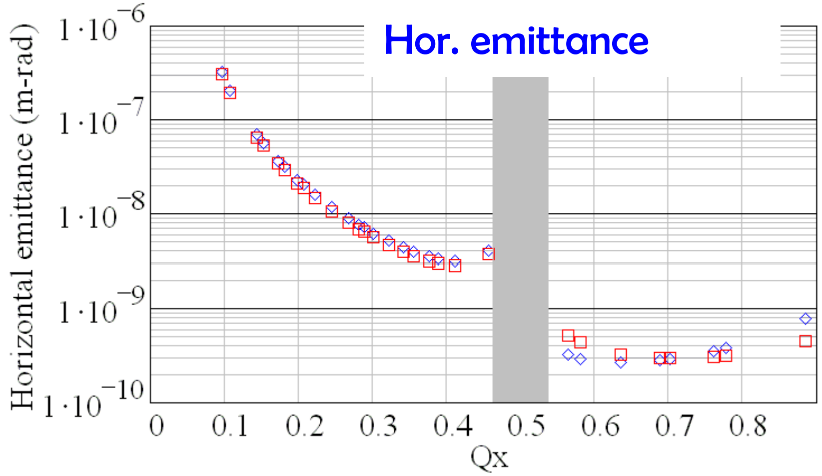
$$a_x \sim k_x(\nu_x, \nu_y)\cdot\frac{\sigma_x}{\xi_x} \qquad a_y \sim k_y(\nu_x, \nu_y)\cdot\frac{\sigma_y}{\xi_y}$$

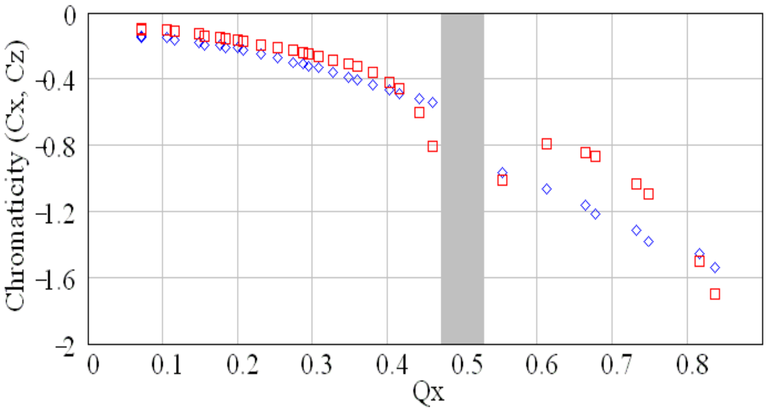
Coefficients $k_{x,y}$ depends weakly on the lattice design

Low emittance lattice (3)

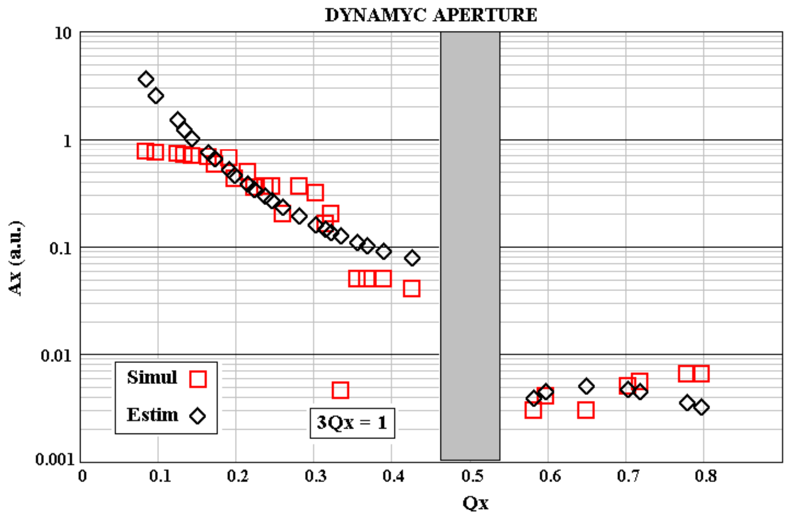
TME example:







Chromaticity per cell



Horizontal DA simulated and fitted according to the main harmonic approximation

Low emittance lattice (4)

- Absolute value of the DA reduces with the emittance decrease and chromaticity growth as

$$a \sim \sigma / \xi \sim \sqrt{\varepsilon} / \xi$$

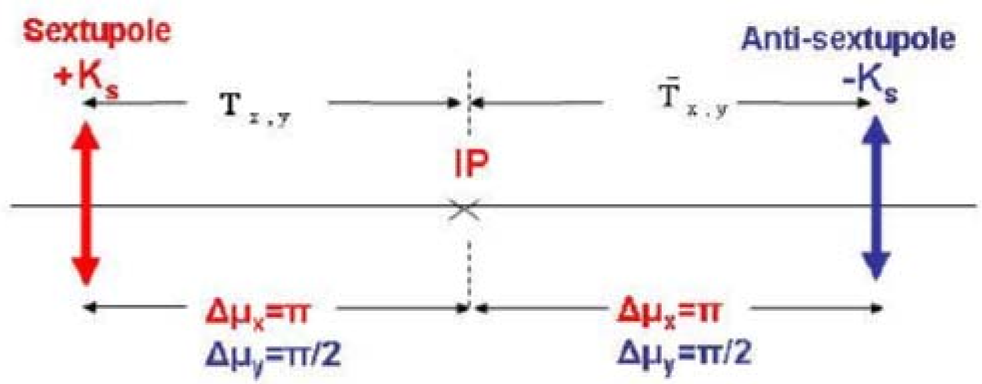
- Relative value of the DA reduces with the chromaticity growth as

$$a / \sigma \sim 1 / \xi$$

- Neither sextupoles strength nor arrangement influence the DA (to some extend, under the considered assumption)

Crab sextupoles (1)

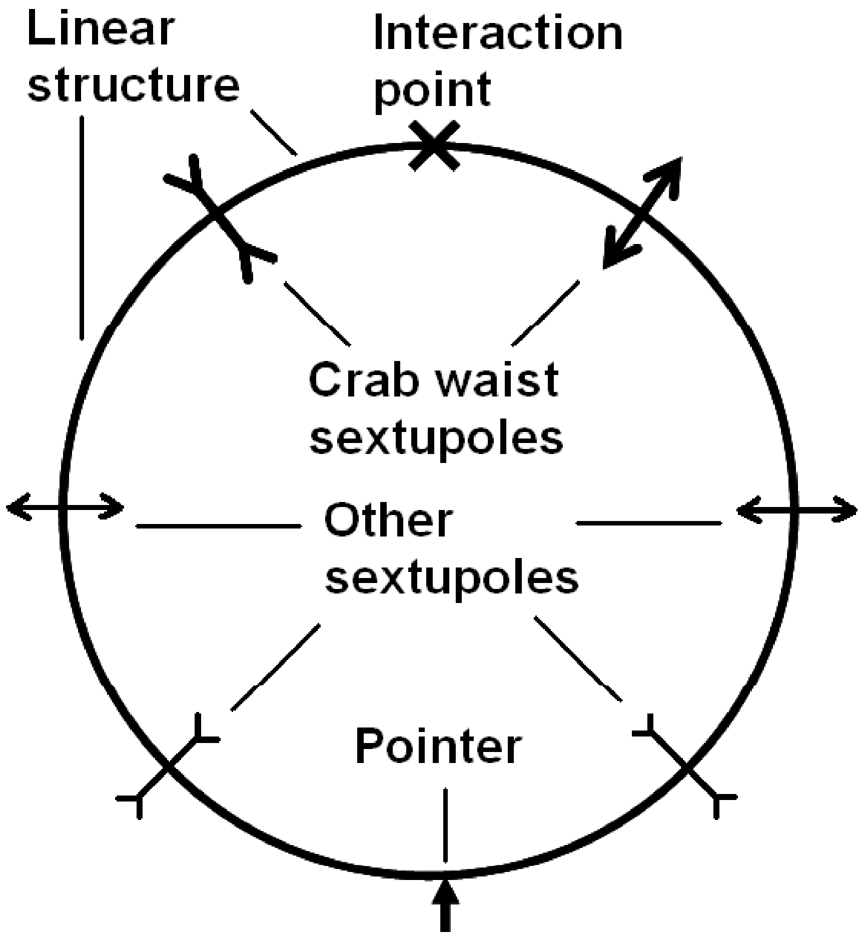
The betatron phase advance between two (point-like) crabbing sextupoles provides exact cancellation of its influence on DA:



But the cancellation condition may be distorted by

- Lattice phase errors
- Non-zero sextupole length
- Chromatic effects
- BB effects

Crab sextupoles (2): ring model

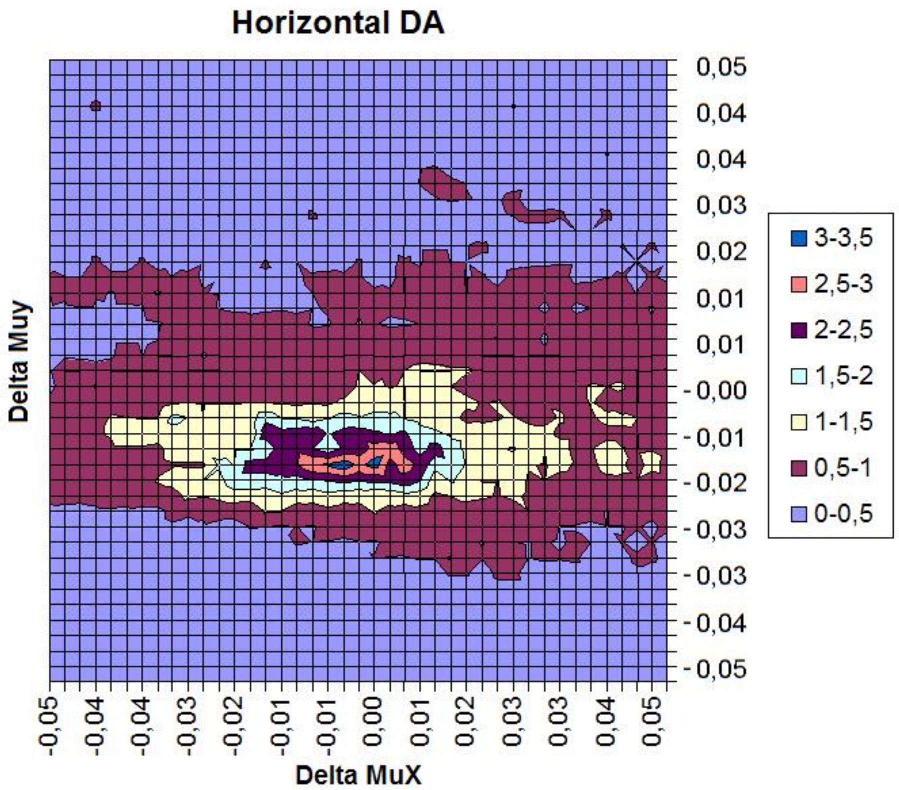


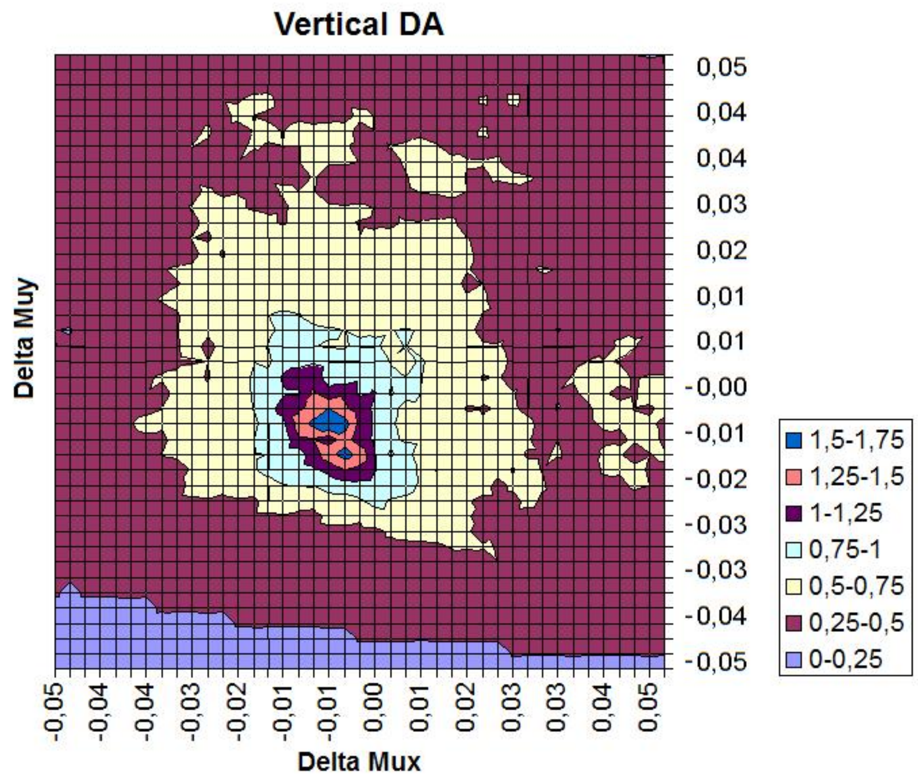
Beta_x^star = 20 mm
Beta_y^star = 0.6 mm

"Other sextupoles" imitate the DA reduction by the ring chromatic sextupoles down to ±20 sigma_x and ±80 sigma_y

Point-like crab sextupoles with the proper phase advance in between do not reduce the DA

Crab sextupoles (3): phase error



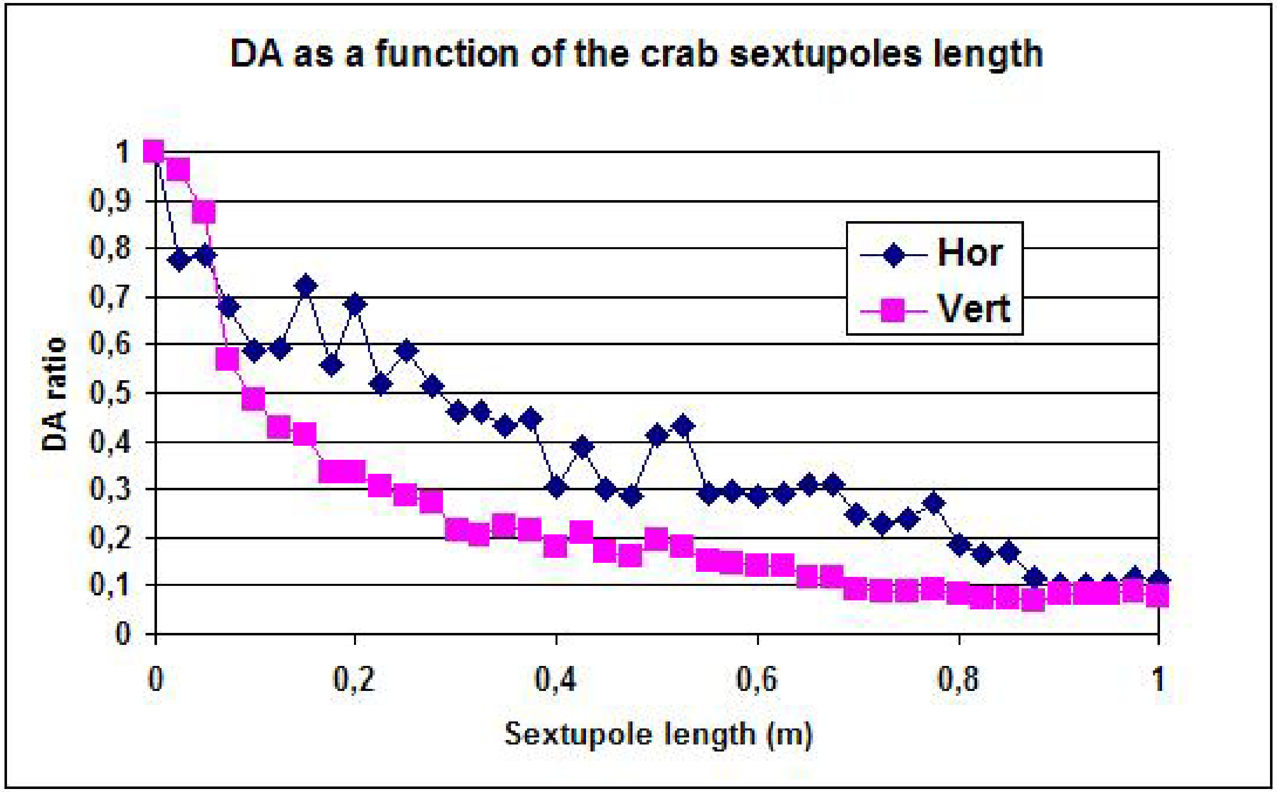


$\Delta\mu_x$=2πn+Delta Mux

$\Delta\mu_y$= πn +Delta Muy

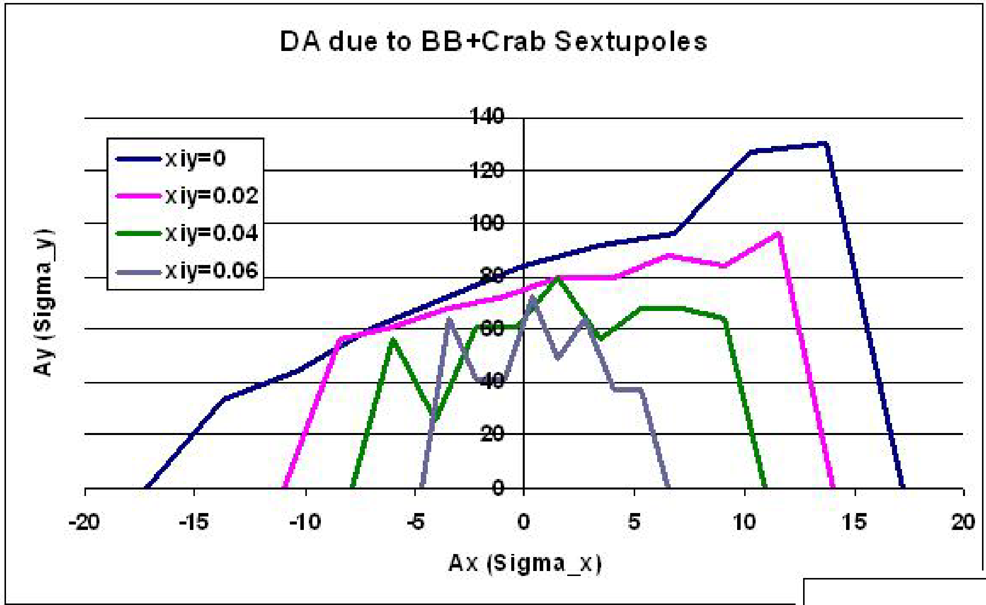
Phase advance error may provide both increasing and decreasing of the DA: Delta Muy=+0.01 DAx × 3; Delta Muy=-0.01 Dax × 0.5

Crab sextupoles (4): finite length



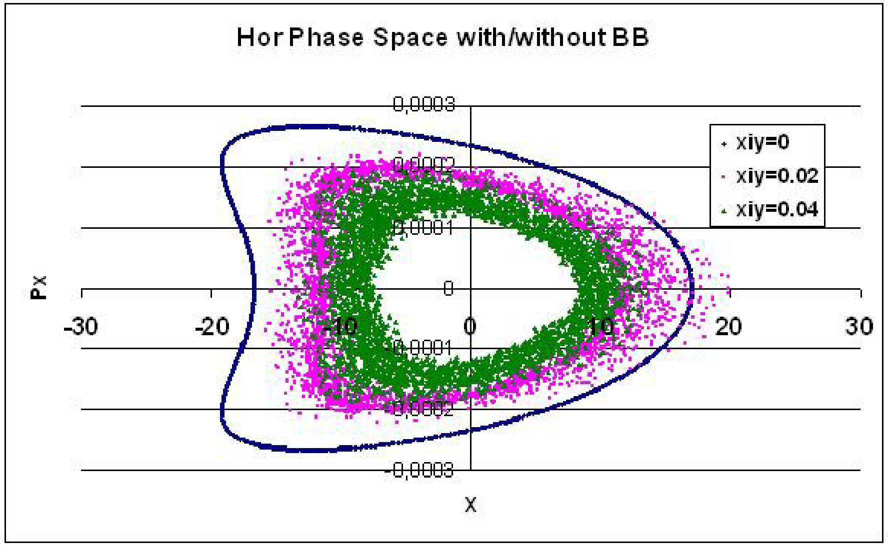
Applying the sextupole length 0.2 m reduce the DA by factor of 2-3 compare to the zero length magnet

Crab sextupoles (5): +BB



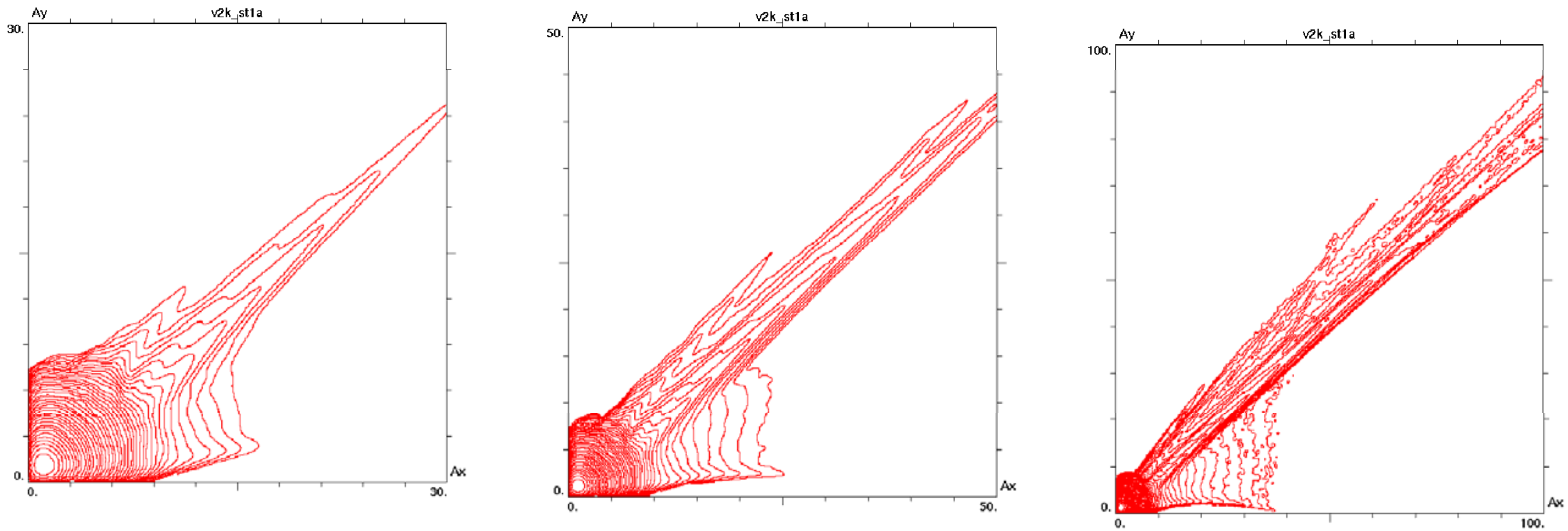
← Model simulation of the combined effect of the crab sextupoles and BB

With increasing of Xi_y the → phase space trajectories became more and more stochastic



Simulation of the BB + realistic lattice (1)

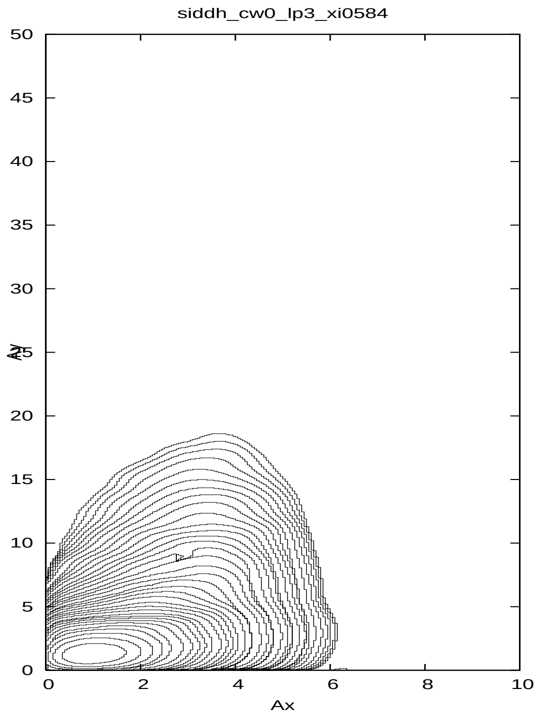
- Motivation: for the strong focusing low emittance and low DA machines combined effects of BB and external nonlinearities can degrade both DA and luminosity
- Tool: new computer code based on LIFETRACK (D.Shatilov) and ACCELERATICUM (P.Piminov) is developed at BINP. The code provides simplectic 6D particles tracking in realistic lattice with BB and radiation (damping and excitation). Including of IBS is under way. Study of particle distribution and loss is available.



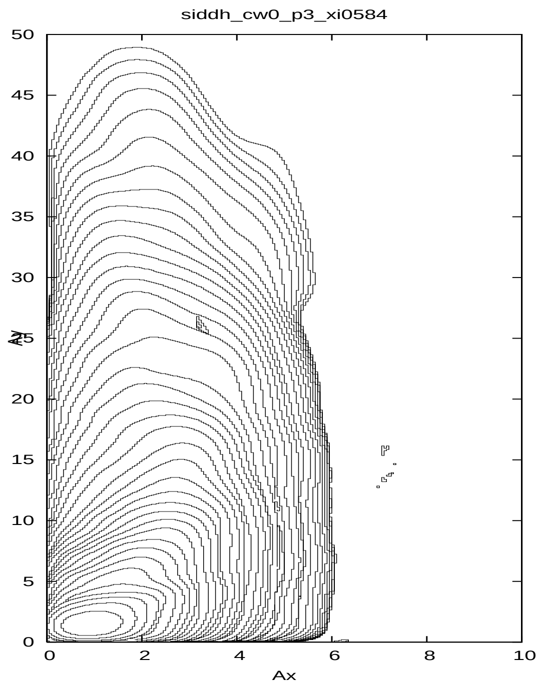
Program test run: “amplitude jet” represents particles loss along the coupling resonance (VEPP-2000, round beams)

Simulation of the BB + realistic lattice (2)

DAΦNE-Siddhatra (Piminov, Shatilov, Zobov)



Lifetrack only, linear lattice



Lifetrack+Acceleraticum, chromatic sextupoles, etc.)

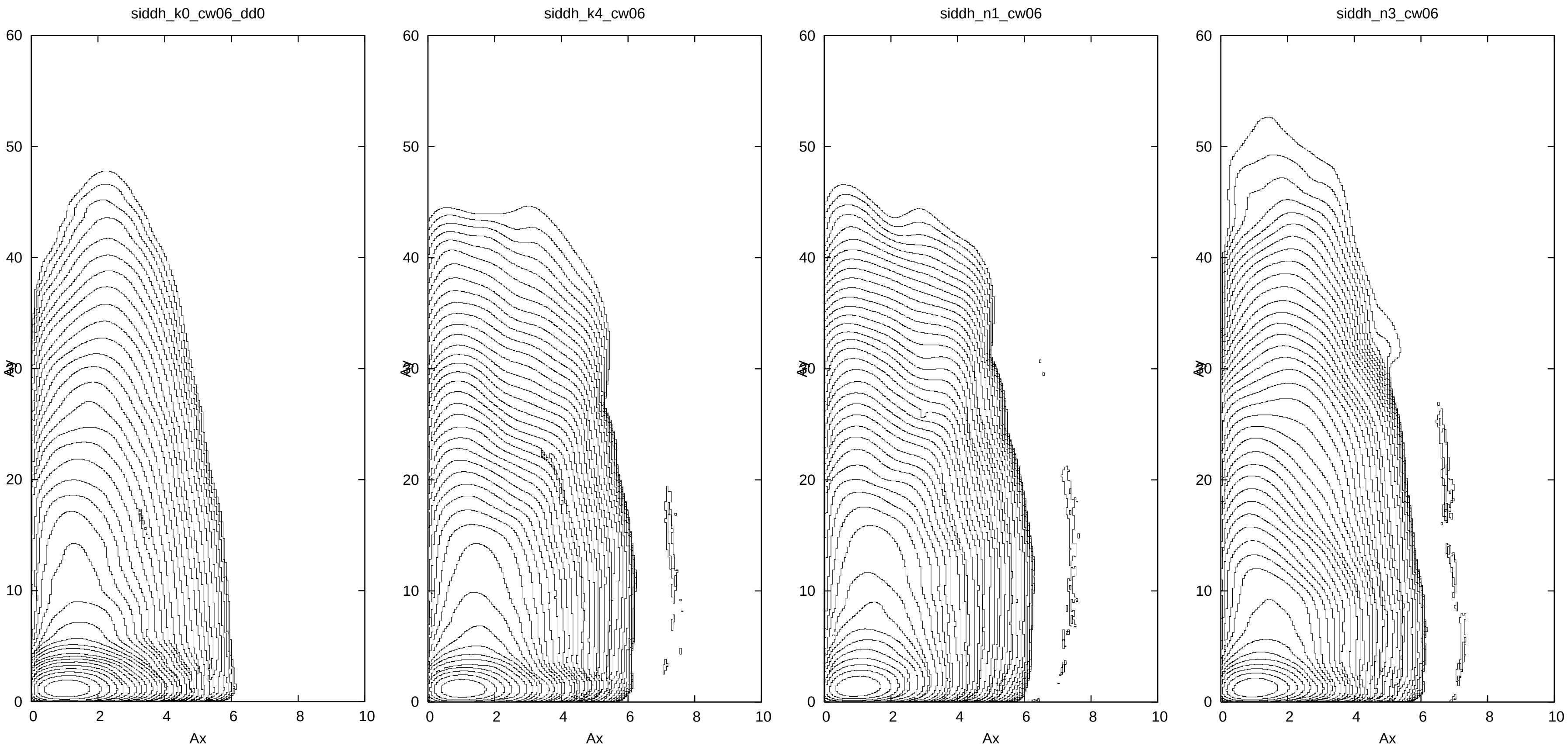
The vertical tail has grown substantively due to the sextupoles but the luminosity remains the same because the aperture is enough to accommodate the beam.

Beam-Beam Simulations for DAFNE-SIDDHARTA

Nominal set of parameters:

β_x^*=20.3cm, β_y^*=0.63 cm, ν_x=0.105, ν_y=0.160, ν_s=0.085

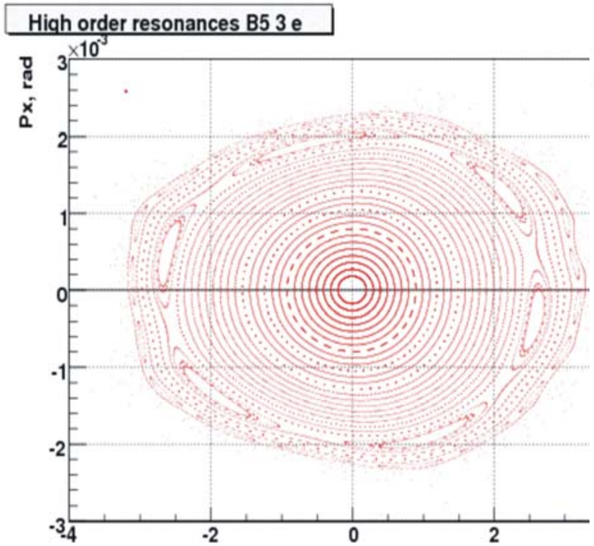
N_b=110, N=2.65 $\cdot 10^{10}$, $\varepsilon_x=2\cdot10^{-9}$m, $\varepsilon_y=10^{-11}$m, Crab Waist =0.6



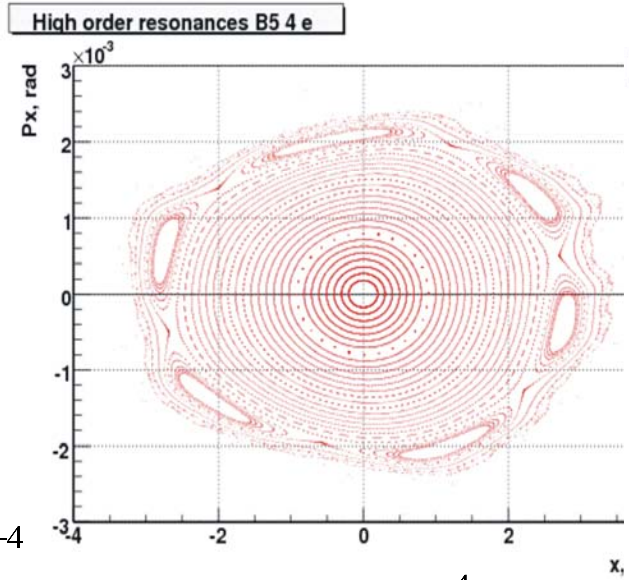
L=1.19·10^{33}	L=1.12·10^{33}	L=0.95·10^{33}	L=1.04·10^{33}
Sext: OFF	**Sext: ON**	**Sext + nonl. FF**	**Sext + old nonl. FF**

DAΦNE FF quadrupole field errors (1)

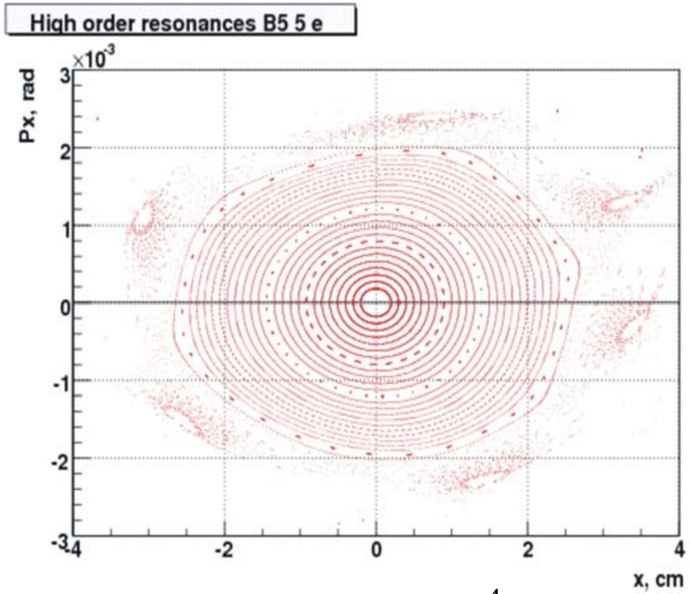
Mechanism:



$$(\Delta B_5 / B_2)_{sys,ran} = 3 \cdot 10^{-4}$$



$$= 4 \cdot 10^{-4}$$



$$= 5 \cdot 10^{-4}$$

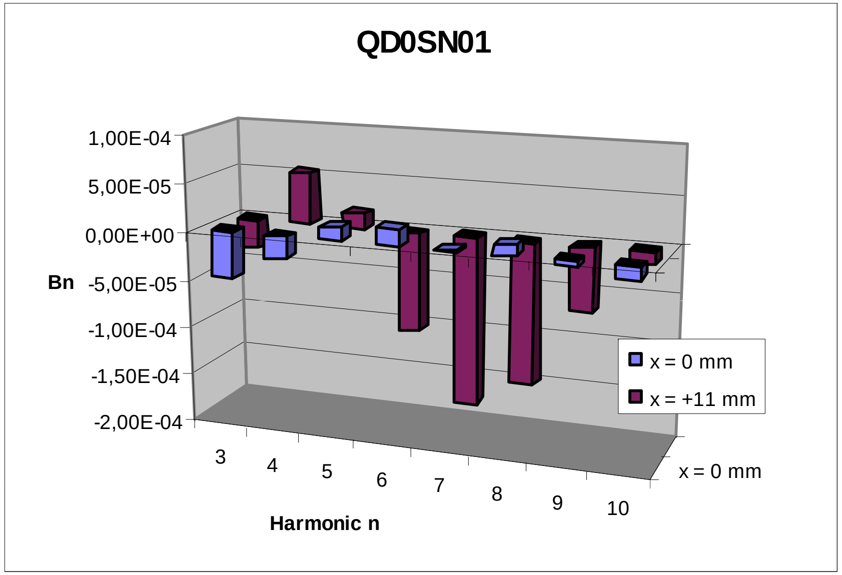
DA reduction by the high order resonance occurs in a stepwise manner

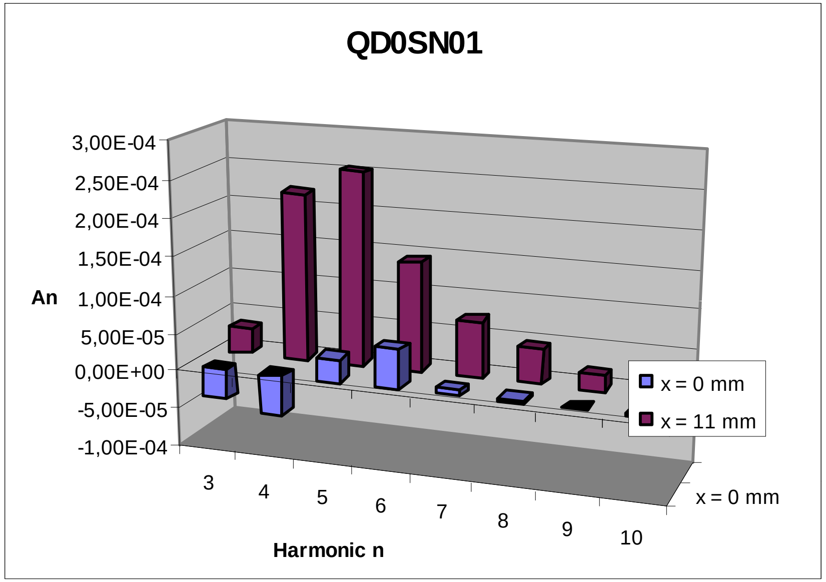
DAΦNE FF quadrupole field errors (2)

Beam offset in the quads x_o = ±11 mm
Measuring radius R_o= 20 mm

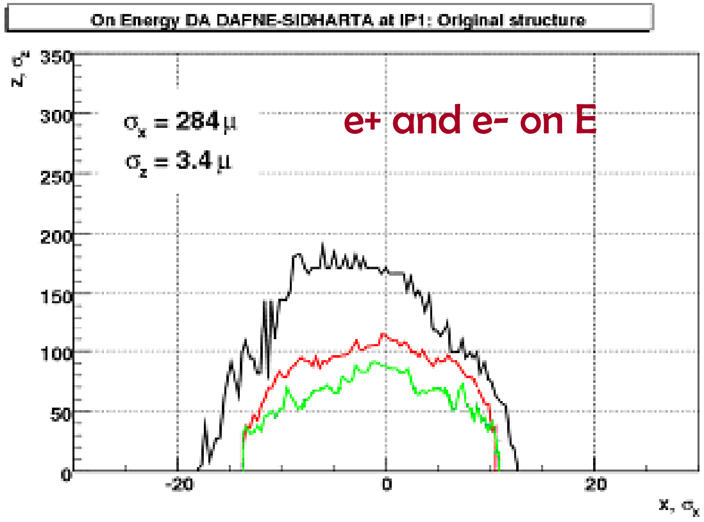
Normal and skew components transformation:

$$\left(B_n' + iA_n'\right) = \sum_{k=n}^{\infty}\left(B_n + iA_n\right)\left[\frac{(k-1)!}{(n-1)!(k-n)!}\right]\left(\frac{x_0}{R_0}\right)^{k-n}$$





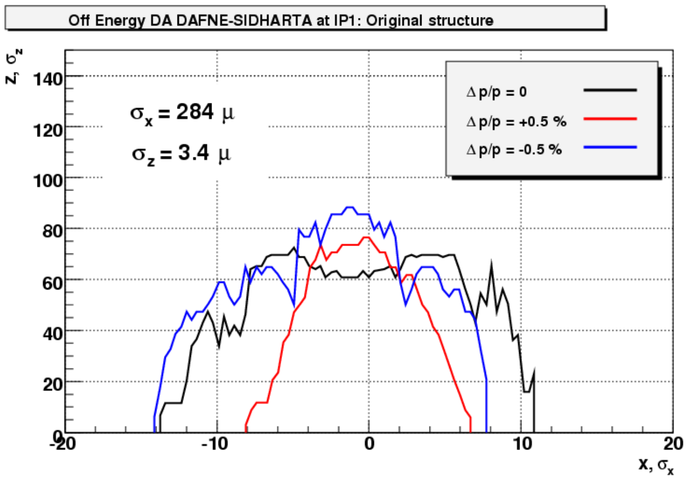
DAΦNE FF quadrupole field errors (3)



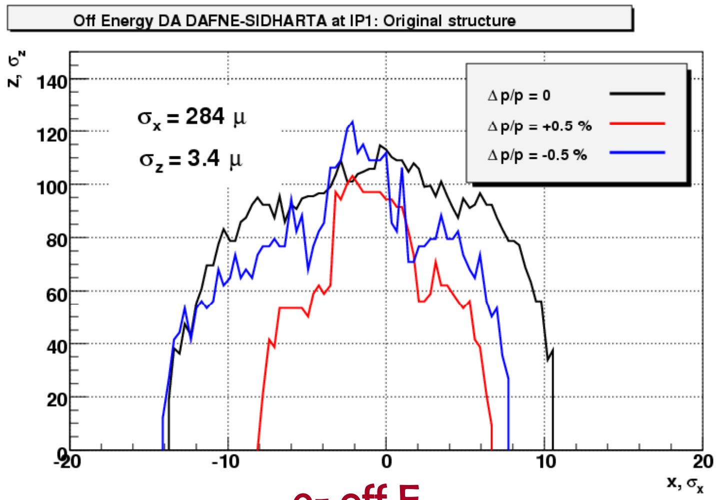
The field errors were inserted in the lattice and beam tracking was performed for e+ and e- beams.

The vertical DA reduces by factor ~2 but is still large enough (~80-100 σ_y).

The horizontal DA reduces by 15%.



e+ off E



e- off E

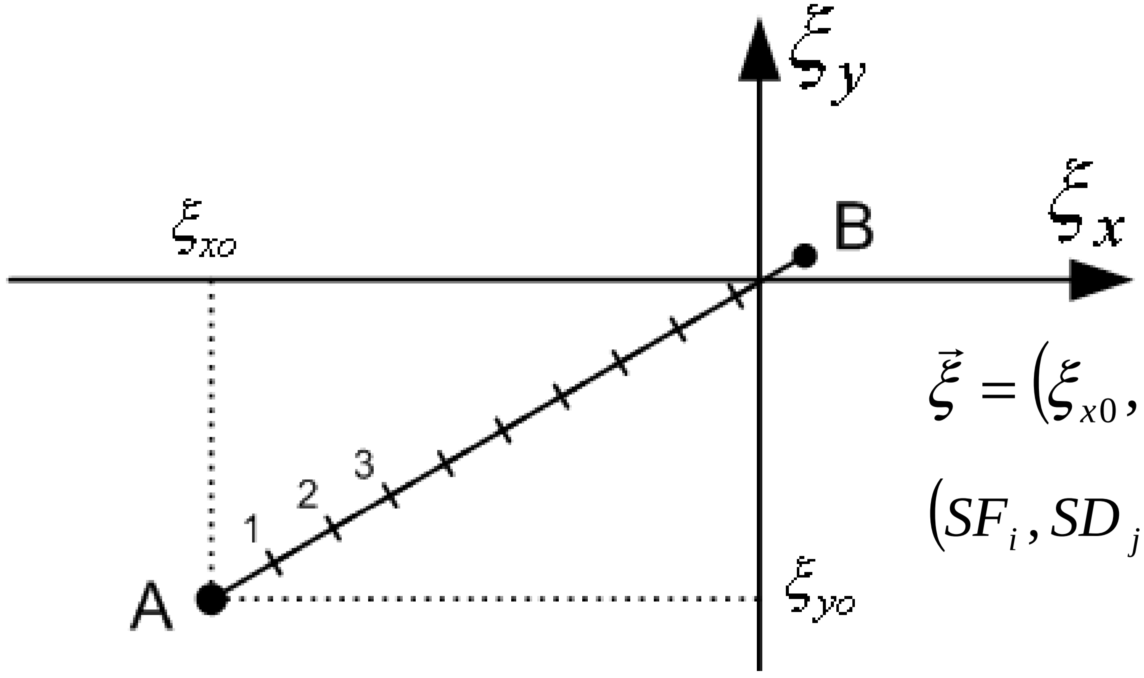
DA increase (1)

Two possible approaches:

- ❖ From theoretical predictions: we chose figures-of-merit (resonance driving terms, detuning coefficients, distortion functions, etc.) and try to optimize them hoping that this allows to open the DA. An example of such approach is the NSLS-II dynamic aperture optimization by a least-square solving of a 52×9 nonlinear system, which includes 27 geometric modes to 3rd order, 12 tune shift coefficients to 6th order and 13 chromatic terms to 6th orders.

- ❖ Phenomenology approach: we know nothing from theory but we can measure the DA rather fast and change something to optimize it

DA increase (2): optimization algorithm

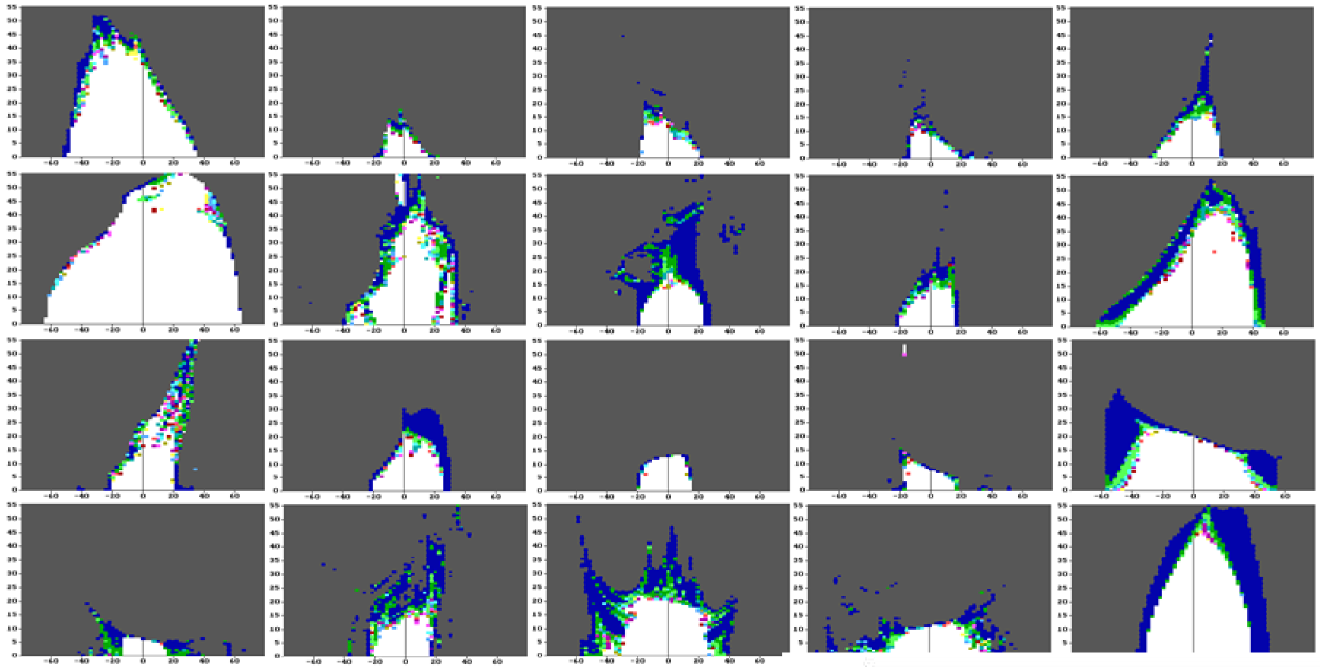


$\vec{\xi} = \left(\xi_{x0}, \xi_{y0}\right)$ - chromaticity vector

$\left(SF_i, SD_j\right)$ - sextupole pair

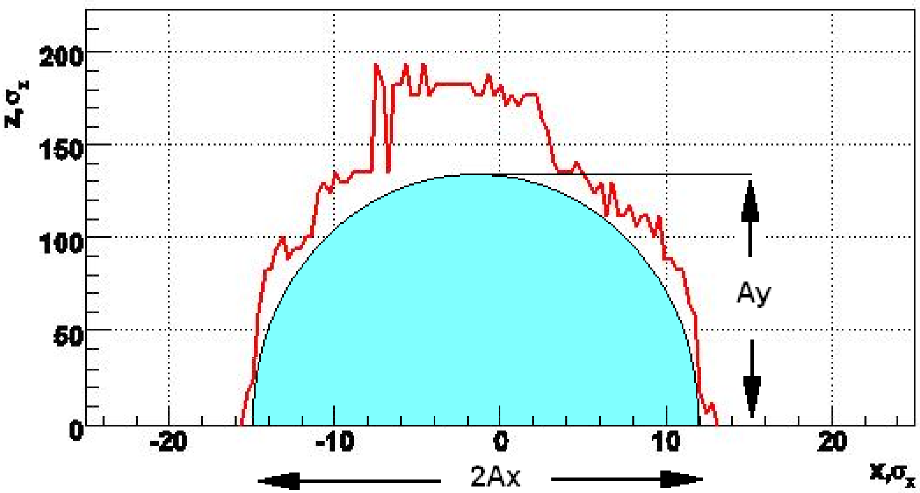
- N small steps for chromaticity correction by every sextupole pair along the chromaticity vector
- The pair providing the largest DA at the step is fixed
- The procedure is repeated until the chromaticity is corrected
- Off energy aperture optimization is available
- Achromatic sextupole (zero dispersion) is included by the gradient search

DA increase (3): a step example

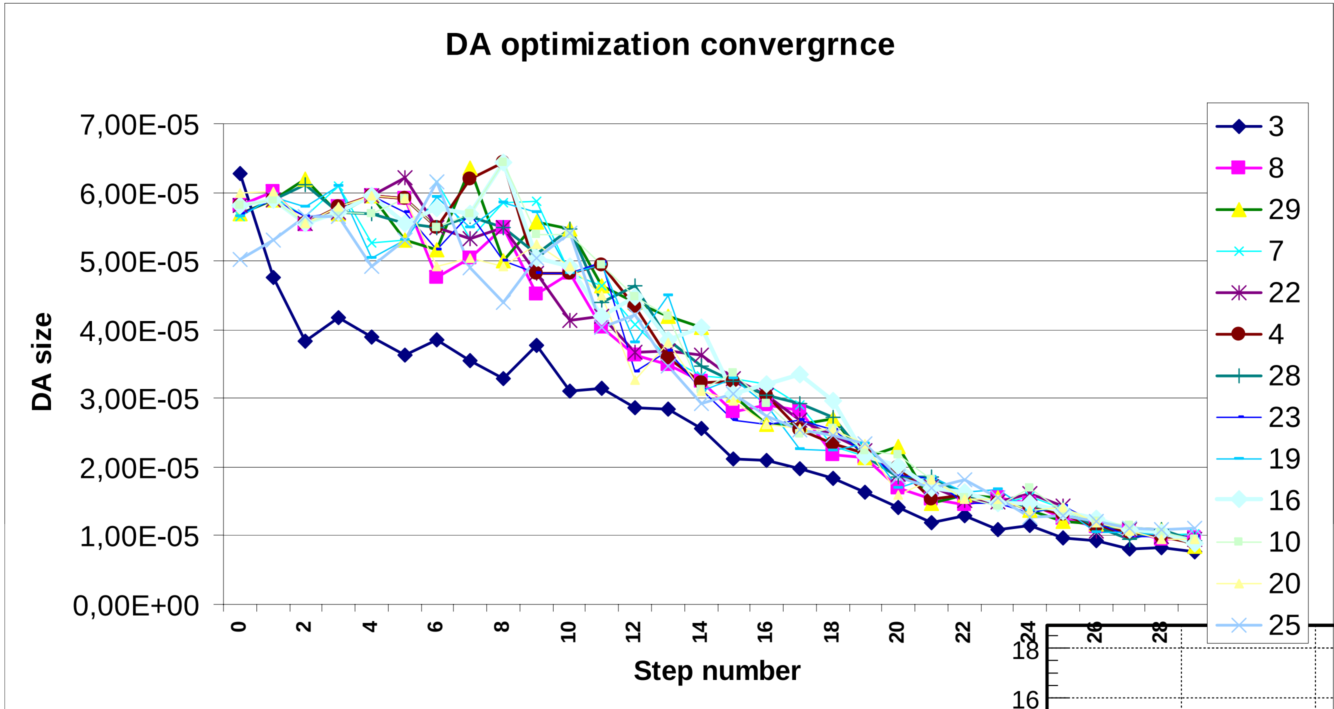


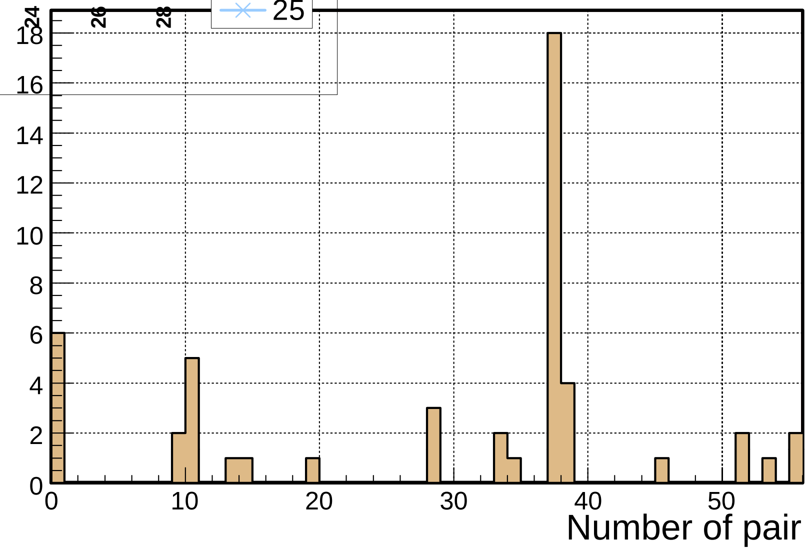
↑ One step of 20 sextupoles pairs

DA definition at →
every step



DA increase (4): method convergence





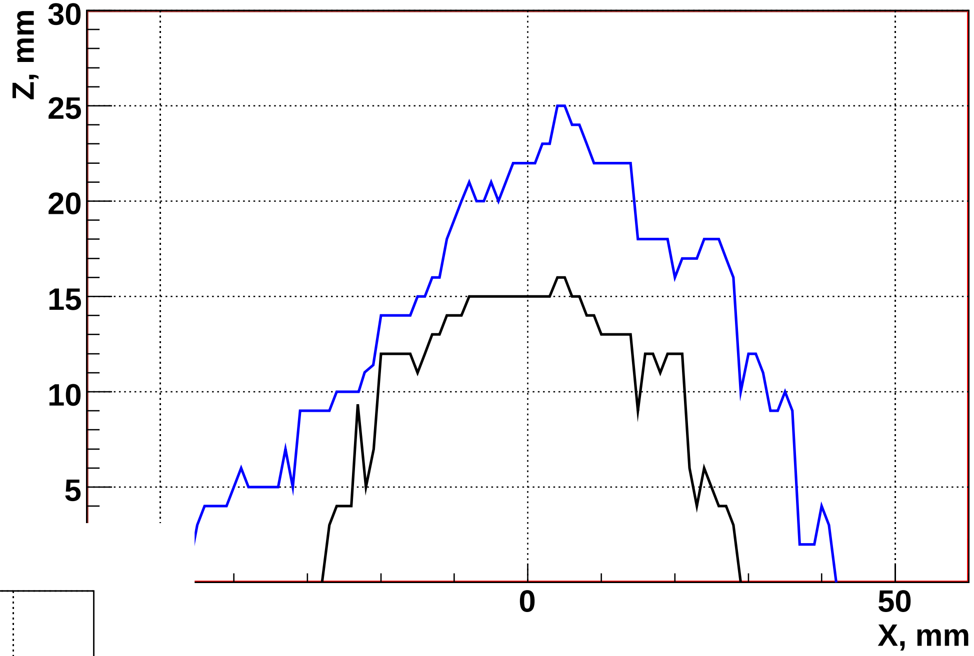
In the example there are 32 sextupole pairs but only 13 of them are effective for chromatic correction and DA optimization

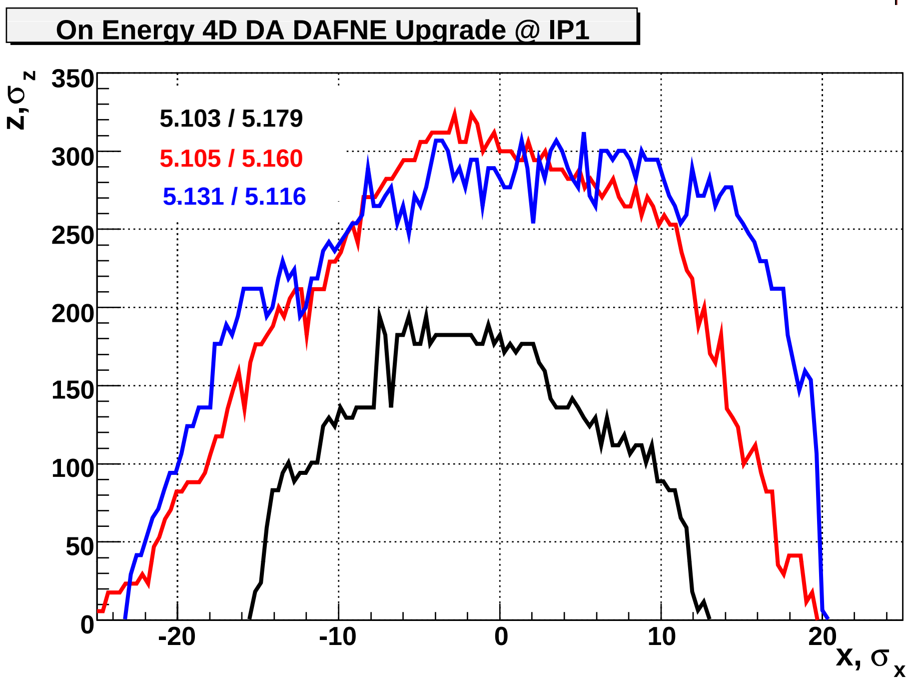
DA increase (5): method effectiveness

ALBA light source →

Black – DA optimized from usual theoretical predictions

Blue – DA optimized by the best sextupole pair method





← DAΦNE Siddhartha

Black – initial DA

Blue – DA optimized by the best sextupole pair method

Red – the same but different tune point

Summary

- In the low emittance lattice the absolute value of DA reduces rather strongly as DA~σ/ξ but the relative value reduces much more moderate DA/σ~$1/\xi$
- Strong crab sextupoles should be studied carefully from a viewpoint of phase advance errors, length, chromatic errors, etc.
- For such strong focusing machines as SuperB and Ctau with powerful crab sextupoles study of joint effects of BB and magnetic nonlinearities is required
- High order field errors in the FF quadrupoles (high betas) is a matter of special care to obtain large DA
- The best sextupole pair method seems simple and effective to optimize the DA