Crab Waist Collision Studies for e+e- Factories

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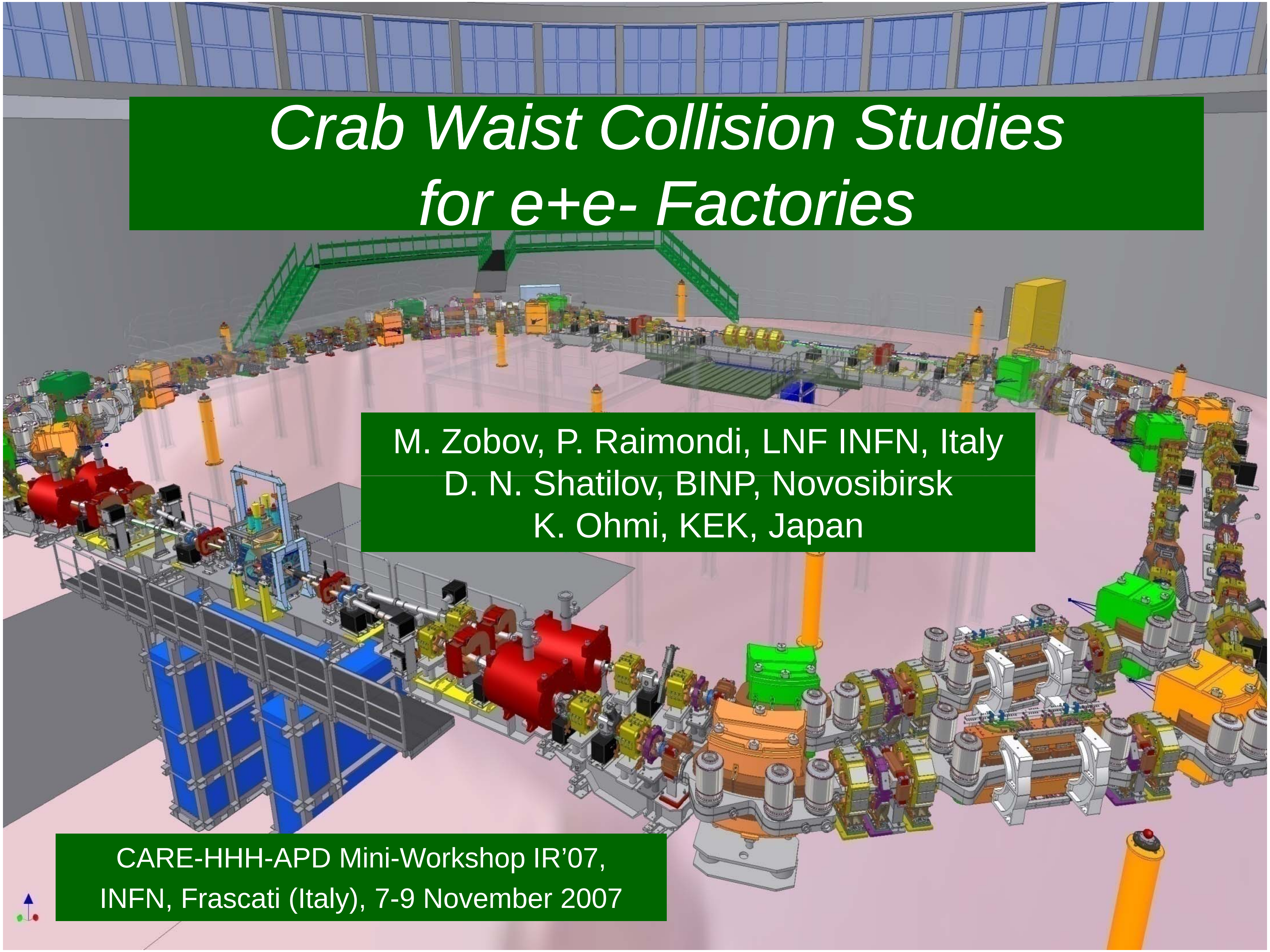


CARE-HHH-APD Mini-Workshop IR'07,
INFN, Frascati (Italy), 7-9 November 2007

OUTLINE

- Crab Waist Concept
- Crab Waist Scheme for DAΦNE Upgrade
- 10^{36} $cm^{-2}s^{-1}$ in SuperB Factory

Numerical Codes Used

Weak-Strong Codes

1. BBC (K. Hirata, Phys.Rev.Lett.74, 2228 (1995))
2. LIFETRAC (D. Shatilov, Part.Accel.52, 65 (1996))
3. BBWS (K. Ohmi)

Strong-Strong Codes

1. BBSS, (K. Ohmi, PRSTAB 7, 104401, (2004))
2. GUINEA-PIG (D. Schulte, CERN-PS-099-014-LP) modified by P. Raimondi for storage rings

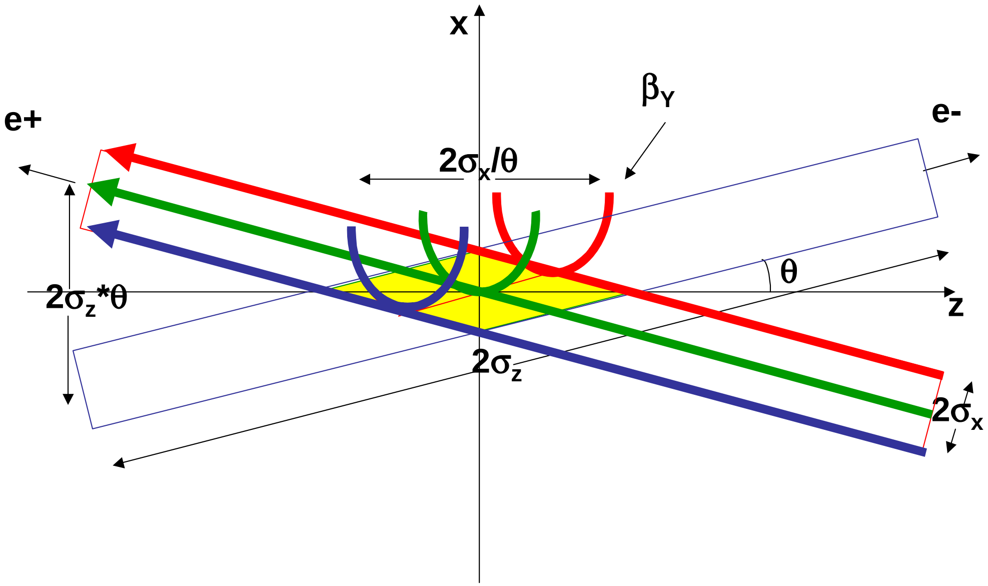
The codes have been successfully used for e+e- factories:
KEKB, DAΦNE, PEP-II, BEPCII and colliders: VEPP4M, VEPP2000.

Crab Waist in 3 Steps

1. Large Piwinski's angle $\Phi = tg(\theta)\sigma_z/\sigma_x$

2. Vertical beta comparable with overlap area $\beta_y \approx \sigma_x/\theta$

3. Crab waist transformation $y = xy'/(2\theta)$

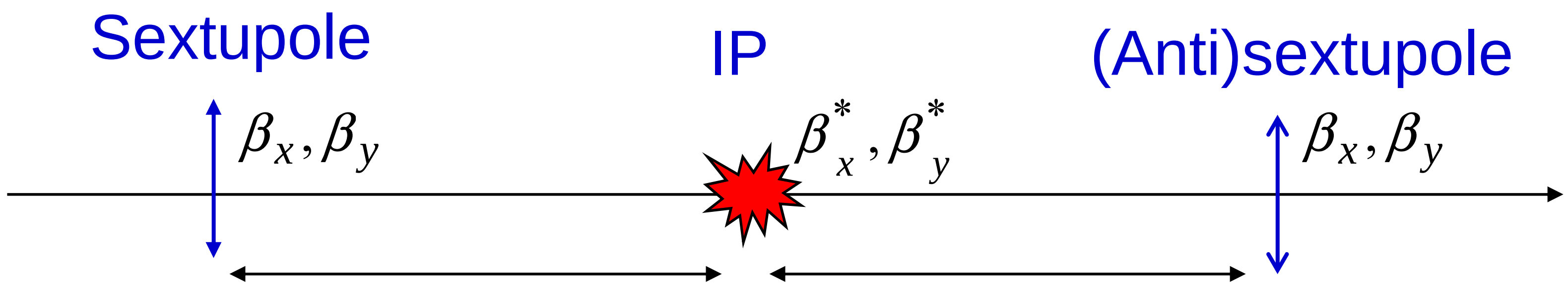


Crabbed waist is realized with a sextupole in phase with the IP in X and at π/2 in Y

1. *P.Raimondi, 2° SuperB Workshop, March 2006*

2. *P.Raimondi, D.Shatilov, M.Zobov, physics/0702033*

Crab Waist Scheme



$$\Delta\mu_y = \frac{\pi}{2} \qquad \Delta\mu_y = \frac{\pi}{2}$$

$$\Delta\mu_x = \pi \qquad \Delta\mu_x = \pi$$

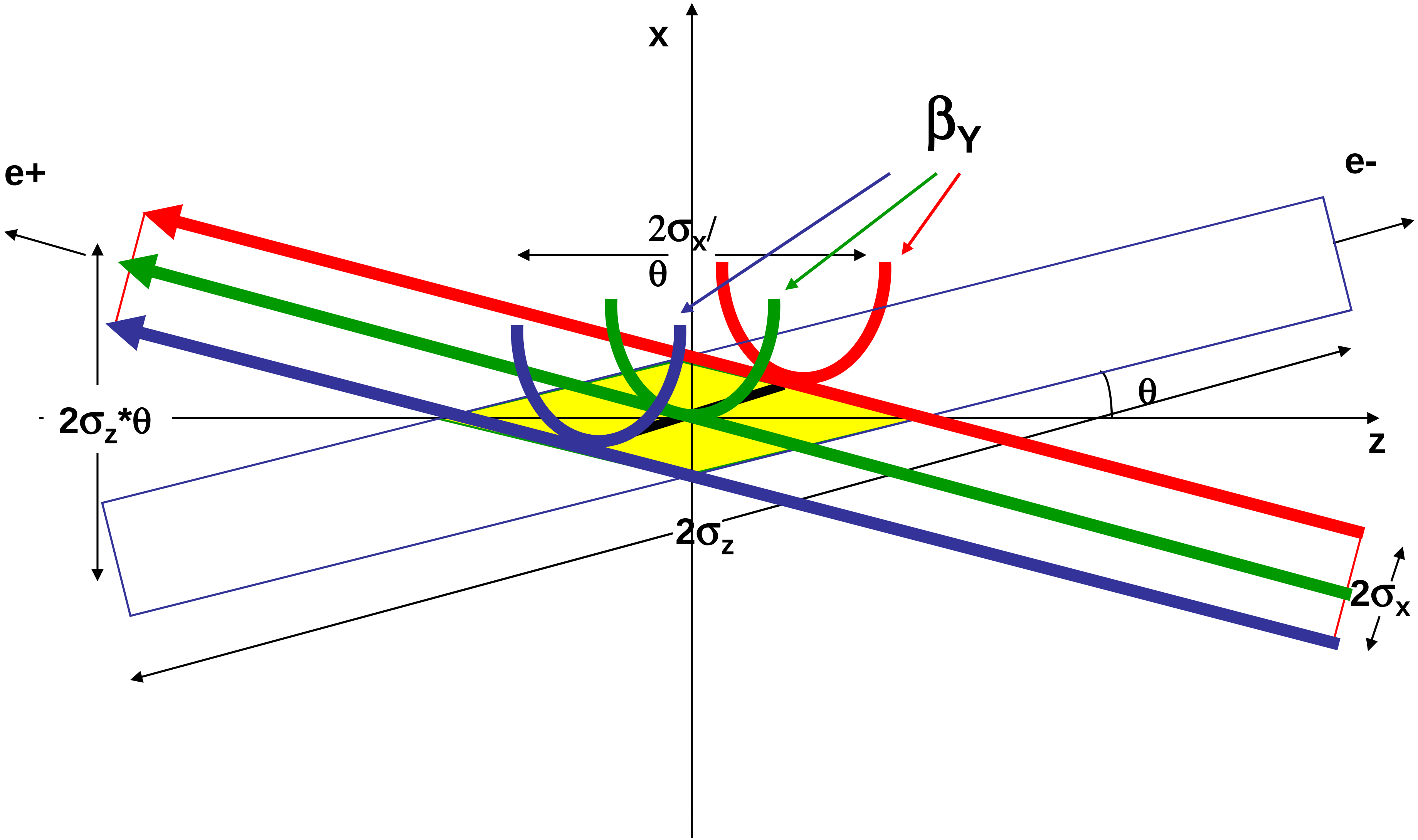
Sextupole strength

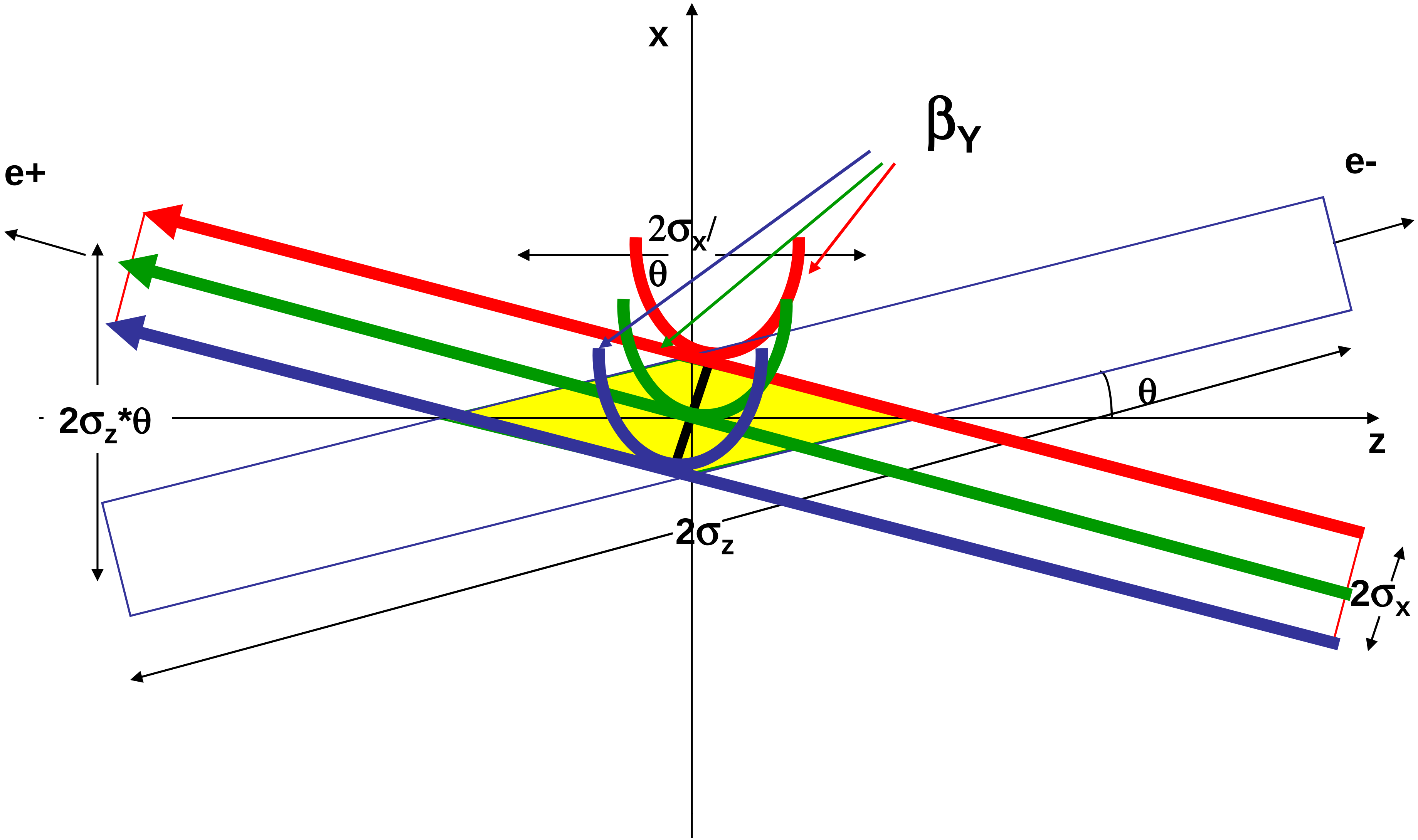
$$K = \frac{1}{2\theta}\frac{1}{\beta^*_y\beta_y}\sqrt{\frac{\beta^*_x}{\beta_x}}$$

Equivalent Hamiltonian

$$H = H_0 + \frac{1}{2\theta} x p_y^2$$

$$\beta_y = \beta^*_y + \frac{(s - x/\theta)^2}{\beta^*_y}$$


x
β_Y
e+
e-
$2\sigma_x/\theta$
$2\sigma_z*\theta$
θ
z
$2\sigma_z$
$2\sigma_x$


x
βY
e+
e-
2σx/
θ
θ
2σz*θ
z
2σz
2σx

Crab Waist Advantages

1. Large Piwinski's angle

$$\Phi = tg(\theta)\sigma_z/\sigma_x$$

- a) Geometric luminosity gain
- b) Very low horizontal tune shift

2. Vertical beta comparable with overlap area

$$\beta_y \approx \sigma_x/\theta$$

- a) Geometric luminosity gain
- b) Lower vertical tune shift
- c) Vertical tune shift decreases with oscillation amplitude
- d) Suppression of vertical synchro-betatron resonances

3. Crabbed waist transformation

$$y = xy'/(2\theta)$$

- a) Geometric luminosity gain
- b) Suppression of X-Y betatron and synchro-betatron resonances

..and besides,

a) There is no need to increase excessively beam current and to decrease the bunch length:

1) Beam instabilities are less severe
2) Manageable HOM heating
3) No coherent synchrotron radiation of short bunches
4) No excessive power consumption

b) The problem of parasitic collisions is automatically solved due to higher crossing angle and smaller horizontal beam size

Large Piwinski's Angle

O. Napoly, Particle Accelerators: Vol. 40, pp. 181-203,1993

P.Raimondi, M.Zobov, DAΦNE Technical Note G-58, April 2003

$$L = n_b f_0 \frac{1}{4\pi\sigma_x\sigma_y}\left[\frac{N^2}{\sqrt{1+\Phi^2}}\right]; \quad \xi_y = \frac{r_e\beta_y}{2\pi\gamma\sigma_y\sigma_x}\left[\frac{N}{\sqrt{1+\Phi^2}}\right]; \quad \xi_x = \frac{r_e\beta_x}{2\pi\gamma\sigma_x^2}\left[\frac{N}{1+\Phi^2}\right]$$

If we can increase *N* proportionally to Φ^*:

1) *L* grows proportionally to Φ;
2) ξ_y remains constant;
3) ξ_x decreases as $1/\Phi$;

$^*\Phi$ is increased by:

a) increasing the crossing angle θ and increasing the bunch length σ_z for LHC upgrade (F. Ruggiero and F. Zimmermann)

b) increasing the crossing angle θ and decreasing the horizontal beam size σ_x in crabbed waist scheme

Low Vertical Beta Function

$$L = n_b f_0 \frac{1}{4\pi\sigma_x\sigma_y}\left[\frac{N^2}{\sqrt{1+\Phi^2}}\right] = n_b f_0 \frac{1}{4\pi\sigma_x\sqrt{\beta_y\varepsilon_y}}\left[\frac{N^2}{\sqrt{1+\Phi^2}}\right] \propto \frac{1}{\sqrt{\beta_y}}$$

$$\xi_y = \frac{r_e\beta_y}{2\pi\gamma\sigma_x\sigma_y}\left[\frac{N}{\sqrt{1+\Phi^2}}\right] = \frac{r_e\beta_y}{2\pi\gamma\sigma_x\sqrt{\beta_y\varepsilon_y}}\left[\frac{N}{\sqrt{1+\Phi^2}}\right] \propto \sqrt{\beta_y}$$

Note that keeping ξ_y constant by increasing the number of particles N proportionally to $(1/\beta_y)^{1/2}$:

$$L \propto \left(\frac{1}{\beta_y}\right)^{3/2}$$

(If ξ_x allows...)

Vertical Synchro-Betatron Resonances

For all (except, maybe, integer and parametric) resonances the values of $(Y_m^{\rm I})_0$ become very small [8] in the region $\beta_z^* \simeq \sigma_x/\phi \ll \sigma_s$, which corresponds to very long bunches, or to a micro-β lattice. In this region the value of the luminosity will be determined by limitations due to horizontal

D.Pestrikov, Nucl.Instrum.Meth.A336:427-437,1993

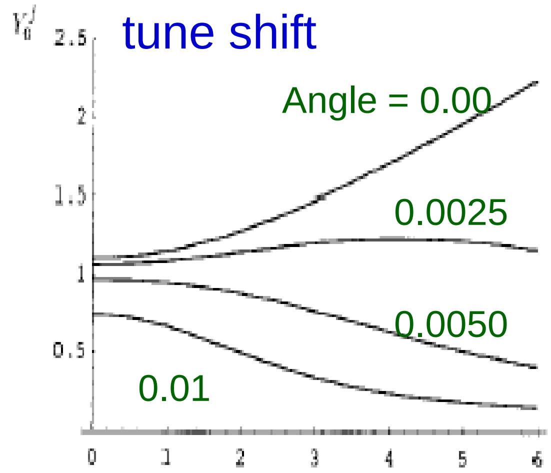
This results in the following expression

$$Y_m^{\rm f} = \frac{\sigma_x}{\phi\sigma_s} \mathcal{I}_{m_s}(a_s/\sigma_s)(Y_m^{\rm f})_0 , \qquad (34)$$

Resonance suppression factor

where the factor $(Y_m^{\rm f})_0$ coincides with a resonance suppressing factor of the synchronous particle ($a_s = 0$), calculated for the bunch length $\sigma_s' = \sigma_x/\phi$:

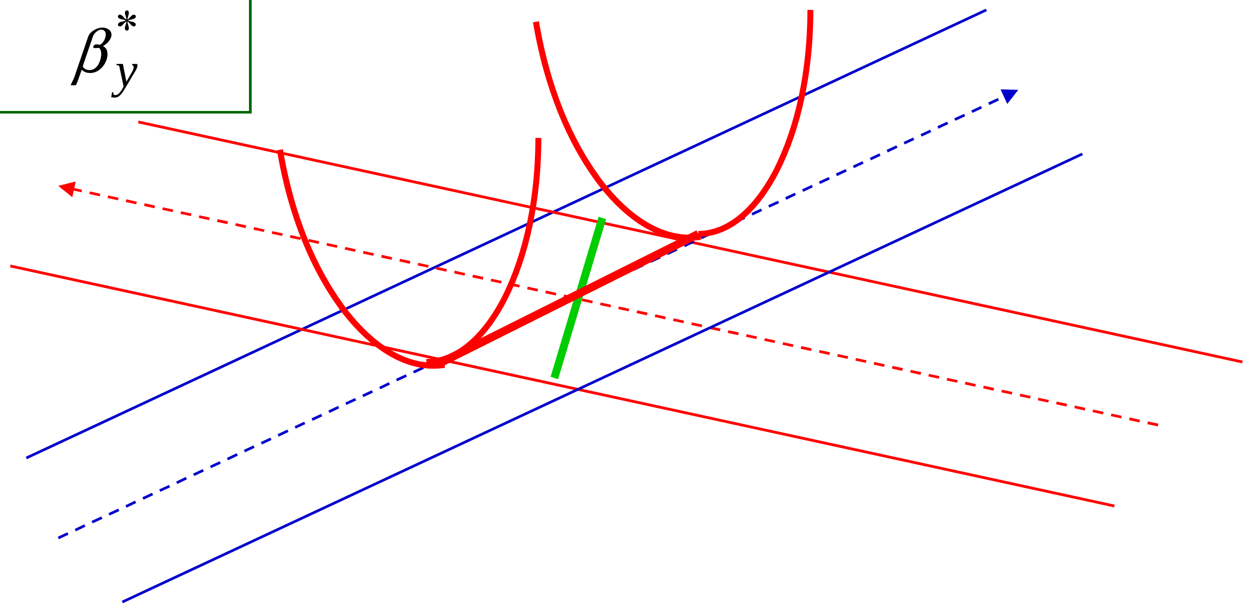
$$(Y_m^{\rm f})_0 = \sqrt{2/\pi} \int_{-\infty}^{\infty} du \exp[-2u^2 + i m_z \psi_z(\zeta_\phi u)] \sqrt{1 + \zeta_\phi^2 u^2}, \quad \zeta_\phi = \frac{\sigma_x}{\phi \beta_z^*}. \qquad (35)$$



Synchrotron amplitude in σ_z

Geometric Factors

$$\beta_y = \beta_y^* + \frac{(s - x/\theta)^2}{\beta_y^*}$$



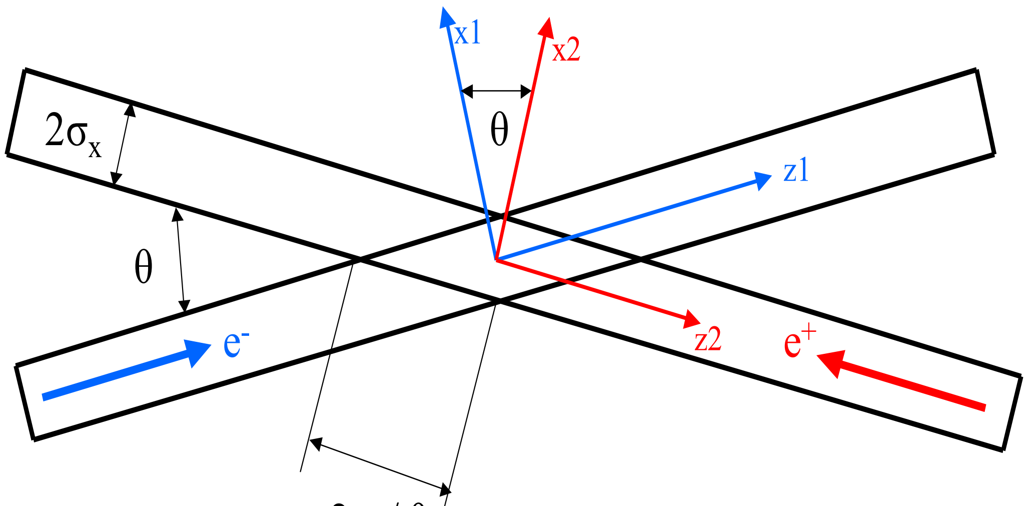
1. Minimum of βy along the maximum density of the opposite beam;

2. Redistribution of βy along the overlap area. The line of the minimum beta with the crab waist (red line) is longer than without it (green line).

$$L = 2cf_0 Cos\left(\frac{\theta}{2}\right)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\rho_1(x,y,z,t)\rho_2(x,y,z,t)dxdydzdt$$

$$\rho_1(x_1,y_1,z_1,t) = \frac{N_1}{(2\pi)^{3/2}\sigma_{x_1}\sigma_{z_1}\sigma_{y_1}(x_1,z_1)}\exp\left\{-\frac{x_1^2}{2\sigma_{x_1}^2} - \frac{(z_1-ct)^2}{2\sigma_{z_1}^2} - \frac{y^2}{2\sigma_{y_1}^2(x_1,z_1)}\right\}$$

$$\rho_2(x,y,z,t) = \frac{N_2}{(2\pi)^{3/2}\sigma_{x_2}\sigma_{z_2}\sigma_{y_2}(x,z)}\exp\left\{-\frac{x^2}{2\sigma_{x_2}^2} - \frac{(z+ct)^2}{2\sigma_{z_2}^2} - \frac{y^2}{2\sigma_{y_2}^2(x,z)}\right\}$$



$$\sigma_{y_1}(x_1,z_1) = \sigma_{y_1}^* \sqrt{1 + \frac{(z_1 - x_1/Tan(\phi_1))^2}{\beta_{y_1}^2}}$$

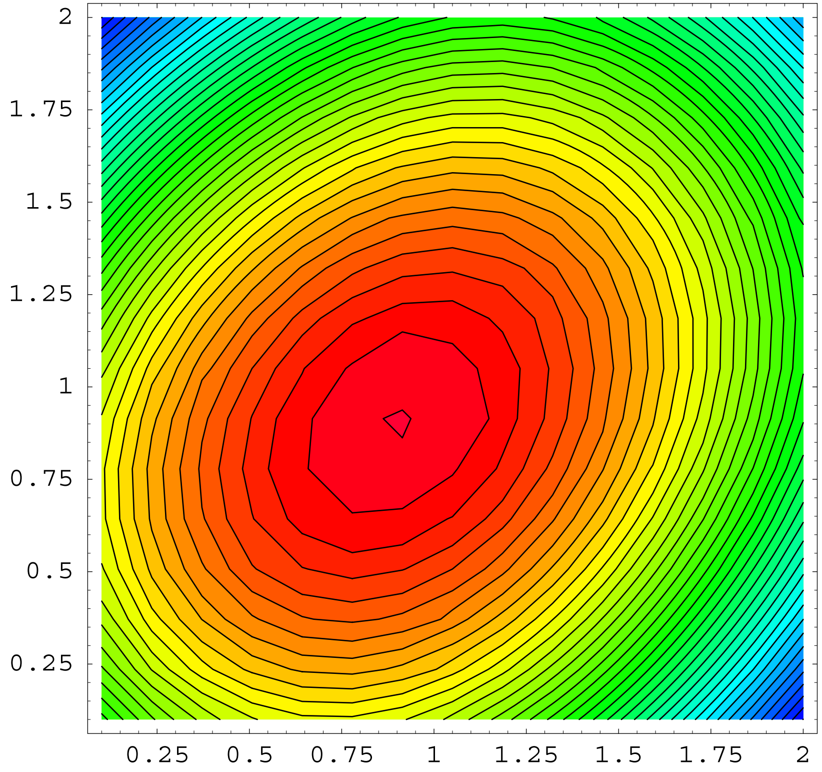
$$\sigma_{y_2}(x,z) = \sigma_{y_2}^* \sqrt{1 + \frac{(z - x/Tan(\phi_2))^2}{\beta_{y_2}^2}}$$

$$x_1 = x\cos(\theta) - z\sin(\theta)$$
$$z_1 = x\sin(\theta) + z\cos(\theta)$$
$$x_2 \equiv x$$
$$z_2 \equiv z$$

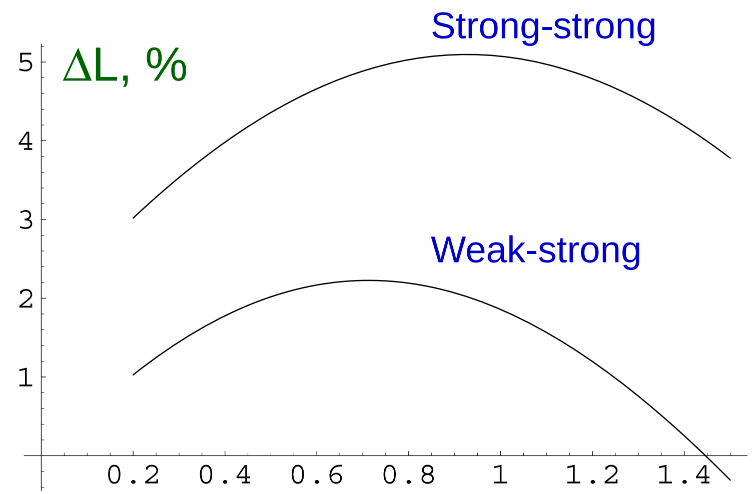
Crab Waist Collisions at $\phi_1 = -\theta$, $\phi_2 = \theta$

Geometric Luminosity Gain due to Crab Sextupoles

(DAΦNE Example)



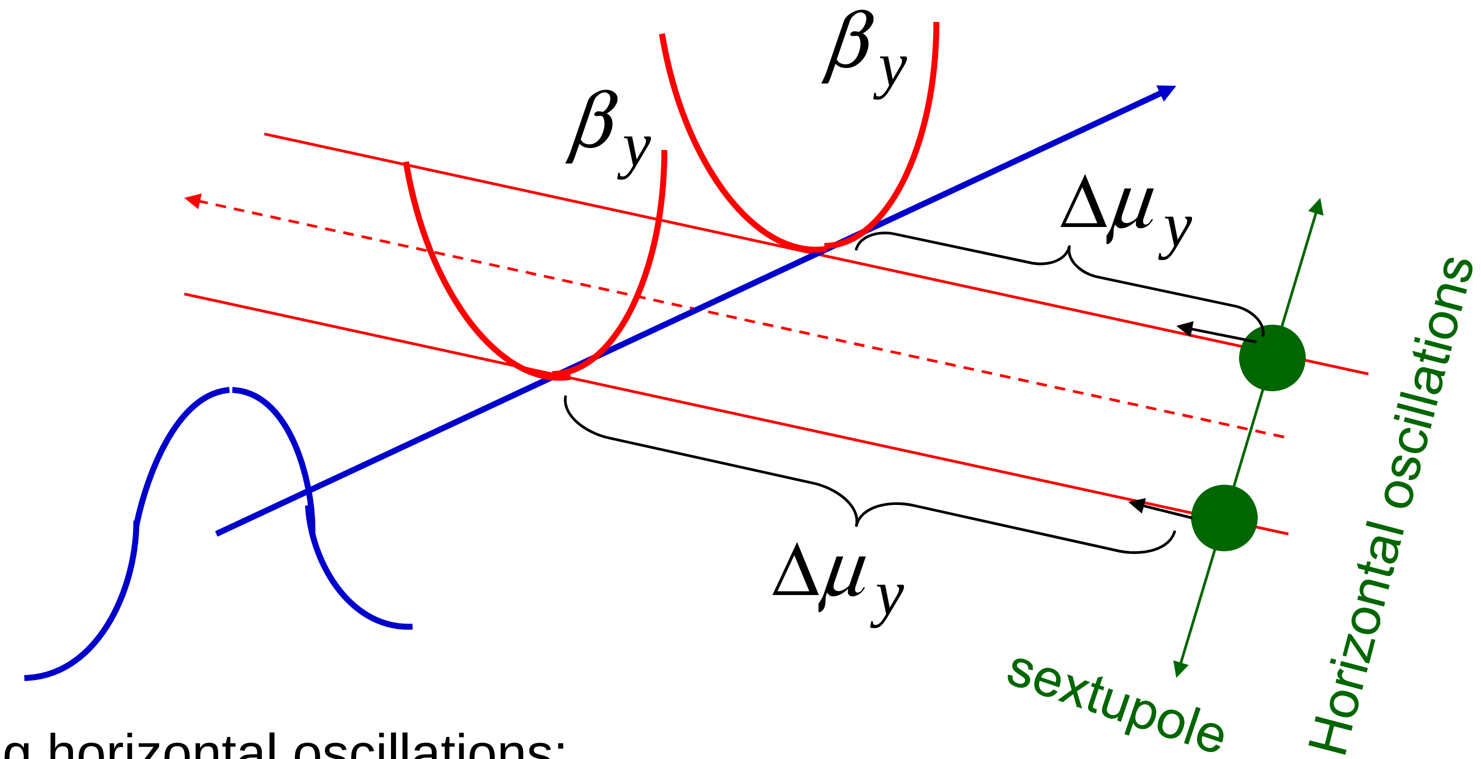
Normalised sextupole strength



Normalised sextupole strength

"..crabbed waist" idea does not provide the significant luminosity enhancement. Explanation could be rather simple: the effective length of the collision area is just comparable with the vertical beta-function and any redistribution of waist position cannot improve very much the collision efficiency…" (I. A. Koop, D.B.Shwatz)

Suppression of X-Y Resonances

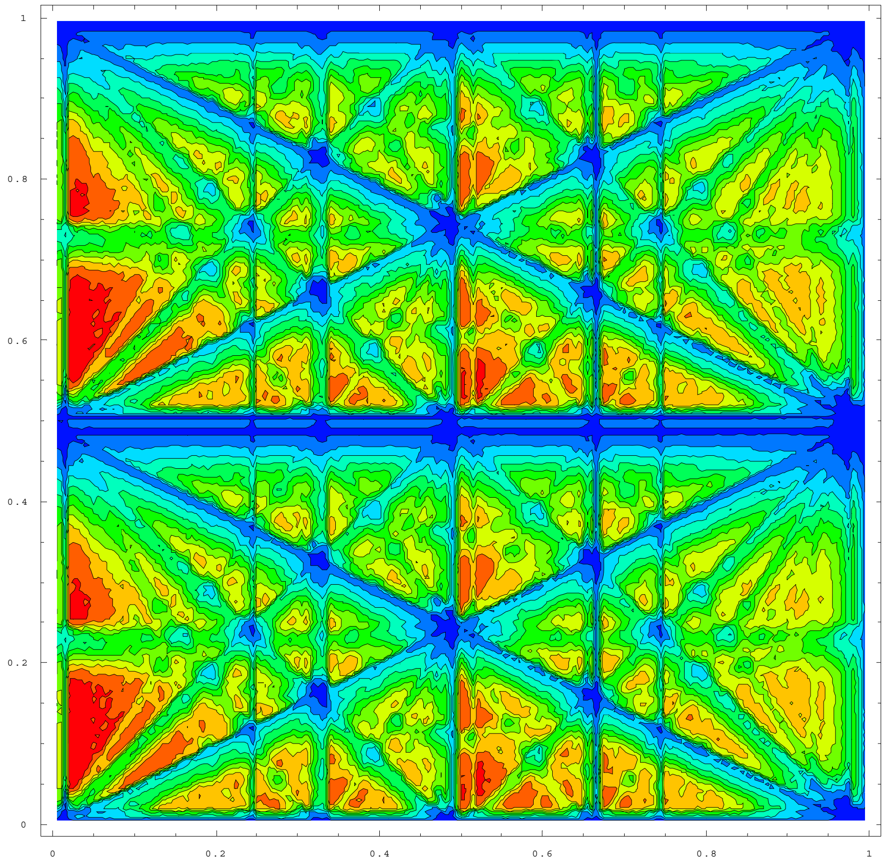


Performing horizontal oscillations:

1. Particles see the same density and the same (minimum) vertical beta function
2. The vertical phase advance between the sextupole and the collision point remains the same ($\pi/2$)

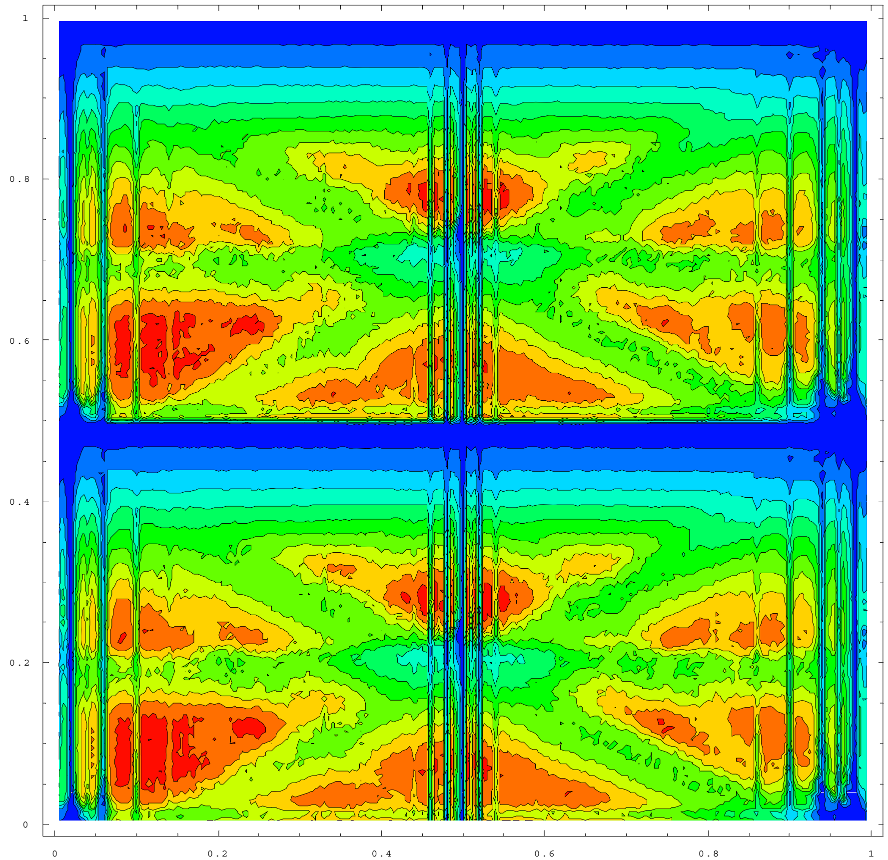
X-Y Resonance Suppression

Much higher luminosity!



Typical case (KEKB, DAΦNE etc.):

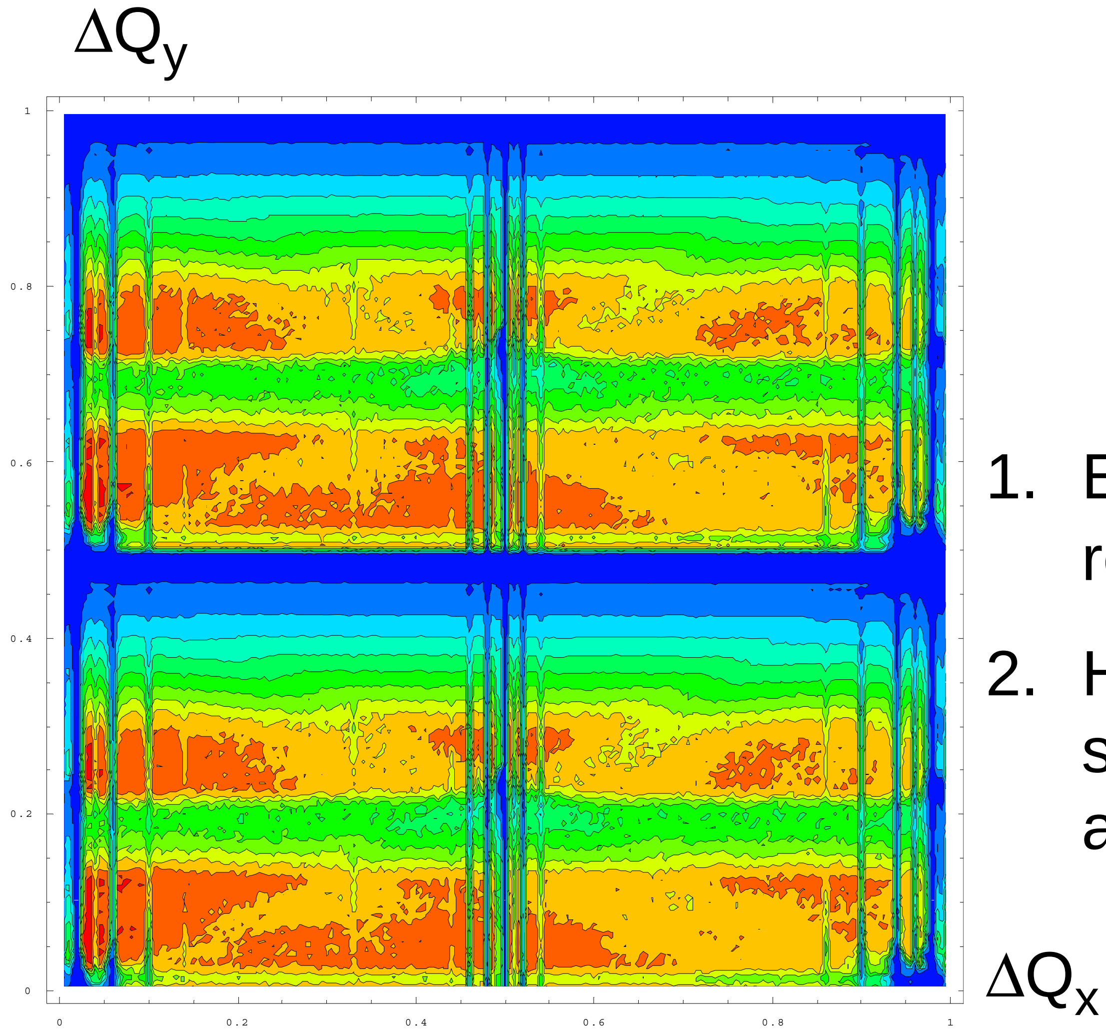
1. low Piwinski angle $\Phi < 1$
2. β_y comparable with σ_z



Crab Waist On:

1. large Piwinski angle $\Phi >> 1$
2. β_y comparable with σ_x/θ

... and in the ideal case



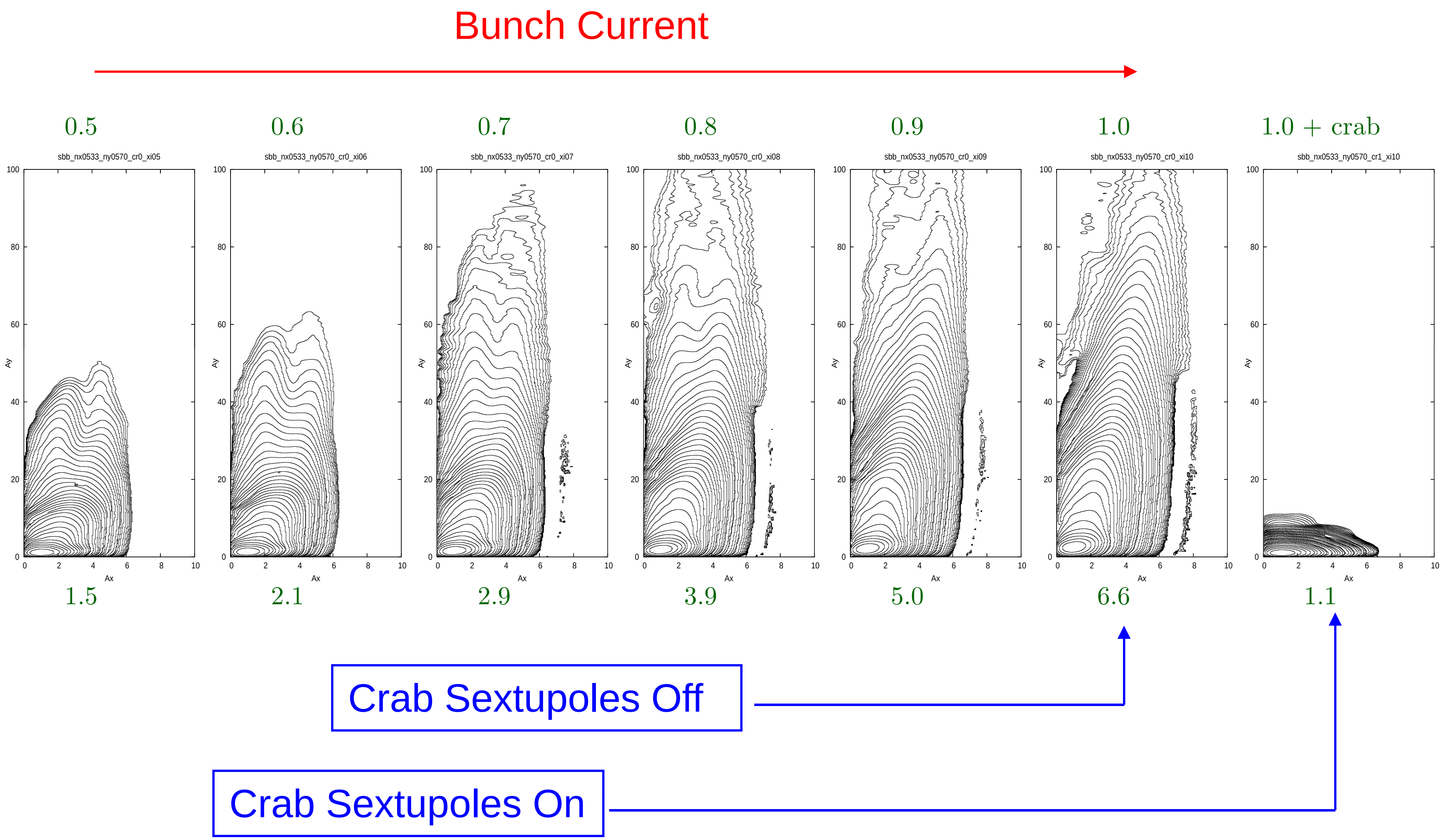
Here strong beam's modulation is excluded

(100 times larger β_y and smaller ε_y)

Crab Waist:

1. Eliminates all (!) X-Y resonances
2. However, some horizontal synchrobetatron resonances appear

Tails in SuperB



DAΦNE Upgrade Parameters

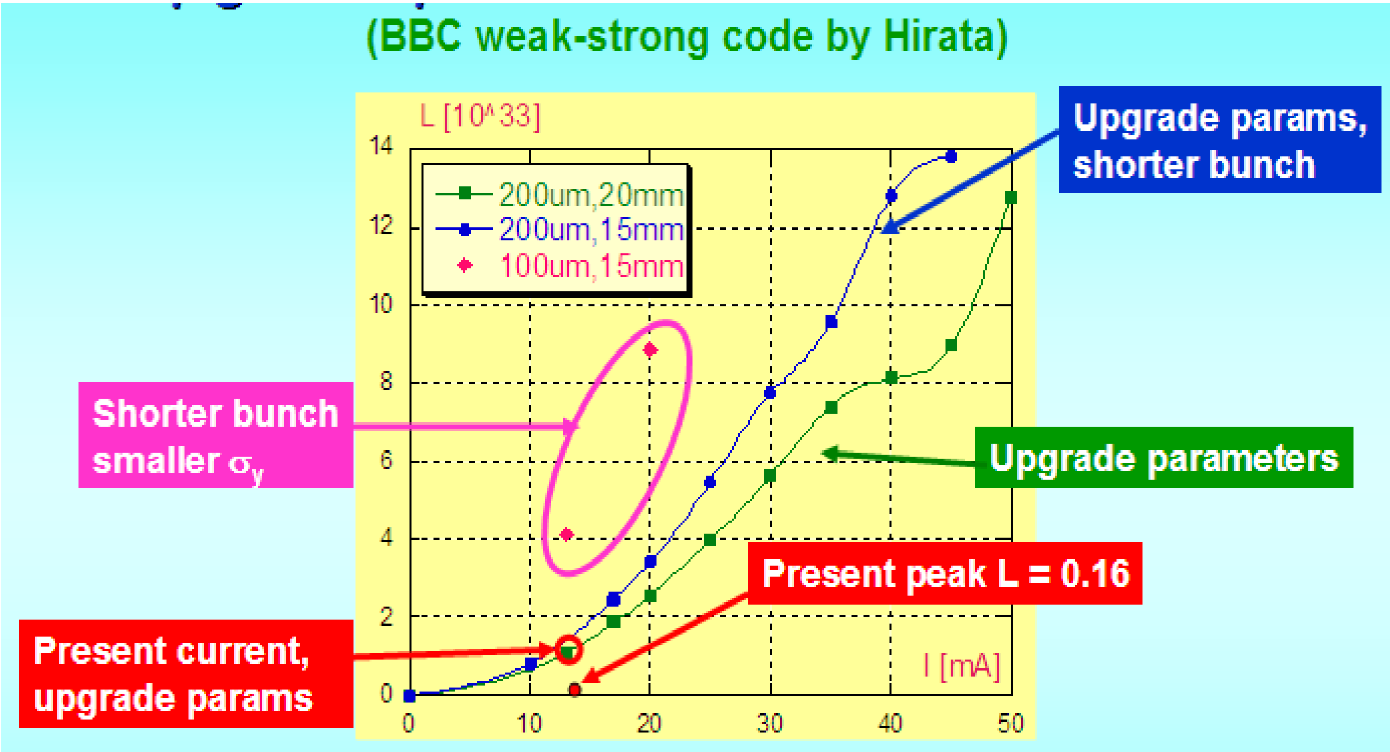
	DAΦNE FINUDA	DAΦNE Upgrade
$\theta_{cross}/2$ (mrad)	12.5	25
ε_x (mm×mrad)	0.34	0.20
β_x^* (cm)	170	20
σ_x^* (mm)	0.76	0.20
$\Phi_{Piwinski}$	0.36	2.5
β_y^* (cm)	1.70	0.65
σ_y^* (μm)	5.4 (low current)	2.6
Coupling, %	0.5	0.5
I_{bunch} (mA)	13	13
N_{bunch}	110	110
σ_z (mm)	22	20
L ($cm^{-2}s^{-1}$) $\times 10^{32}$	1.6	10

Larger Piwinski angle

Lower vertical beta

Already achieved

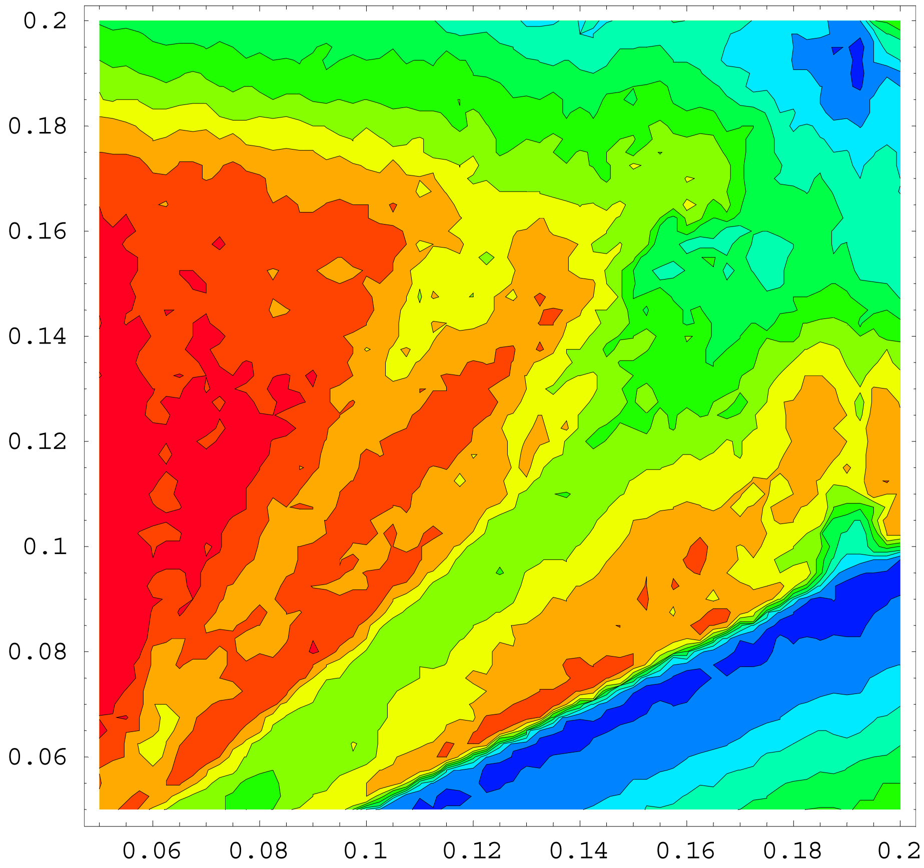
Weak-Strong Beam-Beam Simulation for DAΦNE Upgrade



1. With the present DAΦNE parameters (currents, bunch length etc.) a luminosity in excess of 10^{33} cm^{-2} s^{-1} is predicted
2. With 2A on 2A more than $2x10^{33}$ is possible
3. Beam-beam limit is well above the reacheable currents

Luminosity vs tunes scan

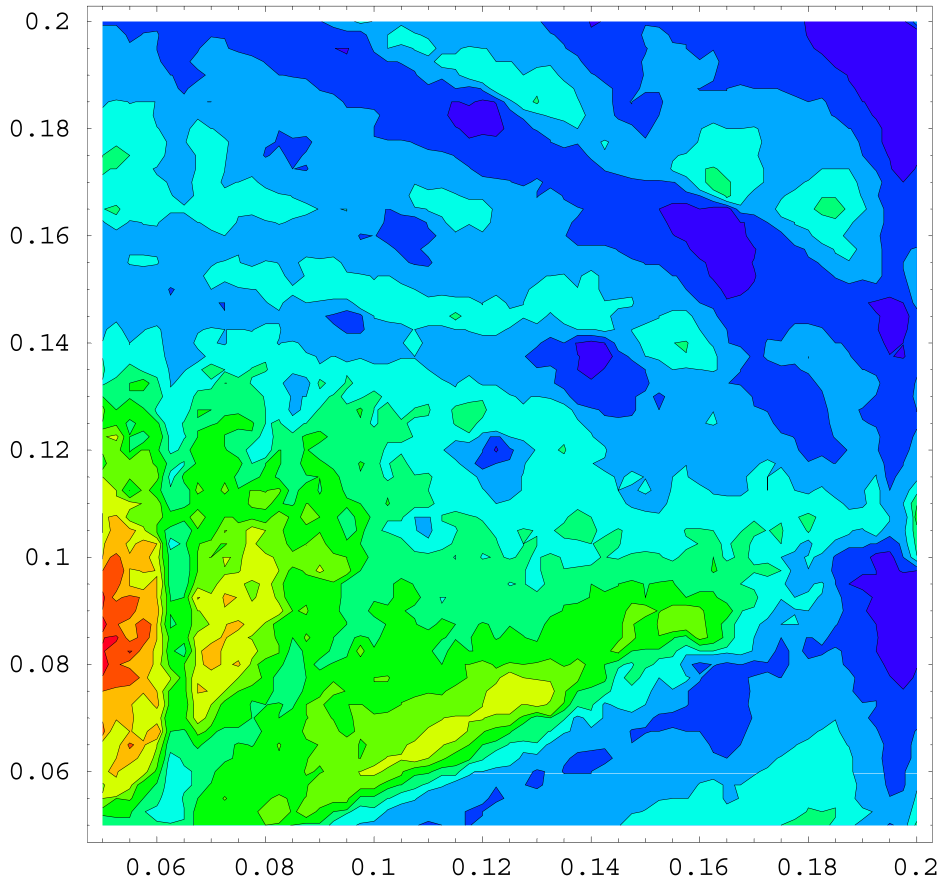
Crab On → 0.6/θ



$L_{max} = 2.97 \times 10^{33}\ cm^{-2}s^{-1}$

$L_{min} = 2.52 \times 10^{32}\ cm^{-2}s^{-1}$

Crab Off

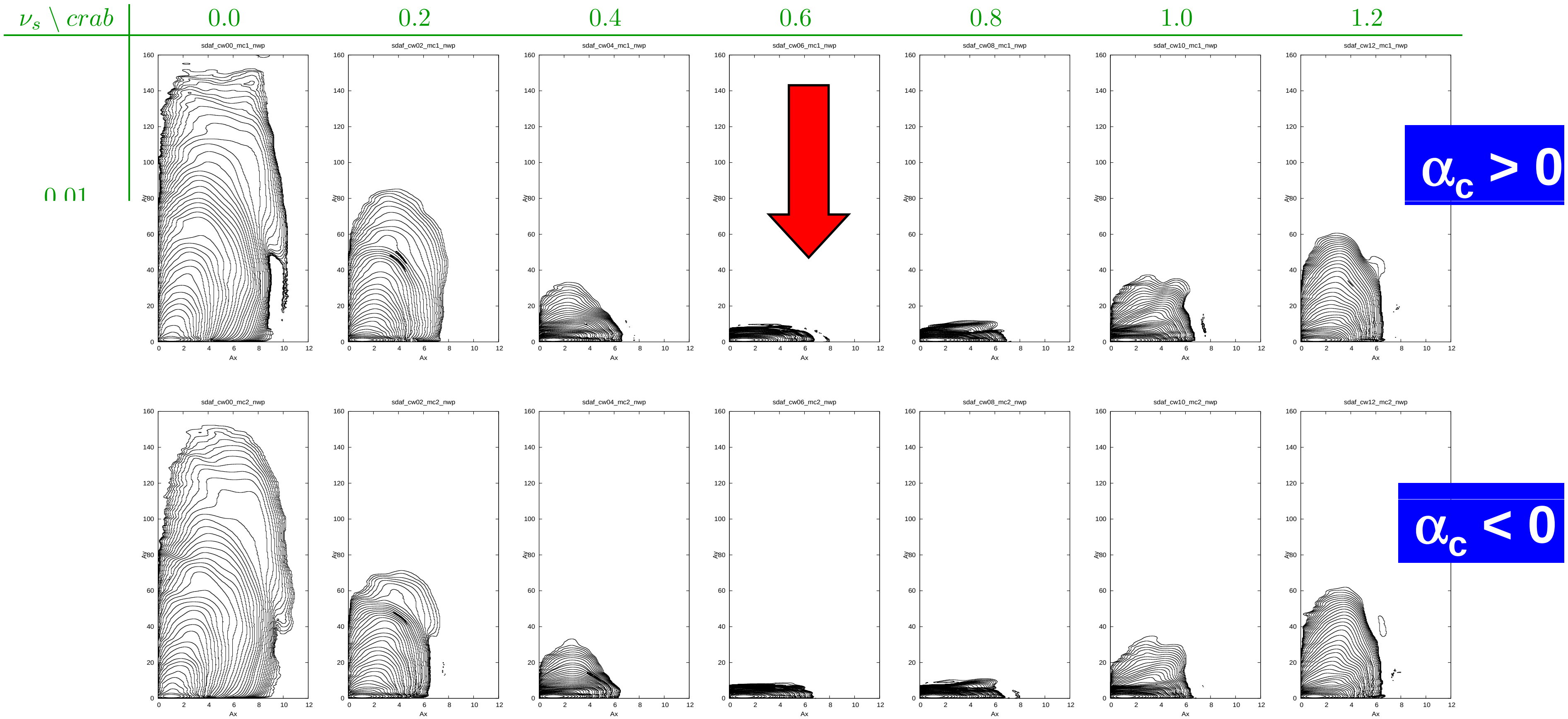


$L_{max} = 1.74 \times 10^{33}\ cm^{-2}s^{-1}$

$L_{min} = 2.78 \times 10^{31}\ cm^{-2}s^{-1}$

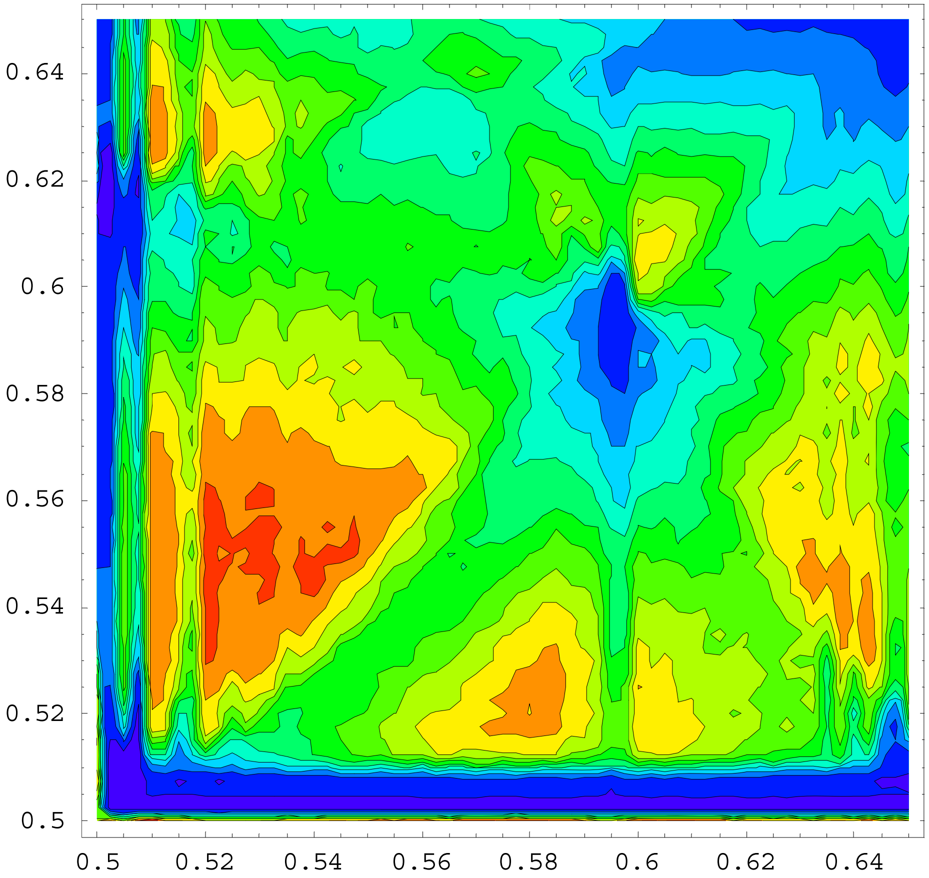
Beam-Beam Tails at (0.057;0.097)

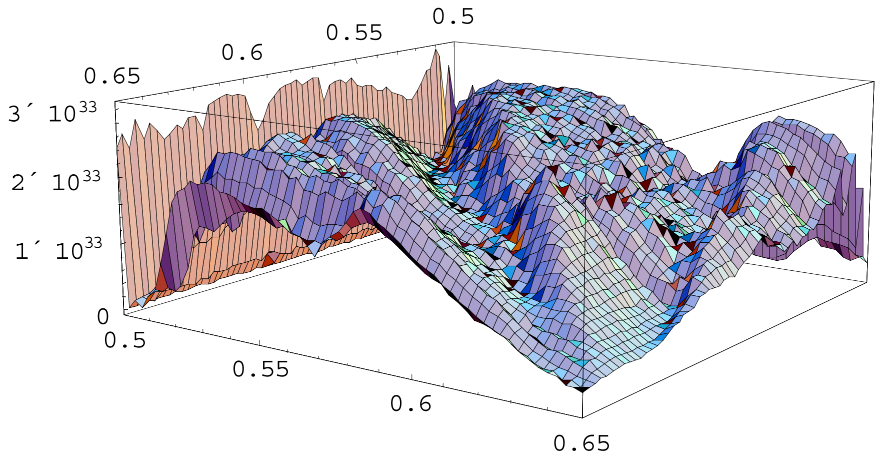
(Lifetrack code by D. Shatilov)



A_x = (0.0, 12 σ_x); A_y = (0.0, 160 σ_y)

Siddharta IR Luminosity Scan above half-integers

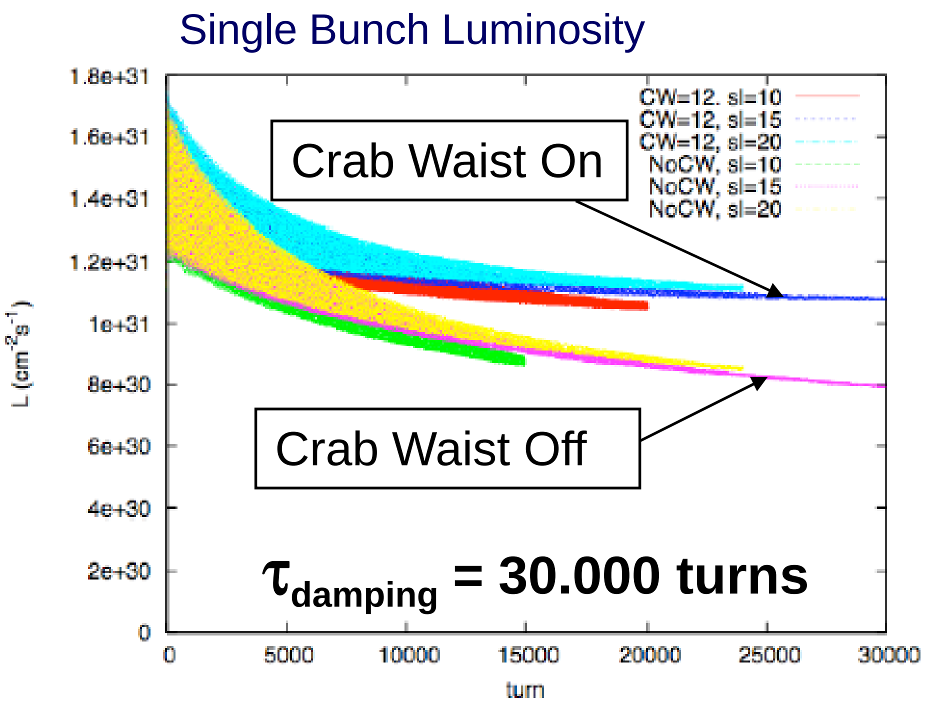


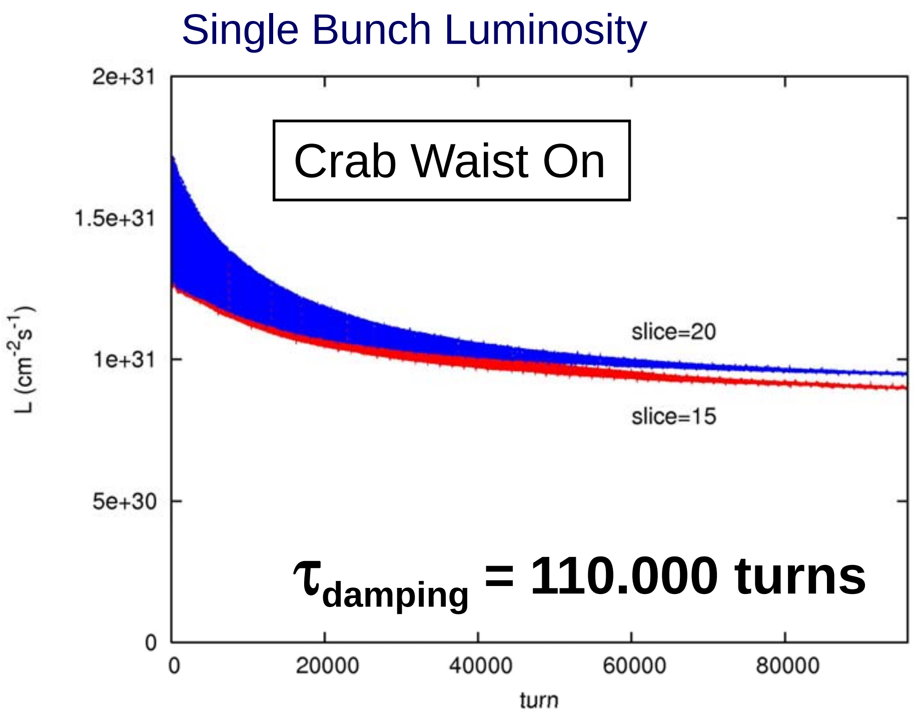


$L_{max} = 3.05 \times 10^{33}\ cm^{-2}s^{-1}$

$L_{min} = 3.28 \times 10^{31}\ cm^{-2}s^{-1}$

Strong-Strong Simulations for DAΦNE Upgrade





x110 bunches = 10^{33} cm^{-2} s^{-1}

(K. Ohmi, BBSS Simulations)

SuperB initial set of parameters

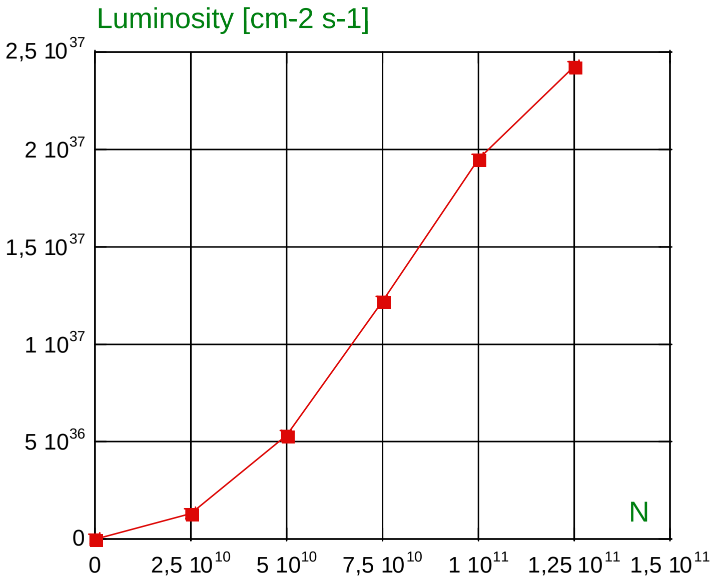
(June 2006)

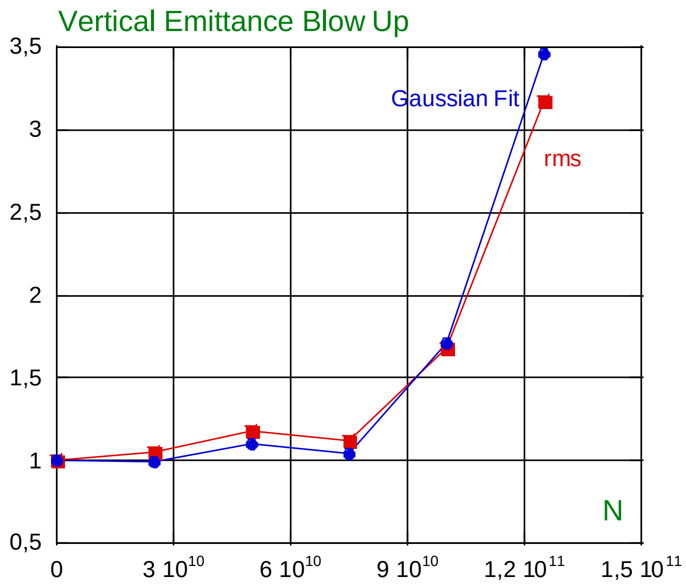
Emit_x	nm	0.8
Emit_y	nm	0.002
Beta_x*	mm	9.0
Beta_y*	mm	0.080
Sigm_x*	μm	2.67
Sigm_y*	nm	12.6
Sigm_z	mm	6.0
Sigm_e		1.0e-3
Cross_angle	mrad	2*25
Np	1e10	2.5
Nb		6000
C	km	3.0
τ_s	msec	10
Collision freq	MHz	600
Luminosity	1e36	1.0

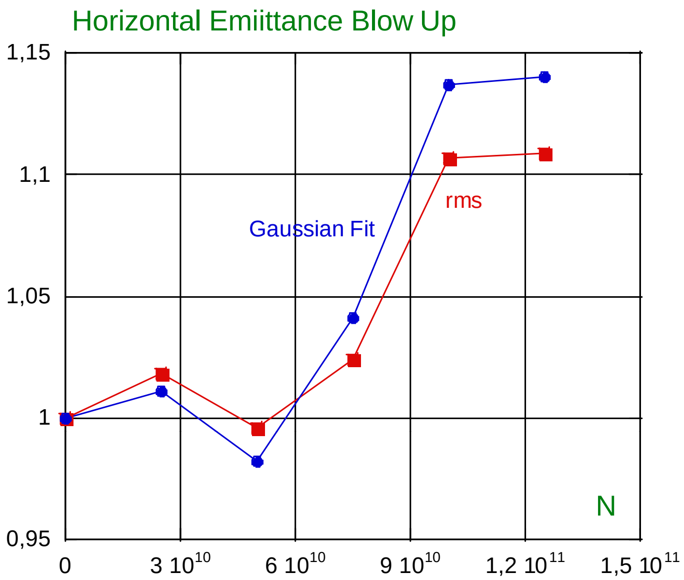
Defined a parameters set based on ILC-like parameters:

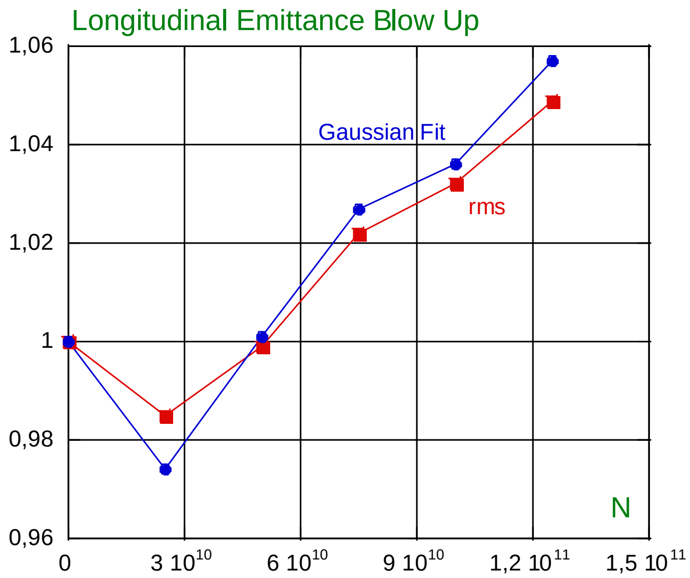
- Same DR bunch length
- Same DR bunch charges
- Same DR damping time
- Same ILC-IP betas
- Same DR emittances
- Crossing Angle and Crab Waist to minimize BB blowup

Luminosity and blowups vs current









To achieve beam-beam limit for the initial set of parameters, N_p should be increased by a factor of 2-3, that gives the luminosity exceeding 10^{37}! Actually it means we have rather big margins to relax some critical parameters, and still get the desired luminosity $L=10^{36}$. The list of parameters to optimize/relax is:

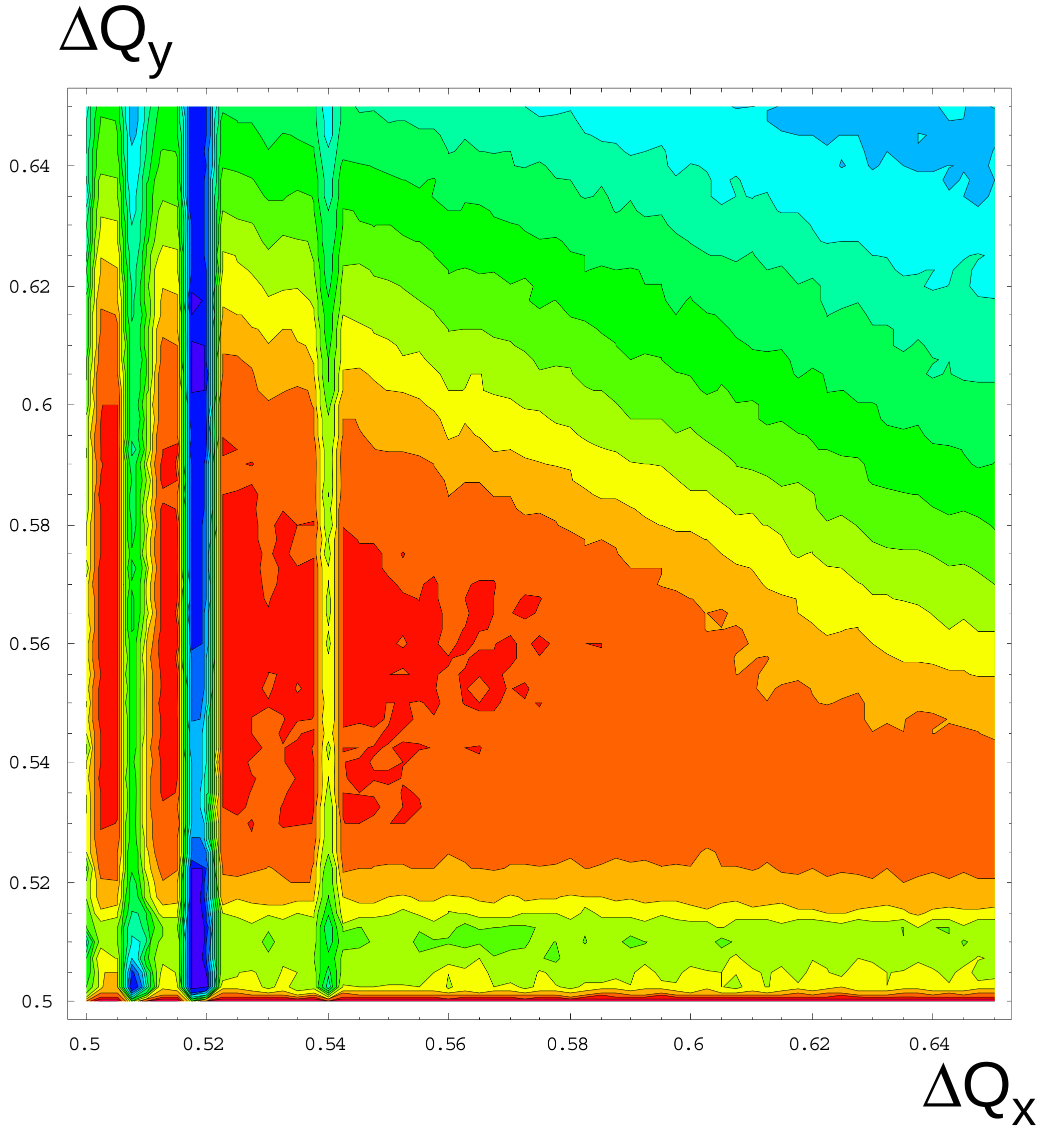
- Damping time
- Crossing angle
- Bunch length
- Bunch current
- Number of bunches
- Emittances
- Betatron coupling
- Beta-functions

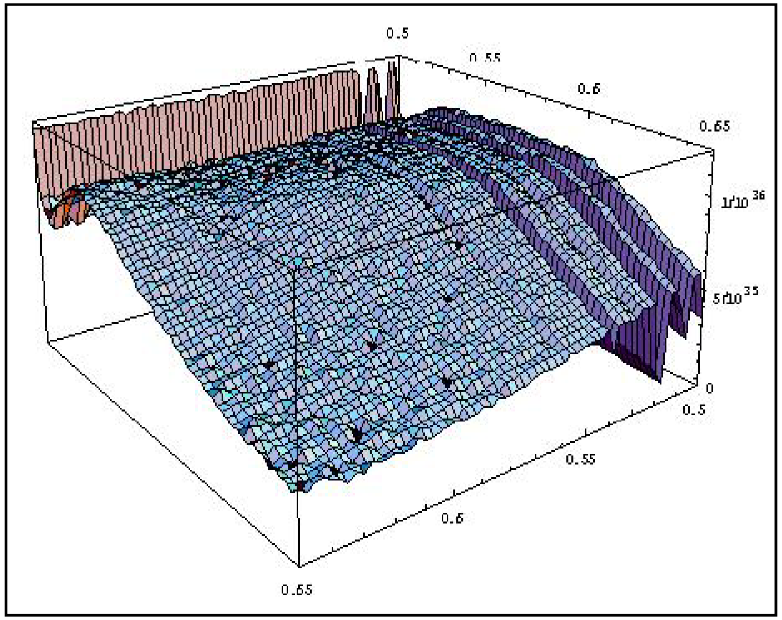
The relation $\beta_y \approx \sigma_x/\theta$ must be satisfied in all optimizations!

Optimization Results

- **Relaxed damping time: 10msec=>16msec**
- **Relaxed y/x IP βs: 80mm/9mm => 300mm/20mm**
- **Relaxed y/x IP σs: 12.6nm/2.67mm => 20nm/4mm**
- **Relaxed crossing angle: 2*25mrad => 2*17mrad**
- **Possible to increase bunch length: 6mm => 7mm**
- **Possible increase in L by further b's squeeze**
- **Possible to operate with half of the bunches and twice the bunch charge (same current), with relaxed requirements on ε_y: 2pm => 8pm (1% coupling)**
- **Possible to operate with half of the bunches and twice the bunch charge (same current), with twice the emittances**

SuperB Luminosity Tune Scan



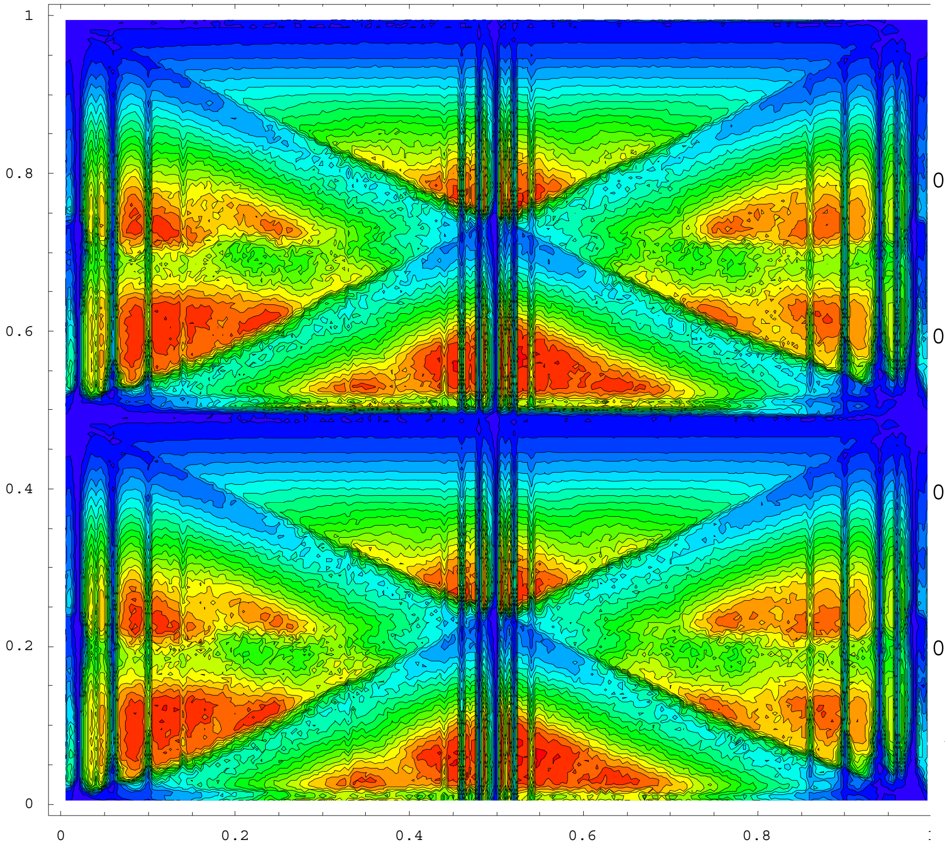


$L_{max} = 1.21 \times 10^{36}\ cm^{-2}s^{-1}$

$L_{min} = 2.25 \times 10^{34}\ cm^{-2}s^{-1}$

SuperB with 2 IP (suggested by A. Variola)

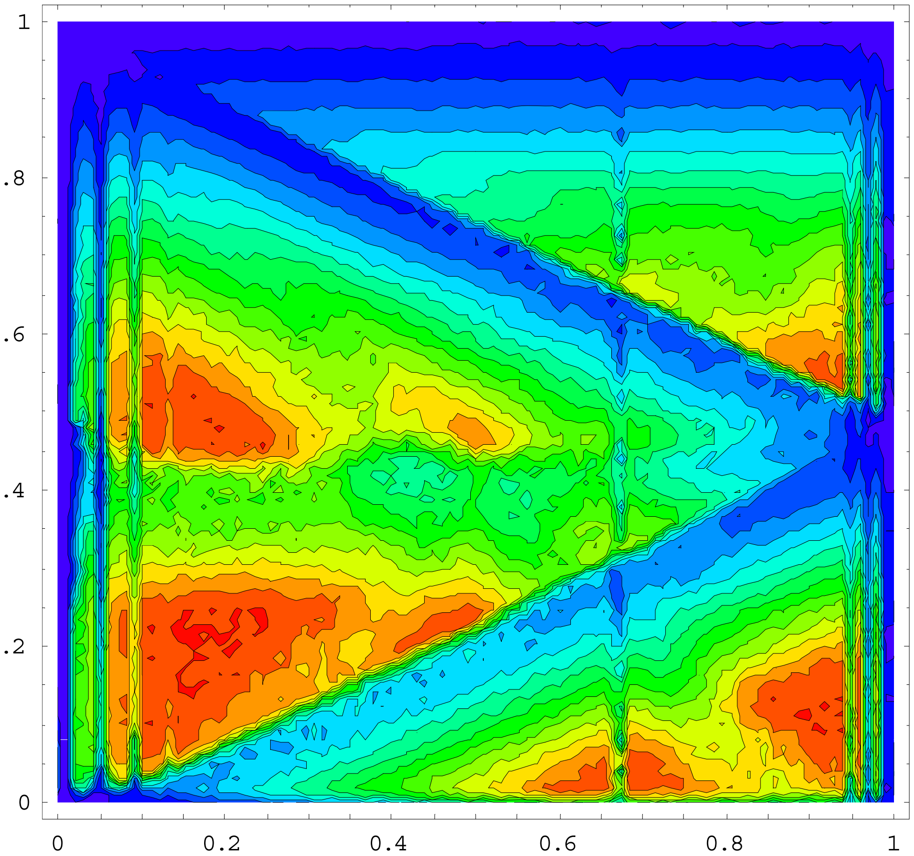
1 IP



$L_{max} = 1.05 \times 10^{36}\ cm^{-2}\ s^{-1}$

$L_{max} = 6.17 \times 10^{33}\ cm^{-2}\ s^{-1}$

2 IP



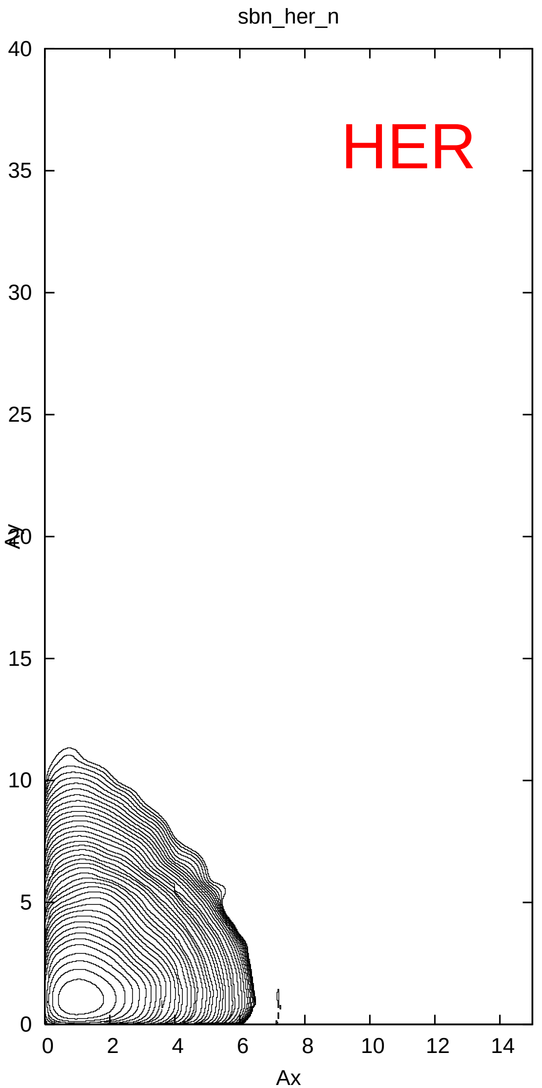
$L_{max} = 1.03 \times 10^{36}\ cm^{-2}\ s^{-1}$

$L_{max} = 7.01 \times 10^{33}\ cm^{-2}\ s^{-1}$

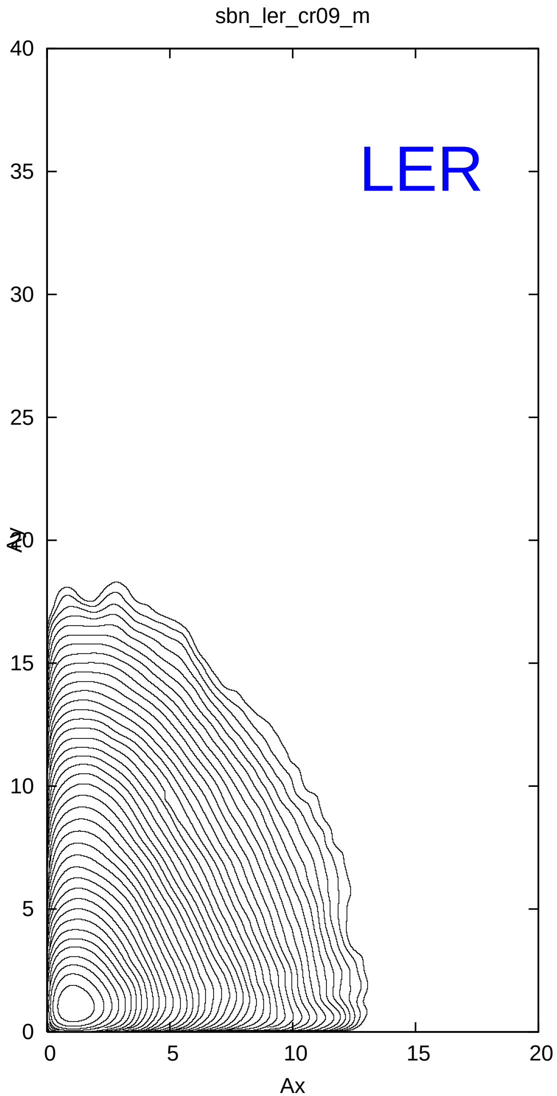
SBF Parameters June,15-2007						
	Nominal Parameters		Upgrade Parameters		Ultimate Parameter	
PARAMETER	LER	HER	LER	HER	LER	HER
Particle type	e+	e-	e+	e-	e+	e-
Energy (GeV)	4	7	4	7	4	7
Luminosity x 10^{36}	1,0		2,0		4,0	
Circumference (m)	1780	1780	1780	1780	1780	1780
Revolution frequency (MHz)	0,169	0,169	0,169	0,169	0,169	0,169
Eff. long. polarization (%)	0	80	0	80	0	80
RF frequency (MHz)	476	476	476	476	476	476
Harmonic number	2824	2824	2824	2824	2824	2824
Momentum spread	7,9E-04	5,6E-04	9,0E-04	8,0E-04	9,0E-04	8,0E-04
Momentum compaction	2,6E-04	3,2E-04	2,4E-04	3,0E-04	2,4E-04	3,0E-04
Rf Voltage (MV)	7	9	10	14	10	14
Energy loss/turn (MeV)	1,09	1,87	1,71	2,74	1,71	2,74
Number of bunches	1342	1342	1342	1342	2684	2684
Particles per bunch x10^{10}	5,52	5,52	5,52	5,52	5,52	5,52
Beam current (A)	2,00	2,00	2,00	2,00	4,00	4,00
Beta y* (mm)	0,22	0,39	0,16	0,27	0,16	0,27
Beta x* (mm)	35	20	35	20	35	20
Emit y (pmr)	7	4	3,5	2	3,5	2
Emit x (nmr)	2,8	1,6	1,4	0,8	1,4	0,8
Sigma y* (microns)	0,039	0,039	0,023	0,023	0,023	0,023
Sigma x* (microns)	9,90	5,66	7,00	4,00	7,00	4,00
Bunch length (mm)	5	5	4,3	4,3	4,3	4,3
Full Crossing angle (mrad)	48	48	48	48	48	48
Wigglers (#) 10 meters each	0	0	4	4	4	4
Damping time (trans/long)(ms)	44/22	44/22	30/15	30/15	30/15	30/15
Luminosity lifetime (min)	7,18	7,18	3,59	3,59	3,59	3,59
Touschek lifetime (min)	13,8	20,6	6,9	10,3	6,9	10,3
Effective beam lifetime (min)	4,7	5,3	2,4	2,7	2,4	2,7
Injection rate pps (100%)	2,6E+11	2,3E+11	5,2E+11	4,6E+11	1,0E+12	9,3E+11
Tune shift y (from formula)	0,15	0,15	0,20	0,20	0,20	0,20
Tune shift x (from formula)	0,0043	0,0025	0,0059	0,0034	0,0059	0,0034
RF Power (MW)	12		18		35	

Beam-Beam Blowup (weak-strong simulations)

Crab=0.8Geom_Crab



Crab=0.9Geom_Crab



$L=10^{36}\ cm^{-2}\ s^{-1}$

Conclusions

1. We hope that now we understand how “Crab Waist” works

2. The expected luminosity increase due to “Crab Waist” is

   a) at least, a factor of 6 for the DAΦNE upgrade

   b) about 2 orders of magnitude for the SuperB project
   (with respect to the existing B-Factories)

3. Let us wait for the first DAΦNE experimental results!

Thank you!