# Exploring SUSY parameter space with MultiNest

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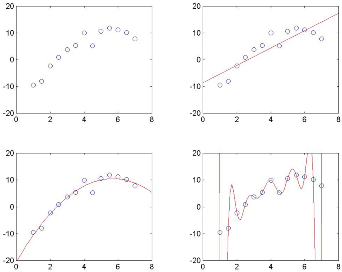
# Outline

# The problem of vast parameter spaces



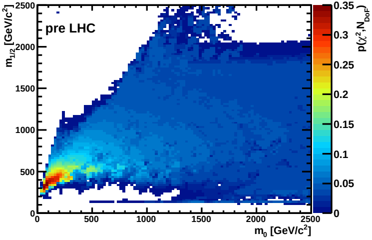
- Theories contain parameters with a priori unknown values
  - – determined from experimental data
- If many free parameters
  - – can accommodate ‘any’ observations
  - – low predictability
- Unbroken SUSY: 1 new parameter
- Broken SUSY: 4 – 105 new parameters
- Need methods for exploring huge parameter spaces
  - – to exclude models
  - – to identify interesting parameter regions



# Parameter scanning basics

UiO : Department of Physics
The Faculty of Mathematics and Natural Sciences

- Given a parameter space
  - e.g. CMSSM: $m_0$, $m_{1/2}$, $A_0$, $\tan\beta$, $\mathrm{sgn}(\mu)$
- Given a set of observations
  - $m_h$, $\Omega h^2$, decay rates, ...
- Pick parameter points according to some algorithm
- For each point:
  - calculate theory predictions
  - compare with observed values
- Quantify the level of agreement in some way
  - e.g. calculate a likelihood value

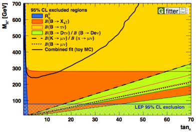


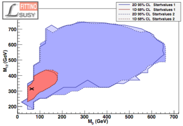

[MasterCode]

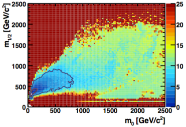
# Oh joy, another scanning tool...

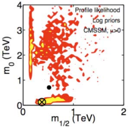


- *Present goal:*
  Construct a scanning tool to explore various SUSY models
- There exist several scanning tools already (*GFitter, Fittino, MasterCode, SuperBayes, . . .*)
  – why make another one?
- For the existing codes:
  – some codes are not public
  – interesting scans already performed by code authors
- Important to have ‘complete’ control of the code
- Easier to tailor the code for future needs
  – mass degenerate models
  – only stau coannihilation regions
  – other special parameter relations



# An aside on Bayesian statistics



- Algorithm for choosing points: *MultiNest*
  – based on Bayesian statistics
- Bayes' theorem:

$$P(\theta|D,H) = \frac{P(D|\theta,H)P(\theta,H)}{P(D|H)} = \frac{L(\theta)\pi(\theta)}{Z}$$

$$\begin{aligned}
H &: \text{model} \\
\theta &: \text{model parameters} \\
D &: \text{data} \\
P(\theta|D,H) &: \text{posterior probability (dist.) of parameters} \\
P(D|\theta,H) = L(\theta) &: \text{likelihood of parameters} \\
P(\theta|H) = \pi(\theta) &: \text{prior probability (dist.) of parameters} \\
P(D|H) = \int L(\theta)\pi(\theta)\,d\theta = Z &: \text{the `Bayesian evidence'}
\end{aligned}$$

- Consistent approach for updating one's *degree of belief* in $\theta, H$ in light of the data $D$ (what theorists are interested in. . . )

# An aside on Bayesian statistics (cont.)



- Bayesian inference:
  - parameter estimation
  - model comparison

- Parameter estimation:
  - use information contained in posterior $P(\theta|D,H)$ (mean, maximum, credible intervals, . . . )
  - $Z$ can often be ignored (only normalization)
  - coincide with frequentist results when data are strong enough

- Model comparison:
  - rank models $H_0$, $H_1$ based on posterior probabilities $P(H_0|D)$, $P(H_1|D)$

$$\frac{P(H_1|D)}{P(H_0|D)} = \frac{P(D|H_1)P(H_1)}{P(D|H_0)P(H_0)} = \frac{Z_1}{Z_0}\frac{P(H_1)}{P(H_0)}$$

  - numerically challenging to calculate $Z = \int L(\theta)\pi(\theta)\,d\theta$ (many parameters, complicated $L(\theta)$)
  - MultiNest is designed to calculate $Z$ efficiently

# The MultiNest algorithm



- MultiNest (Feroz, Hobson) is a Fortran code based on the *nested sampling* algorithm (Skilling)
- Calculates $Z$
- Produces a set of samples of $P(\theta|D,H)$ as a by-product
- Main advantage: good coverage of multi-modal posteriors (important for SUSY)
- Main steps:
  1. draw a set of $N$ points from prior distribution $\pi(\theta)$
  2. calculate likelihoods $L(\theta_1), \ldots, L(\theta_N)$
  3. discard the point with lowest likelihood
  4. pick a new point from $\pi(\theta)$, given that $L(\theta_{\text{new}}) > L(\theta_{\text{discarded}})$
  5. repeat from 3
- Keep the set of *discarded* points $\theta_i$

# The MultiNest algorithm (cont.)

- What happens?
  - – new variable $dX = \pi(\theta)\, d\theta$ (*prior mass*)
  - – integral transformed to 1D integral in $X$:

$$Z = \int_0^1 L(X)\, dX$$

  - – at step $i$: $X_i$ is the prior mass within the parameter region defined by

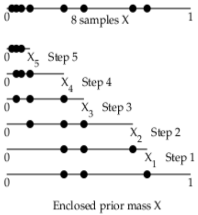
$$L(\theta) > L(\theta_i)$$

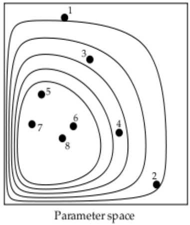
  - – integral approximated by the trapezoidal rule:

$$Z \approx \sum w_i L(\theta_i) = \sum \frac{1}{2}(X_{i-1} - X_{i+1}) L(\theta_i)$$

  - – set of $\theta_i$ is a sample of $P(\theta|D,H)$ if weighted by

$$p_i = \frac{w_i L(\theta_i)}{Z}$$




[Skilling]

# Code overview

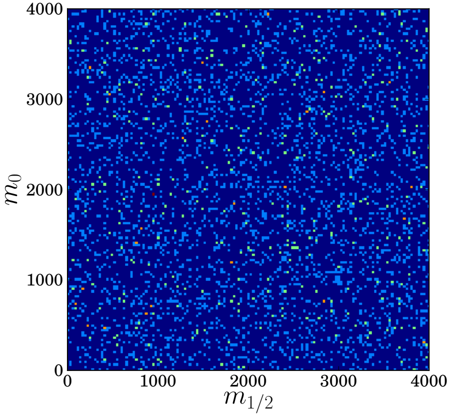
- Current status:
  - main steering code is python based
  - code written for parallel computing
  - complete scan setup (incl. $L(\theta)$ contributions) given in a SLHA file
  - python wrapper for MultiNest (Vallisneri)
  - predictions from calls to SoftSusy, micrOMEGAs, FeynHiggs, HiggsBounds
  - interface for adding user-defined calculations
  - SLHA file stored for each accepted (discarded) point
  - scripts for summarizing and plotting results

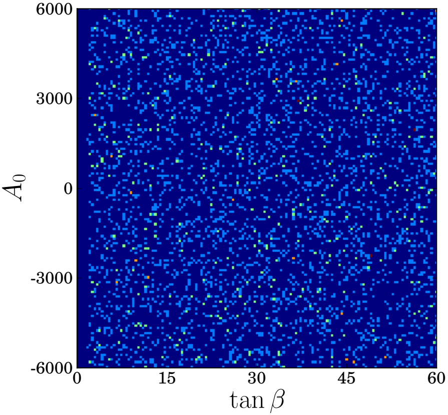
- To-do list (ever expanding):
  - add more observables/predictions
  - separate MultiNest from code for easy replacement
  - move from 4 core laptop to a more powerful system
  - explore some interesting SUSY models!

```
BLOCK ParSetup                      # settings for the parameters in
# Note: At the moment derived parameters can only depend on fixe
    1   m_squarks  MINPAR   1   1    1000.0  4000.0   # -- m_squ
    2   m_eL       EXTPAR   31  1       0.0  1000.0   # -- mL
    3   m_muL      EXTPAR   32  2    1.0*p[2]         #
    4   m_tauL     EXTPAR   33  2    1.0*p[2]         #
    5   m_eR       EXTPAR   34  1       0.0  1000.0   # -- mR
    6   m_muR      EXTPAR   35  2    1.0*p[5]         #
    7   m_tauR     EXTPAR   36  2    1.0*p[5]         #
    8   M1         EXTPAR   1   1       0.0  1000.0   # -- M1
    9   M2         EXTPAR   2   1       0.0  1000.0   # -- M2
   10   M3         EXTPAR   3   1    1000.0  4000.0   # -- M3
   11   tanb       MINPAR   3   1      10.0    50.0   # -- tanb
   12   mu         EXTPAR   23  1       0.0  3000.0   # -- mu
   13   mA0        EXTPAR   26  1    1000.0  4000.0   # -- mA0
   14   A_t/b      MINPAR   5   1   -5000.0  5000.0   # -- At, A
   15   A_tau      EXTPAR   13  1   -5000.0  5000.0   # -- Atau
#
#
#
BLOCK SSsetup                       # settings for SoftSusy (1-7 are
    1   1.000000000e-03             # numerical precision: suggested
    2   0                           # quark mixing parameter: 0..2
    3   0                           # printout level: 0..3
    4   1.0                         # multiplicative factor to chang
    5   1                           # include 2-loop scalar mass squ
    6   8                           # precision -- number of signifi
    7   2                           # number of loops in REWSB/mh ca
    8   _SS_OUTPUT                  # suffix for temporary files
BLOCK FHsetup                       # settings for FeynHiggs (for de
    1   4                           # mssmpart     = 0..4, specifies
    2   0                           # fieldren     = 0..4, determine
    3   0                           # tanbren      = 0..2, determine
    4   2                           # higgsmix     = 1..3, determine
    5   0                           # p2approx     = 0..4, determine
    6   2                           # looplevel    = 0..2, determine
    7   1                           # runningMT    = 0..1, determine
    8   1                           # botResum     = 0..1, determine
    9   0                           # tlCplxApprox = 0..6, determine
```

# Plots with shiny colours

- A few preliminary plots: CMSSM/mSUGRA scan
- $m_0 \in (0, 4000)$ $m_{1/2} \in (0, 4000)$ $A_0 \in (-6000, 6000)$ $\tan\beta \in (2, 60)$ $\mathrm{sgn}(\mu) = +1$
- Flat priors



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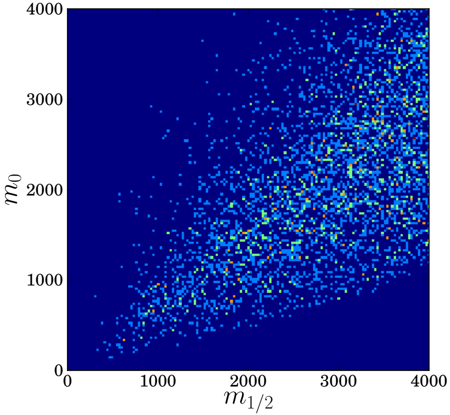
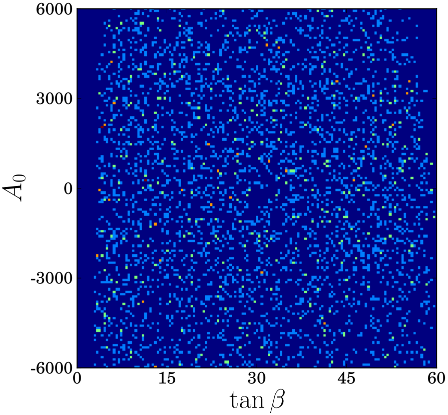
- + discrete constraints: neutral LSP, $m_{\tilde{\tau}_1} < 2\, m_{\tilde{\chi}_1^0}$ , LEP bounds on $m_{\tilde{\tau}_1}$, ...



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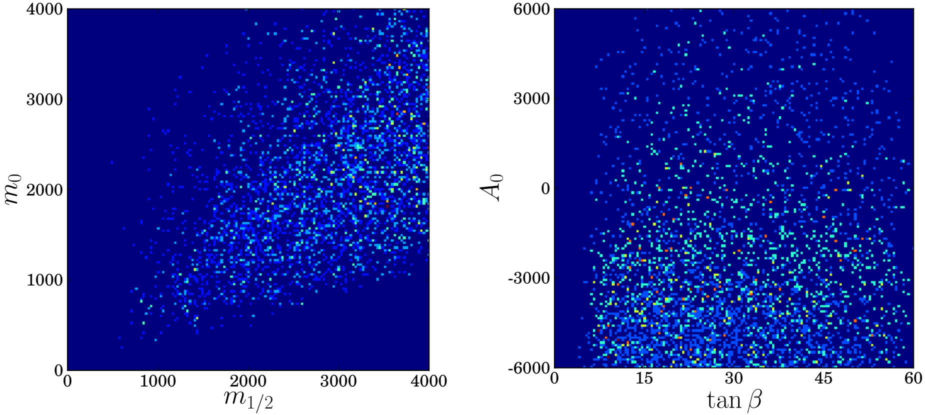
- + gaussian likelihood contribution for $m_{h^0}$



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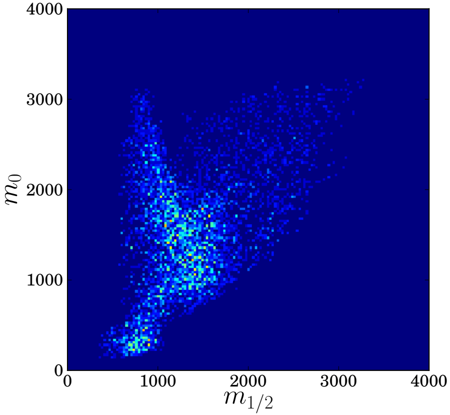
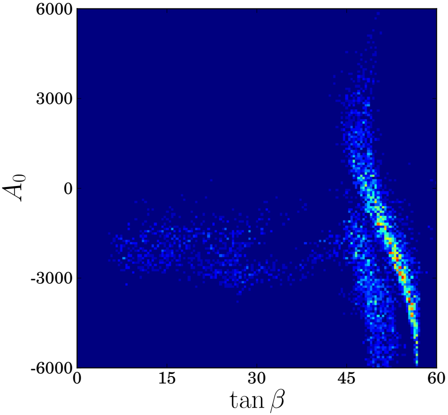
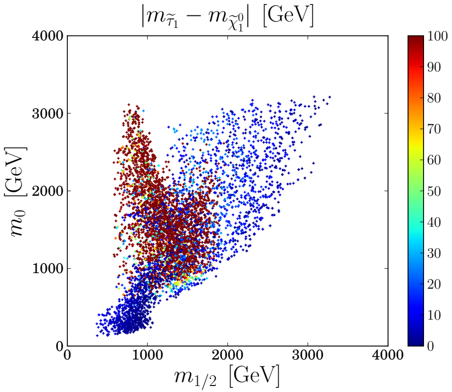
- + gaussian likelihood contribution for $\Omega h^2$



# Plots with shiny colours (cont.)

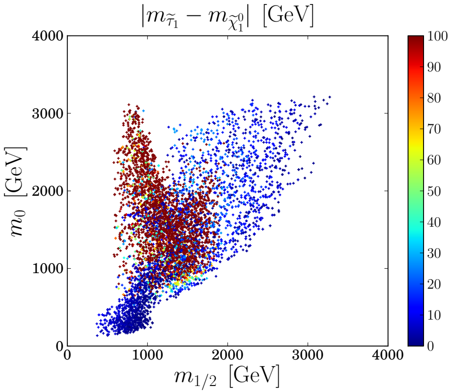
- Found ‘stau coannihilation’ region and ‘Higgs funnel’ region

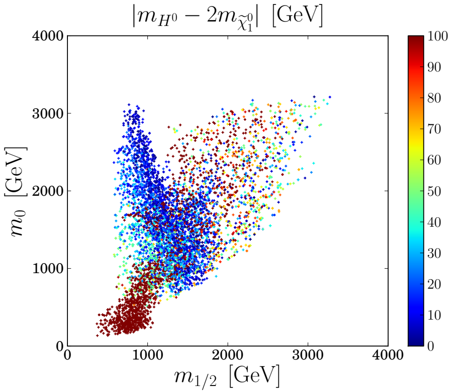


# Plots with shiny colours (cont.)



- Found 'stau coannihilation' region and 'Higgs funnel' region
- This was only an illustration — more interesting results to come (hopefully. . . )



Thank you

# Backup slide 1: Bayesian vs frequentist



- Bayesian: all forms of uncertainty are expressed in terms of probabilities
- Bayesian: 'how probable is a parameter value, given the data?'
  Frequentist: 'how probable is the data, given a parameter value?'
- Very informative data $\rightarrow$ identical results
- F: priors are subjective!
  B: yes, but thinking that high $L \leftrightarrow$ more probable parameter value already assumes some prior
- B: prior dependence give us and estimate of the how robust the inference is
- Model comparison using the Bayesian evidence ($Z$) incorporates 'Occam's razor'
- Choice of method depends on the question you are asking. . .