Tensor meson photoproduction through the final state meson-meson interactions

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Plan

Motivation

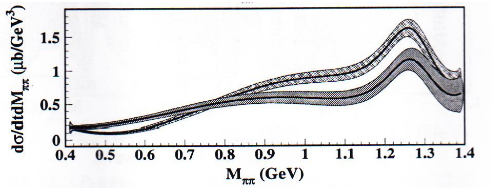
- General motivation: Economical* description of the D-wave $\pi\pi$ photoproduction.
  - Models proposed so far are either theoretically unsatisfactory - often Breit-Wigner intevenes in some point $\Rightarrow$ unitarity broken and/or
  - Difficult in practical application: tensor meson dominance model (TMD).

    Analogy with VMD:
    $$j_\mu(x) \sim \sum_{\rho,\omega,\phi} V_\mu(x)$$
    - hadronic electromagnetic current dominated by vector meson poles
    $$\Theta_{\mu\nu}(x) \sim \phi_{\mu\nu}(x)$$
    - symmetrized, traceless part of the energy-momentum tensor dominated by tensor meson poles

  Problems:
  - many form-factors and coupling constants$\Longrightarrow$difficult to use without futher assumptions,
  - in actual applications TMD is used jointly with VMD and some coupling constants are put $=0$,
  - poor experimental verification

Motivation

- Solution:
- A model where tensor mesons are *dynamically* created through the final state meson-meson interactions
- We use components (Born $\pi\pi$ photoproduction amplitudes, elastic $\pi\pi$ scattering amplitudes) which are (in principle) tested in other applications $\Longrightarrow$no need for extra couplings, form factors, etc.
- (2-particle) unitarity automatically conserved
- Particular motivation:
- The ultimate goal is the phenomenological description of the CLAS' data on the $\pi^+\pi^-$ photoproduction in the D-wave (otherwise the data on the D-wave are scarce)

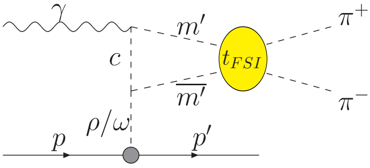


CLAS data:
$E_\gamma = 3.4 - 3.6$ GeV
$0.5 < t < 0.6$ GeV^2.

- Caveat: in this talk the model comparison with data will not be presented
- Excuse: 1. background contribution not included, 2. off-shell effects not included (based on the S-wave calculations one can expect that they affect both the magnitude and phase of the amplitude)

Model

- We assume that the tensor meson photoproduction is a two stage process:

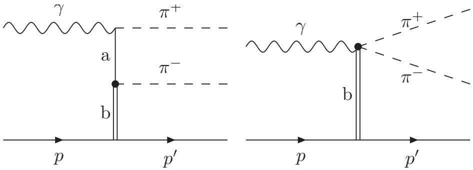


- First a pair of pseudoscalar mesons (pions in this iteration of the model) is produced - described in terms of Born amplitudes
- Then mesons undergo the final state interactions - $\pi\pi$ scattering amplitudes /details, see R. Kamiński talk at this conference/
- $2 \times 2 \times 2 \times 5 = 40$ amplitudes (of which 20 independent) needed to describe the reaction.

Born amplitudes are derived from phenomenological Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\pi\pi\gamma} + \mathcal{L}_{\rho\pi\gamma} + \mathcal{L}_{\omega\pi\gamma} + \mathcal{L}_{\rho\pi\pi\gamma} + \mathcal{L}_{\rho\pi\pi} + \mathcal{L}_{\rho\pi\omega} + \mathcal{L}_{\omega NN} + \mathcal{L}_{\rho NN}.$$

Ch-R Ji *et al.*, PRC 58, 1205 (1998)



$m\overline{m}$	Type I (3 diagrams)	Type II (2 diagrams)
$\pi^+\pi^-$	(a,b)=($\pi^\pm$,ρ^0)	(a,b)=($\rho^\pm$,ω)
$\pi^0\pi^0$		(a,b)=(ρ^0,ω),(ω,ρ^0)

- Structure of the Born amplitude:

$$V_{m\overline{m}} = \sum_{r=I,II} \overline{u}(p',s') J_{r,m\overline{m}} \cdot \epsilon(q,\lambda) u(p,s)$$

where:

$$J^\mu_{r,m\overline{m}} = (\alpha_{r,m\overline{m}} g^{\mu\nu} + k_1^\mu \beta^\nu_{1r,m\overline{m}} + k_2^\mu \beta^\nu_{2r,m\overline{m}})(d_{r,m\overline{m}}\gamma_\nu + e_{r,m\overline{m}}(p+p')_\nu)$$

and $\mathbf{k}_1 = -\mathbf{k}_2 = |k|\hat{\kappa} = |k|(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

($|\mathbf{k}_1| = |\mathbf{k}_2| = k$ from now on)

Partial wave expansion technique

- Partial wave expansion of the $\Omega = (\theta, \phi)$ dependent part of the current:

$$P^{JM,\mu\nu}_{r,m\overline{m}} = \int d\Omega {Y^J_M}^*(\Omega)(\alpha_{r,m\overline{m}} g^{\mu\nu} + k_1^\mu \beta^\nu_{1r,m\overline{m}} + k_2^\mu \beta^\nu_{2r,m\overline{m}})$$

- Q: Why do it analytically ? A: Each integration using standard GAUSS procedure makes the computation longer by a factor of ~ 40 - it matters in fits.

Obervation:
Photon polarization vector: $\epsilon_\mu(\lambda) = -\frac{\lambda}{\sqrt{2}}(0, \cos\theta_q, i\lambda, \sin\theta_q)$ ($\lambda{=}{+}1$)
So the only components of the $P^{LM,\mu\nu}_{r,m\overline{m}}$ tensor which enter the amplitude are: $P^{JM,i0}_{r,m\overline{m}}$ and $P^{JM,ij}_{r,m\overline{m}}$.

Let's take $P^{2M,xy}_{I,m\overline{m}}$:

$$P_I^{2M,ij} = -\frac{1}{\sqrt{4\pi}} \int d\Omega \, {Y_M^2}^*(\Omega) \frac{1}{|\mathbf{q}||\mathbf{k}|} \left[k_1^i \frac{q^j - 2k_1^j}{x - \hat{\mathbf{q}} \cdot \hat{\kappa}} - k_1^i \frac{q^j + 2k_1^j}{x + \hat{\mathbf{q}} \cdot \hat{\kappa}} \right]$$

where: $x = \sqrt{1 + \frac{m_\pi^2}{k^2}} > 1$ and $\hat{\mathbf{q}} \cdot \hat{\kappa} \leqslant 1$

We apply the $\theta \to \pi - \theta, \phi \to \phi + \pi$ transformation to the terms $\sim \frac{1}{x + \hat{\mathbf{q}} \cdot \hat{\kappa}}$

Recall that $Y_M^J \to (-1)^J Y_M^J$ under this transformation

We arrive at the following expresion:

$$P_l^{2M,ij} = \frac{-2}{\sqrt{4\pi}} \int d\Omega {Y_M^2}^*(\Omega)[\underbrace{\frac{\hat{\kappa}^i \hat{q}^j}{x - \hat{\mathbf{q}} \cdot \hat{\kappa}}}_{a} - \underbrace{\frac{2k}{|\mathbf{q}|} \frac{\hat{\kappa}^i \hat{\kappa}^j}{x - \hat{\mathbf{q}} \cdot \hat{\kappa}}}_{b}]$$

For $i = x$ and $j = y$ we expand the $\hat{\kappa}$ versors in terms of spherical harmonics $\hat{\kappa}^x = \frac{1}{\sqrt{2}}(Y^1_{-1} - Y^1_{+1})$, $\hat{\kappa}^y = \frac{i}{\sqrt{2}}(Y^1_{+1} + Y^1_{-1})$ and use the Heine formula to expand the denominators in tems of Legendre polynomials and Legendre functions of the second kind:

$$\frac{1}{x - \hat{\mathbf{q}} \cdot \hat{\kappa}} = \sum_{l=0}^{\infty}(2l+1)P_l(\hat{\mathbf{q}} \cdot \hat{\kappa})Q_l(x)$$

Using the formula $P_l(\hat{\mathbf{q}} \cdot \hat{\kappa}) = \sum_{m=-l}^{+l} \frac{4\pi}{(2l+1)} Y^l_m(\hat{\mathbf{q}})^* Y^l_m(\hat{\kappa})$ we arrive at:

$$P_{la}^{2M,xy} = -\sqrt{\frac{2}{4\pi}} \int d\Omega {Y_M^2}^*(\Omega)(Y^1_{-1} - Y^1_{+1}) 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} Y^l_m(\hat{\mathbf{q}})^* Y^l_m(\hat{\kappa}) Q_l(x) \hat{q}^y$$

For the b term we expand
$\hat{\kappa}^x\hat{\kappa}^y = \sqrt{\frac{2\pi}{3}}[\sqrt{\frac{2}{3}}Y^0_0 - \frac{1}{\sqrt{5}}(Y^2_{-2} + Y^2_{+2} + \sqrt{\frac{2}{3}}Y^2_0)]$ to arrive at:

$$P^{2M,xy}_{lb} = 2\sqrt{\frac{2}{3}}\int d\Omega\, {Y^2_M}^*(\Omega)$$
$$[\sqrt{\frac{2}{3}}Y^0_0 - \frac{1}{\sqrt{5}}(Y^2_{-2} + Y^2_{+2} + \sqrt{\frac{2}{3}}Y^2_0)]4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{+l} Y^l_m(\hat{\mathbf{q}})^* Y^l_m(\hat{\kappa})Q_l(x)$$

- This is a general approach.

The expansion for arbitrary l_1, m_1 is:

$$\int d\Omega\, {Y^2_M}^* Y^{l_1}_{m_1}\sum_{l=0}^{\infty}\sum_{m=-l}^{+l}{Y^l_m}^*(\hat{\mathbf{q}})Y^l_m Q_l(x) = \int d\Omega\, {Y^2_M}^*\sum_{l=0}^{\infty}\sum_{m=-l}^{l}{Y^l_m}^*(\hat{\mathbf{q}})Q_l(x)$$
$$\sum_{j=|l-l_1|}^{l+l_1}\sum_{\mu=-j}^{j}\sqrt{\frac{(2j+1)(2l_1+1)(2L+1)}{4\pi}}\begin{pmatrix} j & l_1 & l \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} j & l_1 & l \\ \mu & m_1 & m\end{pmatrix} Y^j_\mu = \dots$$

$$\ldots = \sum_{l'} \sum_{m'=-l'}^{l'} Y_{m'}^{l'}(\hat{\mathbf{q}}) Q_{l'}(x)$$

$$\sqrt{\frac{(2\cdot 2+1)(2l_1+1)(2l'+1)}{4\pi}} \begin{pmatrix} 2 & l_1 & l' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & l_1 & l' \\ M & m_1 & m' \end{pmatrix}$$

Using spherical harmonics orthogonality we replaced the solid angle integral with angular momentum summation and restricted admissible l' values to those respecting the triangle inequality $|l' - l_1| \leqslant 2 \leqslant l' + l_1$, thus for:

$l_1 = 0$: l'=2

$l_1 = 1$: l'=1, 2, 3

$l_1 = 2$: l'=0, 1, 2, 3, 4

For bosons the sum $2 + l_1 + l'$ in $\begin{pmatrix} 2 & l_1 & l' \\ \mu & m_1 & m' \end{pmatrix}$ must be even. This further restricts admissible l' values to:

$l_1 = 0$: l'=2

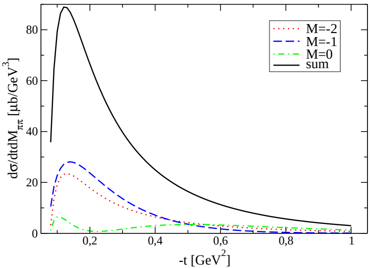
$l_1 = 1$: l'=1, 3

$l_1 = 2$: l'=0, 2, 4

Furthermore, $\mu + m_1 + m' = 0$ which in turn restricts admissible m' values.

Initial conclusion:

- There is finite (however large sometimes) number of terms in D partial wave expansion of the Born amplitudes,
- One needs to expand in terms of spherical hamonics the terms of the amplitude containing: $\hat{\kappa}^i \hat{\mathbf{q}}^j$, $\hat{\kappa}\cdot\hat{\mathbf{q}}\ g^{ij}$, $\hat{\kappa}\cdot\hat{\mathbf{q}}\ \hat{\kappa}^i \hat{\mathbf{q}}^j$, $\hat{\kappa}^i \hat{\kappa}^j$, $(\hat{\kappa}\cdot\hat{\mathbf{q}})^2 g^{ij}, \ldots$
- $P_r^{2M,\mu\nu}$ tensor has no definite symmetry but its off-diagonal elements can be split into non-symmetric and symetric parts which reduces (a little) number of terms involved.
- Strengths of the full (containing the FSI) photoproduction amplitudes corresponding to different $M's$ (angular momentum projections) are *entirely* determined by the Born amplitudes.



D-wave Born cross section - contribution of type II diagrams

Complete photoproduction amplitude

Photoproduction amplitude can be expressed schematically:

$$\hat{T} = \hat{V} + \hat{t}\hat{G}\hat{V}$$

where:

$\hat{T}$-photoproduction amplitude,

$\hat{t}$-$\pi\pi$ scattering amplitude, $\hat{V}$-Born amplitude and

$\hat{G}_{m'\overline{m}'}(\kappa') = \frac{1}{M_{m\overline{m}} - M_{m'\overline{m}'} + i\epsilon}$

For $\pi^+\pi^-$ photoproduction we have:

$$\hat{T}_{\pi^+\pi^-} = \langle \pi^+\pi^- p' | \hat{V} | \gamma p \rangle +$$

$$4\pi \sum_{m'\overline{m}'} \int_0^\infty \frac{\kappa'^2 d\kappa'}{(2\pi)^3} F(\kappa, \kappa') \langle \pi^+\pi^- | \hat{t} | m'\overline{m}' \rangle G_{m'\overline{m}'}(\kappa') \langle m'\overline{m}' p' | \hat{V} | \gamma p \rangle$$

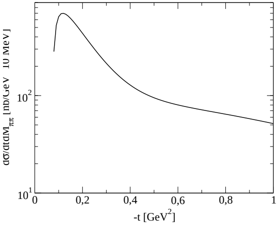
Using the $\pi\pi$ isospin amplitudes and after momentum integration we get:

$$\hat{T}_{\pi^+\pi^-} = \left[1 + i r_\pi \left(\frac{2}{3} t^{I=0} + \frac{1}{3} t^{I=2} \right) + \frac{2}{3} P^{I=0} + \frac{1}{3} P^{I=2} \right] \hat{V}_{\pi^+\pi^-}$$

$$+ \frac{1}{3} [i r_\pi (-t^{I=0} + t^{I=2}) - P^{I=0} + P^{I=2}] \hat{V}_{\pi^0\pi^0}$$

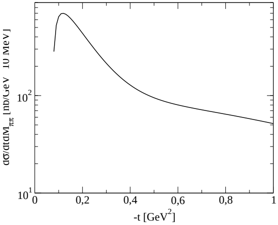
where $r_\pi = -kM_{\pi\pi}/8\pi$.

Born D-wave photoproduction cross section



Born D-wave cross section - this model

Born D-wave photoproduction cross section



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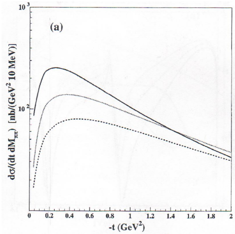

Born S-wave cross section (Ji et al.)

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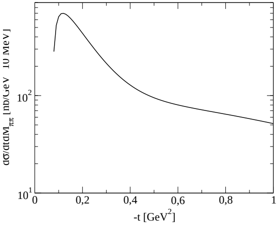

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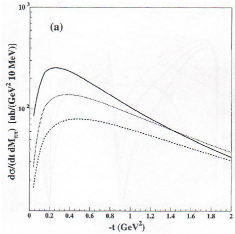

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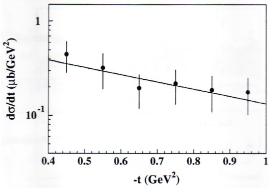
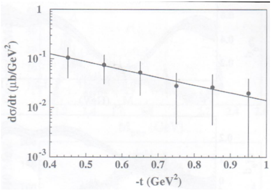


D-wave cross section measured by CLAS



S-wave cross section measured by CLAS

There are hints that the model properly describes the D-wave cross section.

Verification of the model

After taking into account:

- the non-resonant D-wave background,
- S-wave resonances ($f_0(980)$, $f_0(1370)$) and
- P-wave ($\rho(770)$)...

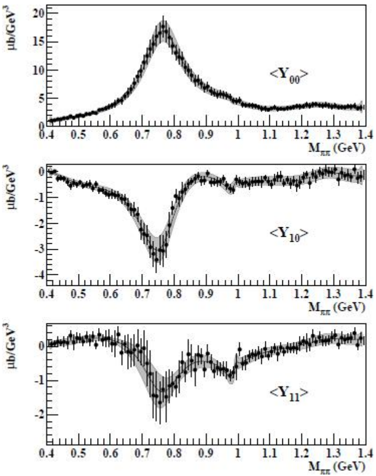
...one can construct the moments of pion angular distribution and check the model against the data.

$$\langle Y_M^L \rangle = N \int d\Omega \, {Y_M^L}^*(\Omega) |A|^2$$

Ex.

$$\begin{aligned}\langle Y_0^1 \rangle = \frac{N}{\sqrt{4\pi}} \Bigg[& 2Re(S_0^* P_0) + 2\sqrt{\frac{3}{5}} Re(P_{-1}^* D_{-1}) + \frac{4}{\sqrt{5}} Re(D_0^* P_0) \\ & + 2\sqrt{\frac{3}{5}} Re(D_{+1}^* P_{+1}) + 2\sqrt{\frac{3}{7}} Re(D_{-2}^* F_{-2}) + 4\sqrt{\frac{6}{35}} Re(D_{-1}^* F_{-1}) \\ & + 6\sqrt{\frac{3}{35}} Re(D_0^* F_0) + 4\sqrt{\frac{6}{35}} Re(D_{+1}^* F_{+1}) + 2\sqrt{\frac{3}{7}} Re(D_{+2}^* F_{+2}) + \ldots \Bigg]\end{aligned}$$

Analysis of moments is a very sensitive method as seen in the case of $f_0(980)$.



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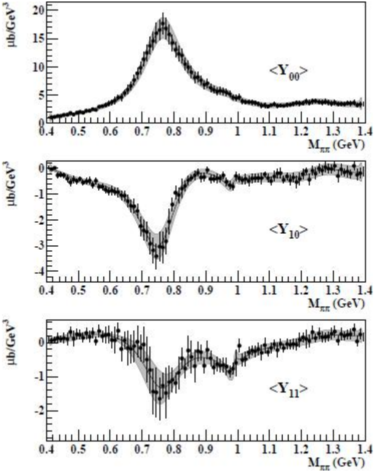

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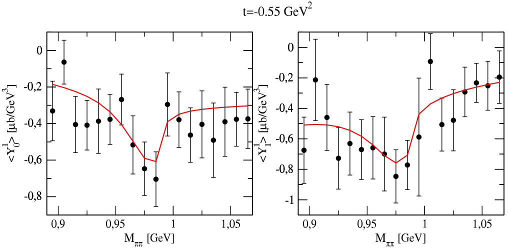
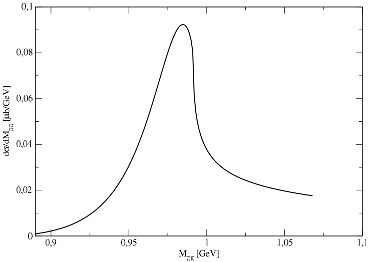


Our fit in the neighbourhood of the $f_0(980)$.



Resulting S-wave mass distibution.

Other estimations: (Marco,Oset,Toki '99 and Donnacie, Kalashnikova '08) predict the mass distribution 5x larger

Conclusions and open problems

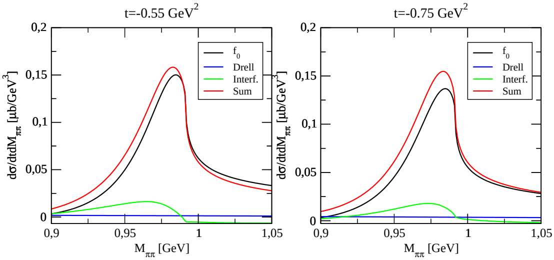
Conclusions

- Based on known form factors and couplings we derived the model to describe the $\pi^+\pi^-$ D-wave photoproduction,
- Supplementing this model with proper D-wave "background" amplitude should enable to describe the mass distribution,
- Furher inclusion of the other partial waves to describe the (moments of) angular distribution of pions.

Conclusions and open problems

Open problems

- What kind of "background" amplitudes we need ?



- Drell amplitudes, usually used to desribe the P-wave background (Söding model) give very small background for the S-wawe
- Are they sufficient for the D-wave ? (unlikely). Suggestions welcome.

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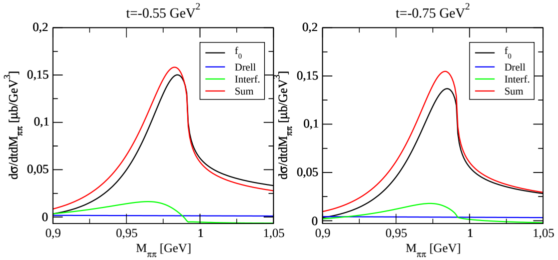

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