# Excited Hadrons, Heavy Quarks and QCD thermodynamics

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# References

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- **Hadron Resonance Gas:** Phys.Rev.Lett. 109 (2012) 151601. arXiv:1207.4875 [hep-ph]. arXiv:1207.7287 [hep-ph].

# Outline

- Quarks and gluons at finite temperature
- Insights from Gluodynamics
- Coupling quarks with Polyakov loops
- Polyakov loops spectroscopy
- Hadron Resonance Gas from Chiral Quark Models
- Conclusions

# QUARKS AND GLUONS AT FINITE TEMPERATURE

# QCD at finite temperature

QCD Lagrangian:

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_f \overline{q}^a_f (i\gamma_\mu D_\mu - m_f) q^a_f \, ;$$

Partition function

$$\begin{aligned} Z_{\rm QCD} &= {\rm Tr} e^{-H/T} = \sum_n e^{-E_n/T} \\ &= \int \mathcal{D}A_{\mu,a} \exp\left[-\frac{1}{4}\int d^4x (G^a_{\mu\nu})^2\right] {\rm Det}(i\gamma_\mu D_\mu - m_f) \end{aligned}$$

Boundary conditions and Matsubara frequencies

$$q(\vec{x},\beta) = -q(\vec{x},0) \qquad A_\mu(\vec{x},\beta) = A_\mu(\vec{x},0) \qquad \beta = 1/T$$

$$\int \frac{dp_0}{2\pi} f(p_0) \to T \sum_n f(w_n)$$

$$w_n = (2n+1)\pi T \qquad w_n = 2n\pi T$$

# Thermodynamic relations

- Statistical mechanics of non-interacting particles

$$\log Z = V\eta g_i \int \frac{d^3p}{(2\pi)^3} \log\left[1+\eta e^{-E_p/T}\right] \qquad E_p = \sqrt{p^2+m^2}$$

$\eta = -1$ for bosons ; $\eta = -1$ for fermions ; $g_i$-number of species

$$\begin{aligned} F &= -T\log Z \qquad P = -T\frac{\partial F}{\partial V} \\ S &= -\frac{\partial(TF)}{\partial T} \qquad E = F + TS \end{aligned}$$

- High temperature limit $\rightarrow$ Free gas of gluons and quarks

$$p \equiv \frac{P}{V} = \left[2(N_c^2-1)+4N_cN_f\frac{7}{8}\right]\frac{\pi^2}{90}T^4$$

Interaction measure (trace anomaly)

$$\Delta \equiv \frac{\epsilon - 3p}{T^4} \rightarrow 0 \qquad (T\rightarrow\infty)$$

# Thermodynamic relations

- Low temperature limit (large $N_c$) gas of hadrons and glueballs

$$p = \sum_i \eta g_i \int \frac{d^3p}{(2\pi)^3} \log\left[1 + \eta e^{-E_p/T}\right]$$

Level density. Hagedorn spectrum for mesons and baryons (Broniowski+Florkowski)

$$\rho(m) = \sum_i g_i \delta(m - m_i) \to A_M e^{m/T_{H,M}} + A_H e^{m/T_{H,M}} \qquad m < 1.5 - 2\mathrm{GeV}$$

Interaction measure

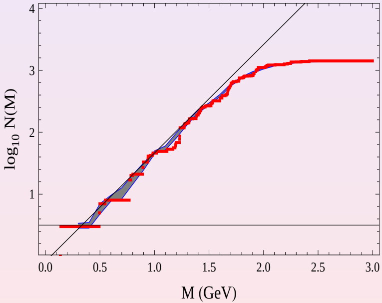
$$\Delta_{HRG} \equiv \frac{\epsilon - 3p}{T^4} \to \sum_i \Delta_i = \Delta_\pi + \ldots \qquad (T \to 0)$$

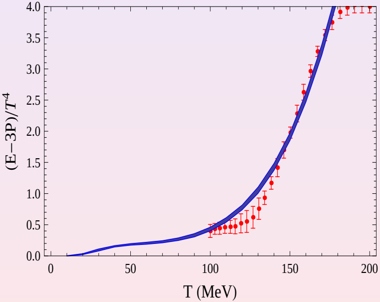
Minimal Hagedorn temperature

$$\Delta_{HRG} \to \frac{A}{T - T_{H,Min}}$$

# The physical resonance spectrum and the half-width rule

- Resonances have a *mass spectrum* (what is the mass?)
- The **half-width rule**: $\Delta M_R = \Gamma_R/2$ or $\Delta M_R^2 = M_R \Gamma_R$



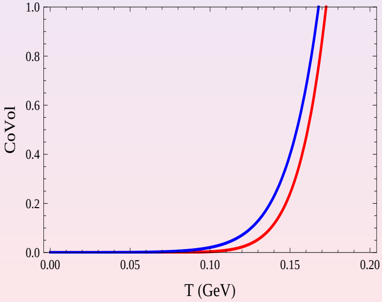



# Excluded Volume constraint

$$\sum V_i N_i \leq V \qquad \sum_i V_i \int \frac{d^3p}{(2\pi)^3} \frac{g_i}{e^{E_i(p)/T} \pm 1} \leq 1$$

MIT bag model

$$V_i = M_i/(4B) \qquad B = (0.166 GeV)^4$$




When hadrons overlapp, excluded volume corrections are important

# Symmetries in QCD

Colour gauge invariance

$$q(x) \to e^{i\sum_a (\lambda_a)^c \alpha_a(x)} q(x) \equiv g(x) q(x)$$
$$A_\mu^g(x) = g^{-1}(x)\partial_\mu g(x) + g^{-1}(x) A_\mu(x) g(x)$$

Only periodic gauge transformations are allowed:

$$g(\vec{x}, x_0 + \beta) = g(\vec{x}, x_0)\,, \qquad \beta = 1/T\,.$$

In the static gauge $\quad \partial_0 A_0 = 0$

$$g(x_0) = e^{i 2\pi x_0 \lambda/\beta}\,, \quad \text{where} \quad \lambda = \mathrm{diag}(n_1, \cdots, n_{N_c})\,, \quad \mathrm{Tr}\lambda = 0\,.$$

**Large** Gauge Invariance: $\Rightarrow$ periodicity in $A_0$ with period $2\pi/\beta$

$$A_0 \to A_0 + 2\pi T\,\mathrm{diag}(n_j) \qquad \text{Gribov copies}$$

**Explicitly Broken in perturbation theory** (non-perturbative finite temperature gluons)

# Symmetries in QCD

In the limit of massless quarks ($m_f = 0$),

- Invariant under scale

$$(\mathbf{x} \longrightarrow \lambda \mathbf{x})$$

Broken by quantum corrections regularization (Trace anomaly)

$$\epsilon - 3p = \frac{\beta(g)}{2g} \langle (G^a_{\mu\nu})^2 \rangle \neq 0 \,,$$

- Chiral **Left** $\leftrightarrow$ **Right** transformations.

$$q(x) \to e^{i \sum_a (\lambda_a)^f \alpha_a} q(x) \qquad q(x) \to e^{i \sum_a (\lambda_a)^f \alpha_a \gamma_5} q(x)$$

Broken by chiral condensate in the vacuum

$$\langle \bar{q} q \rangle \neq 0$$

# INSIGHTS FROM GLUODYNAMICS

# Symmetries in QCD

Gluodynamics: In the limit of heavy quarks ($m_f \to \infty$)

$$Z \to \int \mathcal{D}A_{\mu,a} \exp\left[-\frac{1}{4}\int d^4x (G^a_{\mu\nu})^2\right] \mathrm{Det}(-m_f)$$

Larger symmetry ('t Hooft) Center Symmetry $\mathbb{Z}(N_c)$

$$g(\vec{x}, x_0 + \beta) = z\, g(\vec{x}, x_0)\,, \qquad z^{N_c} = 1, \quad (z \in \mathbb{Z}(N_c))\,.$$

$$g(x_0) = e^{i2\pi x_0 \lambda/(N_c \beta)}\,, \qquad A_0 \to A_0 + \frac{2\pi T}{N_c}\mathrm{diag}(n_j)$$

The Poyakov loop

$$L_T = \frac{1}{N_c}\langle \mathrm{tr}_c e^{iA_0/T}\rangle = e^{-F_q/T} = e^{i2\pi/N_c} L_T = 0$$

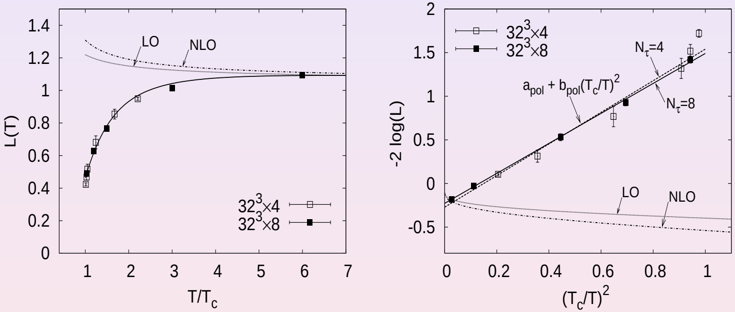
$F_q = \infty$ means CONFINEMENT
At high temperatures $A_0/T << 1$

$$L_T = 1 - \frac{\langle \mathrm{tr}_c A_0^2\rangle}{2N_c T^2} + \cdots = e^{-\frac{\langle \mathrm{tr}_c A_0^2\rangle}{2N_c T^2} + \dots}$$

In full QCD $L_T = \mathcal{O}(e^{-m_q/T}) \neq 0 \ll 1$

# Power temperature corrections in the Polyakov loop

**Renormalized Polyakov Loop $N_c = 3, N_f = 0$**
**O. Kaczmarek et al. PLB543 (2002).**

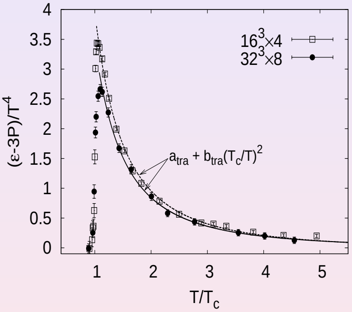



$$-2\log(L) = a_{\rm P} + \frac{a_{\rm NP}}{T^2}\,, \quad a_{\rm NP} = (1.81 \pm 0.13)T_c^2\,, \qquad 1.03T_c < T < 6T_c\,.$$

Perturbative result fails to reproduce lattice data in this regime.

# Power temperature corrections from Lattice data

**Trace Anomaly $N_c = 3, N_f = 0$**
**G. Boyd et al., Nucl. Phys. B469, 419 (1996).**




$$\frac{\epsilon - 3P}{T^4} = a_{\rm P} + \frac{a_{\rm NP}}{T^2}\,, \quad a_{\rm NP} = (3.46 \pm 0.13)T_c^2\,, \qquad 1.13T_c < T < 4.5T_c\,.$$

# COUPLING QUARKS WITH POLYAKOV LOOPS

# Lattice results in full QCD

The chiral-deconfinement cross over is a unique prediction of lattice QCD

- Order parameter of chiral symmetry breaking ($m_q = 0$)
  Quark condensate $SU(N_f) \otimes SU(N_f) \to SU_V(N_f)$

$$\langle \bar{q} q \rangle \neq 0 \quad T < T_c \qquad \langle \bar{q} q \rangle = 0 \quad T > T_c$$

- Order parameter of deconfinement ($m_q = \infty$)
  Polyakov loop: Center symmetry $Z(N_c)$ broken

$$L_T = \frac{1}{N_c} \langle \mathrm{tr}_c e^{iA_0/T} \rangle = 0 \quad T < T_c \qquad L_T = \frac{1}{N_c} \langle \mathrm{tr}_c e^{iA_0/T} \rangle = 1 \quad T > T_c$$

- In the real world $m_q$ is finite but inflexion points nearly coincide (accidental)

$$\frac{d^2}{dT^2} L_T = 0 \qquad \frac{d^2}{dT^2} \langle \bar{q} q \rangle_T = 0$$

  For about the same $T_c = 155(10)$

# Temperature ranges

Momentum scale $p \sim 2\pi T$ (thermal wavelength)

$$T_c = 150\mathrm{MeV} \to p = 1000\mathrm{MeV}$$

- Low temperatures $\to$ Chiral Perturbation theory (ChPT) (pions dominate)

$$\frac{\langle \bar{q} q \rangle_T}{\langle \bar{q} q \rangle_0} = 1 - \frac{T^2}{8 f_\pi^2} - \frac{T^4}{384 f_\pi^4} + \dots$$

$$L(T) = 0$$

- Intermediate temperatures $\to$ Hadron Resonance Gas (HRG) (treats $\pi\pi$ interactions as a $\rho$-resonance, large $N_c$ physics)
- Phase transition (renormalization)
- Above the phase transition (condensates, dim-red)
- Not too high (hard thermal loops) $T \geq 2T_c$
- High temperature $\to$ Perturbation Theory (pQCD)

# Chiral Quark Models at Finite T

- Chiral Quark Models $\longrightarrow$ Dynamics of QCD at low energies (low temperatures).
- Chiral Perturbation Theory $\longrightarrow$ Suppose the non-vanishing of chiral condensate. It cannot describe the QCD phase transition.
- Ogilvie and Meissinger PLB (1995) K. Fukushima, PLB591, 277 (2004). W. Weise et al. PRD73, 014019 (2006), N. Scoccola, D. G. Dumm (2008), S.K. Ghosh et al. PRD73, 114007 (2006), Minimal coupling of Polyakov loop (analogy with chemical potential). **Mean field approximation**.
- E.Megías, E.Ruiz Arriola and L.L.Salcedo, **PRD74**: 065005 (2006). **Quantum and local polyakov loop**

# Minimal coupling of the Polyakov loop

Constituent Quark model:

$$\mathcal{L}_{\rm QC} = \overline{q}\,\mathbf{D}\,q\,, \qquad \mathbf{D} = \not\partial + \not V^f + \not A^f + MU^{\gamma_5} + \hat{m}_0$$

Consider the minimal coupling of the gluons in the model:

$$V_\mu^f \longrightarrow V_\mu^f + gV_\mu^c\,, \quad V_\mu^c = \delta_{\mu 0} V_0^c$$

**Covariant derivative expansion** (E. Megías et al. PLB563(2003), PRD69(2004), Oswald and Dyakonov PRD (2004) ).

$$\mathcal{L}(x) = \sum_n \mathrm{tr}[f_n(\Omega(x))\mathcal{O}_n(x)]\,, \qquad \Omega(\vec{x}, x_0) = \mathbb{P}\, e^{i\int_{x_0}^{x_0+\beta} dx_0' V_0^c(\vec{x}, x_0')}$$

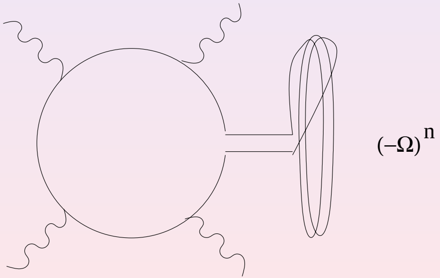
$\Omega$ enters in: $\hat{\omega}_{\mathbf{n}} = \mathbf{2}\pi\mathbf{T}(\mathbf{n} + \mathbf{1}/\mathbf{2} + \hat{\nu})\,, \qquad \Omega = e^{i2\pi\hat{\nu}}$

**The rule to pass from $T = 0$ to $T \neq 0$ is:**

$$\tilde{F}(x;x) \to \sum_{n=-\infty}^{\infty} (-\Omega(\vec{x}))^n \tilde{F}(\vec{x}, x_0 + n\beta; \vec{x}, x_0) \,.$$

The quark condensate writes:

$$\langle \overline{q} q \rangle^* = \sum_n \frac{1}{N_c} \langle \mathrm{tr}_c (-\Omega)^n \rangle \langle \overline{q}(n\beta) q(0) \rangle \,.$$



# Peierls-Yoccoz projection on color singlets

- We introduce a colour source (Polyakov loop).
- We obtain the projection onto the color neutral states by integrating over the $A_0$ field.
- In Quenched approximation: Group integration in SU($N_c$).

$$\langle \mathrm{tr}_c(-\Omega)^n \rangle \equiv \int_{\mathrm{SU}(N_c)} D\Omega \, \mathrm{tr}_c(-\Omega)^n = \begin{cases} N_c \,, & n = 0 \\ -1 \,, & n = \pm N_c \\ 0 \,, & \text{otherwise} \end{cases}$$

There is only contribution from $n = 0, \pm N_c$.

$$\langle \overline{\mathbf{q}}\mathbf{q} \rangle^* \overset{\text{Low T}}{\sim} \langle \overline{q} q \rangle + 4 \left( \frac{MT}{2\pi N_c} \right)^{3/2} e^{-N_c M/T} \,.$$

The $N_c$ suppression is consistent with ChPT.

- Beyond the Quenched approximation:

$$Z = \int DUD\Omega\, e^{-\Gamma_G[\Omega]}\, e^{-\Gamma_Q[U,\Omega]}$$

For any observable: $\langle \mathcal{O} \rangle^* = \frac{1}{Z} \int DUD\Omega\, e^{-\Gamma_G[\Omega]}\, e^{-\Gamma_Q[U,\Omega]}\, \mathcal{O}$ .

1. $\int$ **DU**: Saddle point approximation.
2. $\int$ **D**$\Omega$:
   - Analytically $\longrightarrow$ Expand the exponents and compute correlation functions of Polyakov loops:

$$\int D\Omega\, \mathrm{tr}_c \Omega(\vec{x})\, \mathrm{tr}_c \Omega^{-1}(\vec{y}) = e^{-\sigma|\vec{x}-\vec{y}|/T}$$

   - Numerically $\longrightarrow$ Consider the Polyakov gauge.

# Analytical results in the Unquenched Theory

In the NJL model with Polyakov loop:

$$\langle \overline{q}q \rangle^* \overset{\text{Low T}}{\sim} \langle \overline{q}q \rangle + \frac{N_f V}{\pi^3}(MT)^3 e^{-2M/T} + \mathcal{O}(e^{-N_c M/T})$$

$$L \equiv \left\langle \frac{1}{N_c} \text{tr}_c \Omega \right\rangle \overset{\text{Low T}}{\sim} \frac{N_f}{N_c}\frac{V}{T}\sqrt{\frac{M^3 T^5}{2\pi^3}} e^{-M/T} + \mathcal{O}(e^{-2M/T})$$

$L$-small because Spontaneous Chiral Symmetry Breaking Taking into account the quark binding effects:

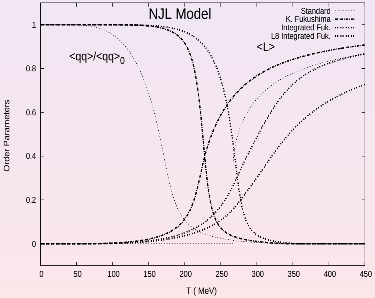
$$\mathcal{O}_q^* = \mathcal{O}_q + \sum_{m_\pi} \mathcal{O}_{m_\pi} \frac{1}{N_c} e^{-m_\pi/T} + \sum_B \mathcal{O}_B \, e^{-M_B/T} + \dots$$

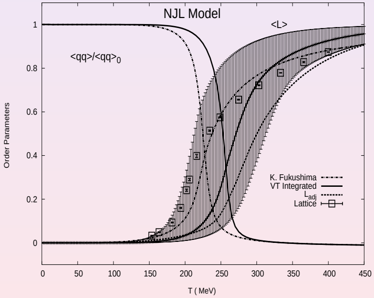
# Phase transition using confinenement domains

$$\langle \mathrm{tr}_c \Omega(\vec{x})\ \mathrm{tr}_c \Omega^{-1}(y) \rangle_{S_G} = e^{-\sigma|\vec{x}-y|/T} ,$$

We take $\Omega$ x-independent in a volume

$$V_\sigma = \int d^3x e^{-\sigma r/T} = \frac{8\pi T^3}{\sigma^3}$$




Polyakov “cooling” : The condensate does not change at low temperatures.

POLYAKOV LOOP SPECTROSCOPY

# Quark-Hadron Duality at Finite Temperature

Partition function for $N_f$-flavours

$$Z_{\rm HRG}(N_f) \equiv \int D\Omega\, e^{-S(N_f)} \qquad S(N_f) = S_q(N_f) + S_G$$

Quark contribution

$$S_q(N_f) = -2N_f \int \frac{d^3x d^3p}{(2\pi)^3} \left( {\rm tr}_c \log \left[ 1 + \Omega(x)\, e^{-E_p/T} \right] + {\rm c.c.} \right)$$

One extra HEAVY QUARK (not anti-quark) with flavour $a$

$$S_q(N_f+1) - S_q(N_f) = -2 \log(1 + \Omega_{aa} e^{-E_h/T}) \approx -2e^{-m_H/T}\Omega_{aa}$$

$$\frac{1}{N_c}\langle {\rm tr}_c \Omega \rangle = \lim_{m_H\to\infty} \frac{1}{2} \left[ \frac{Z_{\rm HRG}(N_f+1)}{Z_{\rm HRG}(N_f)} - 1 \right] e^{m_H/T} = \frac{1}{2N_c} \sum_\alpha g_\alpha e^{-\Delta_\alpha/T}$$

$$\Delta_\alpha = \lim_{m_H\to\infty} (M_{H,\alpha} - m_H)$$

# Polyakov loop in the HRG model

$$L(T) \sim \frac{1}{2N_C} \sum_\alpha g_\alpha e^{-\Delta_\alpha/T}$$

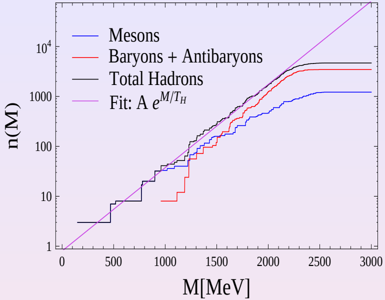
From the PDG

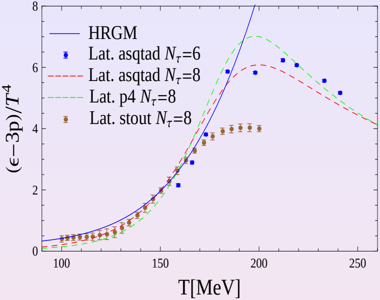
$$\begin{aligned} M_K - m_s \equiv \Delta_s &= 396(24)\,, \\ M_D - m_c \equiv \Delta_c &= 603(81)\,, \\ M_B - m_b \equiv \Delta_b &= 1040(130)\,. \end{aligned} \quad (1)$$

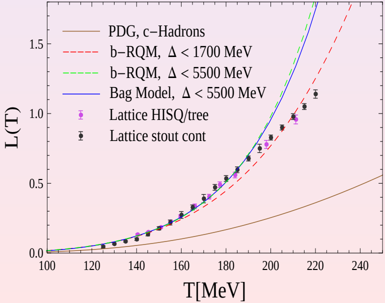
We use single charm states

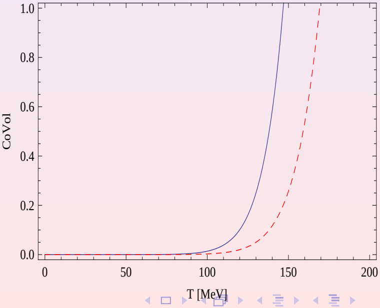
- Mesons $D^0, D^+, D_s^+, D^{*0}, D^{*+}, D_s^{*+}$. A
- Baryons $\Sigma_c^0, \Sigma_c^+, \Sigma_c^{++}, \Xi_c^+, \Xi_c^0, \Omega_c^0, \Lambda_c^+$
- Many $\bar{q}q$ and $qqq$ states needed $\rightarrow$ Relativized-Quark-Model (Isgur) ; MIT Bag Model
- Possibility of discernig exotics $\bar{q}q\bar{q}q$, $\bar{q}qqqq$ from data at finite temperature !!

# Hadron Resonance Gas from Chiral Quark Models



# FROM CHIRAL QUARK MODELS TO THE HADRON RESONANCE GAS

# Quantization of multiquark states

- Multiquark states: Create/Anhiquilate a quark at point $\vec{x}$ and momentum $p$
$$\Omega(x)e^{-E_P/T} \qquad \Omega(x)^{+}e^{-E_P/T}$$
- At low temperatures quark Boltzmann factor small $e^{-E_p/T} < 1$. The action becomes small

$$S_q[\Omega] = 2N_f \int \frac{d^3x d^3p}{(2\pi)^3} \left[\mathrm{tr}_c\Omega(x) + \mathrm{tr}_c\Omega(x)\right] e^{-E_p/T} + \dots$$

$$Z = \int D\Omega\, e^{-S[\Omega]} = \int D\Omega \left(1 - S[\Omega] + \frac{1}{2}S[\Omega]^2 + \dots\right)$$

- $\bar{q}q$ contribution

$$Z_{\bar{q}q} = (2N_f)^2 \int \frac{d^3x_1 d^3p_1}{(2\pi)^3} \int \frac{d^3x_2 d^3p_2}{(2\pi)^3} e^{-E_1/T} e^{-E_2/T} \underbrace{\langle \mathrm{tr}_c \Omega(\vec{x}_1) \mathrm{tr}_c \Omega^\dagger(\vec{x}_2) \rangle}_{e^{-\sigma|\vec{x}_1 - \vec{x}_2|/T}}$$

$$= (2N_f)^2 \int \frac{d^3x_1 d^3p_1}{(2\pi)^3} \frac{d^3x_2 d^3p_2}{(2\pi)^3} e^{-H(x_1,p_1;x_2,p_2)/T}$$

$\bar{q}q$ Hamiltonian

$$H(x_1,p_1;x_2,p_2) = E_1 + E_2 + V_{12} \,.$$

- Quantization in the CM frame $p_1 = -p_2 \equiv p$

$$\left(2\sqrt{p^2 + M^2} + V_{q\bar{q}}(r)\right) \psi_n = M_n \psi_n \,.$$

- Boosting the CM to any frame with momentum $P$

$$Z_{\bar{q}q} \to \sum_n \int \frac{d^3R d^3P}{(2\pi)^3} e^{-E_n(P)/T}$$

A GAS OF NON INTERACTING MESONS ! (valid to $\bar{q}q\bar{q}q$)

# Polyakov loop in the quark model

$$L_T = 2N_f \int \frac{d^3x\, d^3p}{(2\pi)^3} e^{-E_p/T} \frac{1}{N_c} \underbrace{\langle \mathrm{tr}_c \Omega(\vec{x}_0) \mathrm{tr}_c \Omega^\dagger(\vec{x}) \rangle}_{e^{-\sigma|\vec{x}_0 - \vec{x}|/T}} + \cdots$$

$$= \frac{2N_f}{N_c} \int \frac{d^3x\, d^3p}{(2\pi)^3} e^{-H(\vec{x},\vec{p})/T}$$

Heavy-light ground state system

$$(\sqrt{p^2 + m_q^2} + \sigma r)\psi_n = \Delta_n \psi_n$$

In the limit $m_q \to 0$ we make $p \sim 1/r$ and $\Delta \sim 2\sqrt{\sigma} \sim 900\mathrm{MeV}$

$$N_c L(T) \sim 2N_f e^{-\Delta_M/T} + (2N_f^2 + N_f) e^{-\Delta_B/T} + \cdots = 21 e^{-\bar{\Delta}/T} \qquad (N_f = 3)$$

# CONCLUSIONS

# Conclusions:

- Chiral quark models coupled to Polyakov loops are useful as a first estimate of QCD features. The reproduce qualitatively the cross-over observed in QCD.
- Chiral quark models at finite temperature have much better properties when the Polyakov loop (colour source) is projected a la Peierls-Yoccoz onto singlet colour states.
- With simple modifications the transition to the hadronic degrees of freedom becomes possible.
- Polyakov loops in fundamental and higher representations allow to deduce multiquark quark states, gluelumps etc. containing one or several heavy quark states. This goes beyond the models and opens up the possibility of a Polyakov loop spectroscopy in cluding exotics.