

Scalar mesons and tetraquarks from twisted mass lattice QCD

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Introduction, motivation (1)

- The nonet of light scalar mesons ($J^P = 0^+$)
 - $\sigma \equiv f_0(500)$, $I = 0$, 400 ... 550 MeV,
 - $\kappa \equiv K_0^*(800)$, $I = 1/2$, 682 ± 29 MeV,
 - $a_0(980)$, $f_0(980)$, $I = 1$, 980 ± 20 MeV, 990 ± 20 MeV

is poorly understood:

- All nine states are unexpectedly light (should rather be close to the corresponding $J^P = 1^+, 2^+$ states around 1200 ... 1500 MeV).
- The ordering of states is inverted compared to expectation:
 - * E.g. in a $q\bar{q}$ picture the $I = 1$ states $a_0(980)$, $f_0(980)$ must necessarily be formed by two u/d quarks, while the $I = 1/2$ κ states are made from an s and a u/d quark; since $m_s > m_{u/d}$ one would expect $m(\kappa) > m(a_0(980)), m(f_0(980))$.

Introduction, motivation (2)

- * In a tetraquark picture the quark content could be the following:
 $\kappa \equiv \bar{s}l\bar{l}l$, while $a_0(980), f_0(980) \equiv \bar{s}l\bar{l}s$; this would naturally explain the observed ordering.
- Certain decays also support a tetraquark interpretation: e.g. $a_0(980)$ readily decays to $K + \bar{K}$, which indicates that besides the two light quarks required by $I = 1$ also an $s\bar{s}$ pair is present.
- Study these states by means of twisted mass lattice QCD to confirm or to rule out their interpretation in terms of tetraquarks.
- Examples of heavy mesons, which are tetraquark candidates:
 - $D_{s0}^*(2317)^\pm (I(J^P) = 0(0^+))$, $D_{s1}(2460)^\pm (I(J^P) = 0(1^+))$,
[\[talk by Martin Kalinowski on Friday\]](#)
 - charmonium states $X(3872)$, $Z(4430)^\pm$, $Z(4050)^\pm$, $Z(4250)^\pm$, ...

Lattice QCD hadron spectroscopy (1)

- Lattice QCD: discretized version of QCD,

$$S = \int d^4x \left(\sum_{\psi \in \{u,d,s,c,t,b\}} \bar{\psi} \left(\gamma_\mu (\partial_\mu - iA_\mu) + m^{(\psi)} \right) \psi + \frac{1}{2g^2} \text{Tr} \left(F_{\mu\nu} F_{\mu\nu} \right) \right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

- Let \mathcal{O} be a suitable “hadron creation operator”, i.e. an operator formed by quark fields ψ and gluonic fields A_μ such that $\mathcal{O}|\Omega\rangle$ is a state containing the hadron of interest ($|\Omega\rangle$: QCD vacuum).
- More precisely: ... an operator such that $\mathcal{O}|\Omega\rangle$ has the same quantum numbers (J^{PC} , flavor) as the hadron of interest.
- Examples:
 - Pion creation operator: $\mathcal{O} = \int d^3x \bar{u}(\mathbf{x}) \gamma_5 d(\mathbf{x})$.
 - Proton creation operator: $\mathcal{O} = \int d^3x \epsilon^{abc} u^a(\mathbf{x}) (u^{b,T}(\mathbf{x}) C \gamma_5 d^c(\mathbf{x}))$.

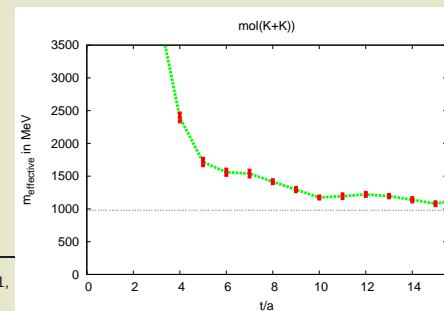
Lattice QCD hadron spectroscopy (2)

- Determine the mass of the ground state of the hadron of interest from the exponential behavior of the corresponding correlation function \mathcal{C} at large Euclidean times T :

$$\begin{aligned}\mathcal{C}(t) &= \langle \Omega | (\mathcal{O}(t))^\dagger \mathcal{O}(0) | \Omega \rangle = \langle \Omega | e^{+Ht} (\mathcal{O}(0))^\dagger e^{-Ht} \mathcal{O}(0) | \Omega \rangle = \\ &= \sum_n \left| \langle n | \mathcal{O}(0) | \Omega \rangle \right|^2 \exp \left(- (E_n - E_\Omega) t \right) \approx \quad (\text{for } "t \gg 1") \\ &\approx \left| \langle 0 | \mathcal{O}(0) | \Omega \rangle \right|^2 \exp \left(- \underbrace{(E_0 - E_\Omega)}_{m(\text{hadron})} t \right).\end{aligned}$$

- Usually the exponent is determined by identifying the “plateaux-value” of a so-called effective mass:

$$\begin{aligned}m_{\text{effective}}(t) &= \frac{1}{a} \ln \left(\frac{\mathcal{C}(t)}{\mathcal{C}(t+a)} \right) \approx \quad (\text{for } "t \gg 1") \\ &\approx E_0 - E_\Omega = m(\text{hadron}).\end{aligned}$$



Tetraquark creation operators

- At the moment we study
 - $a_0(980)$, mass 980 ± 20 MeV, quantum numbers $I(J^{PC}) = 1(0^{++})$;
 - $\kappa \equiv K_0^*(800)$, mass 682 ± 29 MeV, quantum numbers $I(J^P) = 1/2(0^+)$.
- Tetraquark operators for $a_0(980)$ (quantum numbers $I(J^{PC}) = 1(0^{++})$):

- Needs **two light quarks** due to $I = 1$, e.g. $u\bar{d}$.
- $a_0(980)$ decays to $K\bar{K}$... suggests an $s\bar{s}$ component.
- **Molecule type** (models a bound $K\bar{K}$ state):

$$\mathcal{O}_{a_0(980)}^{K\bar{K} \text{ molecule}} = \int d^3x \left(\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x}) \right) \left(\bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x}) \right).$$

- **Diquark type** (models a bound diquark-antidiquark):

$$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \int d^3x \left(\epsilon^{abc} \bar{s}^b(\mathbf{x}) C \gamma_5 \bar{d}^{c,T}(\mathbf{x}) \right) \left(\epsilon^{ade} u^{d,T}(\mathbf{x}) C \gamma_5 s^e(\mathbf{x}) \right).$$

Lattice setup (1)



- Gauge link configurations generated by ETMC.
- 2+1+1 dynamical quark flavors, i.e. u , d , s and c sea quarks.
- Lattice spacing $a = 0.086$ fm (rather fine, computations at even finer lattice spacings planned).
- Various lattice volumes:
 - Small volume $L^3 \times T = 20^3 \times 48$ lattice sites, spatial extension 1.73 fm
→ rather easy to identify momentum excitations.
(Most of the numerical results shown in the following were obtained with this volume.)
 - ...
 - Large volume $L^3 \times T = 32^3 \times 64$ lattice sites, spatial extension 2.75 fm
→ less finite size effects.
 - Different volumes needed to study resonances in a rigorous way.
(Not done yet ... will be one of our next steps.)

Lattice setup (2)

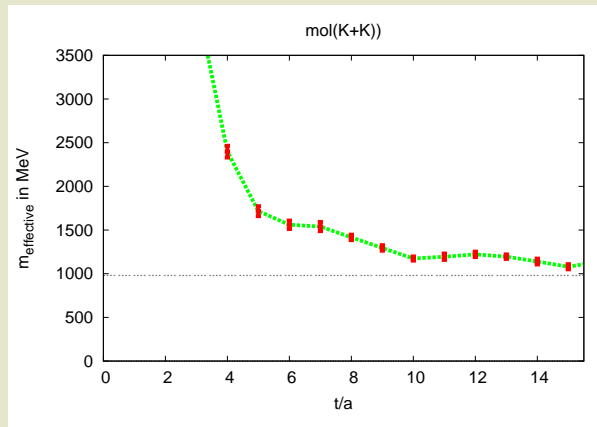
- Various light u/d quark masses, corresponding pion masses $m_{\text{PS}} \approx 280 \dots 460$ MeV (physical light u/d quark masses [$m_{\text{PS}} = m_{\pi} \approx 140$ MeV] are technically extremely challenging; because of that in lattice QCD one usually studies several heavier quark masses and extrapolates to the “physical point”).

Numerical results $a_0(980)$ (1)

- Effective mass, molecule type operator:

$$\mathcal{O}_{a_0(980)}^{K\bar{K} \text{ molecule}} = \sum_{\mathbf{x}} \left(\bar{s}(\mathbf{x}) \gamma_5 u(\mathbf{x}) \right) \left(\bar{d}(\mathbf{x}) \gamma_5 s(\mathbf{x}) \right).$$

- The effective mass plateau indicates a state, which is roughly consistent with the experimentally measured $a_0(980)$ mass 980 ± 20 MeV.
- Conclusion: $a_0(980)$ is a tetraquark state of $K\bar{K}$ molecule type ...?

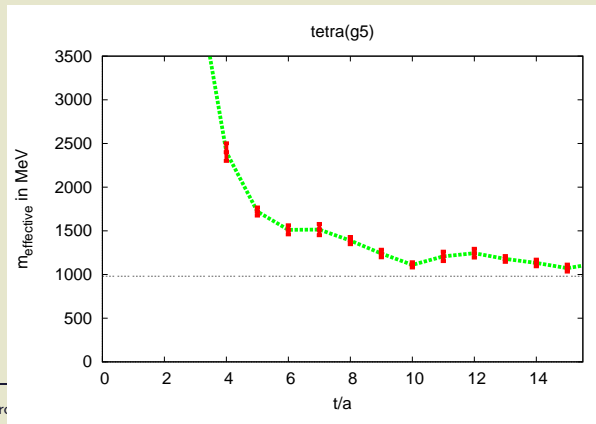


Numerical results $a_0(980)$ (2)

- Effective mass, diquark type operator:

$$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \sum_{\mathbf{x}} \left(\epsilon^{abc} \bar{s}^b(\mathbf{x}) C \gamma_5 \bar{d}^{c,T}(\mathbf{x}) \right) \left(\epsilon^{ade} u^{d,T}(\mathbf{x}) C \gamma_5 s^e(\mathbf{x}) \right).$$

- The effective mass plateaux indicates a state, which is roughly consistent with the experimentally measured $a_0(980)$ mass 980 ± 20 MeV.
- Conclusion: $a_0(980)$ is a tetraquark state of diquark type ...? Or a mixture of $K\bar{K}$ molecule and a diquark-antidiquark pair?



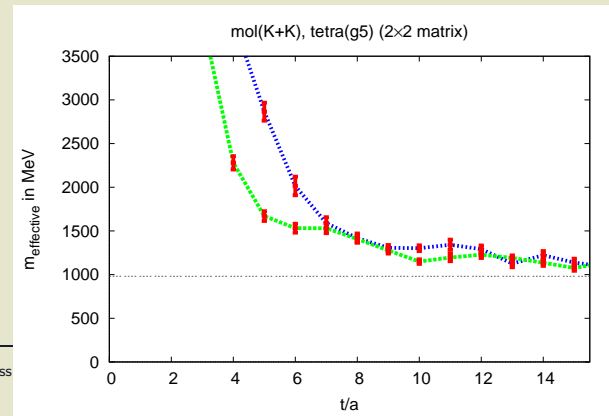
Numerical results $a_0(980)$ (3)

- Study both operators at the same time, extract the two lowest energy eigenstates by diagonalizing a 2×2 correlation matrix (“generalized eigenvalue problem”):

$$\mathcal{O}_{a_0(980)}^{K\bar{K} \text{ molecule}} = \sum_{\mathbf{x}} \left(\bar{s}(\mathbf{x}) \gamma_5 u(\mathbf{x}) \right) \left(\bar{d}(\mathbf{x}) \gamma_5 s(\mathbf{x}) \right)$$

$$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \sum_{\mathbf{x}} \left(\epsilon^{abc} \bar{s}^b(\mathbf{x}) C \gamma_5 \bar{d}^{c,T}(\mathbf{x}) \right) \left(\epsilon^{ade} u^{d,T}(\mathbf{x}) C \gamma_5 s^e(\mathbf{x}) \right).$$

- Now two orthogonal states roughly consistent with the experimentally measured $a_0(980)$ mass 980 ± 20 MeV ...?



Two-particle creation operators (1)

- Explanation: there are two-particle states, which have the same quantum numbers as $a_0(980)$, $I(J^{PC}) = 1(0^{++})$,
 - $K + \bar{K}$ ($m(K) \approx 500$ MeV),
 - $\eta_s + \pi$ ($m(\eta_s \equiv \bar{s}\gamma_5 s) \approx 700$ MeV, $m(\pi) \approx 300$ MeV in our lattice setup),

which are both around the expected $a_0(980)$ mass 980 ± 20 MeV.

- What we have seen in the previous plots might actually be two-particle states (our operators are of tetraquark type, but they nevertheless generate overlap [possibly small, but certainly not vanishing] to two-particle states).
- To determine, whether there is a bound $a_0(980)$ tetraquark state, we need to resolve the above listed two-particle states $K + \bar{K}$ and $\eta_s + \pi$ and check, whether there is an additional 3rd state in the mass region around 980 ± 20 MeV; to this end we need operators of two-particle type.

Two-particle creation operators (2)

- Two-particle operators with quantum numbers $I(J^{PC}) = 1(0^{++})$:

– Two-particle $K + \bar{K}$ type:

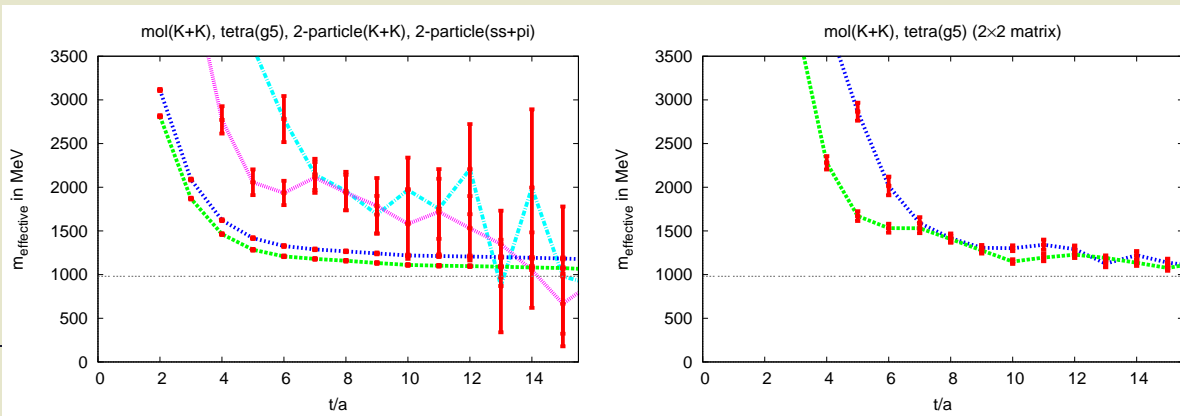
$$\mathcal{O}_{a_0(980)}^{K+\bar{K} \text{ two-particle}} = \left(\sum_{\mathbf{x}} \bar{s}(\mathbf{x}) \gamma_5 u(\mathbf{x}) \right) \left(\sum_{\mathbf{y}} \bar{d}(\mathbf{y}) \gamma_5 s(\mathbf{y}) \right).$$

– Two-particle $\eta_s + \pi$ type:

$$\mathcal{O}_{a_0(980)}^{\eta_s+\pi \text{ two-particle}} = \left(\sum_{\mathbf{x}} \bar{s}(\mathbf{x}) \gamma_5 s(\mathbf{x}) \right) \left(\sum_{\mathbf{y}} \bar{d}(\mathbf{y}) \gamma_5 u(\mathbf{y}) \right).$$

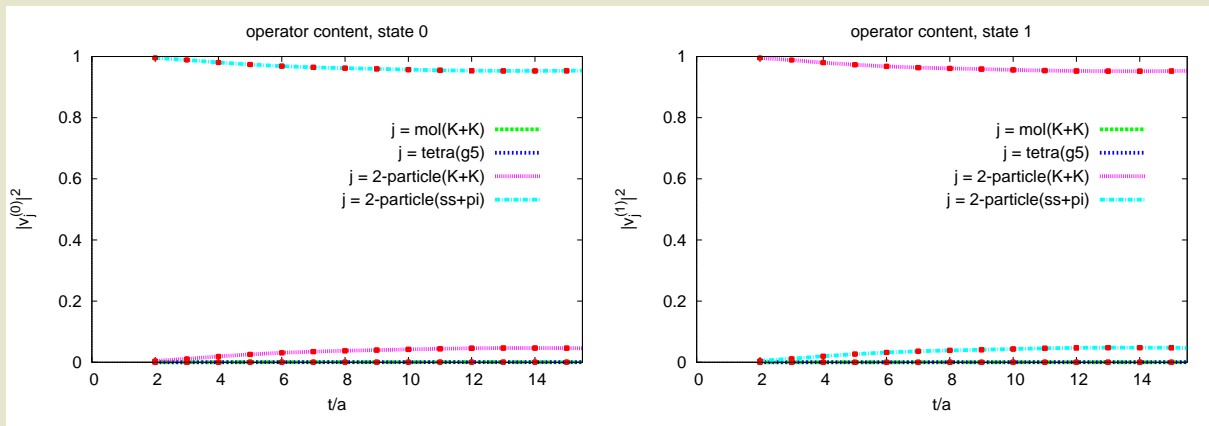
Numerical results $a_0(980)$ (4)

- Study all four operators ($K\bar{K}$ molecule, diquark, $K + \bar{K}$ two-particle, $\eta_s + \pi$ two-particle) at the same time, extract the four lowest energy eigenstates by diagonalizing a 4×4 correlation matrix (left plot).
 - Still only two low-lying states around 980 ± 20 MeV, the 2nd and 3rd excitation are ≈ 750 MeV heavier.
 - The signal of the low-lying states is of much better quality than before (when we only considered tetraquark operators)
 - suggests that the observed low-lying states have much better overlap to the two-particle operators and are most likely of two-particle type.



Numerical results $a_0(980)$ (5)

- When determining low-lying eigenstates from a correlation matrix, one does not only obtain their mass, but also information about their operator content, i.e. which percentage of which operator is present in an extracted state:
 - The ground state is a $\eta_s + \pi$ state ($\gtrsim 95\%$ two-particle $\eta_s + \pi$ content).
 - The first excitation is a $K + \bar{K}$ state ($\gtrsim 95\%$ two-particle $K + \bar{K}$ content).

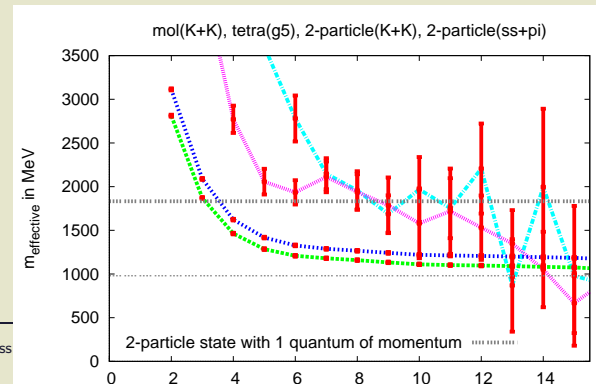


Numerical results $a_0(980)$ (6)

- What about the 2nd and 3rd excitation? ... Are these tetraquark states? ... What is their nature?
- Two-particle states with one relative quantum of momentum (one particle has momentum $+p_{\min} = +2\pi/L$ the other $-p_{\min}$) also have quantum numbers $I(J^{PC}) = 1(0^{++})$; their masses can easily be estimated:
 - $p_{\min} = 2\pi/L \approx 715$ MeV (the results presented correspond to the small lattice with spatial extension $L = 1.73$ fm);
 - $m(K(+p_{\min}) + \bar{K}(-p_{\min})) \approx 2\sqrt{m(K)^2 + p_{\min}^2} \approx 1750$ MeV;
 - $m(\eta(+p_{\min}) + \pi(-p_{\min})) \approx \sqrt{m(\eta)^2 + p_{\min}^2} + \sqrt{m(\pi)^2 + p_{\min}^2} \approx 1780$ MeV;

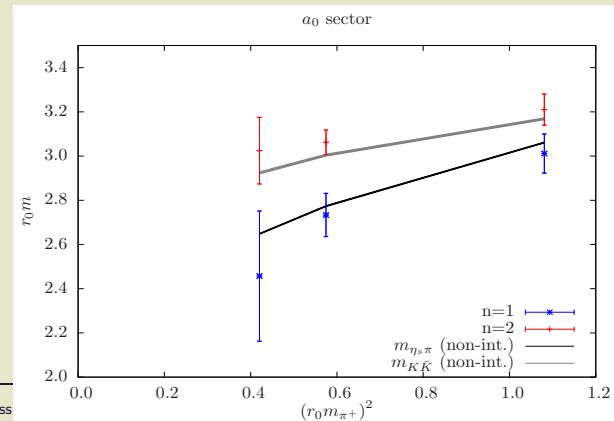
these estimated mass values are consistent with the observed mass values of the 2nd and 3rd excitation

→ suggests to interpret these states as two-particle states.



Numerical results $a_0(980)$ (7)

- Summary regarding the presented “ $a_0(980)$ results”:
 - In the $a_0(980)$ sector (quantum numbers $I(J^{PC}) = 1(0^{++})$) we do not observe any low-lying (mass $\lesssim 1750$ MeV) tetraquark state, even though we employed operators of tetraquark structure ($K\bar{K}$ molecule, diquark).
 - The experimentally measured mass for $a_0(980)$ is 980 ± 20 MeV.
 - Conclusion: $a_0(980)$ does not seem to be a strongly bound tetraquark state (either of molecule or of diquark type) ... maybe an ordinary quark-antiquark state (unlikely, lattice results indicate the opposite) or a rather unstable resonance.
- Similar results for the range of light quark masses investigated ($m_{PS} \approx 280 \dots 460$ MeV).



Numerical results κ

- Tetraquark operators for κ (quantum numbers $I(J^P) = 1/2(0^+)$):

– Molecule type (models a bound $K\pi$ state):

$$\mathcal{O}_{\kappa}^{K\pi \text{ molecule}} = \sum_{\mathbf{x}} \left(\bar{s}(\mathbf{x}) \gamma_5 q(\mathbf{x}) \right) \left(\bar{q}(\mathbf{x}) \gamma_5 u(\mathbf{x}) \right), \quad q\bar{q} = u\bar{u} + d\bar{d}$$

– Diquark type (models a bound diquark-antidiquark):

$$\mathcal{O}_{\kappa}^{\text{diquark}} = \sum_{\mathbf{x}} \left(\epsilon^{abc} \bar{s}^b(\mathbf{x}) C \gamma_5 \bar{q}^{c,T}(\mathbf{x}) \right) \left(\epsilon^{ade} q^{d,T}(\mathbf{x}) C \gamma_5 u^e(\mathbf{x}) \right).$$

- An analysis yields only the expected low-lying two-particle $K + \pi$ energy levels.
- Conclusions: κ does not seem to be a strongly bound tetraquark state (either of molecule or of diquark type) ... maybe an ordinary quark-antiquark state (unlikely, lattice results indicate the opposite) or a rather unstable resonance.

Conflict with existing lattice results

- In a similar recent lattice study of $\sigma \equiv f_0(500)$ and $\kappa \equiv K_0^*(800)$ bound tetraquark states have been observed in both sectors.

[S. Prelovsek, T. Draper, C. B. Lang, M. Limmer, K. -F. Liu, N. Mathur and D. Mohler,
Phys. Rev. D **82**, 094507 (2010) [arXiv:1005.0948 [hep-lat]]]

- In particular for κ this conflict has to be resolved.

$a_0(980)$ and κ as resonances

- A lattice study of $a_0(980)$ and κ as resonances requires rather precise computations of the masses of the two-particle states $K + \bar{K}$, $\eta + \pi$ and $K + \pi$ for various spatial volumes.
- Technically very challenging.
- No results yet.

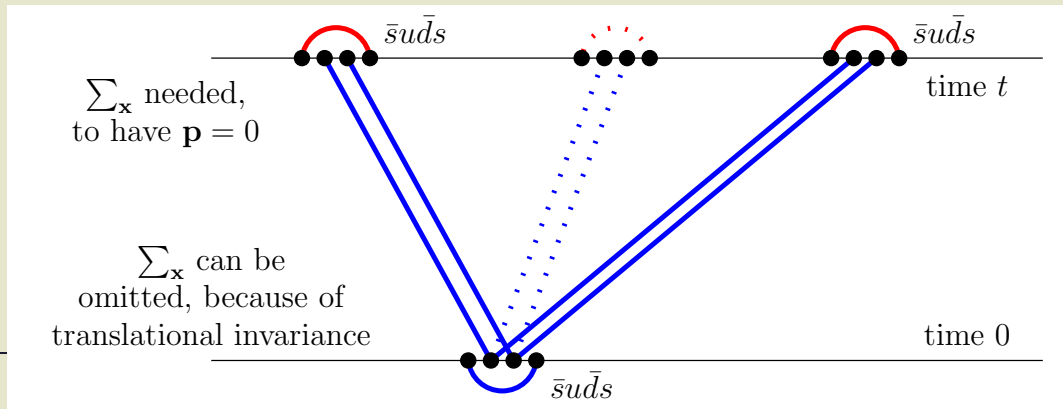
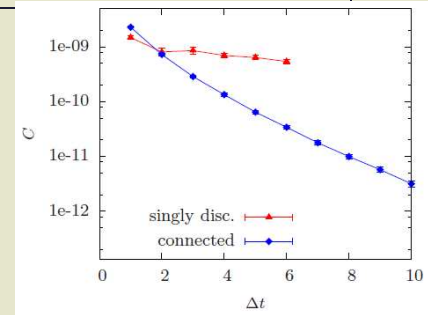
Sources of systematic error, outlook (1)

- The computations presented are technically rather challenging; there are several possible sources of systematic error, which have not yet been studied, but which need to be addressed in the future:
 - Inclusion of (singly) disconnected diagrams.
 - Include also $q\bar{q}$ creation operators (implies singly disconnected diagrams), e.g. for $a_0(980)$

$$\mathcal{O}_{a_0(980)}^{q\bar{q}} = \sum_{\mathbf{x}} \bar{d}(\mathbf{x})u(\mathbf{x}).$$

Singly disconnected diagrams

- Missing diagrams for e.g. $a_0(980)$, κ , $D_{s0}^*(2317)^\pm, \dots$
- Blue and red lines represent quark propagators:
 - Blue: point-to-all propagators applicable.
 - Red: due to $\sum_{\mathbf{x}}$, all-to-all propagators needed.
 - All-to-all propagators can only be estimated stochastically; using several stochastic all-to-all propagators results in a poor signal-to-noise ratio.
 - combine three point-to-all (blue) and one stochastic all-to-all (red) propagator.



Sources of systematic error, outlook (2)

- Continuum limit (at the moment only a single value of the lattice spacing, $a = 0.086$ fm, has been considered).
- Finite volume studies (extrapolate the here presented results to infinite spatial volume, determine resonance properties).
- The techniques and codes developed can be used with only minor modifications to study other tetraquark candidates, e.g.
 - $\sigma \equiv f_0(500), f_0(980),$
 - $D_{s0}^*(2317)^\pm, D_{s1}(2460)^\pm,$
 - charmonium states $X(3872), Z(4430)^\pm, Z(4050)^\pm, Z(4250)^\pm, \dots$