Hadronic resonances in Lattice QCD

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Excited QCD

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Outline

- ❑ Motivation
- ❑ mass and width of hadronic resonance from lattice
- ❑ Results from lattice simulation:
  - ρ resonance
  - charm resonances
  - strange resonances
- ❑ Conclusions

Almost all hadrons are hadronic resonances (decay strongly)

$\bar{u}u$	$\bar{s}u$	$\bar{c}u$	uud
$\pi^{\pm}$	$K^{\pm}$	$D^{\pm}$	p
π^{0}	K^{0}	D^{0}	n
η	K_S^{0}	$D^{*}(2007)^{0}$	$N(1440)\ 1/2^{+}$
$f_0(500)$ or σ was $f_0(600)$	K_L^{0}	$D^{*}(2010)^{\pm}$	$N(1520)\ 3/2^{-}$
$\rho(770)$	$K_0^{*}(800)$ or κ	$D_0^{*}(2400)^{0}$	$N(1535)\ 1/2^{-}$
$\omega(782)$	$K^{*}(892)$	$D_0^{*}(2400)^{\pm}$	$N(1650)\ 1/2^{-}$
$\eta'(958)$	$K_1(1270)$	$D_1(2420)^{0}$	$N(1675)\ 5/2^{-}$
$f_0(980)$	$K_1(1400)$	$D_1(2420)^{\pm}$	$N(1680)\ 5/2^{+}$
$a_0(980)$	$K^{*}(1410)$	$D_1(2430)^{0}$	$N(1685)\ ?^{?}$
$\phi(1020)$	$K_0^{*}(1430)$	$D_2^{*}(2460)^{0}$	$N(1700)\ 3/2^{-}$
$h_1(1170)$	$K_2^{*}(1430)$	$D_2^{*}(2460)^{\pm}$	$N(1710)\ 1/2^{+}$
$b_1(1235)$	$K(1460)$	$D(2550)^{0}$	$N(1720)\ 3/2^{+}$
$a_1(1260)$	$K_2(1580)$	$D(2600)$	$N(1860)\ 5/2^{+}$
$f_2(1270)$	$K(1630)$	$D^{*}(2640)^{\pm}$	$N(1875)\ 3/2^{-}$
$f_1(1285)$	$K_1(1650)$	$D(2750)$	$N(1880)\ 1/2^{+}$
$\eta(1295)$	$K^{*}(1680)$		$N(1895)\ 1/2^{-}$
$\pi(1300)$	$K_2(1770)$		$N(1900)\ 3/2^{+}$
$a_2(1320)$	$K_3^{*}(1780)$		$N(1990)\ 7/2^{+}$
$f_0(1370)$	$K_2(1820)$		$N(2000)\ 5/2^{+}$
$h_1(1380)$	$K(1830)$		$N(2040)\ 3/2^{+}$
$\pi_1(1400)$			$N(2060)\ 5/2^{-}$
$\eta(1405)$			$N(2100)\ 1/2^{+}$
$f_1(1420)$			
$\omega(1420)$			
$f_2(1430)$			
$a_0(1450)$			
$\rho(1450)$			

- ❑ stable on strong decay: ab-initio OK
- ❑ others decay strongly; hadronic resonances

up tp now ab intio (first simulation: CP-PACS 2007)

our coll. (Lang et al) (2011,2012)

Verduci, Lang (2012)

Hadronic resonances appear in scattering

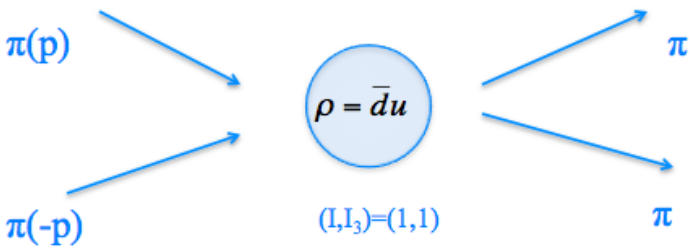
Decay quickly through strong int.

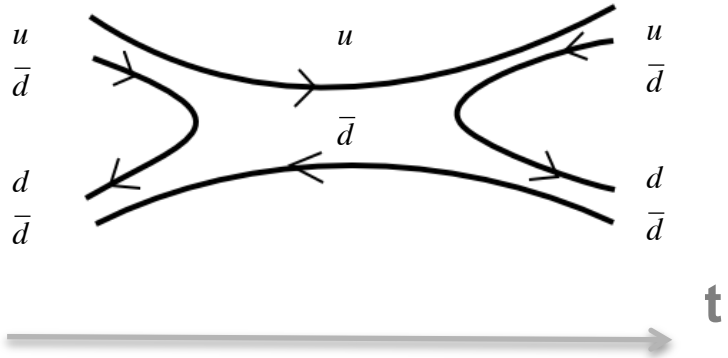
decay time: τ

uncertainty in E: $\Gamma = \hbar / \tau$

Γ=10−300 MeV

Example: ρ

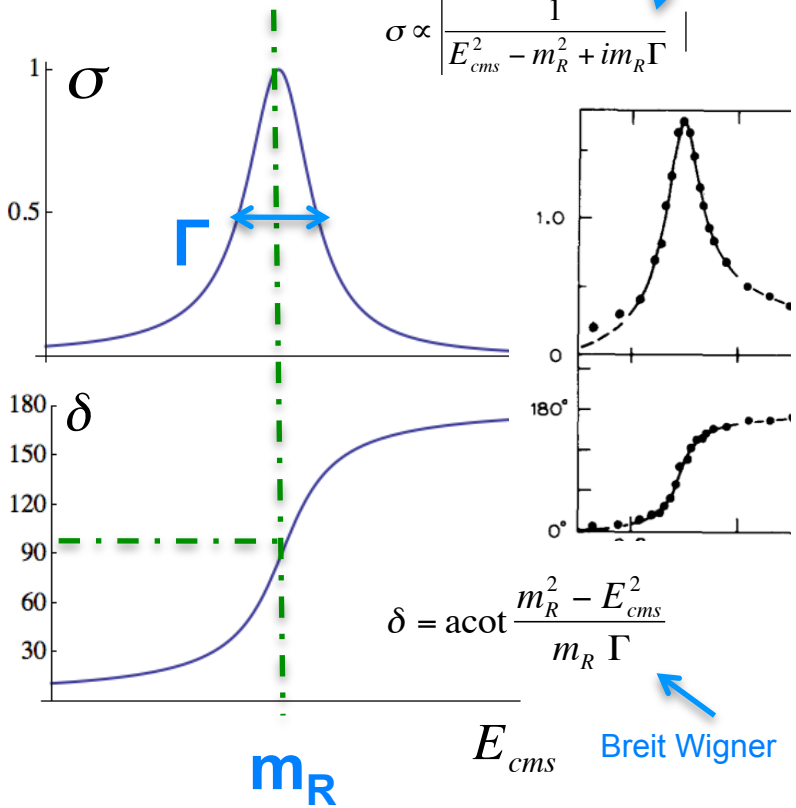




experiment measures

Breit Wigner

$$\sigma \propto \left| \frac{1}{E_{cms}^2 - m_R^2 + i m_R \Gamma} \right|^2$$

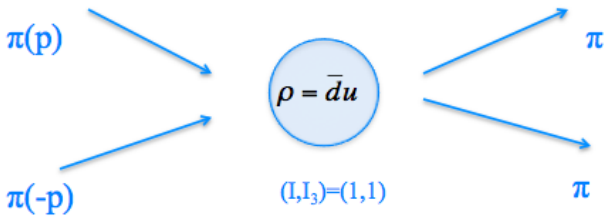


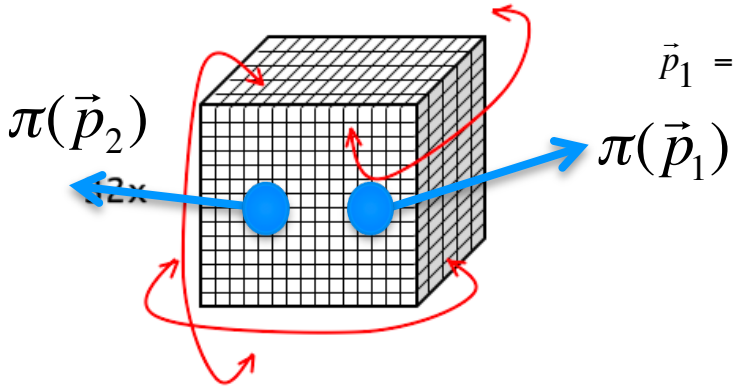
$$\delta = \mathrm{acot} \frac{m_R^2 - E_{cms}^2}{m_R \, \Gamma}$$

Breit Wigner

Our goal: determine m and Γ from lattice QCD !

Resonances in scattering on the lattice





$$\vec{p}_1 = \vec{n}_1 \frac{2\pi}{L}$$

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^{+}(0) | 0 \rangle$$

$$\mathcal{O} = \pi(\vec{p}_1)\pi(\vec{p}_2) = [\bar{u}\gamma_5 d]\ [\bar{d}\gamma_5 d]$$

$$\rho \qquad = \bar{u}\gamma_i d$$

- numerically demanding to evaluate such C(t)
- therefore mostly unexplored ab-initio till now
- our advantage: so-called "distillation method" (Peardon et al. 2009)

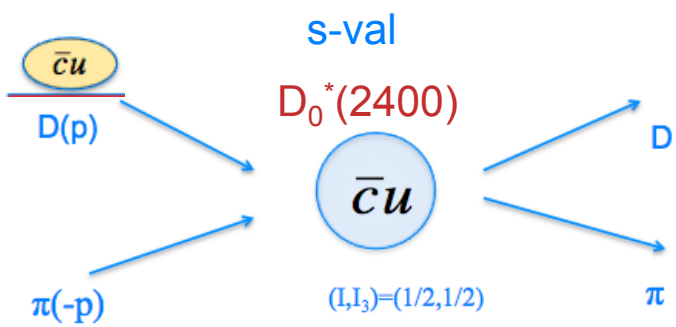
$$C_{ij}(t) = \sum_n A_n^{ij}\, e^{-E_n t}$$

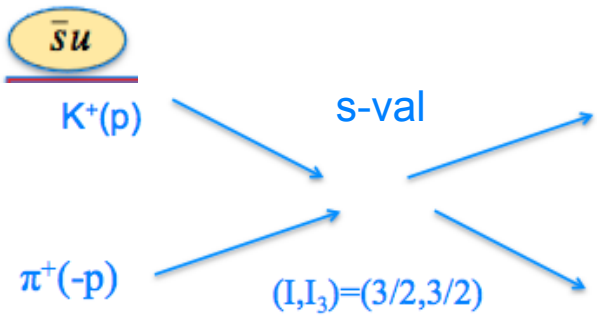
$$E(L) = \sqrt{m_1^2 + \vec{p}_1^{\,2}} + \sqrt{m_2^2 + \vec{p}_2^{\,2}} + \Delta E$$

due to strong int.

$$\vec{p}_1 = \tfrac{2\pi}{L}\vec{n}_1 \qquad \vec{p}_2 = \tfrac{2\pi}{L}\vec{n}_2$$

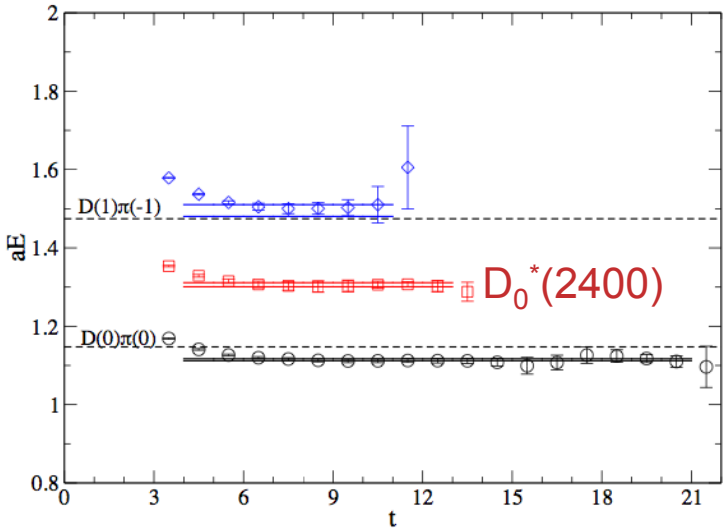
Examples of energy spectra

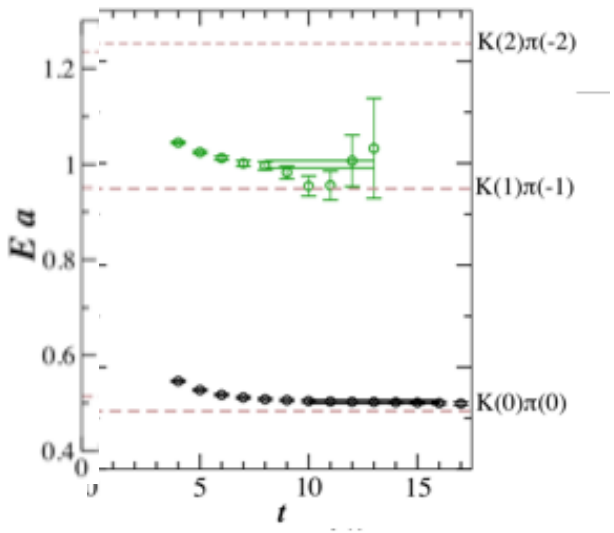




resonant channel

non-resonant channel



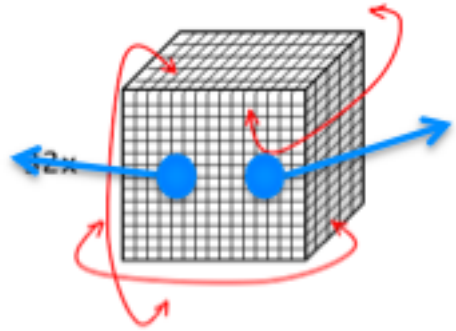


$$E(L) = \sqrt{m_1^2 + \vec{p}_1^{\,2}} + \sqrt{m_2^2 + \vec{p}_2^{\,2}} + \Delta E$$

due to strong int.

$$a = 0.124\ fm = \frac{1}{1.59\ GeV} \qquad \vec{p}_1 = \tfrac{2\pi}{L}\vec{n}_1 \qquad \vec{p}_2 = \tfrac{2\pi}{L}\vec{n}_2$$

Energy shift ΔE renders δ(E)

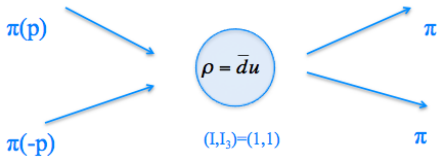


$$E(L) = \sqrt{m_1^2 + \vec{p}_1^{\,2}} + \sqrt{m_2^2 + \vec{p}_2^{\,2}} + \Delta E$$

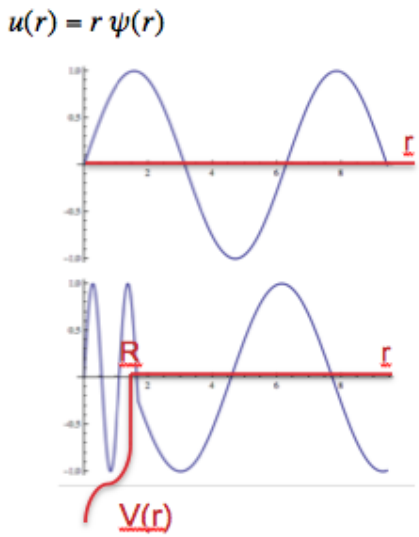
E(L) ➔ δ (E) [Luscher 1986] , for p1+p2=0

E(L) ➔ δ (E) [Leskovec, S.P., PRD 2012] , for p1+p2≠0, m1≠m2

What is δ(E) ?



$$\sigma \propto \frac{\sin^2\delta}{p_{cms}^2}$$



primer : $l = 0$

rdece : $V(r > R) = 0$

$$\psi(r) \propto \frac{\sin(kr)}{r}$$

$$\psi(r) \propto \frac{\sin(kr+\delta)}{r} \qquad r > R$$

neznan $\qquad r < R$

Lattice simulation

- 280 gauge config with dynamical u,d quarks (generated by A. Hasenfratz)

  thanks !!

$$N_f = 2 \quad a = 0.1239 \pm 0.0013\ fm \quad a^{-1} = 1.58 \pm 0.02\ GeV$$

$$N_L^3 \times N_T = 16^3 \times 32 \qquad L \approx 2\ fm \qquad T = 4\ fm \qquad m_\pi \approx 266\ MeV$$

- dynamical u, d , valence u,d,s : Improved Wilson Clover

  valence c: Fermilab method [El-Khadra et al. 1997]

  a set using r0

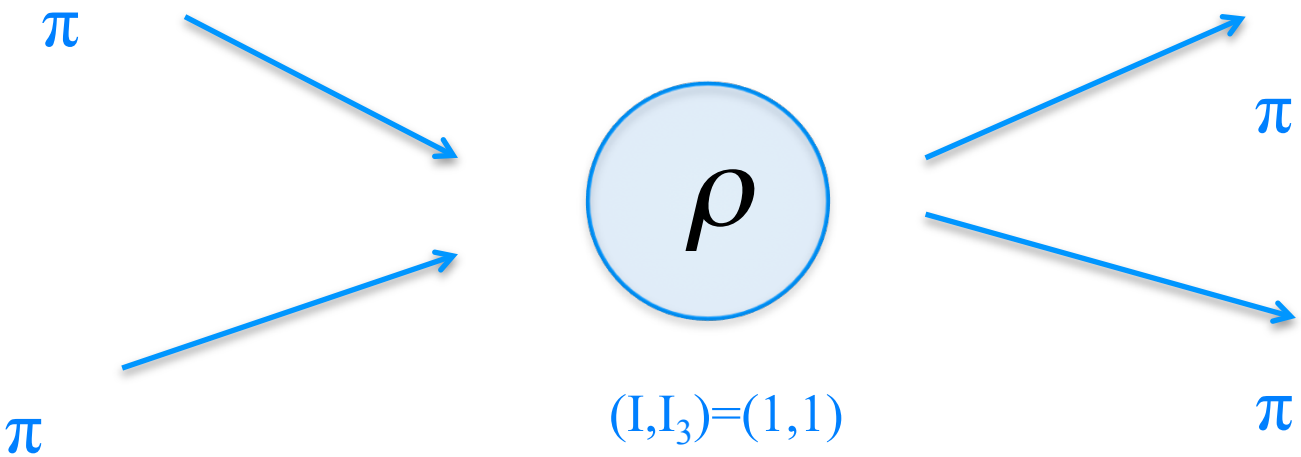
  m_s set using ϕ

  m_c set using $\frac{1}{4}[M_2(\eta_c) + 3M_2(J/\psi)]_{lat} = \frac{1}{4}[M(\eta_c) + 3M(J/\psi)]_{\exp}$

- heavy quark treatment tested on

  <u>charmonium</u> with satisfactory results

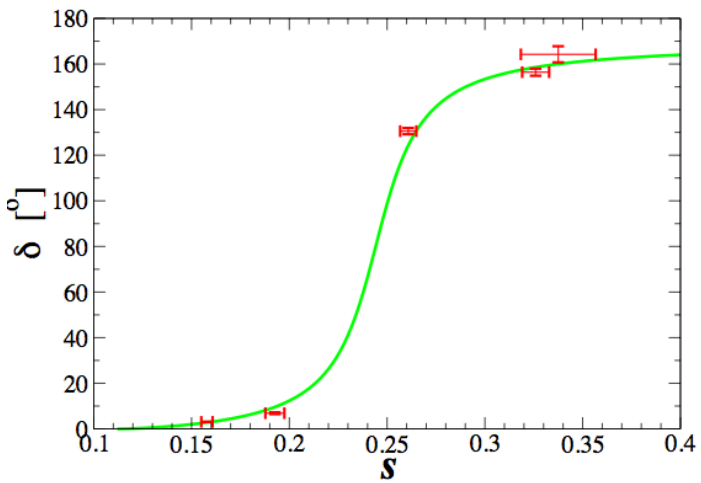
π π scattering



ρ resonance @ m_π~266 MeV

[Lang, Mohler, S. P., Vidmar, PRD 2011

π π scattering with L=1 (p-wave) and isospin I=1



$$s = E_{cms}^2 = E^2 - P^2$$

$$a = \frac{-\sqrt{s}\,\Gamma(s)}{s - m^2 + i\sqrt{s}\,\Gamma(s)} = \frac{1}{2i}\left(e^{2i\delta} - 1\right)$$

equivalent to real equation

$$\sqrt{s}\,\Gamma(s)\cot\delta(s) = m_\rho^2 - s$$

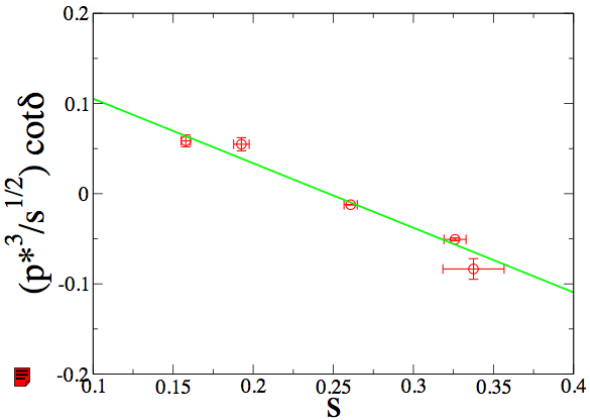
$$\Gamma(s) = \frac{p^3}{s}\frac{g_{\rho\pi\pi}^2}{6\pi}$$

The final relation has two parameters: m(ρ) and g(ρππ)

$$L_{\text{eff}} = g_{\rho\pi\pi}\sum_{abc}\epsilon_{abc}(k_1 - k_2)_\mu\ \rho_\mu^a(p)\pi^b(k_1)\pi^c(k_2)$$

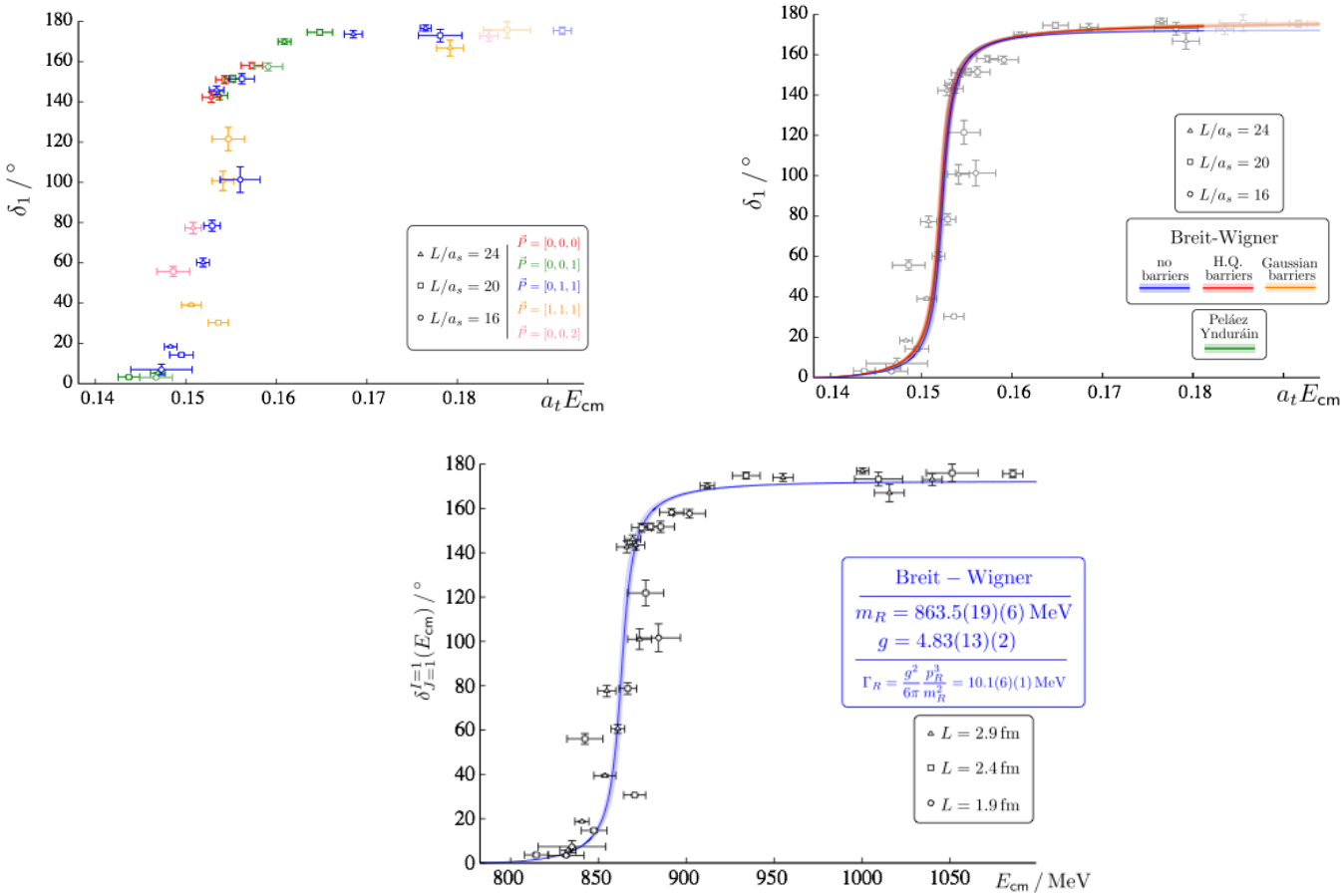
our final result

lat ($m_\pi = 266\,MeV$)	exp ($m_\pi = 140\,MeV$)
$m_\rho \approx 792 \pm 12$ MeV	$m_\rho = 775$ MeV
$g_{\rho\pi\pi} = 5.13 \pm 0.20$	$g_{\rho\pi\pi} = 5.97$



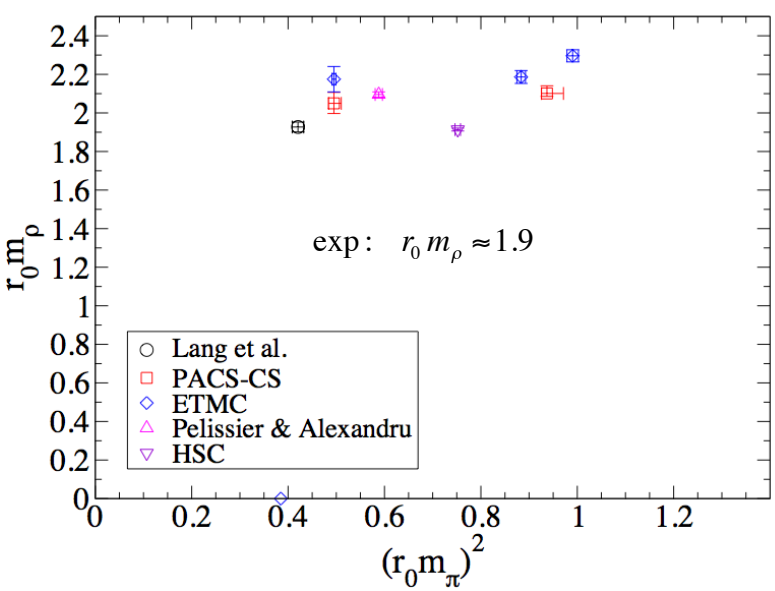
[HSC: Dudek, Edwards, Thomas, 2013]

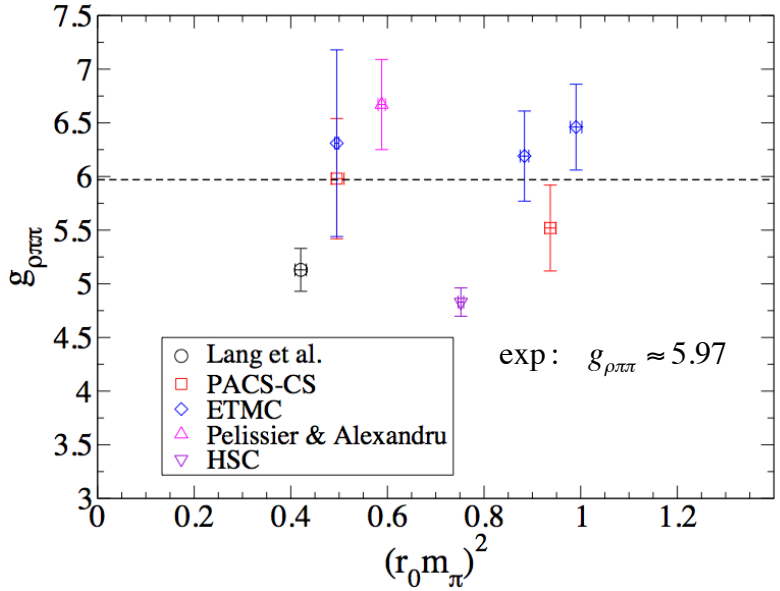
ρ resonance @ m_π~400 MeV



[thanks to D. Mohler for the plot]

Compilation of rho simulations





Why the following results have less points on phase-shift curve ?

previous result : $\delta(s)$ at relatively many s

$\pi\pi \rightarrow \rho \rightarrow \pi\pi$

$s = E_{cms}^2 = E^2 - P^2$ several $P = p_1 + p_2$ simulated

the following results : $\delta(s)$ at few s

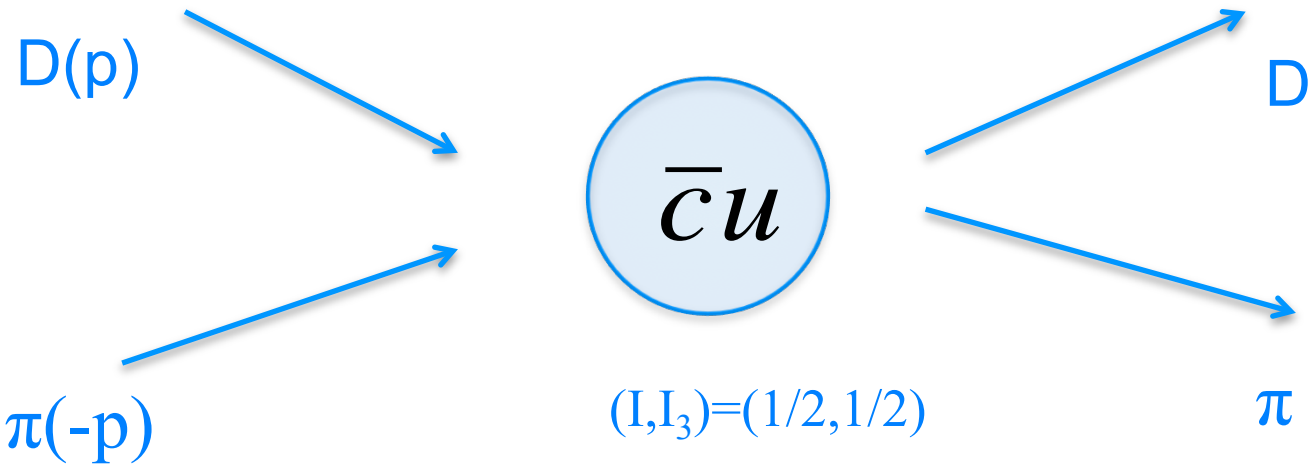
$K\pi \rightarrow K^* \rightarrow K\pi$

$D\pi \rightarrow D^* \rightarrow D\pi$

$s = E_{cms}^2 = E^2$ only $P = 0$ simulated

$m_1 \neq m_2$ & $P \neq 0$: even and odd L mix [Leskovec, S.P., PRD 2012]

Dπ and D*π scattering & charm-light resonances



D-meson resonances: brief introduction

- only 1S and 1P $\bar{c}u$ states well established in exp

  for $m_c=\infty$ [Isgur & Wise, 1991] :

  - two 1P states decay only in S-wave ➔ broad

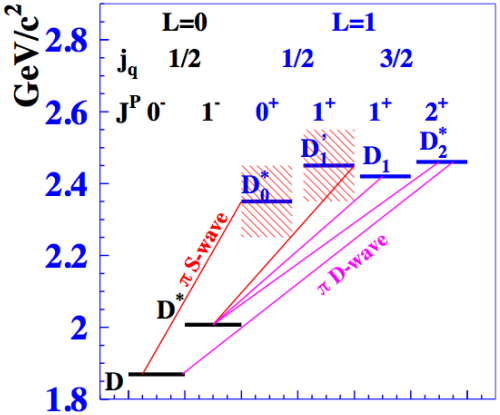
    we treat those two as resonances

  - two 1P states decay only in D-wave ➔ narrow in exp

    ➔ "stable" on our lat

    below D-wave th. D(1)π(-1)

    we treat those as stable: M=E(L)

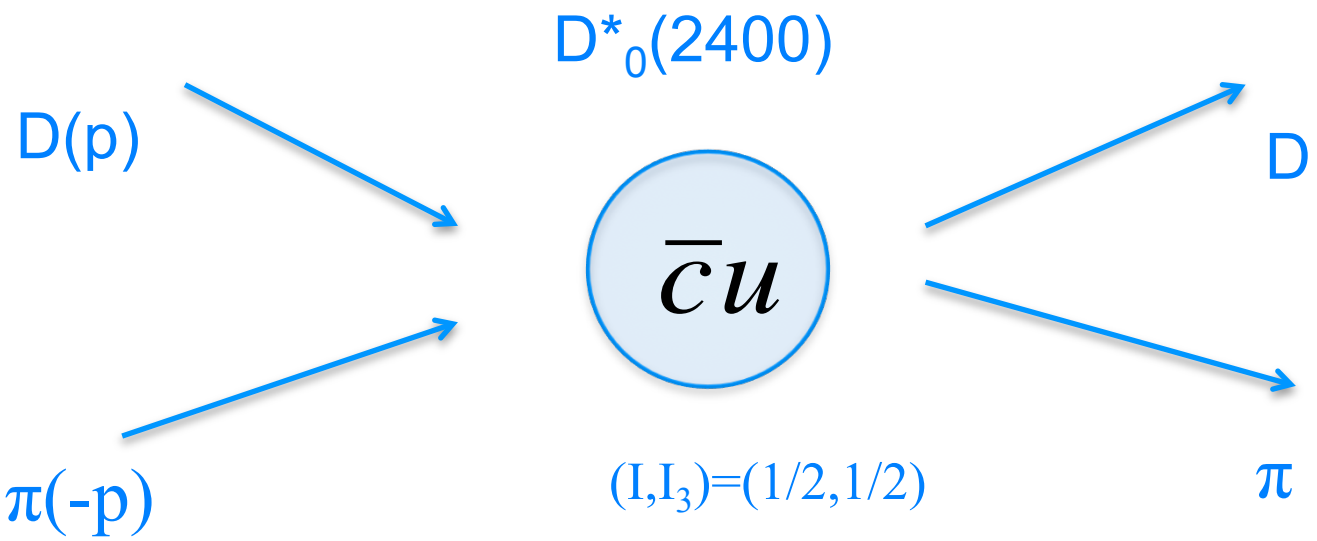


taken from
Belle PRD(2004)

- radial and orbital excitations [Babar 2010]:

  poorly known in exp (need confirmation), O=quark-antiquark

Dπ scattering



Dπ scattering : I=1/2, s-wave, $J^P=0^+$

$D^*_0(2400)$ $\exp: M \approx 2318\ MeV \quad \Gamma \approx 267\ MeV \quad \bar{c}u$?

$\bar{c}\bar{s}su \pm \bar{c}\bar{d}du$?

$D_{s0}(2317)$ $\exp: M \approx 2318\ MeV \quad \Gamma \approx 0\ MeV \quad \bar{c}s$?

$\bar{c}\bar{u}us \pm \bar{c}\bar{d}ds$?

degeneracy between non-strange and strange partners
not naively expected for conventional quark-antiquark

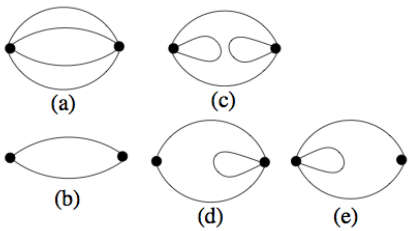
interesting to see if lattice QCD reproduces correct masses and widths of these two states

[Mohler, S. P., Woloshyn, arXiv:1208.4059]

Dπ scattering: I=1/2, s-wave, $J^P=0^+$

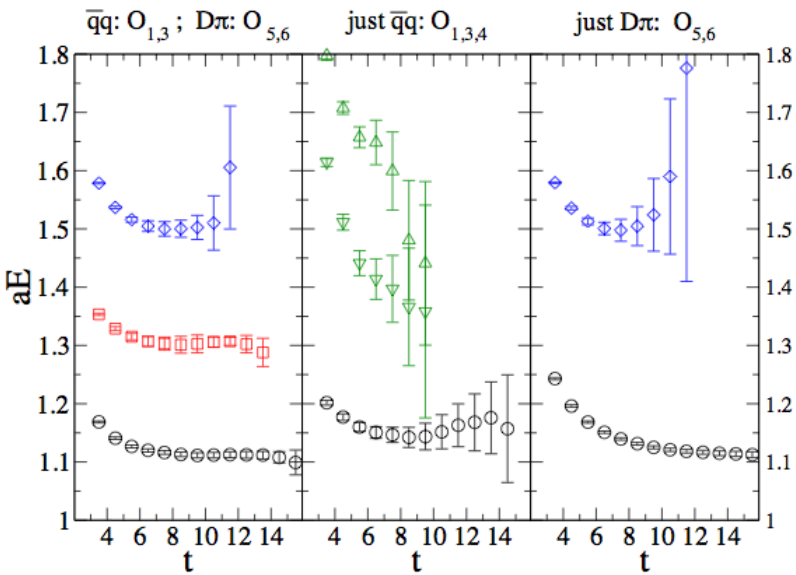
exp:
$D_0^*(2400)$

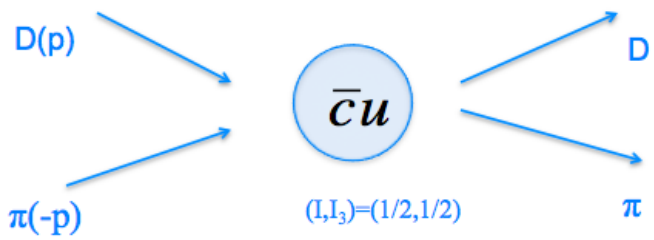
interpolators : 4 quark-antiquark, 2 meson-meson



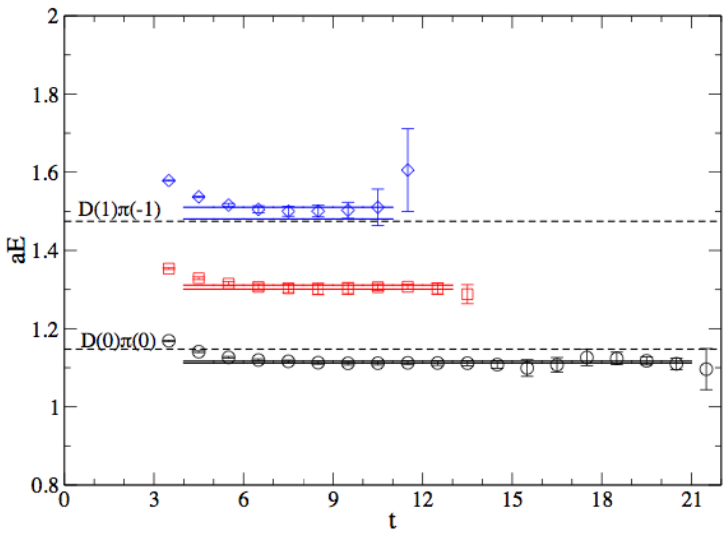
$$\bar{q}q$$
$$\bar{q}\gamma_i\overrightarrow{\nabla}_i q$$
$$\bar{q}\gamma_t\gamma_i\overrightarrow{\nabla}_i q$$
$$\bar{q}\overleftarrow{\nabla}_i\overrightarrow{\nabla}_i q$$

$$\sqrt{\tfrac{2}{3}}D^-(0)\pi^+(0)+\sqrt{\tfrac{1}{3}}\bar{D}^0(0)\pi^0(0)\,,$$
$$\sum_i \sqrt{\tfrac{2}{3}}D^-(\mathbf{e}_i)\pi^+(-\mathbf{e}_i)+\sqrt{\tfrac{1}{3}}\bar{D}^0(\mathbf{e}_i)\pi^0(-\mathbf{e}_i)$$





Dπ scattering: resulting levels and phase shifts (P=pD+pπ=0)



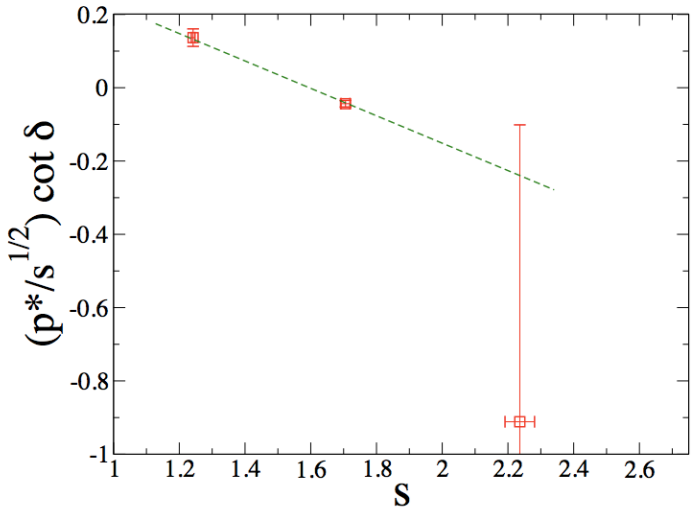
δ~173±12^0

δ~103^0 → indication for reson.

δ~ 41^0 i → $a_{D\pi}^{I=1/2} = \lim_{p\to 0} \frac{\tan\delta(p)}{p} = 0.81 \pm 0.14\ fm$

[Mohler, S. P., Woloshyn, arXiv:1208.4059]

Dπ scattering: extracting resonance parameters for $D_0(2400)$



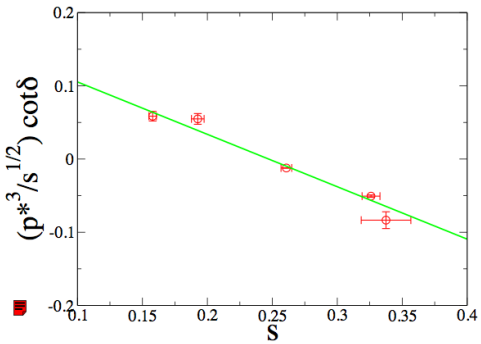
$$s = E^2$$

$$a = \frac{-\sqrt{s}\,\Gamma(s)}{s - m^2 + i\sqrt{s}\,\Gamma(s)} = \frac{1}{2i}\left(e^{2i\delta} - 1\right)$$

$$\sqrt{s}\,\Gamma(s)\cot\delta(s) = m^2 - s, \qquad \Gamma(s) = \frac{p}{s}g^2$$

$$\frac{p}{\sqrt{s}}\cot\delta = \frac{1}{g^2}(m^2 - s)$$

For comparison, our result for rho: there one can check linear behavior.



	m - 1/4(mD+3 mD*)	g
lat	351 ± 21 MeV	2.55 ± 0.21 GeV
exp	347 ± 29 MeV	1.92 ± 0.14 GeV

it would be great to have δ at more values of s, but I will speak about challenges concerning this at the end of my talk

[Mohler, S. P., Woloshyn, arXiv:1208.4059]

Dπ scattering : I=1/2, s-wave, $J^P=0^+$

$D^*_0(2400)$ $\exp: M \approx 2318\ MeV \quad \Gamma \approx 267\ MeV \quad \bar{c}u$?

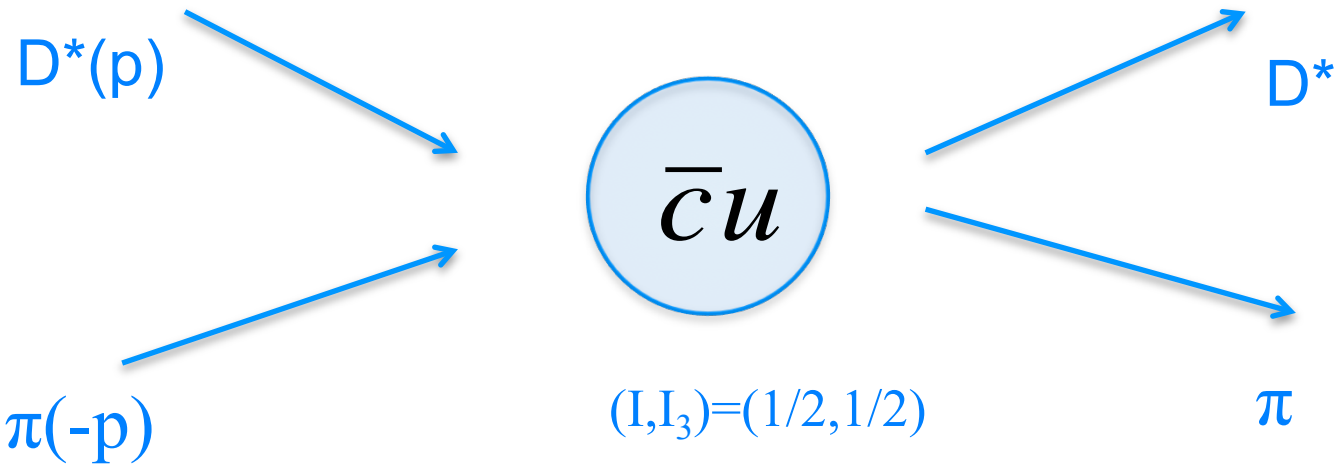
$\bar{c}\bar{s}su \pm \bar{c}\bar{d}du$?

$D_{s0}(2317)$ $\exp: M \approx 2318\ MeV \quad \Gamma \approx 0\ MeV \quad \bar{c}s$?

$\bar{c}\bar{u}us \pm \bar{c}\bar{d}ds$?

Our resulting D0*(2400) mass is in favorable agreement with exp without valence $\bar{s}s$ pair.

D*π scattering



[Mohler, S. P., Woloshyn, arXiv:1208.4059]

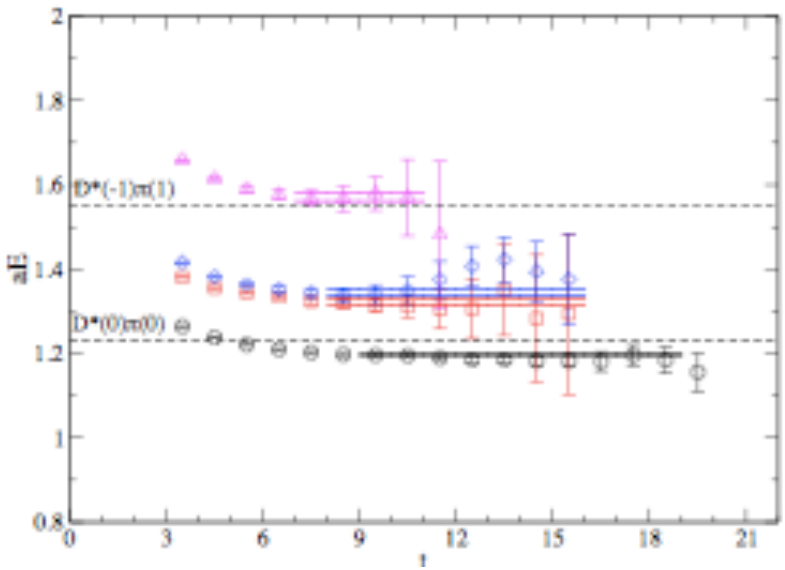
D*π scattering: I=1/2, s-wave, $J^P=1^{++}$

exp:
$D_1(2430)$ broad
$D_1(2420)$ narrow

interpolators : 8 quark-antiquark, 2 meson-meson

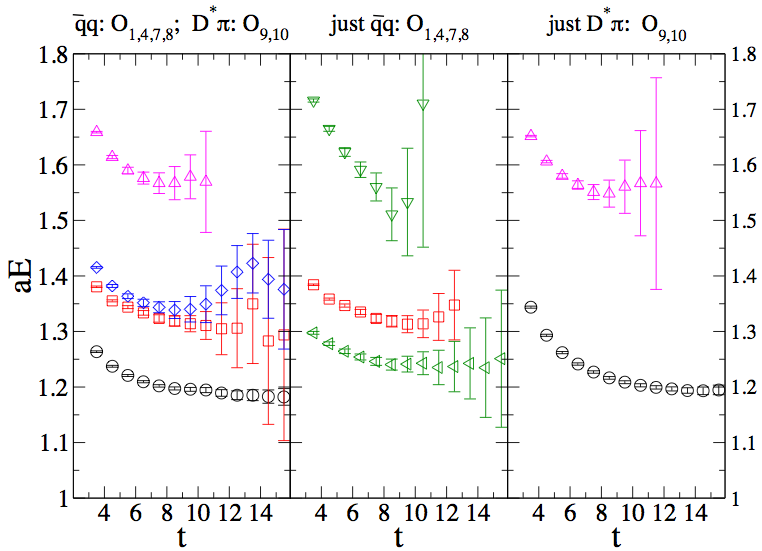
$$\bar{q}\gamma_i\gamma_5 q$$
$$\bar{q}\epsilon_{ijk}\gamma_j\overrightarrow{\nabla}_k q$$
$$\bar{q}\epsilon_{ijk}\gamma_t\gamma_j\overrightarrow{\nabla}_k q$$
$$\bar{q}\overleftarrow{\nabla}_i\gamma_i\gamma_5\overrightarrow{\nabla}_i q$$
$$\bar{q}\overleftarrow{\Delta}\gamma_i\gamma_5\overrightarrow{\Delta} q$$
$$\bar{q}\overleftarrow{\Delta}\epsilon_{ijk}\gamma_j\overrightarrow{\nabla}_k q$$
$$\bar{q}\overleftarrow{\Delta}\epsilon_{ijk}\gamma_t\gamma_j\overrightarrow{\nabla}_k q$$
$$\bar{q}|\epsilon_{ijk}|\gamma_5\gamma_j\overrightarrow{D}_k q$$

$$\mathcal{O}_9 = \sqrt{\tfrac{2}{3}}D^{*-}(0)\pi^+(0) + \sqrt{\tfrac{1}{3}}\bar{D}^{*0}(0)\pi^0(0)\,,$$
$$\mathcal{O}_{10} = \sum_i \sqrt{\tfrac{2}{3}}D^{-*}(\mathbf{e}_i)\pi^+(-\mathbf{e}_i) + \sqrt{\tfrac{1}{3}}\bar{D}^{0*}(\mathbf{e}_i)\pi^0(-\mathbf{e}_i)$$



$$a^{I=1/2}_{D^*\pi} = \lim_{p\to 0}\frac{\tan\delta(p)}{p} = 0.81 \pm 0.17\ fm$$

analysis/approximation inspired by $m_c=\infty$ limit



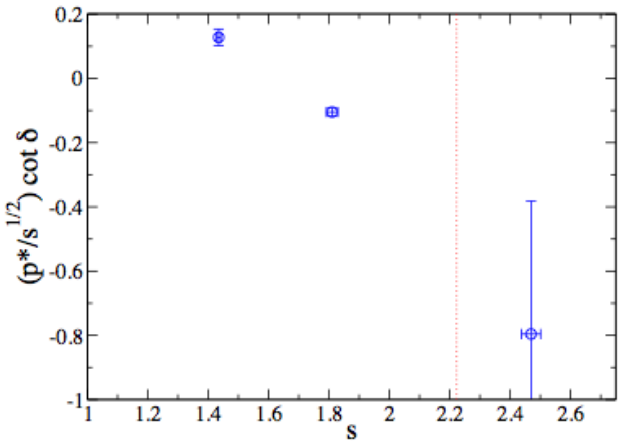
[Isgur & Wise, 1991]

- Blue expected to decay only in S-wave since present only when D(0)pi(0) in the basis.
- Then red expected to decay only in D-wave: stable on our lattice (in HQ limit) as below D-wave threshold

We assume that narrow red state does not affect phase shift of other three levels: BW fit through those three:

blue level: broad $D_1(2430)$

red level: "stable" $D_1(2420)$



$$\Gamma(s) = \frac{p}{s} g^2 \qquad \frac{p}{\sqrt{s}} \cot\delta = \frac{1}{g^2}(m^2 - s)$$

results for $D_1(2430)$

	m - 1/4(mD+3 mD*)	g
lat	381 ± 20 MeV	2.01 ± 0.15 GeV
exp	456 ± 40 MeV	2.50 ± 0.40 GeV

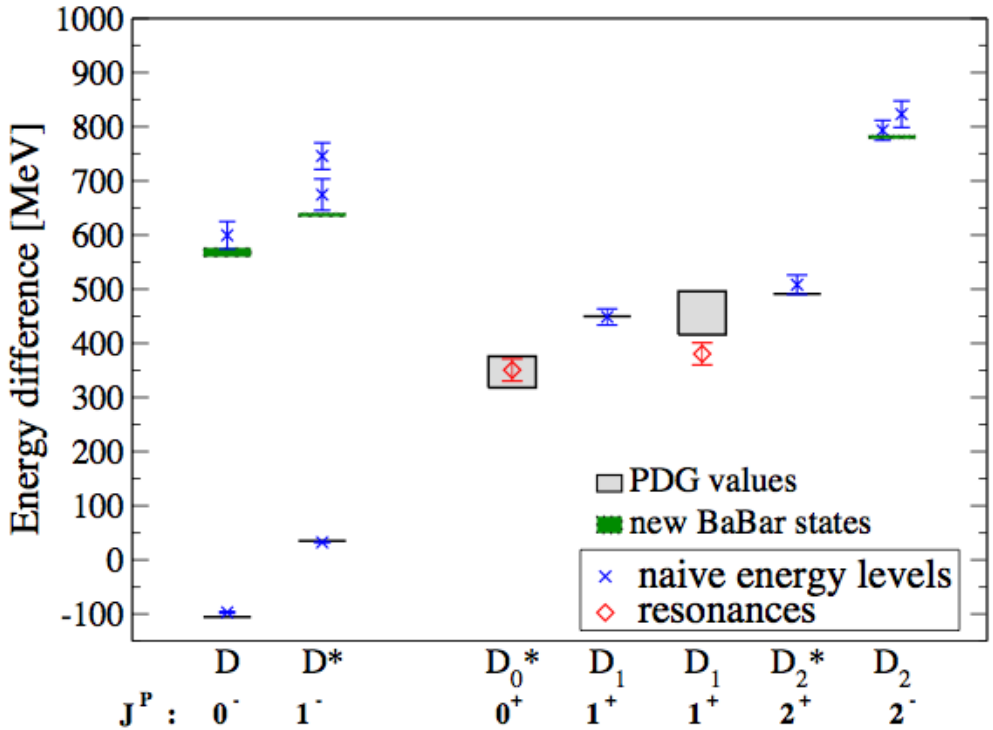
[Mohler, S. P., Woloshyn, arXiv:1208.4059, PRD 2013]

resulting D-meson spectrum

energy difference ≡

m - $\frac{1}{4}[m(D) + 3m(D^*)]$ =

exp : $\frac{1}{4}[m(D) + 3m(D^*)] = 1.97\ GeV$

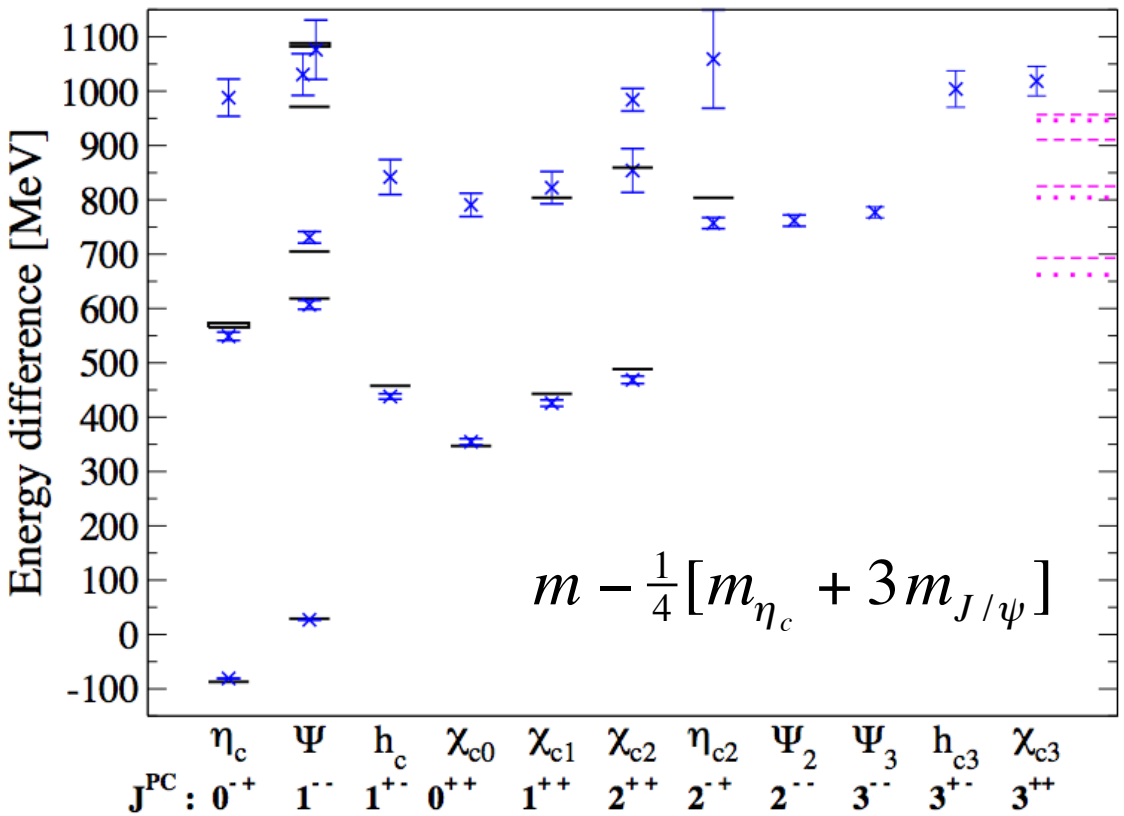


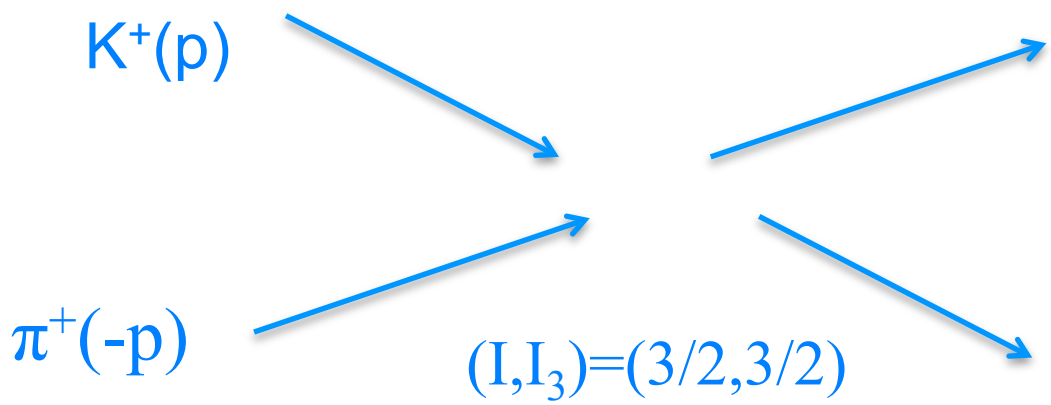
red diamonds: our lat results for resonance masses from scattering study
blue crosses: our lattice results for other resonances: m=E(L), O= qbar q

[Mohler, S. P., Woloshyn, arXiv:1208.4059, PRD 2013]

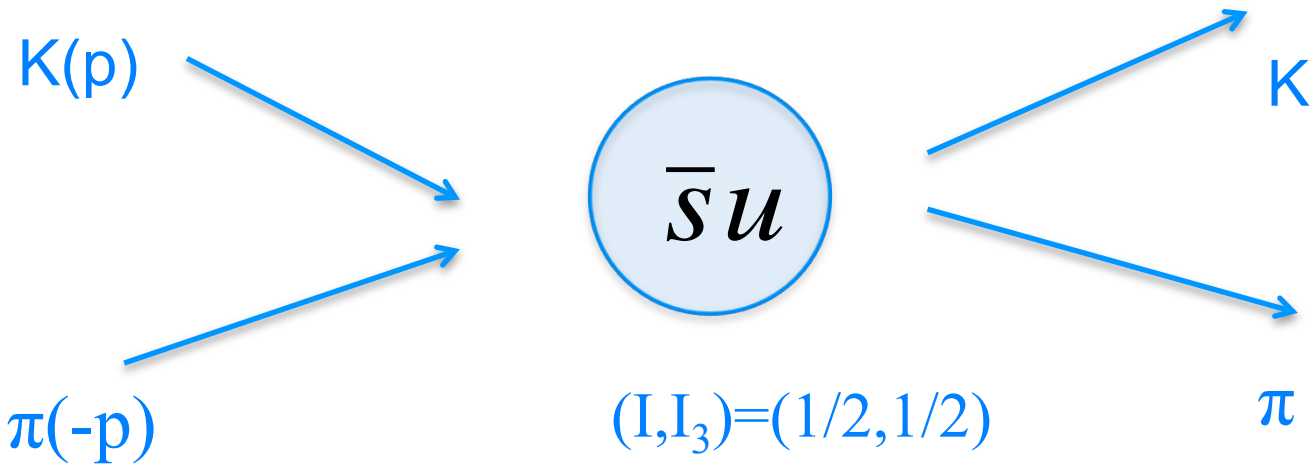
For those interested in charmonium

(widths not determined in this case)



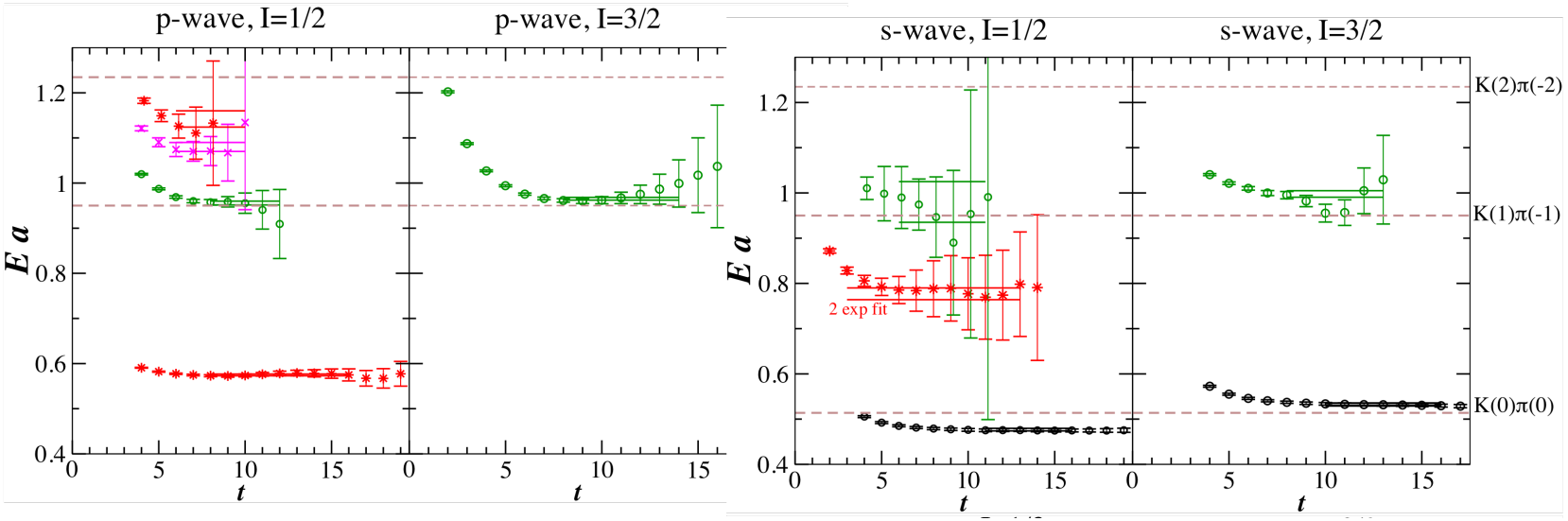


Kπ scattering & strange resonances



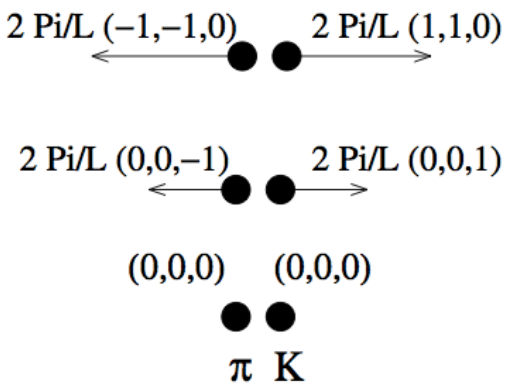
[Lang, Leskovec, Mohler, S. P., arXiv:1207.3204, PRD 2012]

K π : energy levels below inelastic threshold



K*(1680)
K*(1410)
K*(892)

K_0*(1430)
no level near κ: discussed later on



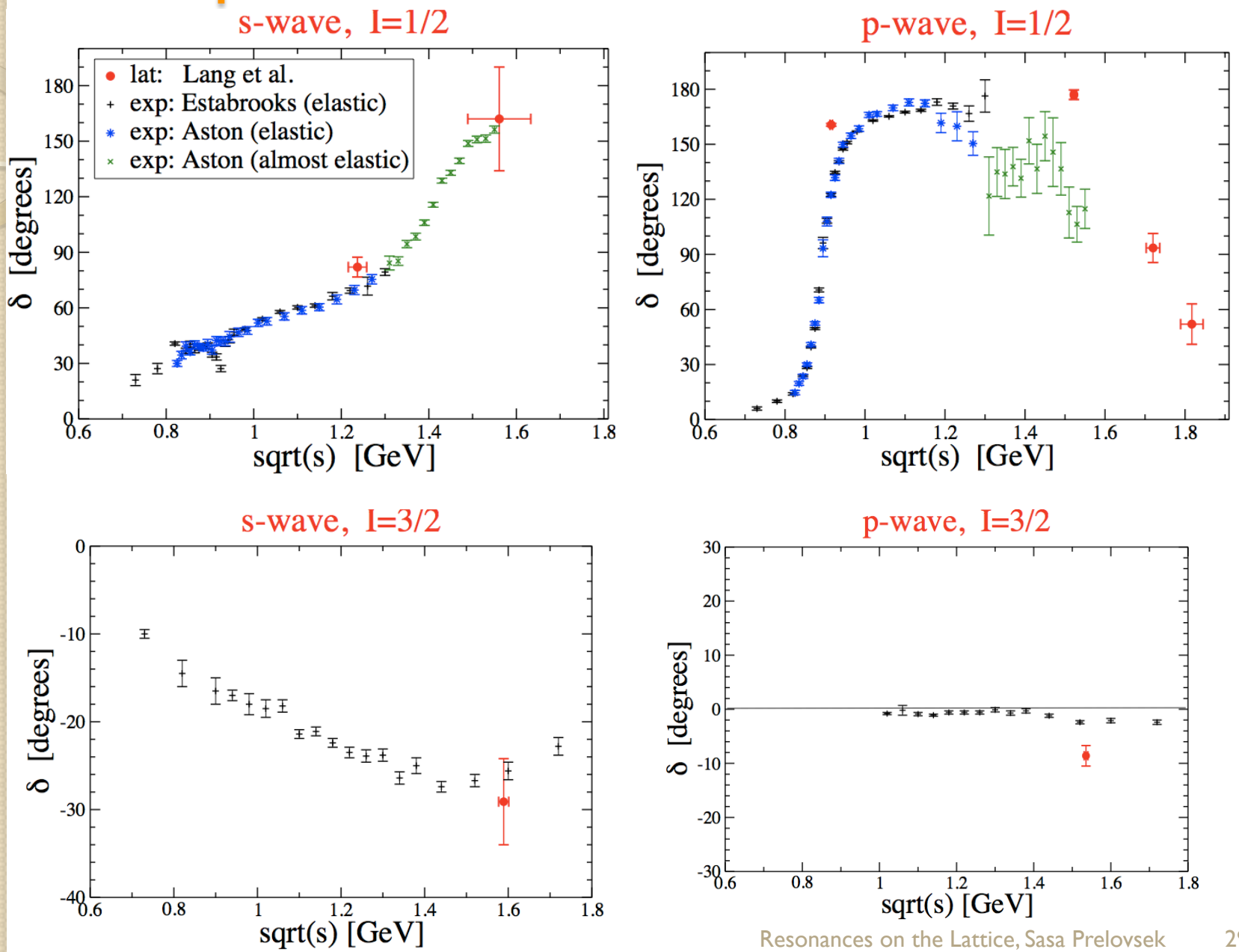
1/a ~ 1.6 GeV

K π phase shifts

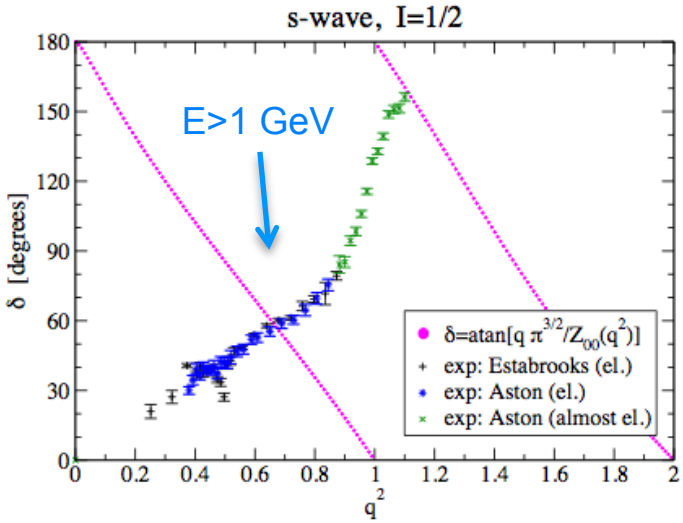
[Lang, Leskovec, Mohler, S. P., arXiv:1207.3204, PRD 2012]

$m_\pi \approx 266\ MeV \quad m_K \approx 552\ MeV$

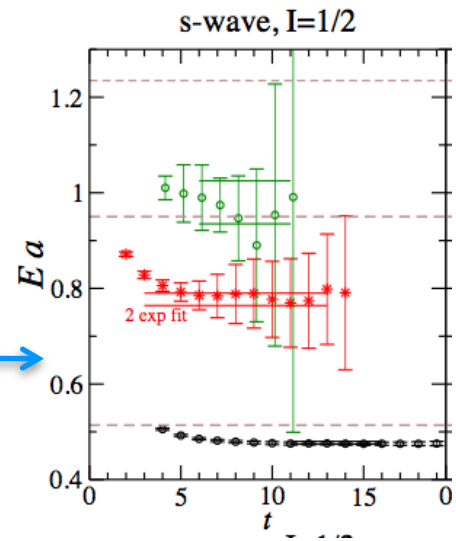
$\sqrt{s} = \sqrt{E^2 + P^2} = M_{K\pi}$

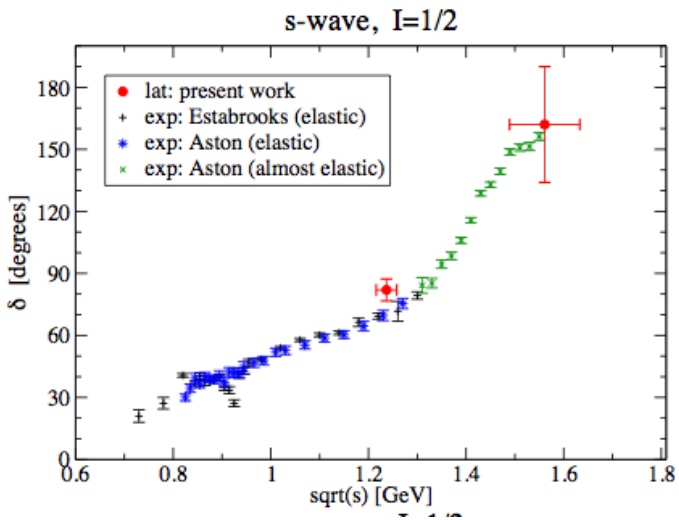


cautionary remarks on $K_0^*(800)$ or κ



1 GeV





- we do not see any other level below 1 GeV except for K(0)pi(0)
- so we do not see additional level related to kappa
- this is expected for our lattice L~2 fm assuming experimental δ, since experimental δ does not reach 90^0 below 1 GeV
- conclusion: we qualitatively agree with experimental phase shift but we can not conclude whether kappa pole exists or not

$$a_0 \equiv \lim_{p \to 0} \frac{\tan \delta(p)}{p}$$

s-wave scattering lengths a_0 for

Kπ , Dπ , D*π

[Lang, Leskovec, Mohler, S. P., arXiv:1207.3204, PRD 86]

[Mohler, S. P., Woloshyn, arXiv:1208.4059]

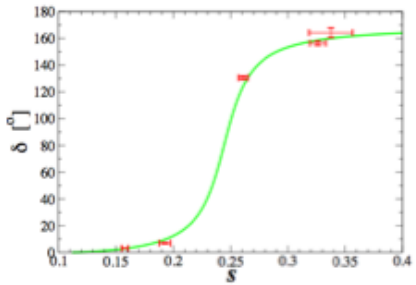
Conclusions

- We simulated QCD based on fundamental theory (L_{QCD}) : lattice QCD

- **stable hadrons:**
  well explored, good agreement with exp.

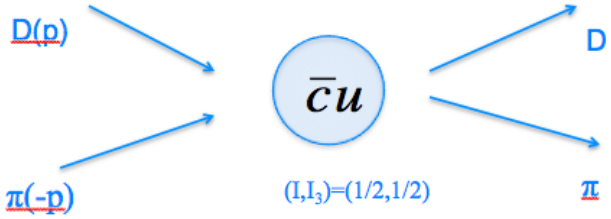
- **hadronic resonances** (most of hadrons)
  almost unexplored
  we presented pioneering, exploratory results

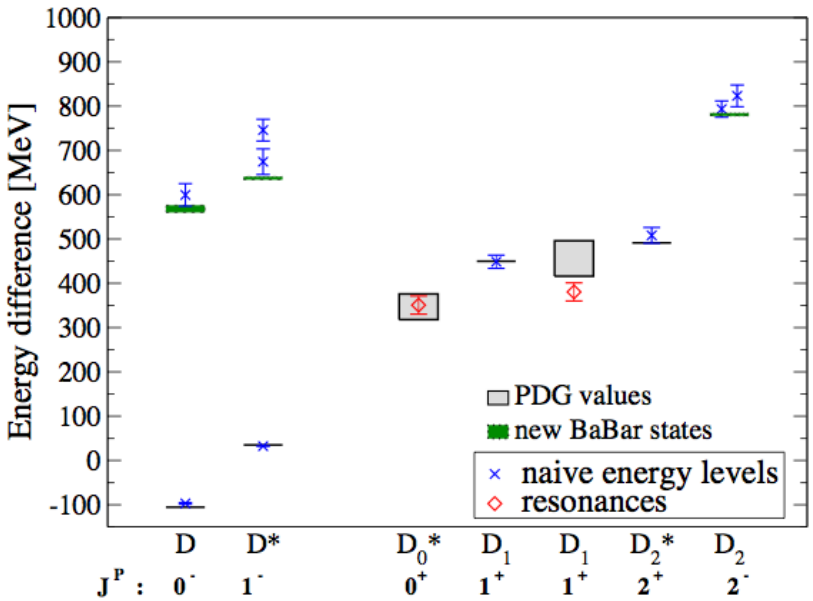
(1) <u>π π scattering and ρ resonance</u> (only one simulated up to now)



Conclusions (continued)

(1) Dπ and D*π scattering and D-meson resonances





$$\Gamma(s) = \frac{p}{s} g^2$$

$D_0^*(2400)$ resonance
$J^P=0^+$

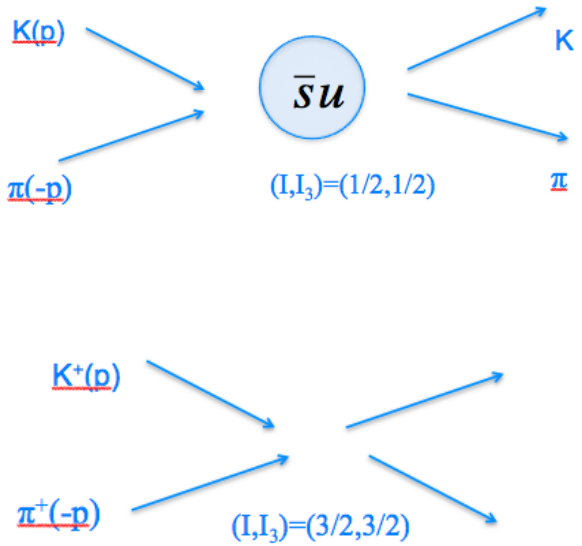
	m - 1/4(mD+3 mD*)	g
lat	351 ± 21 MeV	2.55 ± 0.21 GeV
exp	347 ± 29 MeV	1.92 ± 0.14 GeV

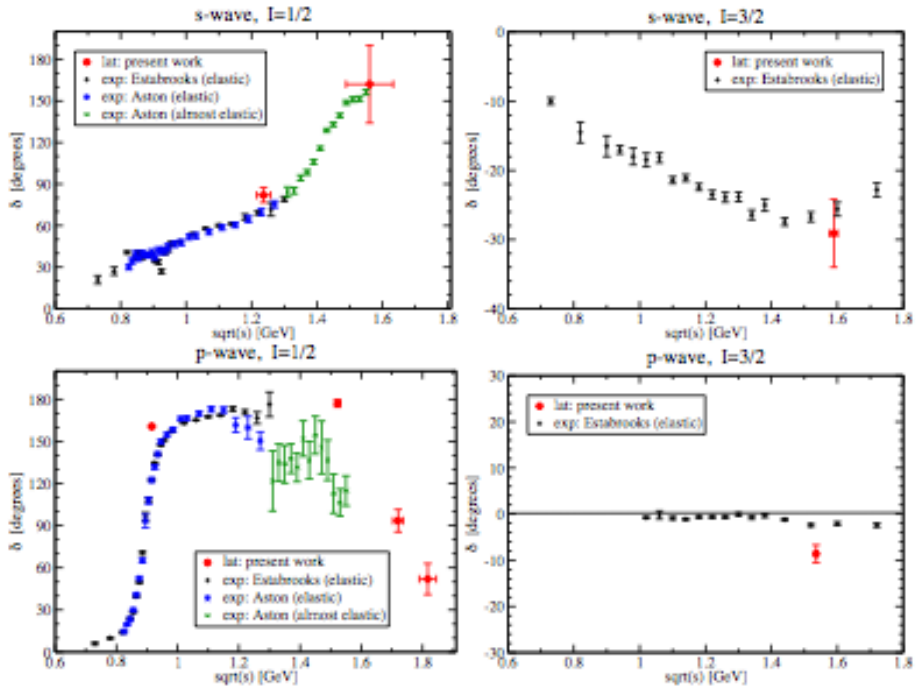
$D_1(2430)$ resonance
$J^P=1^+$

	m - 1/4(mD+3 mD*)	g
lat	381 ± 20 MeV	2.01 ± 0.15 GeV
exp	456 ± 40 MeV	2.50 ± 0.40 GeV

Conclusions (continued)

Kπ and strange resonances





- lots of resonances remain to be studied
  looking forward to simulate exotic resonances that have been observed

Backup slides

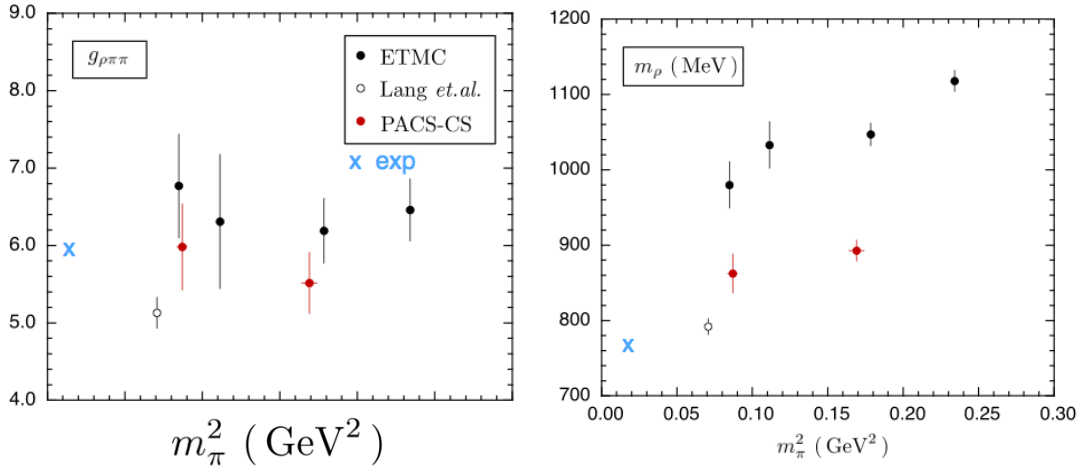
Comparison with other lattice studies

Previous simulations:
- Michael & McNeil [PLB 2003]:ρ➔ππ amp; not Luscher m.
- CP-PACS [PRD 2007]: first study with Luscher method
- QCDSF [Latt proc 2008]
- BMW [Latt proc 2010]

Recent simulations:

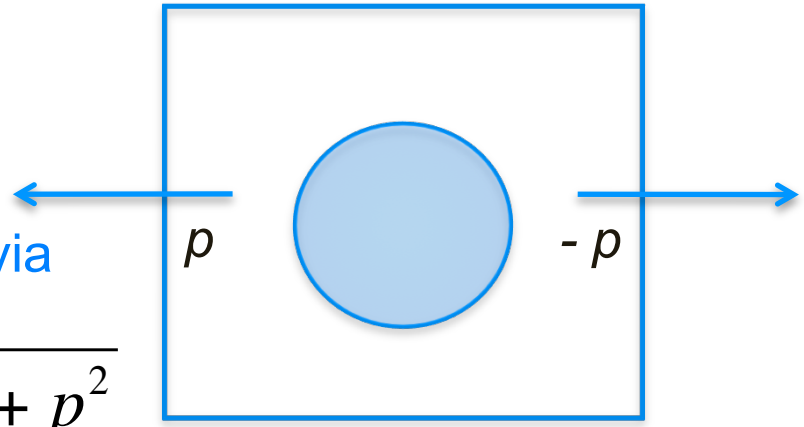
	authors	ref	date	Nf	mπ [MeV]	L [fm]	Quark a.	interp.
[1]	ETMC (Feng et al.)	PRD	Nov 10	2	290 - 480	1.9 , 2.5	Twisted m.	2
[2]	Lang, Mohler, S.P.	1103.5506	May 11	2	266	2	Clover	16
[3]	PACS-CS	1106.5356	June 11	2+1	300 , 410	2.9	Wilson	2

comparison taken from PACS-CS paper [3]



Possible suspects for m(ρ):
- flavor breaking [1]
- small L [2]
- large systematic errors for m(π)=300 MeV [3]
- using just 2 interp. [1,3]
- missing dyn. strange [1,2]
- discretisation errors
- left for future simulations

Extracting δ(p) from E_n at $p_1+p_2=0$ [Luscher]



- extract $E_n(L)$
- E_n renders p in "outside" region via

$$E = \sqrt{s} = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$$

- p contains info on δ(p)

$$\tan\delta(s) = \frac{\pi^{3/2} q}{Z_{00}(1;q^2)} \qquad q \equiv \frac{L}{2\pi} p$$

$$Z_{00}(1;q^2) \equiv \sum_{\vec{n}\in N^3} \frac{1}{\vec{n}^2 - q^2}$$

effective range for K-pi I=3/2 s-wave

p cot(δ)= $$\frac{1}{a_\ell^I} + \tfrac{1}{2} r_\ell^I p^{*2} + \mathcal{O}(p^{*4})$$

from ground state

$$a_0^{I=3/2} = -1.13 \pm 0.15\, a = -0.140 \pm 0.018 \text{ fm} \qquad (18)$$

$$\frac{a_0^{I=3/2}}{\mu_{K\pi}} = -3.94 \pm 0.52 \text{ GeV}^{-2} \quad \text{at} \quad m_\pi \simeq 266 \text{ MeV} .$$

from two states

$$a_0^{3/2} = -1.12 \pm 0.15\, a = -0.139 \pm 0.018 \text{ fm}$$

$$r_0^{I=3/2} = 1.5 \pm 2.0\, a = 0.19 \pm 0.25 \text{ fm}$$

[Lang, Leskovec, Mohler, S. P., arXiv:1207.3204, PRD 86]

s-wave scattering lengths

$$a_0 \equiv \lim_{p \to 0} \frac{\tan \delta(p)}{p}$$

at our mpi=266 MeV, mK, mD

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

our lat. sim.	a_0 [fm]	a_0 / μ [GeV^{-2}]
K π , I=3/2	-0.140 ± 0.018	-3.94 ± 0.52
K π , I=1/2	0.636 ± 0.090	17.9 ± 2.5
D π , I=1/2	0.81 ± 0.14	17.7 ± 3.1
D*π , I=1/2	0.81 ± 0.17	17.6 ± 3.6

→ $r_{eff} \sim 0$

[Weinberg's current algebra 1966]
scattering of pion on any particle

$$\frac{a_0^{I=1/2}}{\mu} = \frac{1}{2\pi F_\pi^2} \approx 10\ GeV^{-2}$$

$$\frac{a_0^{I=3/2}}{\mu} = -\frac{1}{2} \times \frac{a_0^{I=1/2}}{\mu}$$

a_0 : comparison with others

$$a_0 \equiv \lim_{p\to 0} \frac{\tan\delta(p)}{p}$$

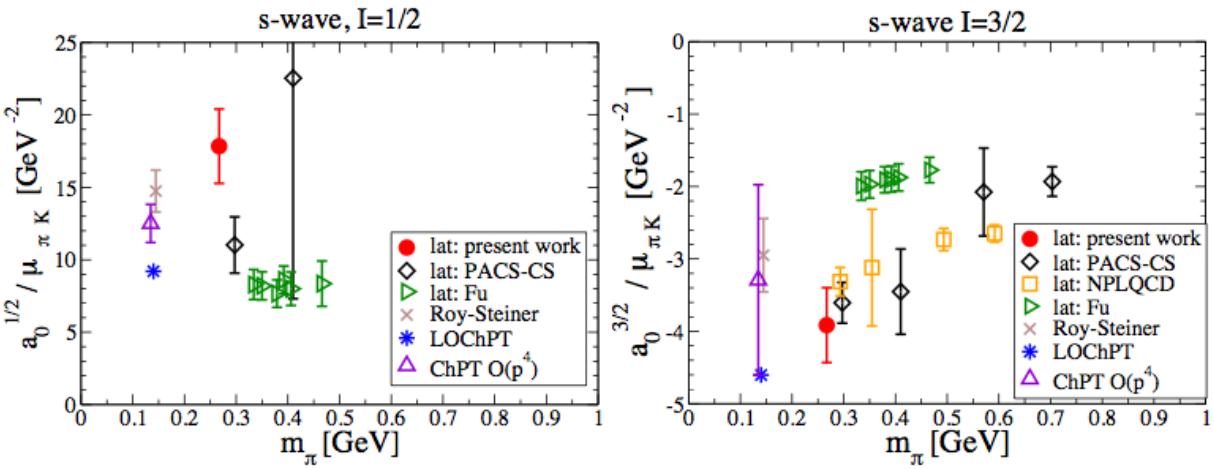
a_0/μ compared as not dependent of mpi in LOChPT

$$\frac{a_0^{I=1/2}}{\mu} = \frac{1}{2\pi F_\pi^2} \approx 10\ GeV^{-2}$$

$$\frac{a_0^{I=3/2}}{\mu} = -\frac{1}{4\pi F_\pi^2}$$

[Weinberg's current algebra 1966]
scattering of pion on any particle

- Kπ



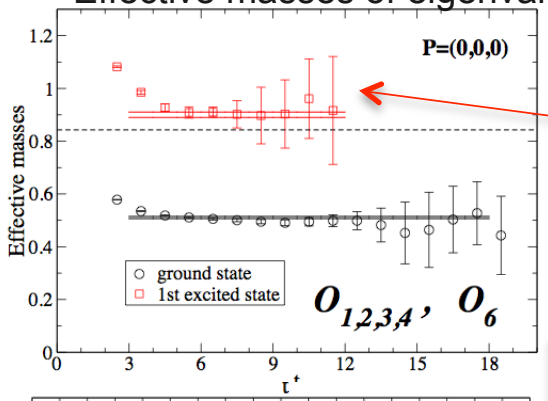
- Dπ

only indirect lattice determination from D➔π semileptonic form factors [Flynn, Nieves 2007]

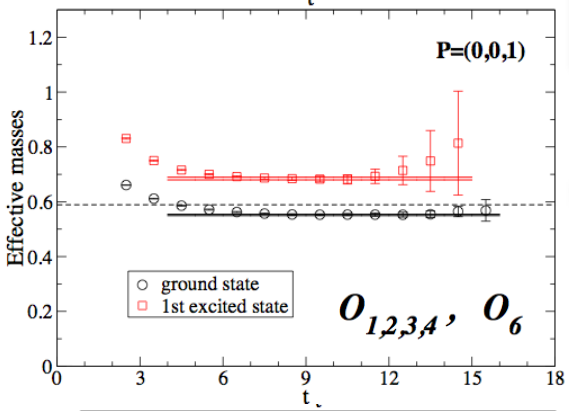
I=1/2	our result	Flynn & Nieves
a_0/μ [GeV^{-2}]	17.7 ± 3.1	15.9 ± 2.2

π π scattering : energies for three different P

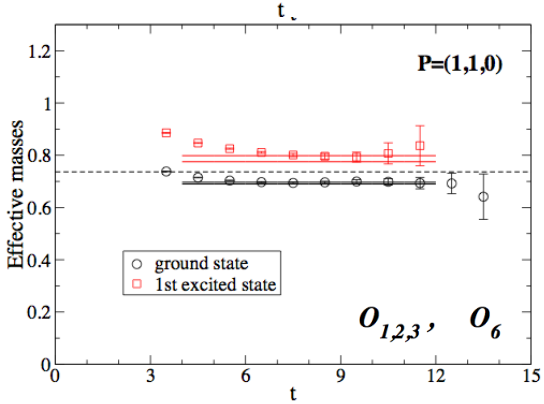
Effective masses of eigenvalues



E_n above 4π (dismissed)



E(π π) noninter.



$s=E^2-P^2$

6 different values of s from one L !