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Dirac Equation in Curved Spacetime

Dag-Morten Sjøstrøm

NTNU, Institute of Physics

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The Dirac Equation in flat spacetime

$$(\mathrm{i}\gamma^a \partial_a - m)\psi = 0$$

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}$$

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Going curved

-Same procedure as last year?

- Standard replacements:

$$\eta_{\mu\nu} \to g_{\mu\nu}, \qquad \partial_\mu \to \nabla_\mu \equiv \partial_\mu + \Gamma_\mu.$$

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- ψ is not a four-vector.

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The tetrad formalism

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- Can use them to find connection coefficients for local vectors.
- These give the spin connection.

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The connection for spinors

$$\Omega_\mu = \frac{1}{8}\omega_{ab\mu}[\gamma^a, \gamma^b]$$

Where $\omega_{ab\mu}$ is the spin connection.

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The Dirac equation in curved spacetime

$$[\mathrm{i}\mathrm{e}_a^\mu \gamma^a (\partial_\mu + \Omega_\mu) - m]\psi = 0$$

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