Restoration of axial symmetry at finite temperature and possible relations with restoration of chiral symmetry and deconfinement

Maria C. Ruivo, Pedro Costa and C. A. de Sousa

Centro de Física Computacional
Departamento de Física, Universidade de Coimbra

Bjelasnica Montain, Sarajevo, 7 February 2013

M. C. Ruivo et al, Phys. Rev **D79** (2012) 116003; **D77** (2012) 096009

This work is funded by FEDER, through the Programa Operacional Factores de Competitividade - COMPETE and by National funds through FCT-Fundação para a Ciência e Tecnologia in the frame of project CERN/FP/123620/2011.

Contents

- Introduction/Motivation
  - Restoration of χ_s and deconfinement;
  - Restoration of $U_A(1)$ symmetry and its effects;
  - Overview of models, concepts and results;
- Models and formalism;
  - SU(2) PNJL model with anomaly;
  - SU(2) EPNJL model with anomaly;
- Calculations and results
  - Temperature dependence of the coupling coefficients;
  - Results
- Summary

Introduction/ Motivation

- **Some important queries in QCD thermodynamics:**
  - Proximity or coincidence of the two phase transitions characterized by restoration of chiral symmetry and deconfinement in Lattice QCD .
  - In pure gauge theory the Polyakov loop is the parameter associated to deconfinement. Is there an entanglement between the quark condensate and the Polyakov field?
  - The $U_A(1)$ symmetry, broken in the vacuum at the quantum level, is restored in the chiral symmetric phase?
  - Is there any connection between the two last phase transitions and restoration of axial symmetry?

Bazavov et al. (2012PRD), Bornyakov et al.(2010PRD, arXiv:1102.4461).

Restoration of axial symmetry - possible effects

- Consequences for the nature of the phase transition depending on the degree of anomaly present at T_C .
- Behaviour of the observables: the topological susceptibility, the meson axial chiral partners, the η' mass.
- Lattice calculations indicate a decrease of the topological susceptibility with increasing T [1] .
- The meson correlators of chiral and axial partners should become degenerate.
- Recent experimental results [2] in Au+Au collisions are compatible with a decrease η' mass of about 200 MeV.
  Return 9th "prodigal" Goldstone boson?

1– Alles, et al.(1997NPB)Bazavov, et al. (arXiv:1205.3535), Aoki, et al.(arXiv:1209.2061), Cossu et al. (PoS LATTICE2011)

2– T. Gsörgö et al. (2010PRL)

Introduction/ Motivation

- **Overview of models, concepts and results**
  - **Concepts in Polyakov-Nambu-Jona-Lasinio [PNJL] type models**
    - The NJL part of the model describes the chiral phase transition and its associate order parameter, the quark condensate.
    - The inclusion of the Polyakov loop [1] allows a description of deconfinement
    - In order to account for the proximity of the two last phase transitions, an extension of the PNJL model has been proposed, the Entangled Polyakov–Nambu–Jona-Lasinio (EPNJL) [2], where the four quark interaction has a dependence on the Polyakov field, Φ .

1– Meisinger & Olgive(1996 PLB),Fukushima(2004 PLB), Ratti et al(2005PRD), Megias et al.(2006PRD)

2– Sakai et al (2010PRD, 2011JPG), Kouno et al(2011PRD)

Introduction/Motivation

- **Overview of models, concepts and results**
- **Breaking and restoration of the $U_A(1)$symmetry**
  - The explicit breaking of the $U_A(1)$ symmetry in the vacuum is assumed to be done by instantons through the 't Hooft interaction
  - The Polyakov loop leads to a faster tendency to restoration of axial symmetry [1]
  - Several models assume a temperature dependence of the 't Hooft coupling in order to account for the restoration of $U_A(1)$ symmetry [2]

1– Costa et al,(2009PRD)

2– Fukushima (2001PRC) Costa et al (2004PRD), (2005PRD)

Introduction/Motivation

- **Main topics investigated:**
  - Restoration of axial symmetry exploring different schemes of temperature dependence of the coupling vertices in SU(2) PNJL and EPNJL models.
  - The relevant order-like parameters are analyzed as well as thermodynamic quantities, such as the pressure, the behavior of the topological susceptibility and the convergence of meson axial and chiral partners;
  - Discussion about possible relations between deconfinement, restoration of chiral and axial symmetries.

The PNJL SU(2) Lagrangian with anomaly:

$$\mathcal{L}_{\mathcal{PNJL}} = \bar{q}(\, i\,\gamma^{\mu}\, D_{\mu} - \hat{m})q + \mathcal{L}_1 + \mathcal{L}_2 - \mathcal{U}\,(\Phi[A], \bar{\Phi}[A]; T)\,,$$

with two different interacting parts:

$$\mathcal{L}_1 = g_1\Big[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2 + (\bar{q}\vec{\tau}q)^2 + (\bar{q}i\gamma_5 q)^2\Big],$$

$$\mathcal{L}_2 = g_2\Big[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2 - (\bar{q}\vec{\tau}q)^2 - (\bar{q}i\gamma_5 q)^2\Big].$$

- $q = (u, d)$ are the quark fields; we assume $m_u = m_d = m$.
- $\mathcal{L}_1$ and $\mathcal{L}_2$ are invariant upon $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)$ transformations; $\mathcal{L}_2$ is non-covariant upon $\mathrm{U}_A(1)$ transformations.

Model and formalism

- Quarks are coupled simultaneously to the chiral condensate and to the Polyakov loop through the covariante derivative: $D^\mu = \partial^\mu - iA^\mu$. The Polyakov field is given by:

$$\Phi(\vec{x}) = \frac{1}{N_c} \mathrm{Tr}_c << \mathcal{P} \exp \left[i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right] >> \quad (1)$$

- The coupling of the quarks to the Polyakov loop allows to describe the (statistical) confinement/deconfinement phase transition. In pure gauge theory he Polyakov loop is an order parameter for the restoration of the $\mathbb{Z}_3$ center symmetry of QCD:
- $\mathbb{Z}_3$ is broken in the deconfined phase ($\Phi \to 1$)
- $\mathbb{Z}_3$ is restored in the confined one ($\Phi \to 0$)
- At $T = 0$: $\Phi = \bar{\Phi} = 0 \longmapsto$ both sectors decouple.

Ratti et al.(2006PRD), Hansen et al, (PRD2007)

Model and formalism

The pure gauge sector is described by an effective potential $\mathcal{U}\left(\Phi[A],\bar{\Phi}[A];T\right)$ chosen to reproduce at the mean-field level the results obtained in lattice calculations:

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{a\left(T\right)}{2}\bar{\Phi}\Phi + b(T)\ln[1-6\bar{\Phi}\Phi+4(\bar{\Phi}^3+\Phi^3)-3(\bar{\Phi}\Phi)^2], \quad (2)$$

where

$$a\left(T\right) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 \text{ and } b(T) = b_3\left(\frac{T_0}{T}\right)^3. \quad (3)$$

a_0	a_1	a_2	b_3
3.51	-2.47	15.2	-1.75

Kaczmarek et al 2002, 2007)

The Lagrangian density can be rewritten as:

$$\begin{aligned}\mathcal{L}_{PNJL} &= \bar{q}\,(i\,\gamma^{\mu}\,D_{\mu} - \hat{m})\,q + G_s\,[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2] + G_a\,[(\bar{q}\vec{\tau}q)^2 + (\bar{q}i\gamma_5 q)^2] \\ &- \mathcal{U}\,(\Phi[A],\bar{\Phi}[A];T)\,,\end{aligned}$$

$G_s = g_1 + g_2$ and $G_a = g_1 - g_2$. The grand canonical potential density is:

$$\begin{aligned}\Omega(\Phi,\bar{\Phi},M;T,\mu) &= \mathcal{U}\,(\Phi,\bar{\Phi},T) + 4G_s N_f\,\langle\bar{q}_i q_i\rangle^2 - 2N_c\,N_f\int_{\Lambda}\frac{\mathrm{d}^3p}{(2\pi)\,3}\,E_i \\ &- 2N_f\,T\int\frac{\mathrm{d}^3p}{(2\pi)^3}\,\left(z_{\Phi}^{+}(E_i) + z_{\Phi}^{-}(E_i)\right),\end{aligned}$$

where E_i are the quasi-particle energies and z_{Φ}^{+} and z_{Φ}^{-} are the partition function densities.

We can redefine the coupling constants such as the set (g_1, g_2) or (G_s, G_a) will be replaced by (G, c) as:

$$G_s = g_1 + g_2 = G, \qquad G_a = g_1 - g_2 = G\,(1 - 2\,c), \tag{4}$$

$g_1 = G\,(1 - c), \quad g_2 = c\,G$, $c \in \{0, 1\}$ specifies the degree of $U_A(1)$ symmetry breaking.

The Entangled Polyakov-Nambu-Jona-Lasinio (EPNJL) [1]

An explicit dependence of the fourk-quark vertex G on the Polyakov field, Φ, is considered:

$$G(\Phi) = G\left[1 - \alpha_1\,\Phi\bar{\Phi} - \alpha_2\,(\Phi^3 + \bar{\Phi}^3)\right],$$

which respect chiral, P, C and the extended $\mathbb{Z}_3$ symmetries. We use $\alpha_1 = \alpha_2 = 0.2$ and $T_0 = 170$ MeV as in **1**.

1– Sakai et al (2010PRD, 2011JPG), Kouno et al(2011PRD)

Model and formalism

	f_π [MeV]	$\bar{q}q^{1/3}$ [MeV]	m_π [MeV]	m_σ [MeV]	m_η [MeV]	m_{a_0} [MeV]	$\chi^{1/4}$ [MeV]
Model	93	-241	140.2	803.7	704.5	919.8	180.8
Exp. /Latt.	92.4	-267	135.0	400-1200	547.3	984.7	180

Table 1: Numerical values for the calculated observables compared with experimental and lattice results, obtained with $\Lambda = 590$ MeV, $G_s\Lambda^2 = G\Lambda^2 = 2.435$, $c = 0.2$ and $m = 6$ MeV.

Calculations and results

- gap equation:

$$M_i = m_i - 4G \langle \bar{q}q \rangle_i \,,$$

where $\langle \bar{q}q \rangle_i$ is the quark condensate.

- meson propagator:

$$1 - 4G_{s,a}\,\Pi_{\mathcal{M}}(q^2 = M^2_{\mathcal{M}}) = 0,$$

G_s is associated the channel with π and σ quantum numbers and G_a with η and a_0.

- Topological susceptibility $\chi = \int \mathrm{d}^4x \, \langle 0|TQ(x)Q(0)|0\rangle_c$:
  - in the SU(2) model:

$$\chi = 4N_f \, g_2^2 \, \langle \bar{q}q \rangle^2 \, \frac{4I_1}{1 - 16G_a I_1}, \tag{5}$$

where $I_1 = -\langle \bar{q}q \rangle_i / 4M_i$.

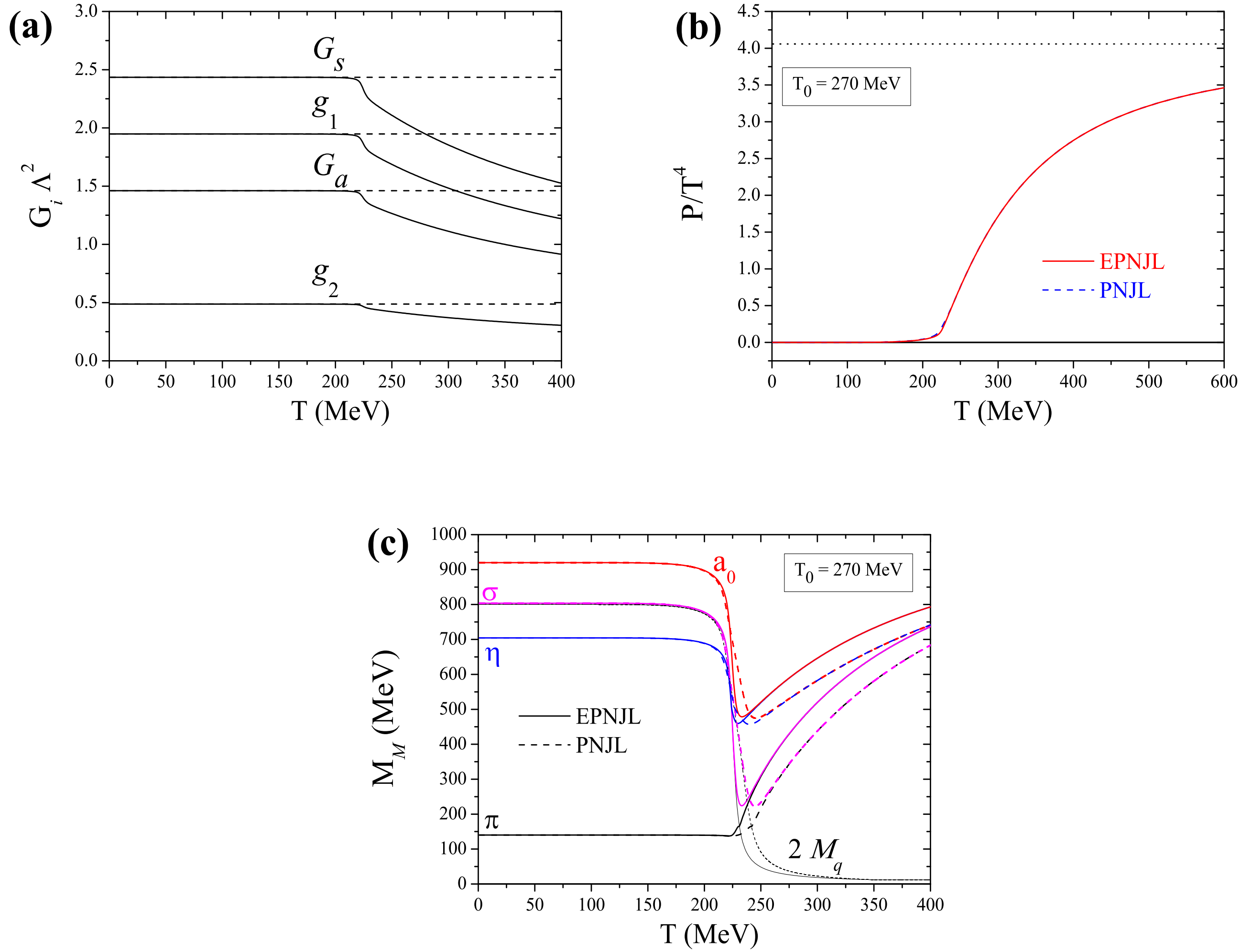
Regularization at finite temperature

- We use a finite cutoff, Λ, in the vacuum, to regularize integrals. At finite T, due to the presence of the partition functions, we make $\Lambda \longrightarrow \infty$.
- for $T > T_{eff}$ (T_{eff} is the temperature at which the symmetry dynamically broken is fully restored, that is $M_i = m_i$), the condensates are set to zero.
- This procedure allows the presence of high momentum quarks at finite T, ensuring that the pressure goes to the Stefan–Boltzmann limit and a better description of several thermodynamic quantities [1].

1– Costa et al. PRD (2007,2008)

Calculations and results

Results without explicit dependence of g_1 and g_2 on temperature; $T_0 = 270$



Calculations and results

- In the SU(3) NJL and PNJL models we get effective restoration of axial symmetry only as a consequence of the restoration of chiral symmetry (due the use of the present type of regularization).
- In the SU(2) models an independent mechanism of instanton suppression is necessary.

Inclusion of temperature dependence on the coupling coefficients

- In the PNJL model

$$g_1(T) = G(1 - c(T)), \quad g_2(T) = G\,c(T).$$

with:

$$c(T) = 0.2 f(T), \text{ where } f(T) = 1/\left(1 + \exp\left((T - T_0)/10\right)\right).$$

$G_s = G$ is always constant; on the contrary, G_a varies.

- In the EPNJL model we have equivalent temperature dependence with the replacement:
  $g_1(T) \to g_1(\Phi, T)$ and $g_2(T) \to g_2(\Phi, T)$.

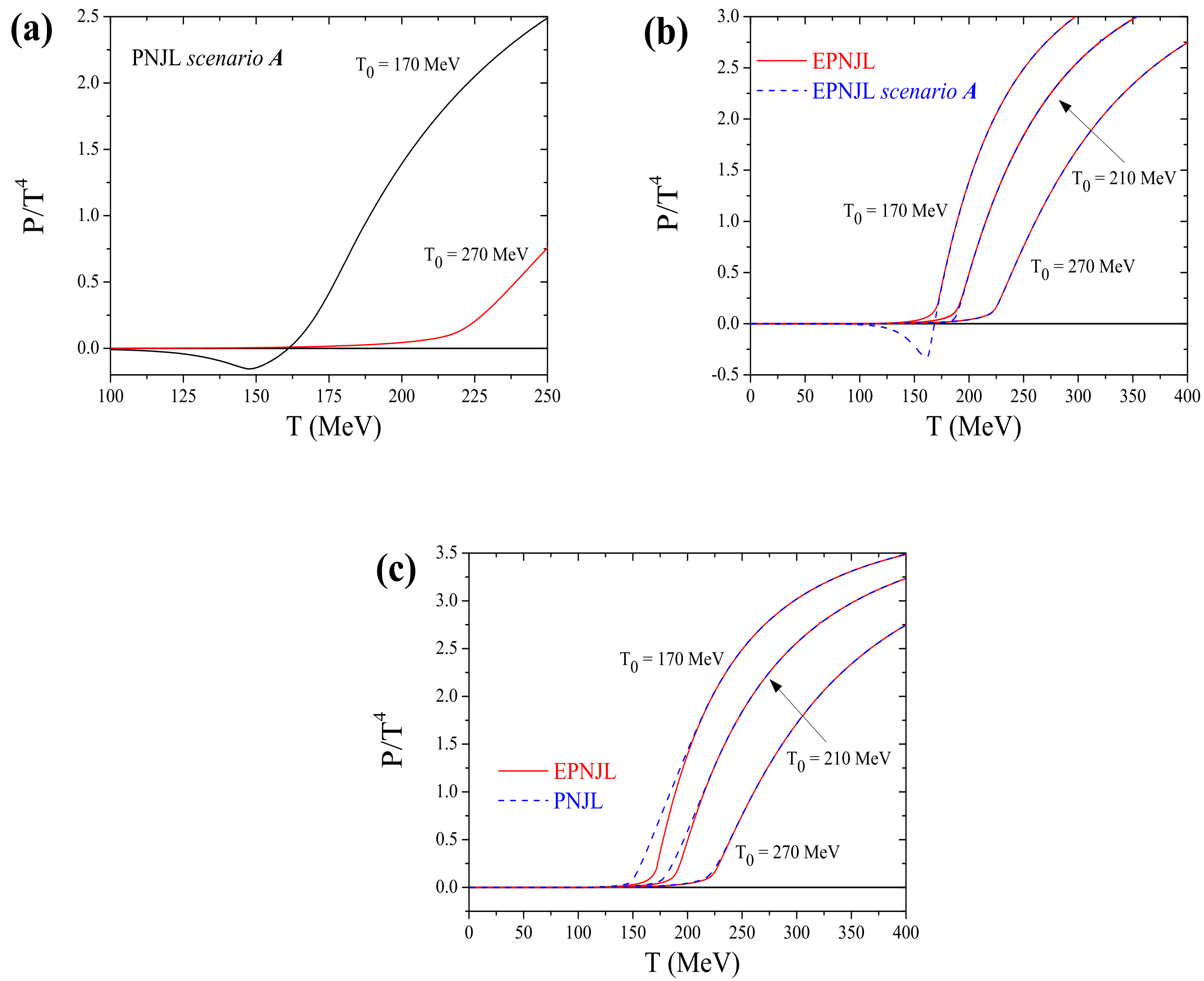
Calculations and results

Characteristic temperatures different values of T_0 ($\Delta = (T_\chi - T_d)/T_\chi$)

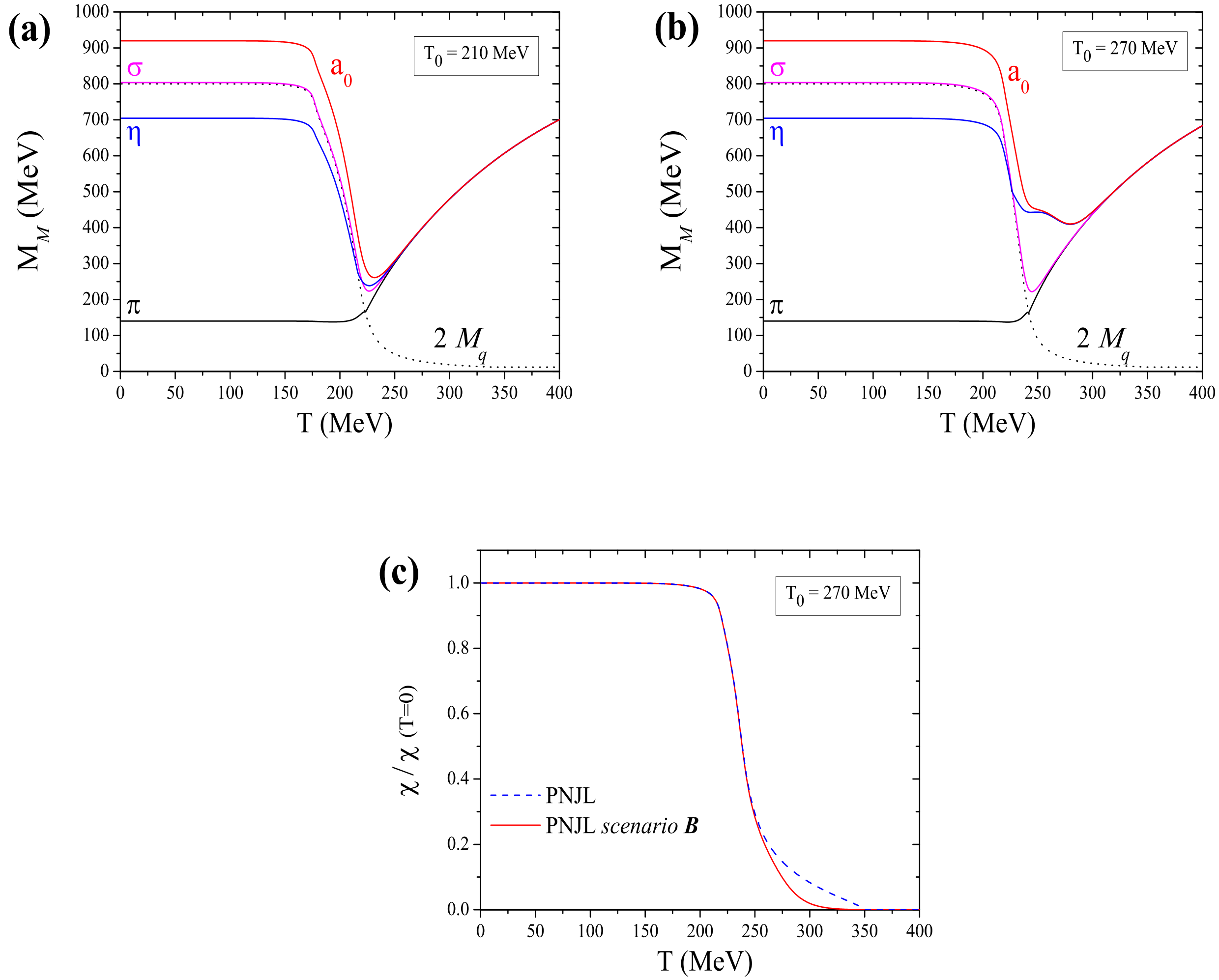
Scenario B	T_0 [MeV]	T_χ [MeV]	T_d [MeV]	Δ –	T_{eff} [MeV]
PNJL	210	215	177	18%	~ 250
	270	237	219	8%	~ 300
EPNJL	170	173	173	–	~ 200
	270	223	223	–	~ 300

Calculations and results

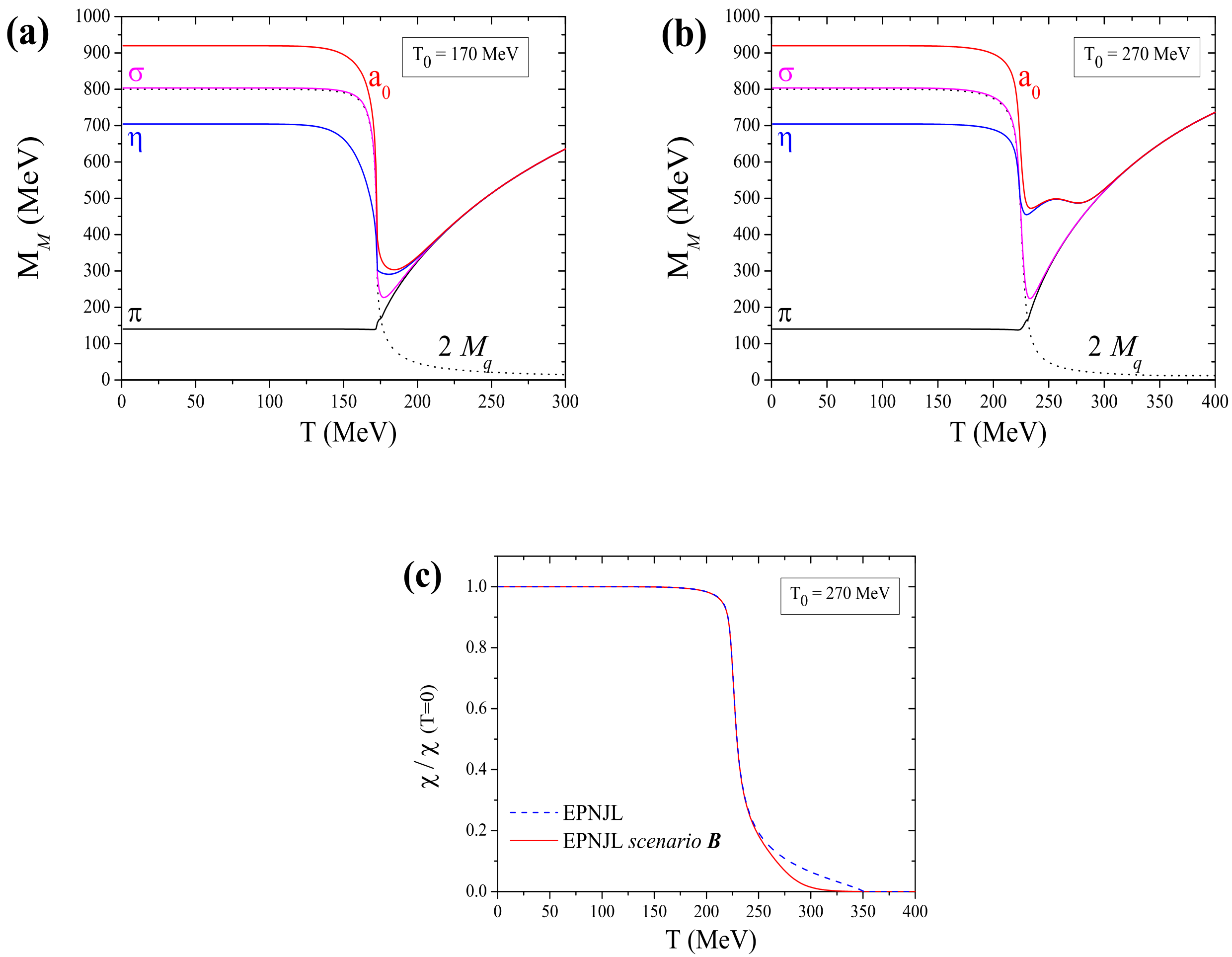
Pressure for different values of T_0



Meson masses and topological susceptibility in the PNJL model



Meson masses and topological susceptibility in the EPNJL model



Summary

- A comparative study of deconfinement, restoration of chiral and axial symmetries in the PNJL and EPNJL models was performed.
- EPNJL model: coupling coefficients depend on Φ, $\longrightarrow T_d = T_\chi = 170$ MeV
- In both models the masses of chiral partners (σ, π) and (η, a_0) converge and the χ vanishes.
- Without explicit temperature dependence of g_2, axial partners do not converge.
- We consider g_1 and g_2 as explicit functions of temperature $(g_2 \to 0,\ g_1 \to \text{maximum}) \longrightarrow$ convergence of the axial partners (π, a_0) and (σ, η).
- Effective restorations of the two symmetries get closer for lower values of T_0
- Some effects of the breaking of both symmetries remain above the critical temperature until both are effectively restored.