

[199/118]



eQCD,  
Sarajevo

Feb, 2013

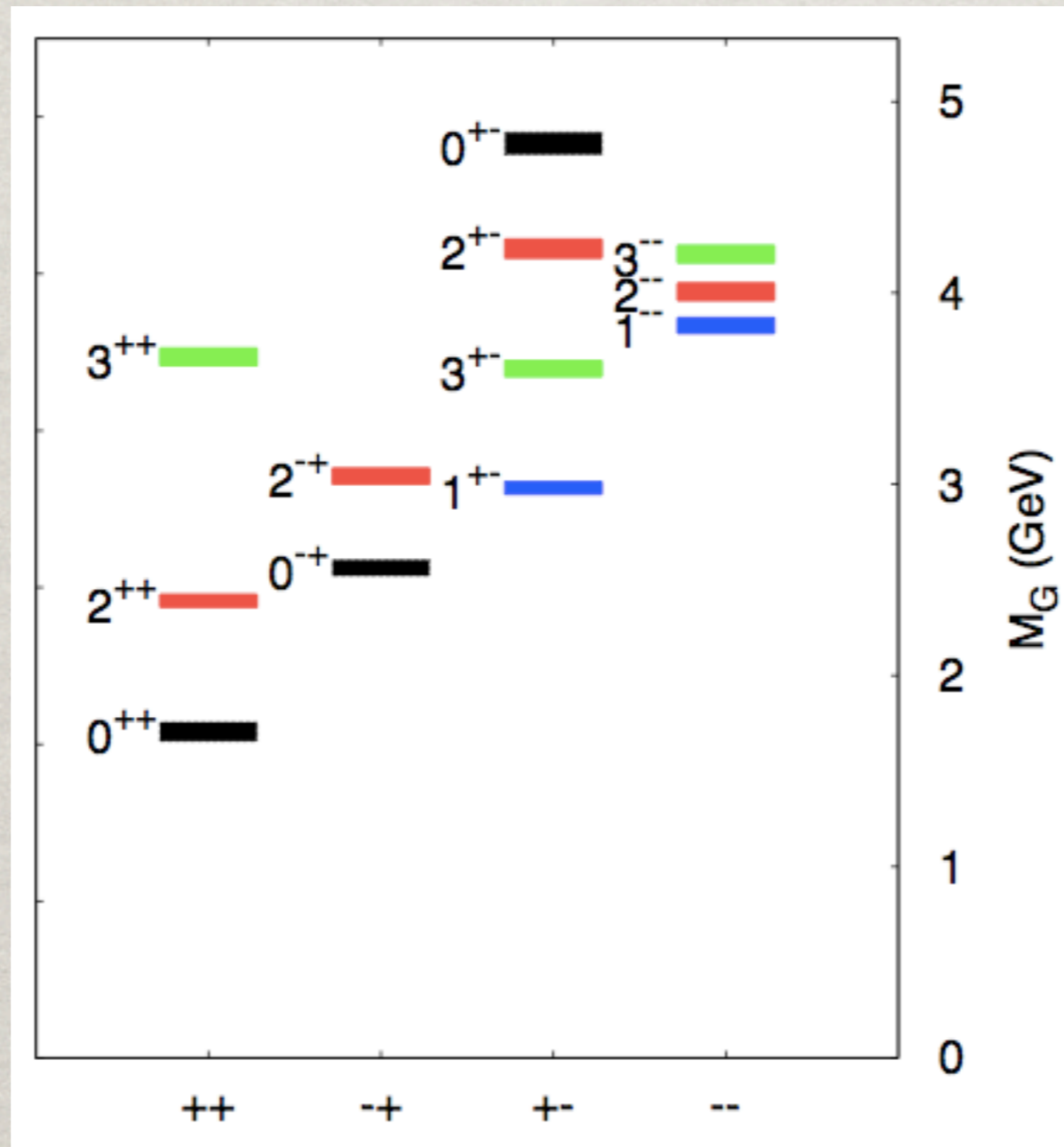
# GLUEBALLS IN THE BETHE-SALPETER FORMALISM

Eric Swanson



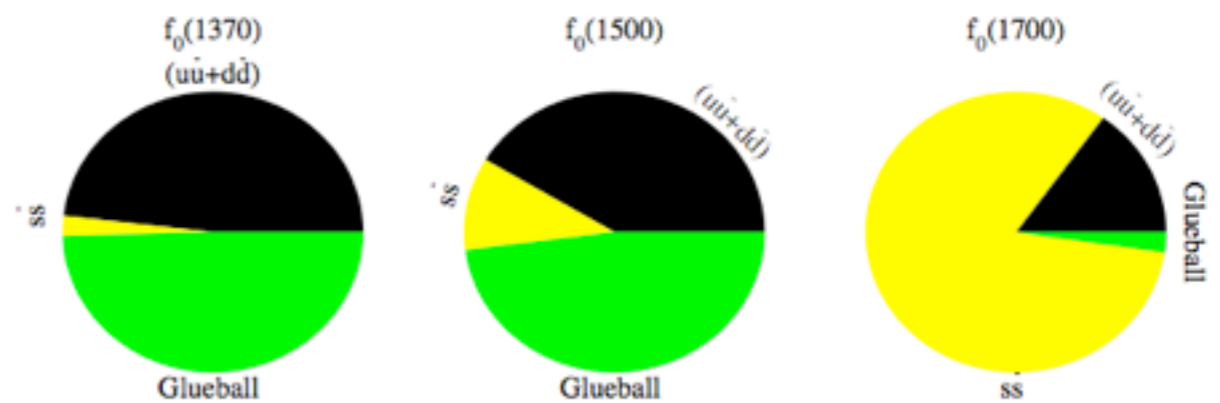
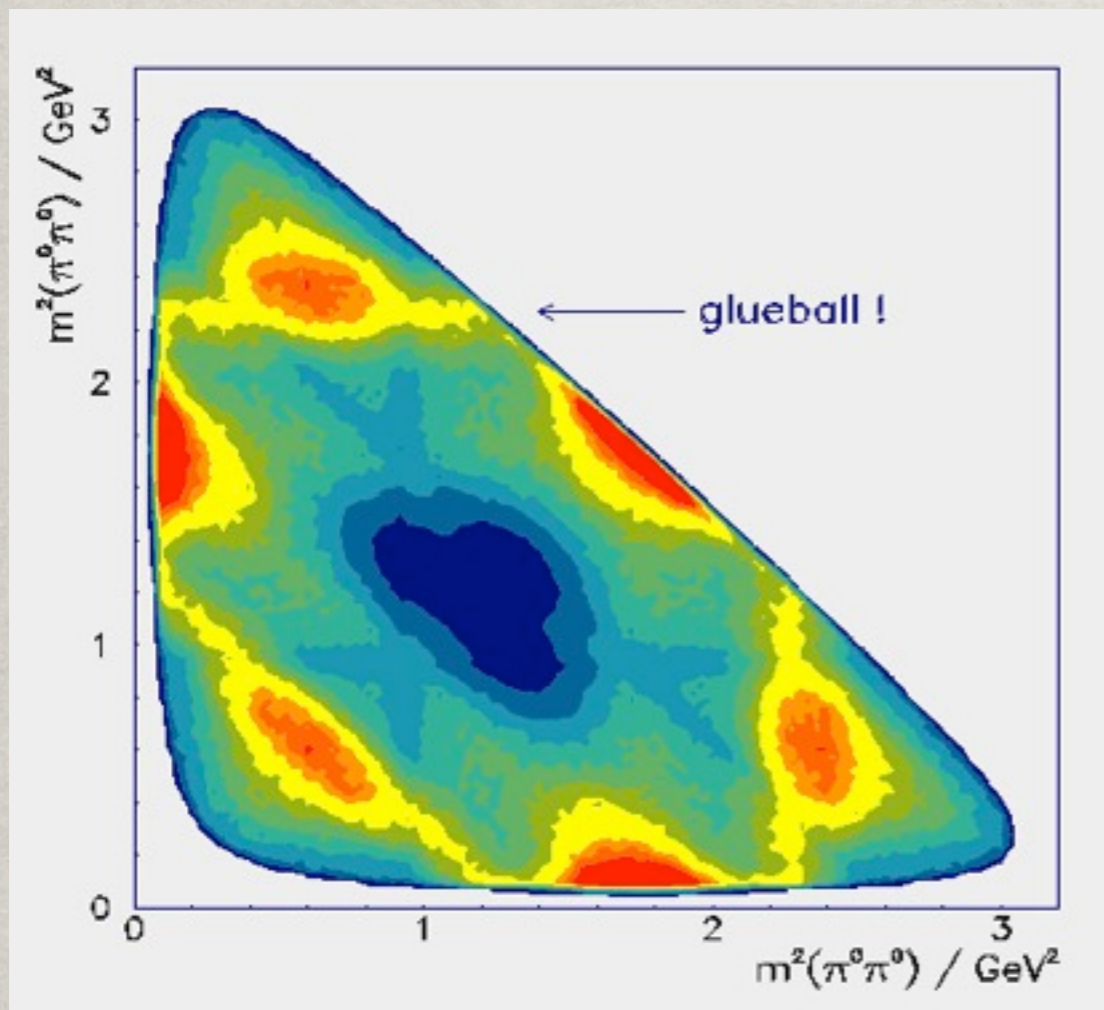
# LATTICE RESULTS

Y. Chen et al., Phys. Rev. D73, 014516 (2006).



# EXPERIMENT

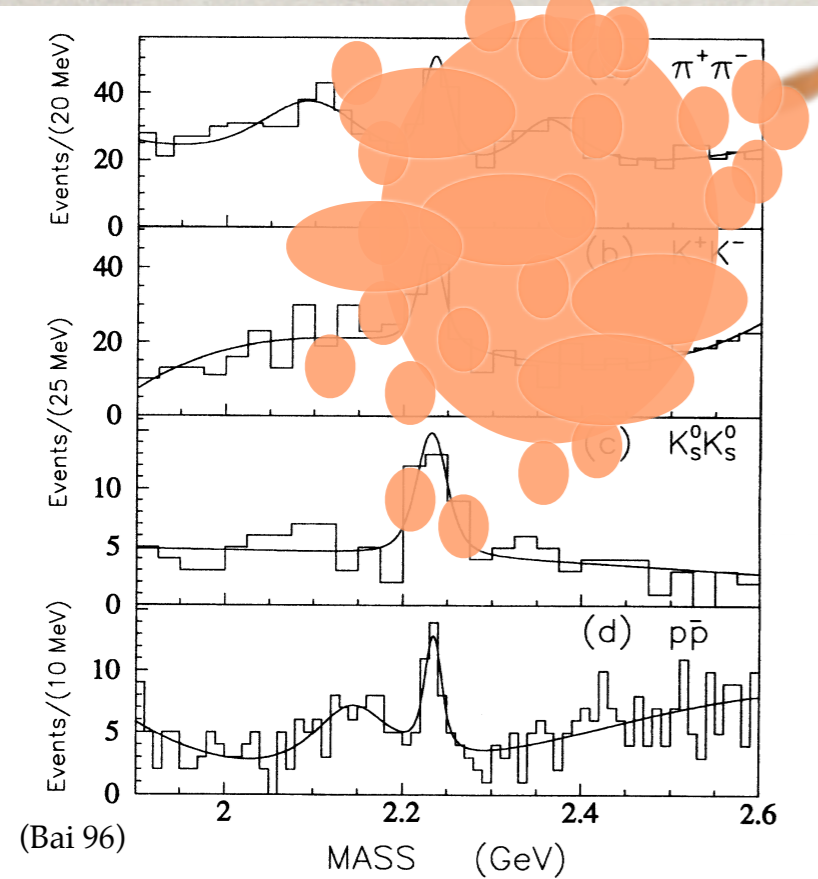
$$f_0(1500) \quad IJ^{PC} = 00^{++}$$



# EXPERIMENT

$$\xi(2230) \quad J = 2$$

$$J/\psi \rightarrow \gamma h$$

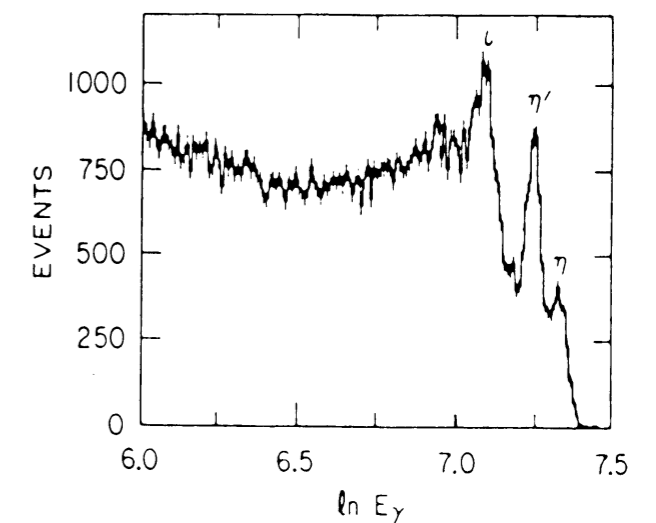


$$D \rightarrow \pi\pi, \eta\eta$$



Crystal Barrel

$$J/\psi \rightarrow \gamma \xi$$



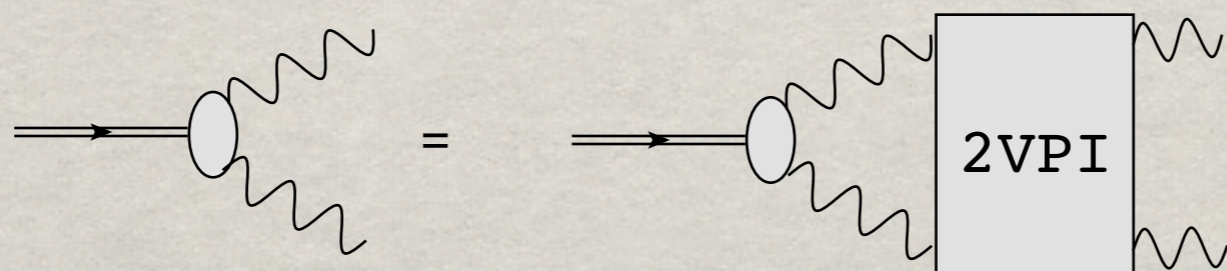
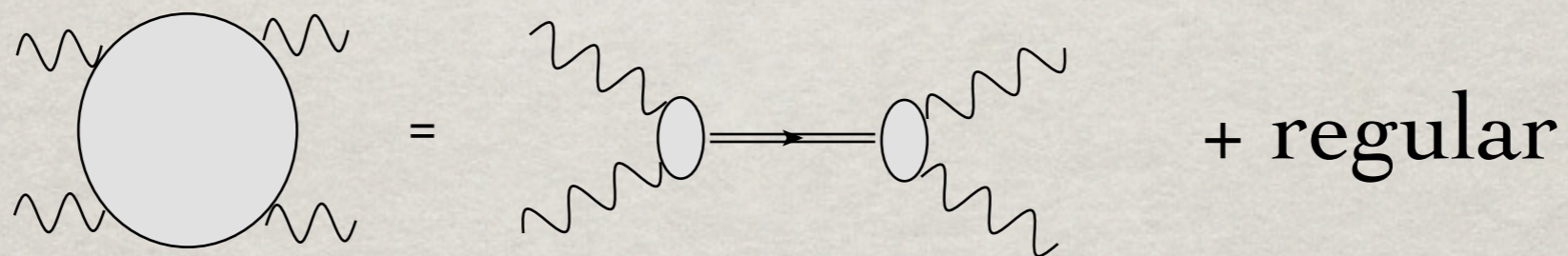
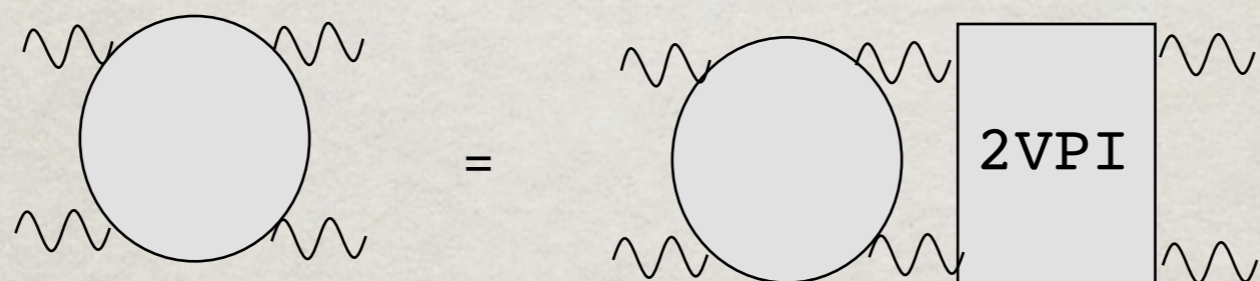
Crystal Ball

# BETHE-SALPETER FORMALISM

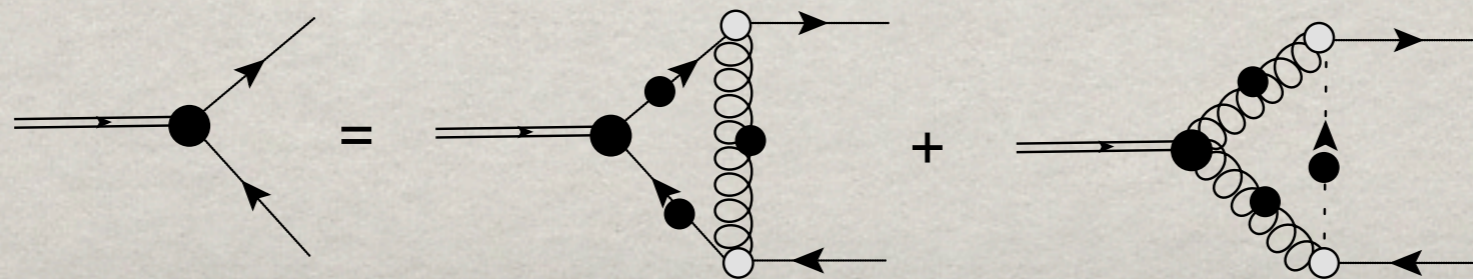
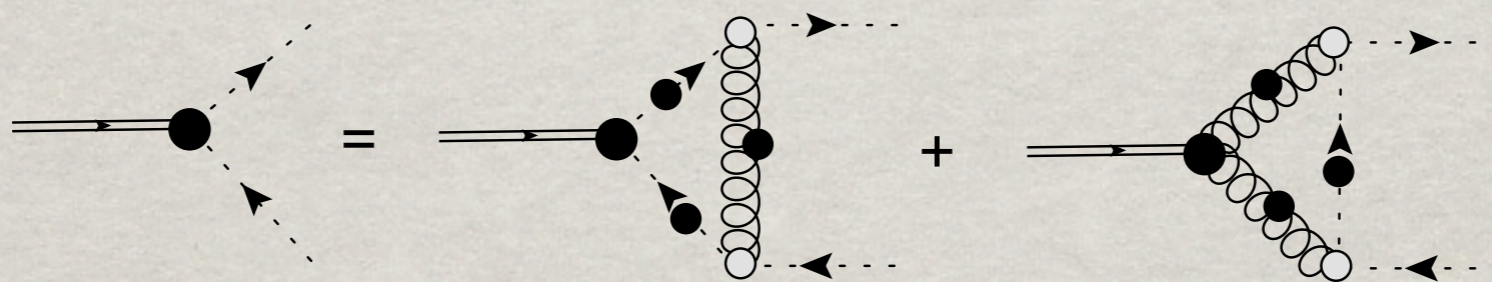
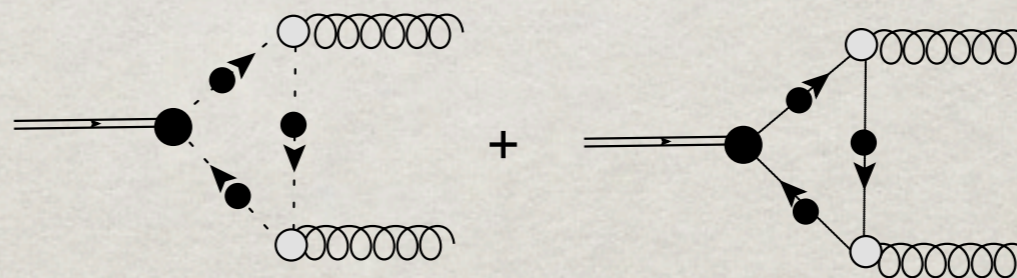
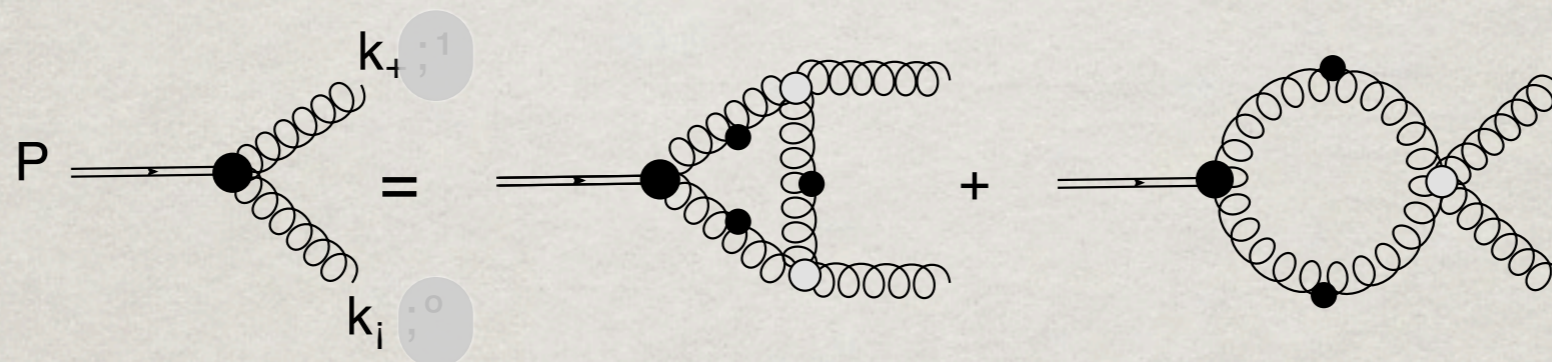
J. Meyers and E.S. Swanson, PRD [[arxiv:1211.4648](#)]

R. Alkofer and L. von Smekal, Phys. Rept. **353**, 281 (2001).

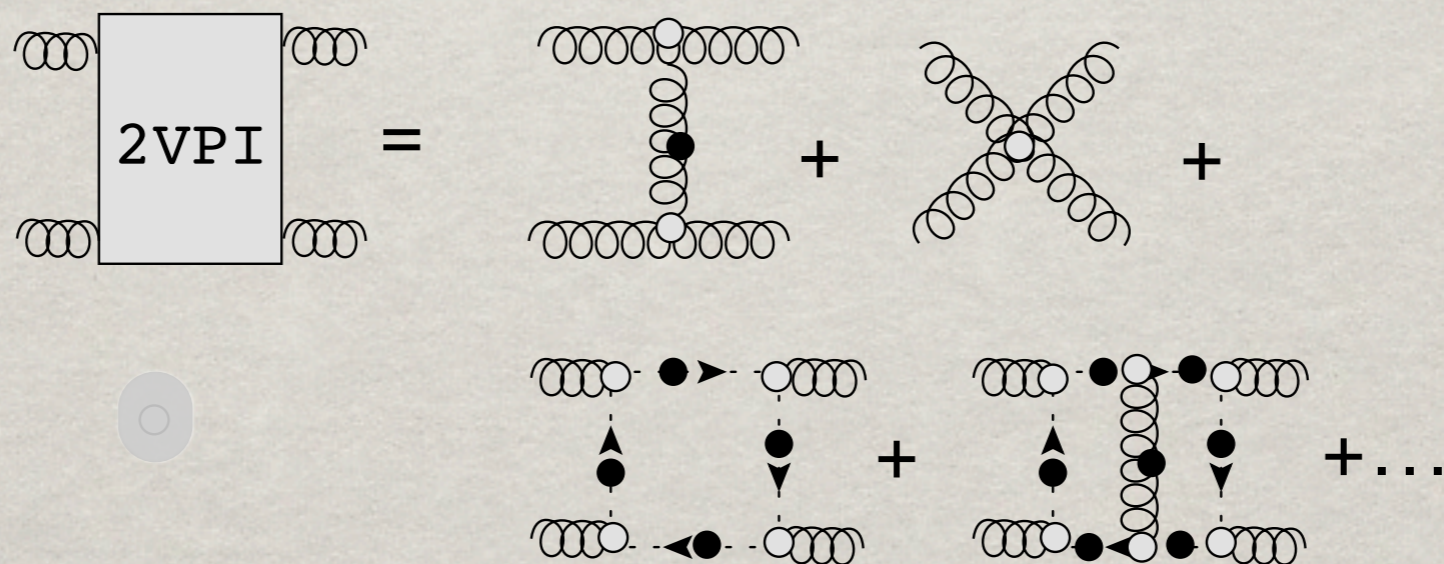
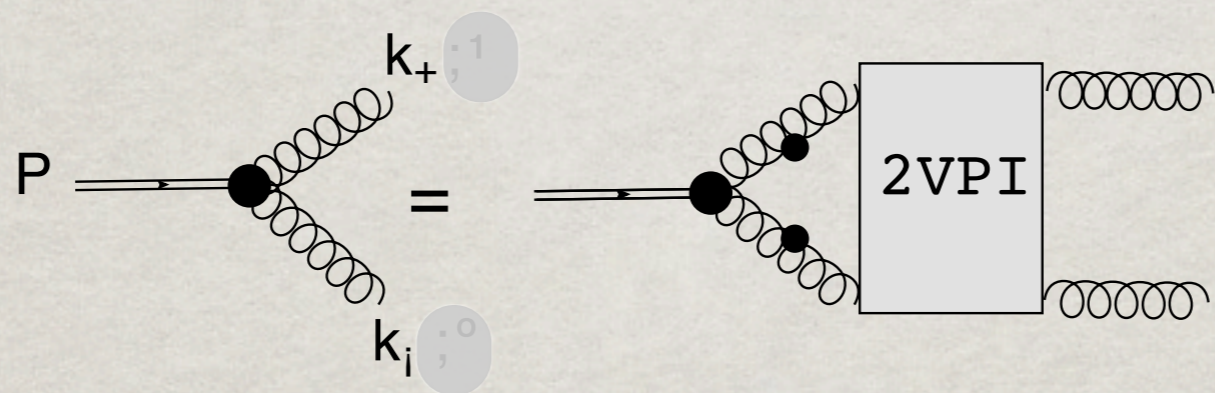
# BETHE-SALPETER FORMALISM



# BETHE-SALPETER FORMALISM



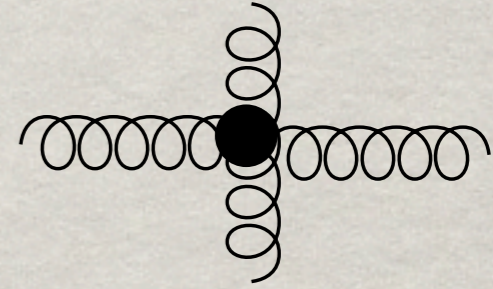
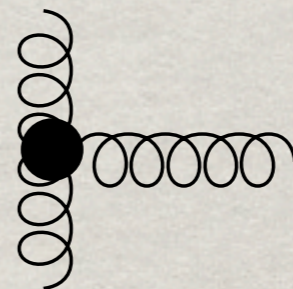
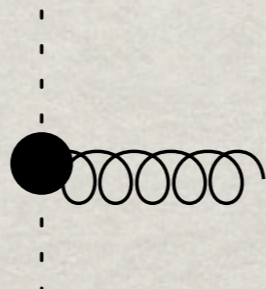
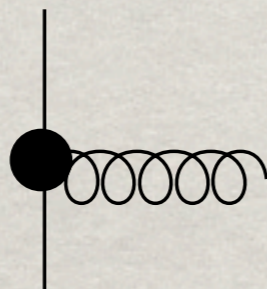
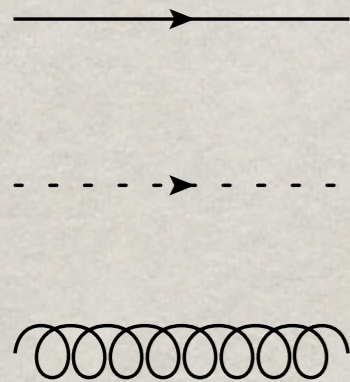
# BETHE-SALPETER FORMALISM





# BETHE-SALPETER FORMALISM

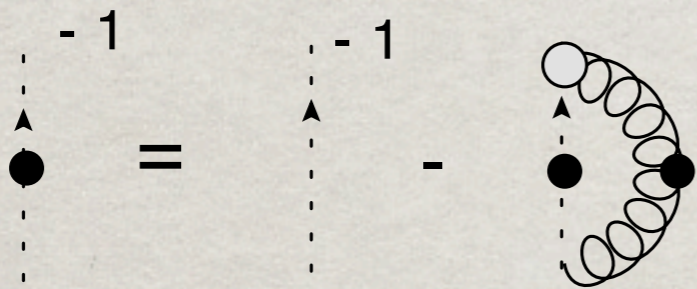
required elements:



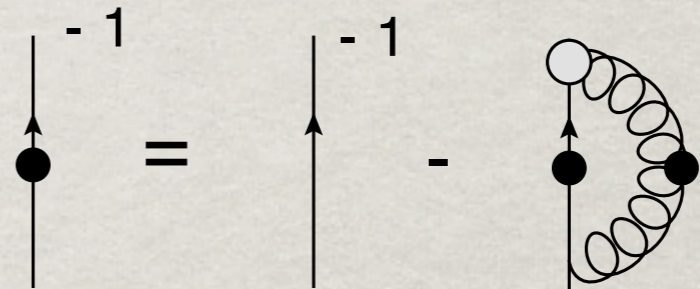
# GAP EQUATIONS

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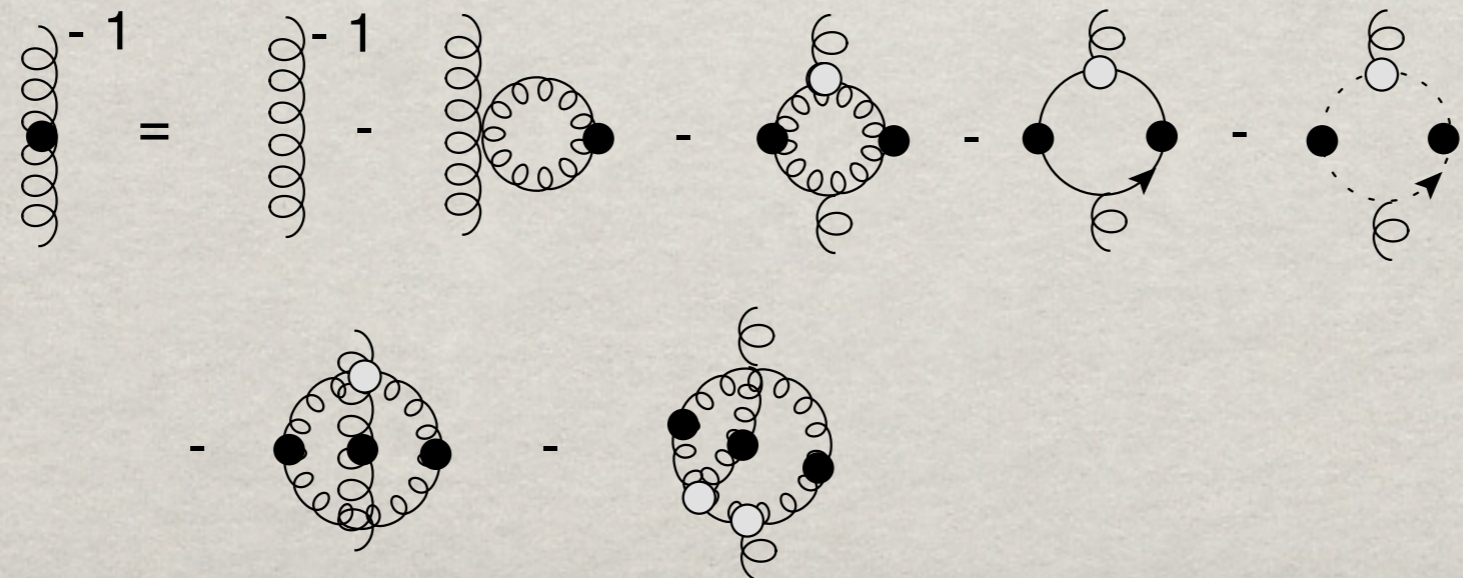
ghost



quark



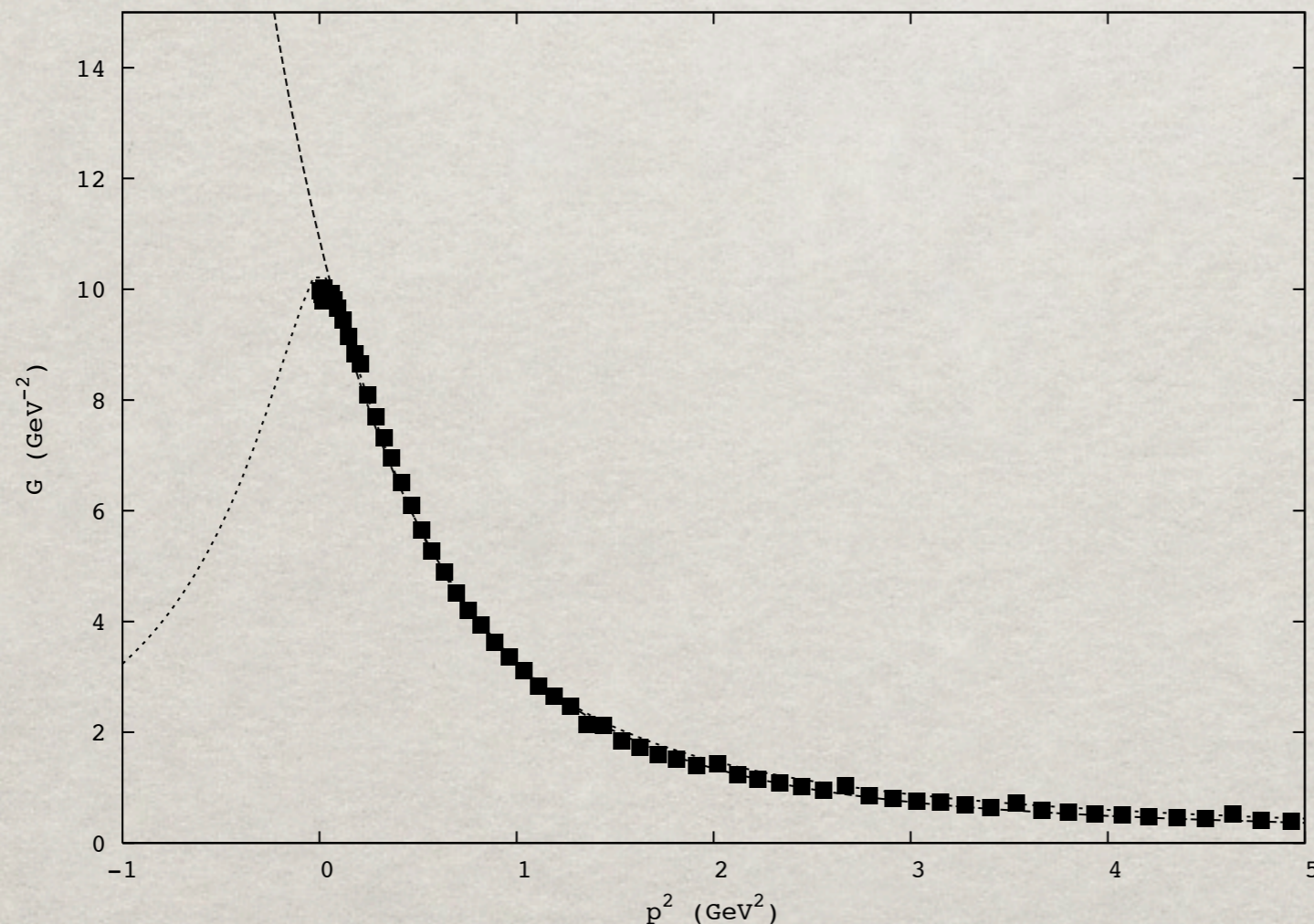
gluon



# GAP EQUATIONS

gluon propagator:

$$D_{\mu\nu}(k) \equiv -i \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \cdot G(k) - i\xi \frac{k^\mu k^\nu}{k^4 + i\epsilon}$$



I.L. Bogolubsky, E.M. Ilgenfritz, M. Muller-Preussker and  
A. Sternbeck, Phys. Lett. **B676**, 69 (2009)

# GAP EQUATIONS

vertices:

various models taken from

A.C. Aguilar and J. Papavassiliou, *Phys. Rev.* **D83**, 014013 (2011)

L. von Smekal, A. Hauck, and R. Alkofer, *Ann. Phys.* **267**, 1 (1998)

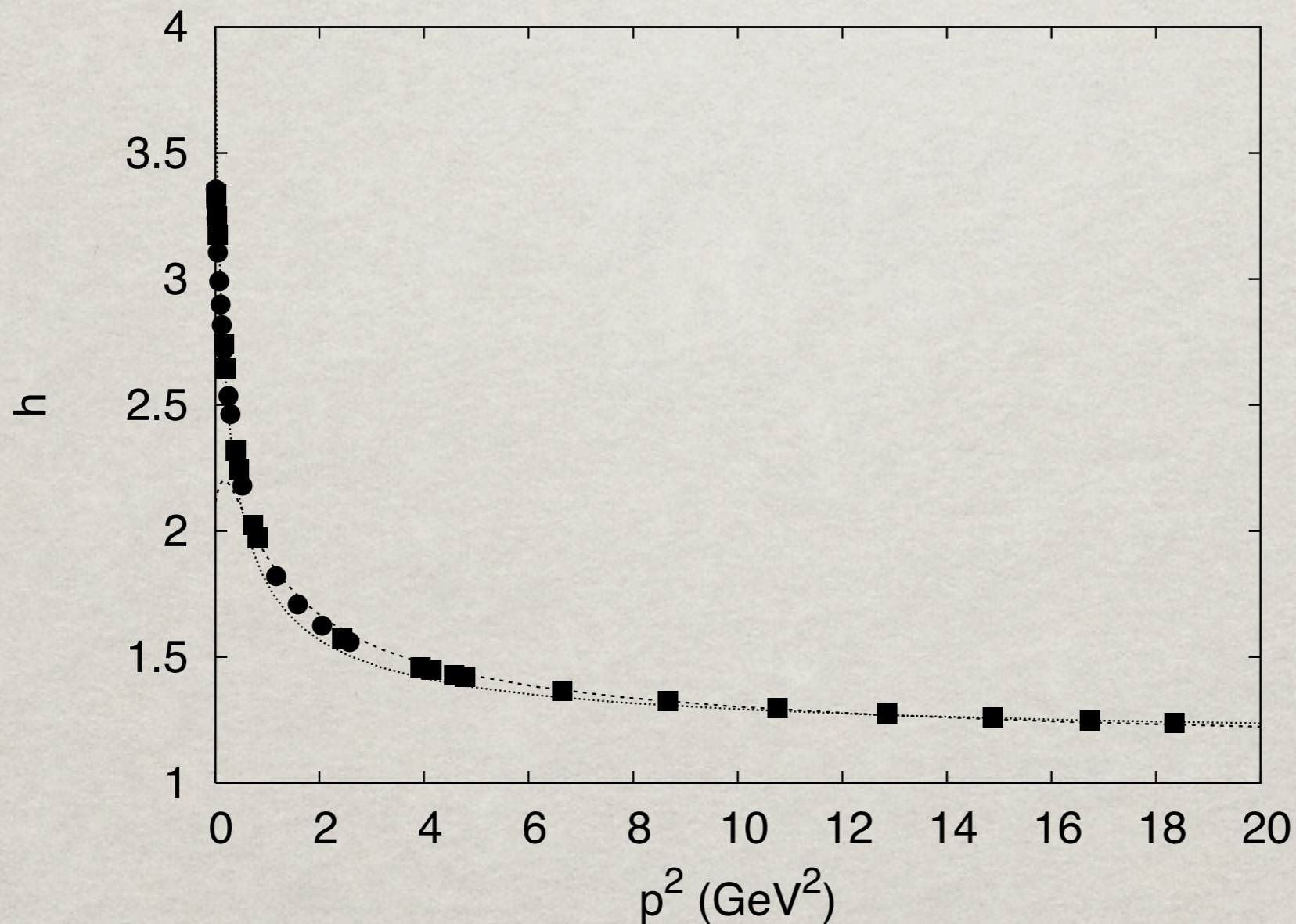
M.R. Pennington and D.J. Wilson, *Phys. Rev.* **D84**, 119901 (2011)

attempt to take into account Slavnov-Taylor identities, multiplicative renormalisability, etc

# GAP EQUATIONS

results: ghost

$$H(q) \equiv i\delta^{ab} \frac{h(q)}{q^2}$$



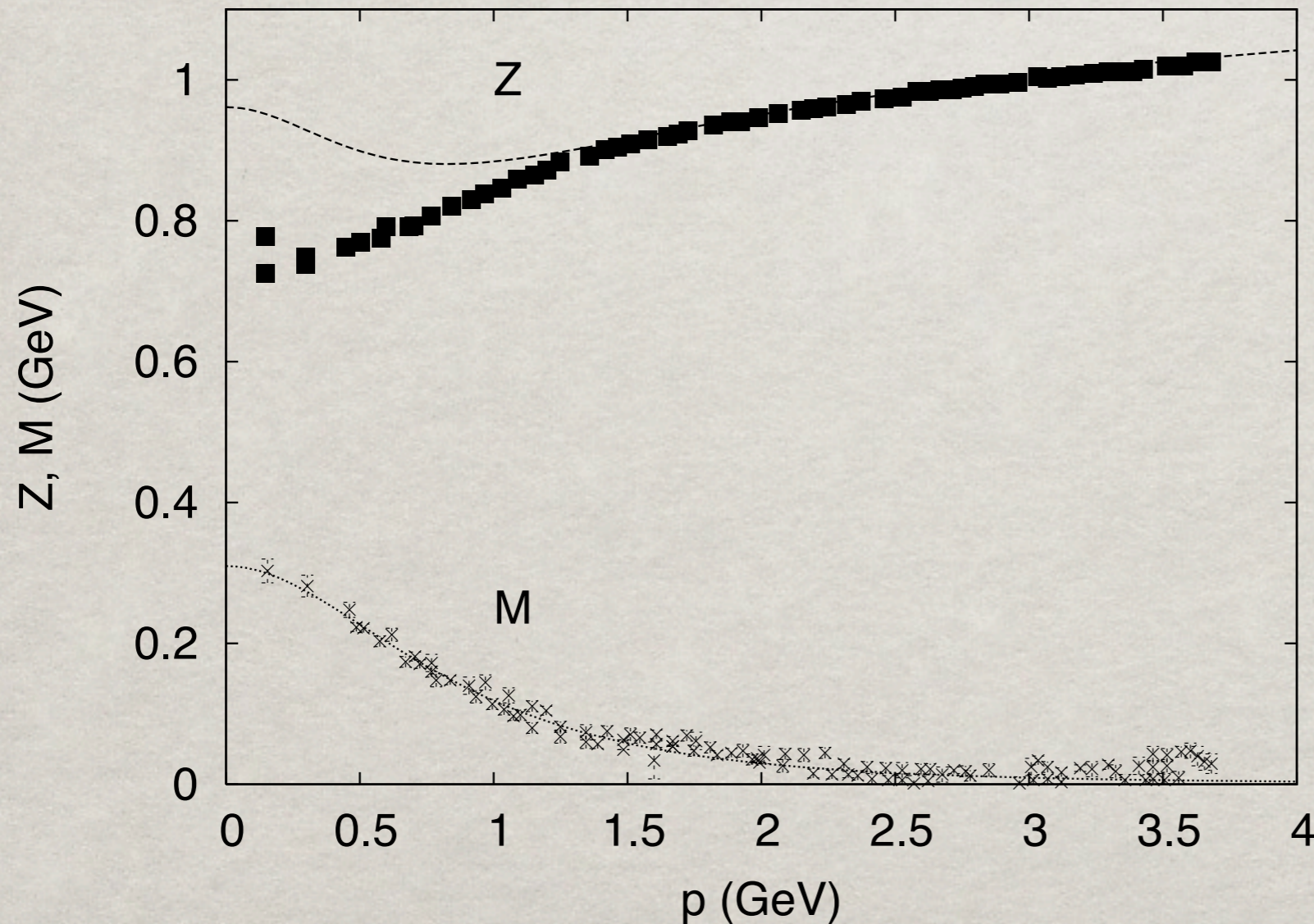
# GAP EQUATIONS

results: quark

$$f_\pi = 240 \text{ MeV}$$

$$\langle \bar{\psi}\psi \rangle(1 \text{ GeV}) = (-251 \text{ MeV})^3$$

$$S(k) = i \frac{Z(k)}{\not{k} - M(k)}$$



# GLUEBALL EQUATIONS



# BETHE-SALPETER EQUATIONS

$$\begin{aligned}
 \chi_{\mu\nu}(k_+, k_-) = & ig^2 N \int \frac{d^4 q}{(2\pi)^4} \chi_{\alpha\beta}(q_+, q_-) \mathcal{C}_{\dots\mu\nu}^{\alpha\beta} G(q_+) G(q_-) \\
 & + ig^2 N \int \frac{d^4 q}{(2\pi)^4} \chi_{\alpha\beta}(q_+, q_-) \mathcal{T}_{\dots\mu\nu}^{\alpha\beta}(q, k; P) G(q_+) G(q_-) G(Q) \\
 & + ig^2 N \int \frac{d^4 q}{(2\pi)^4} \chi(q_+, q_-) \mathcal{G}_{\mu\nu}(q, k; P) H(q_+) H(q_-) H(Q) \\
 & - \frac{g^2}{2} \int \frac{d^4 q}{(2\pi)^4} \text{tr} [\gamma_\mu S(q_+) \mathbb{X}(q_+, q_-) S(-q_-) \gamma_\nu S(Q)] h^n(k_+) h^n(k_-) \bar{A}(Q, q_+) \bar{A}(Q, q_-) \\
 & + \text{crossed}
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 \chi(k_+, k_-) = & ig^2 N \int \frac{d^4 q}{(2\pi)^4} \chi(q_+, q_-) \mathcal{H}(q, k; P) H(q_+) H(q_-) G(Q) \\
 & + ig^2 N \int \frac{d^4 q}{(2\pi)^4} \chi_{\alpha\beta}(q_+, q_-) \mathcal{B}^{\alpha\beta}(q, k; P) G(q_+) G(q_-) H(Q)
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 \mathbb{X}(k_+, k_-) = & g^2 C_F \int \frac{d^4 q}{(2\pi)^4} \gamma_\alpha S(-Q) \gamma_\beta G(q_+) G(q_-) \chi_{\alpha\beta}(q_+, q_-) h^n(q_+) h^n(q_-) \bar{A}(Q, k_+) \bar{A}(Q, k_-) \\
 & + ig^2 C_F \int \frac{d^4 q}{(2\pi)^4} \gamma_\mu S(q_+) \mathbb{X}(q_+, q_-) S(-q_-) \gamma_\nu G(Q) P_{\mu\nu}(Q) h^{2n}(Q) \bar{A}(k_+, q_+) \bar{A}(k_-, q_-)
 \end{aligned} \tag{3}$$

# BETHE-SALPETER EQUATIONS

$$C^{\alpha\beta\mu\nu} = 2g^{\mu\nu}g^{\alpha\beta} - g^{\alpha\nu}g^{\beta\mu} - g^{\alpha\mu}g^{\beta\nu}$$

$$T_{\alpha\beta\mu\nu} = P^{\gamma\gamma'}(Q) V_{\alpha\mu\gamma}(q_+, k_+) V_{\beta\nu\gamma'}(q_-, k_-)$$

$$P^{\gamma\gamma'}(Q) = g^{\gamma\gamma'} - \frac{Q^\gamma Q^{\gamma'}}{Q^2}$$

$$\begin{aligned} V_{\alpha\mu\gamma}(q_+, k_+) = & A_+(q_+, k_+, q_+ - k_+)g^{\alpha\mu}(q_+ + k_+)^\gamma + \\ & + A_+(k_+, q_+ - k_+, q_+)g^{\mu\gamma}(q_+ - 2k_+)^\alpha + \\ & + A_+(q_+ - k_+, q_+, k_+)g^{\gamma\alpha}(k_+ - 2q_+)^\mu. \end{aligned}$$

# BETHE-SALPETER EQUATIONS

$$\mathcal{G}^{\mu\nu} = \left( q_+^\mu \frac{h(k_+)}{h(q_+)} + Q^\mu \left( \frac{h(k_+)}{h(Q)} - 1 \right) \right) \left( Q^\nu \frac{h(k_-)}{h(Q)} - q_-^\nu \left( \frac{h(k_-)}{h(q_-)} - 1 \right) \right)$$

$$\mathcal{H} = P_{\mu\nu}(Q) k_+^\mu k_-^\nu \cdot \left( \frac{h(Q)}{h(k_+)} + \frac{h(Q)}{h(q_+)} - 1 \right) \cdot \left( \frac{h(Q)}{h(k_-)} + \frac{h(Q)}{h(q_-)} - 1 \right)$$

$$\mathcal{B}^{\alpha\beta} = -k_+^\alpha k_-^\beta \cdot \left( \frac{h(q_+)}{h(Q)} + \frac{h(q_+)}{h(k_+)} - 1 \right) \cdot \left( \frac{h(q_-)}{h(Q)} + \frac{h(q_-)}{h(k_-)} - 1 \right)$$

# BETHE-SALPETER EQUATIONS

impose transversality and parity

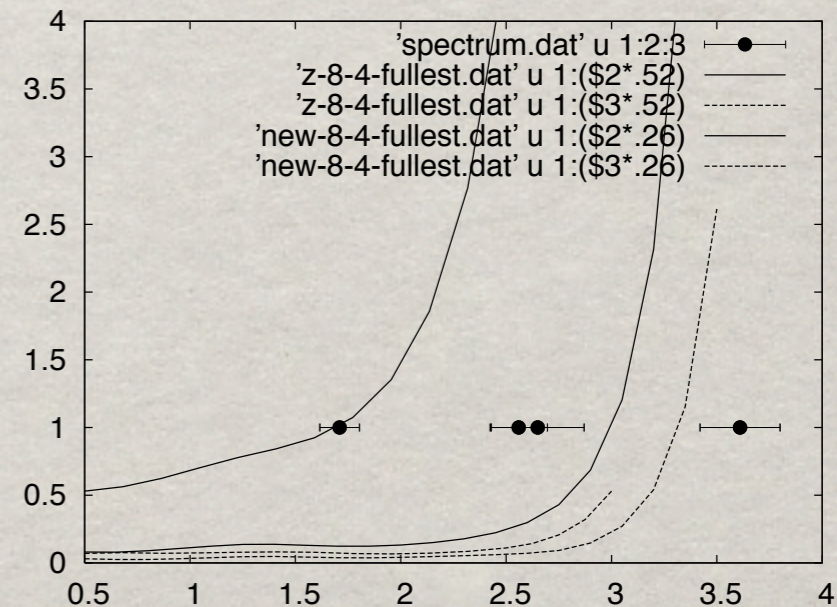
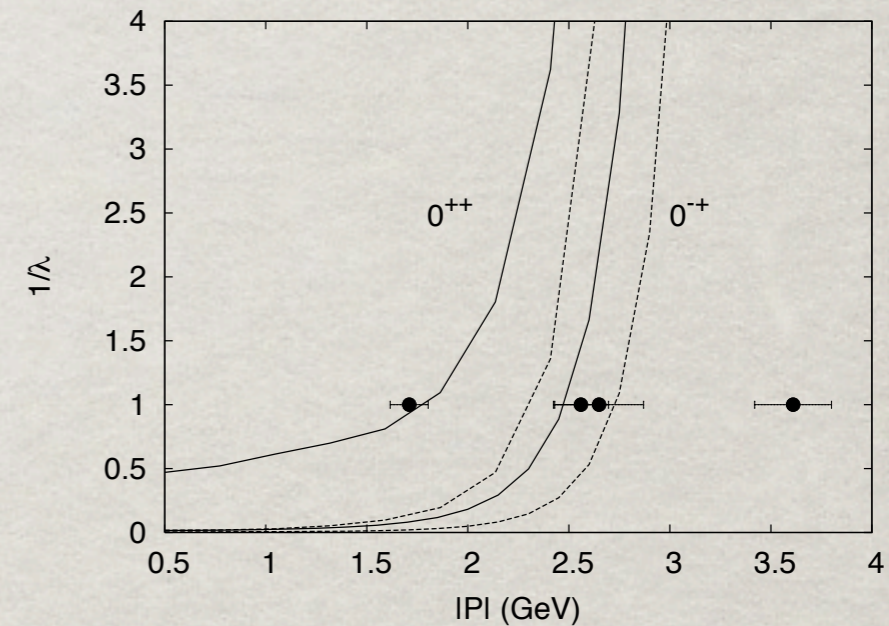
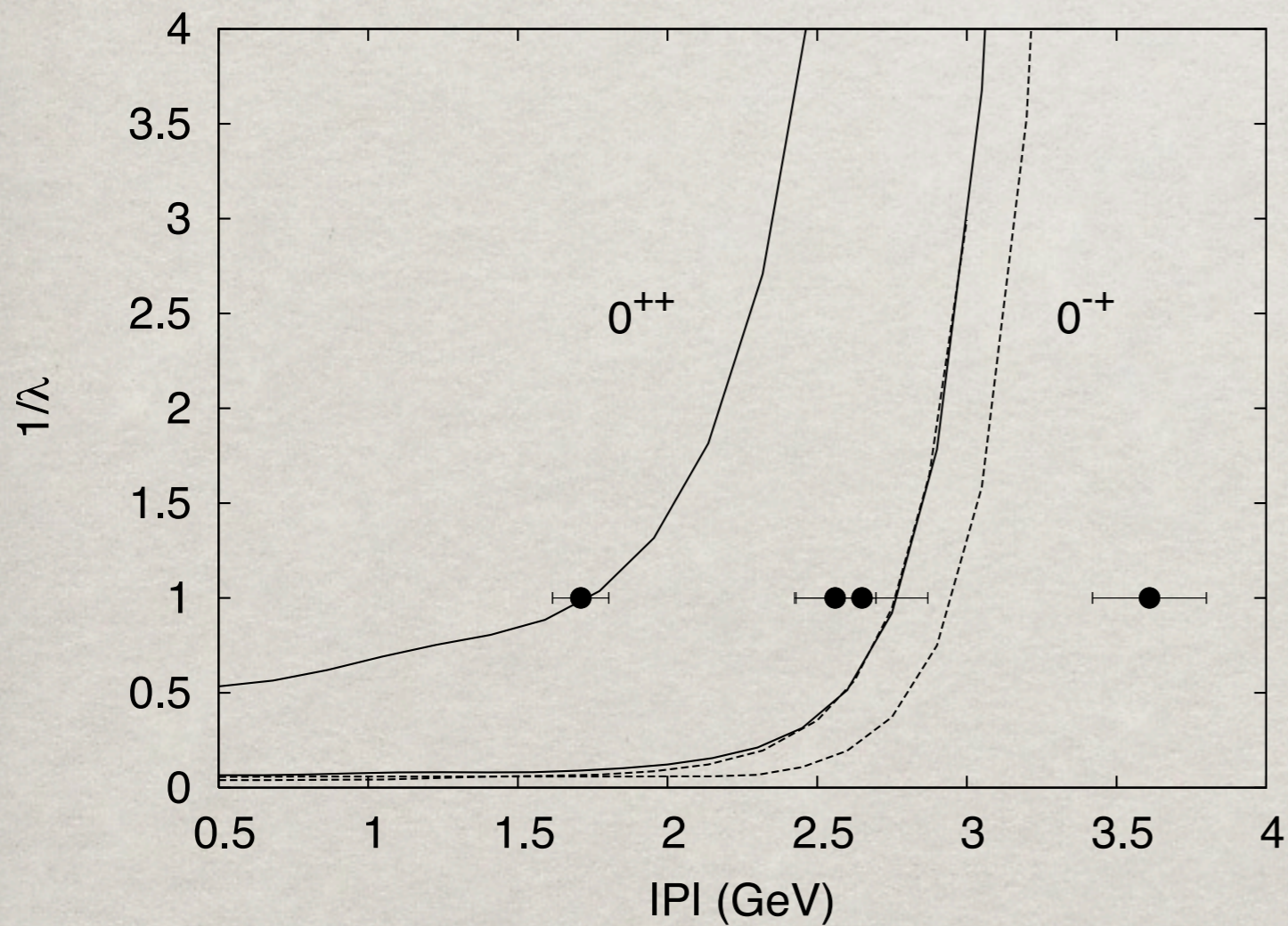
$$\chi_R^{\mu\nu}(k_+, k_-) = \epsilon(\mu, \nu) \cdot \Pi_R \cdot \chi_R^{\mu\nu}(\tilde{k}_+, \tilde{k}_-)$$

$$\chi^{\mu\nu}(k_+, k_-) = A_0 g^{\mu\nu} + A_1 k_+^\mu k_+^\nu + A_2 k_-^\mu k_-^\nu + A_3 k_+^\mu k_-^\nu + A_4 k_-^\mu k_+^\nu + A_5 \epsilon^{\mu\nu\alpha\beta} k_{+\alpha} k_{-\beta}$$

$$\chi^{\mu\nu}(k_+, k_-) = \epsilon^{\mu\nu\alpha\beta} k_\alpha P_\beta F(k, P) \quad [0^{-+}]$$

$$\chi^{\mu\nu}(k_+, k_-) = A(k, P) A^{\mu\nu} + B(k, P) B^{\mu\nu} \quad [0^{++}]$$

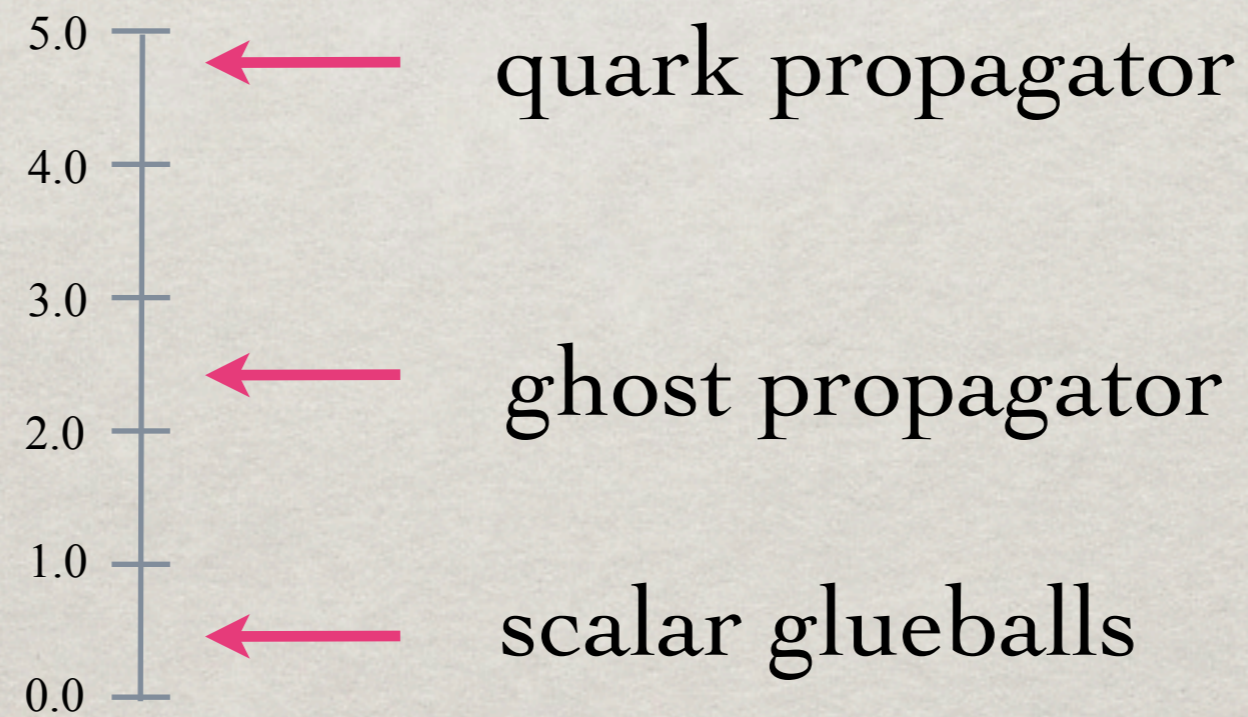
# BETHE-SALPETER EQUATIONS



# CONCLUSIONS

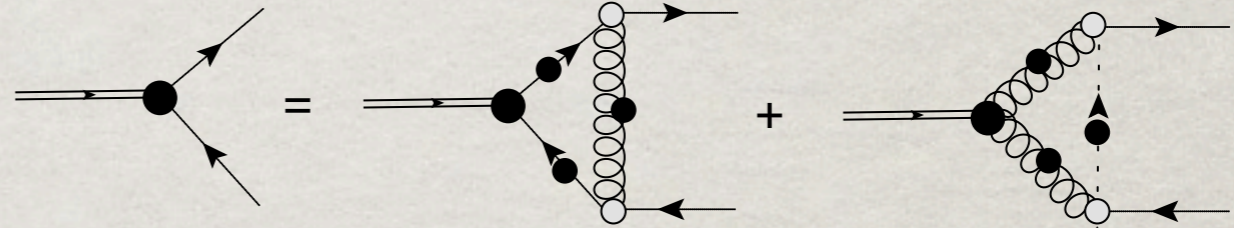
# CONCLUSIONS

$g(\mu = 1)$



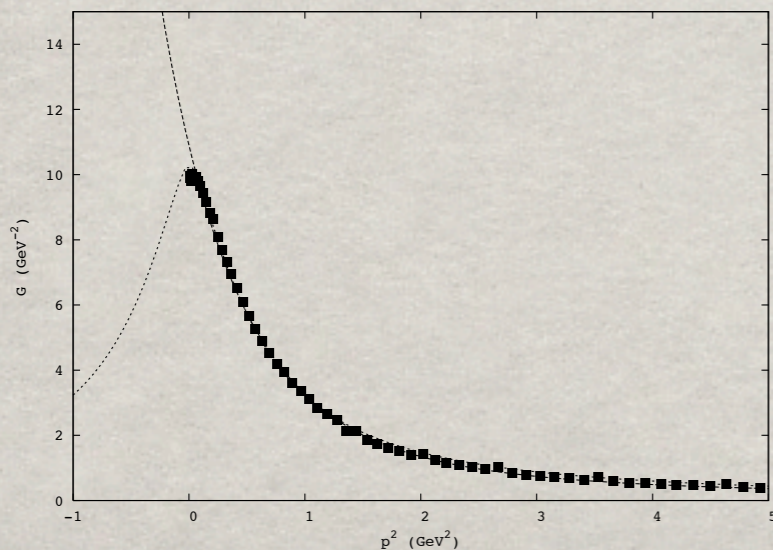
# CONCLUSIONS

## Wilson loop



the Bethe-Salpeter equation simplifies in the heavy quark limit

$$V \rightarrow -iG(-\vec{p}^2)$$

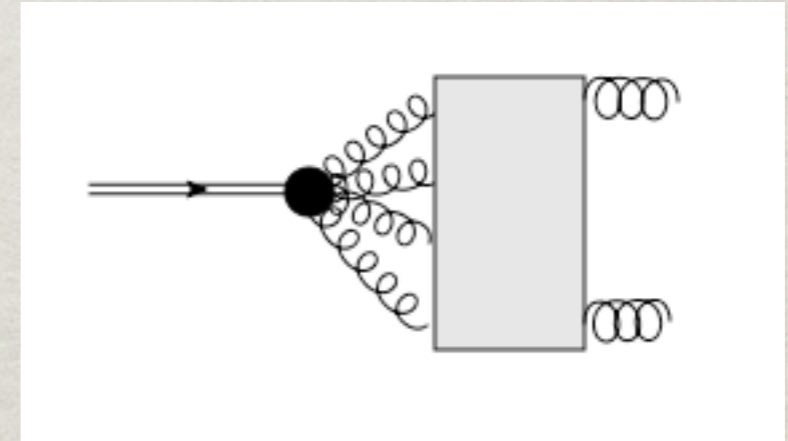


[screened Coulomb with log corrections]



# CONCLUSIONS

- kernel structure
- instantons
- reggeons
- flux tubes
- N-ality confinement
- pomeron



# CONCLUSIONS

- this methodology is in its infancy -- it will likely require sophisticated modelling
- the importance of topological vacuum effects and multigluon dynamics may be revealed.
- PANDA@GSI, JLab, BESIII, superB, LHCb provide hope for progress

# + ÆRIC MEC HEHT GEWYRCAN

