DYNAMIC APERTURE STUDIES IN e^+e^- FACTORIES WITH CRAB WAIST

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Abstract

Crab Waist collision scheme being applied to the electron-positron collider may limit a dynamic aperture essentially. In the paper we discuss some aspects of such limitation including low emittance lattice, strong crab sextupoles, crosstalk between beam-beam nonlinear force and lattice nonlinearities, and small multipole errors in the final focus quadrupoles.

INTRODUCTION

Crab Waist (CW) collision approach was proposed recently [1-3] to obtain extremely high luminosity in e^+e^- colliders. This approach exploits two main potentially advantageous ideas.

According to the first idea, the Piwinski angle is increased by decreasing the horizontal beam size (low emittance lattice) and increasing the crossing angle. The most important effect here relates to the reduction of the overlap length of colliding bunches (much smaller than the bunch length) allowing us to obtain an ultra-low β_y at IP (fraction of mm).

However, a large Piwinski angle introduces new beam-beam resonances and may limit the maximum achievable tune shifts. This is where the second advantageous idea – the CW innovation – is required. The CW transformation boosts the luminosity, mainly by suppression of betatron and synchrobetatron resonances.

The CW correction scheme is realized in practice by two sextupole magnets in phase with the IP in the *x* plane and at $\pi/2$ in the *y* plane, on both sides of the IP.

The CW scheme features can reduce collider dynamic aperture through the following mechanisms:

- A low-emittance strong-focusing lattice requires a set of powerful sextupole magnets for chromaticity correction.
- The crab sextupoles phased as described above cancel each other exactly in a kick approximation limit. In reality the finite sextupole length and inevitable lattice errors break the cancellation condition.
- An extremely low beta-star at IP provides very large betatron amplitudes in the final focus quadrupoles making them sensitive to the magnetic multipole errors.
- The increased particles density at the interaction point (beam-beam effects) together with the reduced dynamic aperture emphasizes the importance of joint study of these two effects more realistically than before.

Below we consider these sources of the dynamic aperture limitation in the Crab Waist machines in details.

LOW EMITTANCE LATTICE

Some general features of the DA in the low emittance lattice can be found by simple analytic estimation using a well-known sextupole Hamiltonian in harmonic form

$$H = \nu_x J_x + \nu_y J_y$$
$$+(2J_x)^{3/2}\sum_n [3A_{1n}\cos(\varphi_x - n\theta) + A_{3n}\cos(3\varphi_x - n\theta)]$$
$$-3(2J_x)^{1/2}(2J_y)\sum_n [2B_{1n}\cos(\varphi_x - n\theta) + B_{+n}\cos(\varphi_+ - n\theta)$$
$$+B_{-n}\cos(\varphi_- - n\theta)],$$

where $\theta = s/R$ is the azimuthal angle (an independent variable), R is the average orbit radius and the five types of harmonics ($j = 1,3$)

$$A_{jn} = \frac{1}{48\pi}\sum_m \beta_{xm}^{3/2}(k_2 l)_m \cos(j\psi_x - \nu\theta + n\theta)_m,$$

$$B_{1n} = \frac{1}{48\pi}\sum_m \beta_{xm}^{1/2}\beta_{ym}(k_2 l)_m \cos(\psi_x - \nu\theta + n\theta)_m,$$

$$B_{\pm n} = \frac{1}{48\pi}\sum_m \beta_{xm}^{1/2}\beta_{ym}(k_2 l)_m \cos(\psi_\pm - \nu_\pm\theta + n\theta)_m$$

represent the main structural resonances. Sextupoles are considered as kicks with the normalized integrated strength $(k_2 l)_m$, and the values subscribed by "±" have the form $\psi_\pm = \psi_x \pm 2\psi_y$, etc.

After some manipulation [4, 5] the following simple estimation of the DA size can be found

$$A_x = k_x(\nu_x,\nu_y)\cdot\frac{\sigma_{x0}}{\xi_x}, \quad A_y = k_y(\nu_x,\nu_y)\cdot\frac{\sigma_{y0}}{\xi_y}, \qquad (1)$$

where σ is the beam size at the DA observation point and ξ is the natural chromaticity. Coefficient k depends weakly on the tune point and the lattice details.

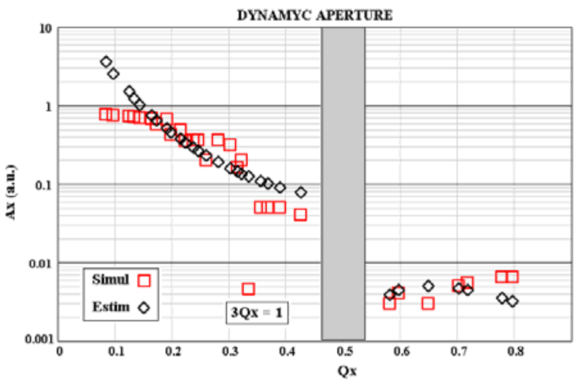


Fig. 1 Horizontal DA size obtained analytically (red) and numerically (black). Grey strip shows the optically unstable area near the half integer resonance.

To verify the above expressions we performed a computer simulation of the DA size as a function of the horizontal tune that unambiguously represents the lattice focusing strength (emittance). The results as they are shown in Fig.1 demonstrate good correspondence between analytic and numeric calculation.

CRAB WAIST SEXTUPOLES

Betatron phase advance between two (point-like) crabbing sextupoles (see Fig.2) provides an exact cancellation of its influence on DA.

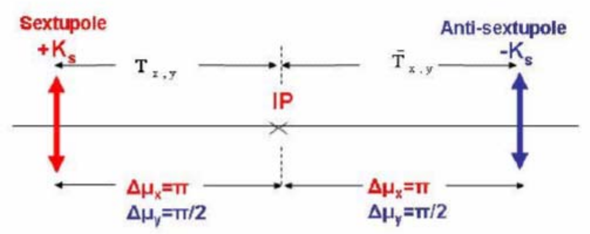


Fig.2 Crabbing sextupoles arrangement

However, due to different reasons (lattice errors, finite length of the sextupoles, chromatic effects, beam-beam effects, etc.) the exact phase tuning breaks and the residue aberration (small but applied for strong sextupoles) influence the dynamic aperture.

We studied these effects numerically for a simple collider model which includes a set of nonlinear beam-beam kicks, two CW sextupoles, two "other" sextupoles, which imitate chromatic sextupoles and a set of matrices providing the betatron phase tuning between the nonlinear elements. The results of the simulation are presented below.

Fig.3 shows a scan of the horizontal DA ratio with and without the linear phase mismatch between the crab sextupoles.

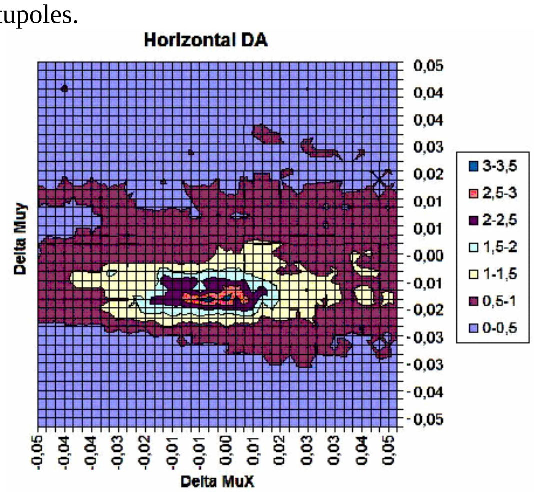


Fig.3 A horizontal DA changes as a function of the betatron phase error between crab sextupoles

The color in the Fig.3 indicates the ratio DA_{error}/DA_{ideal} as a function of the betatron phase error between the crab sextupoles: $\Delta\mu_x = 2\pi n - \mu_x(S_1 - S_2)$, $\Delta\mu_y = \pi n - \mu_y(S_1 - S_2)$. One can see that the residue perturbation of the mismatched crabbing sextupoles interferes with the perturbation from all other (chromatic) sextupoles and can either increase (twice for $\Delta\mu_y = -0.01$) or decrease (twice for $\Delta\mu_y = +0.01$) the dynamic aperture.

The fact that the real sextupole is not a kick-like object but has finite length yields another effect on the DA. Fig.4 shows how the DA reduction depends on the crab sextupole length: for $L = 0.2$ m the vertical DA shrinks by factor of 3 and the horizontal one twice.

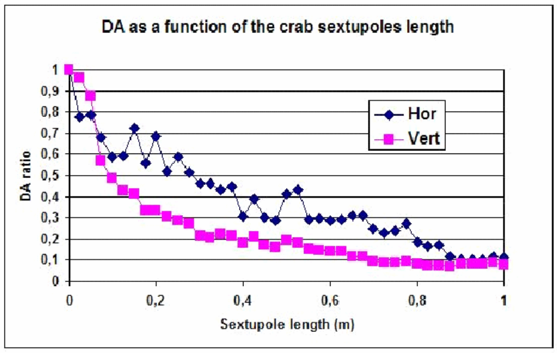
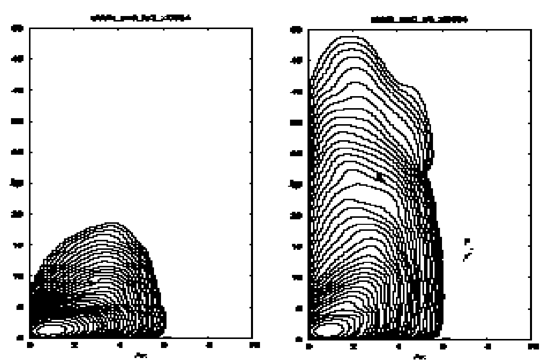


Fig.4 DA reduction vs. the crab sextupole length

BEAM-BEAM AND SEXTUPOLES

Strong sextupole effect (both chromatic and crab) can interfere with the intensive beam-beam interaction and produce a crosstalk in a self-consistent manner: beam-beam interaction reduces a DA initially limited by the sextupoles and the reduced DA gives the beam lifetime degradation through the beam tail growing.

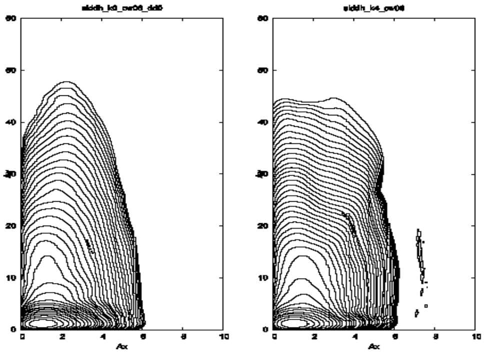
To investigate these phenomena we have combined a beam-beam computer code LIFETRACK [6] with the general tracking code ACCELERATICUM [7] and applied new software to the DAΦNE e^+e^- collider in the Siddharta Crab Waist operation mode [8].



Sextupole off *Sextupole on*

Fig.5 Vertical tail grows due to the joint effect of the BB and chromatic sextupole nonlinearities

The results are given in Figs. 5 and 6. The plots in Figures demonstrate the contour lines for the particle density distribution in the betatron amplitude space. In Fig.5 the beam-beam effects are studied with the chromatic sextupoles on and off. One can see that the sextupoles induce the vertical tail growth, which, in case of DA deficiency, would degrade the beam lifetime. In Fig.6 the crab sextupoles are added to the chromatic ones and again the chromatic sextupoles are on (right plot) and off (left plot).



Sextupole off *Sextupole on*

Fig.6 Vertical tail growth due to the joint effect of the BB and chromatic sextupoles is cured by the crab sextupole switched on for both plots

But this time there is no vertical amplitude growth because the crab sextupoles improve the situation for the beam-beam effects in the case of the Crab Waist collision scheme.

MULTIPOLE ERRORS IN THE FF QUADS

High beta values in the final focus quadrupoles and possible offset of the beam orbit in the first quad (due to the large crossing angle) can emphasize the influence of the high multipoles content in the FF quads to the beam dynamics.

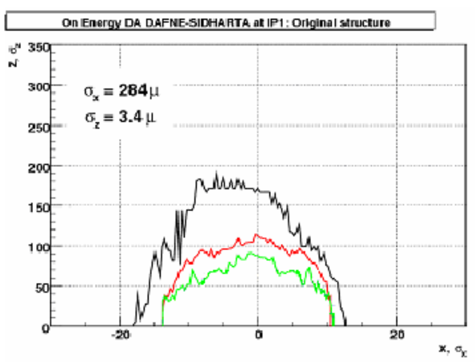


Fig.7 The ideal DA (black) vs. the DA with errors in the FF quadrupoles for the electron (red) and positron (grin) beams

For the Siddharta experiment at DAΦNE new permanent magnet FF quadrupoles were produced by Aster Enterprises Inc. and the magnetic field components were carefully measured by rotating coils. We introduced the harmonic coefficients in the machine lattice and provided particles tracking by the ACCELERATICUM code. As the beams orbit is shifted in the FF quadrupoles by x_0 = ±11 mm, the field expansion coefficients have to be transformed to the shifted coordinate frame according to

$$\left(B_n' + iA_n'\right) = \sum_{k=n}^{\infty}\left(B_n + iA_n\right)\left[\frac{(k-1)!}{(n-1)!(k-n)!}\right]\left(\frac{x_0}{R_0}\right)^{k-n},$$

where R_0 is the coil measuring radius.

The results of the DA calculation with the field errors in the FF quadrupoles are depicted in Fig.7 and one can see that when the multipole errors are taken into account, the vertical DA reduces almost twice as compared to the ideal case.

CONCLUSIONS AND OUTLOOK

Several sources of the DA limitations in the Crab Waist collider have been considered. The conclusions are:

- A low emittance lattice provides general DA reduction according to $A \sim \sigma / \xi$ where σ and ξ are the beam size and natural chromaticity, respectively.
- In spite of the fact that crab sextupoles are properly phased to cancel combined aberrations but lattice errors, finite length, etc. can detune the phasing and cause the DA deterioration.
- Common influence of the BB and lattice nonlinearities should be studied carefully by the special codes, which consider all above effects realistically.
- Crab Waist scheme provides tough constrains to the FF quadrupole multipole errors tolerance.

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