The Role of the Quark-Gluon Vertex in the QCD Phase Transition

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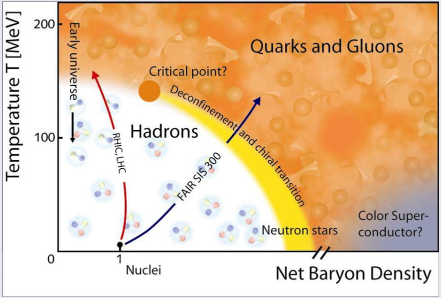
University of Graz

(A. Windisch, R. Alkofer)

Outline

The Phase Diagram of QCD

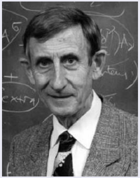
- no free quarks observed in nature → ***Confinement***
- how can three light quarks constitute a heavy proton? → ***Dynamical Breaking of Chiral Symmetry*** ($D_\chi SB$)
- quarks behave massless for high T → chiral symmetry restored
- **possible relation between the two phenomena?**



Why studying the Quark-Gluon Vertex?

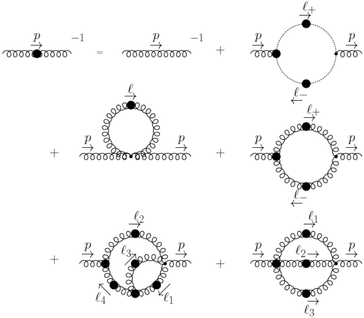
- **quark-gluon vertex** ⇔ **interaction between quarks/gluons**
- **no full self-consistent solution** of this object at $T = 0$
- **only models available for** $T \neq 0$ **and/or** $\mu \neq 0$

A Theoretical Description - The Formalism

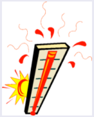
The  -  Approach

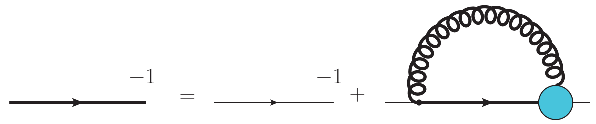
(F.J. Dyson) (J. Schwinger)

- we work in:
  - Landau gauge
  - Euclidean space-time



Dyson-Schwinger Equation for the Quark Propagator





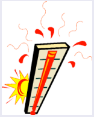
$$S^{-1}(p) = Z_2 S_0^{-1}(p) - C_F g^2 T \sum_{\omega_{\boldsymbol{k}}(\phi)} \int \frac{d^3k}{(2\pi)^3} Z_{1F} \gamma^\mu S(k) \boldsymbol{\Gamma}^\nu(\boldsymbol{p},\boldsymbol{k}) D_\gamma^{\mu\nu}(q)$$

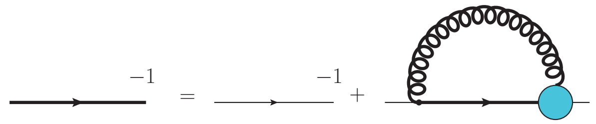
Dyson-Schwinger Equations

- non-perturbative approach
- $\mu - T$-plane accessible
- infinite tower $\rightsquigarrow$ truncations
- **solve self-consistently $\rightsquigarrow$ further input needed**
    - **gluon propagator & quark-gluon vertex**

Dyson-Schwinger Equations and Matsubara Formalism

Dyson-Schwinger Equation for the Quark Propagator





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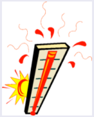
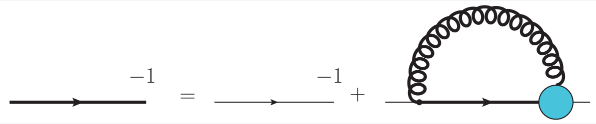
Input for the Gluon Propagator

- $D_\gamma^{\mu\nu}(q) \rightarrow D_{\gamma,T}^{\mu\nu}(\omega_q, \vec{q}) + D_{\gamma,L}^{\mu\nu}(\omega_q, \vec{q})$
- take **from lattice** results, [Fischer, Maas and Mueller, EPJ C68 (2010)]
- also **unquechend results** available, [Aouane et al., arXiv:1212.1102]

Dyson-Schwinger Equation for the Quark Propagator





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Models for the Quark-Gluon Vertex

- **vertex model** taken from [Fischer, PRL103 (2009)], [Fischer and Mueller, PRD80 (2009)]

$$\boldsymbol{\Gamma^\nu(\omega_p, \vec{p}; \omega_k, \vec{k})} = \left\{ kinematical \;\; term \right\}^\nu \times \left\{ \frac{d_1}{d_2 + q^2} + \boldsymbol{\Delta}_{UV} \right\}$$

- blue term → **STI motivated kinematical expression**
- red term → **phenomenological model** → new parameters: d_1, d_2
- proper **perturbative running**: $\Delta_{UV} \equiv \dfrac{k^2}{\Lambda^2 + q^2} \left(\dfrac{\beta_0 \alpha(\mu) \ln[\frac{q^2}{\Lambda^2} + 1]}{4\pi} \right)^{2\delta}$

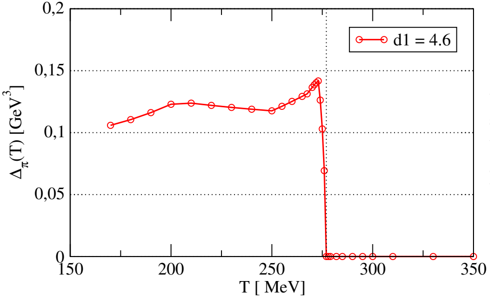
Order Parameters

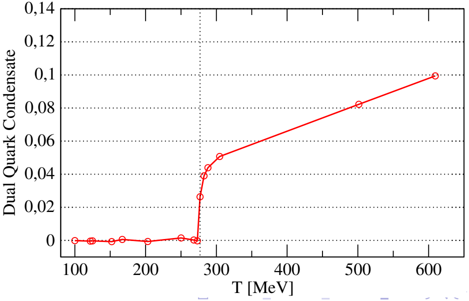
- quark condensate $\langle\bar{\psi}\psi\rangle_\phi$
- dual quark condensate, [Gattringer, PRL97 (2006)], [Bilgici et al., PRD77 (2008)], [Fischer, Maas and Mueller, EPJ C68 (2010)], ...
- quark-gluon vertex model:

$$\Gamma^\nu(\omega_p,\vec{p};\omega_k,\vec{k}) = \Big\{\textit{kinematical term}\Big\}^\nu \times \left\{\frac{d_1}{d_2+q^2} + \Delta_{UV}\right\}$$

  - fix d_1 and d_2 to physical observables
  - strong parameter dependence

- $SU(3)$ calculation with **quenched gluon input**





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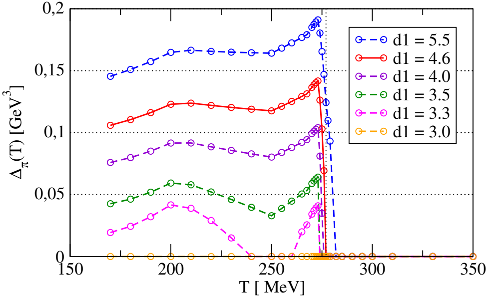
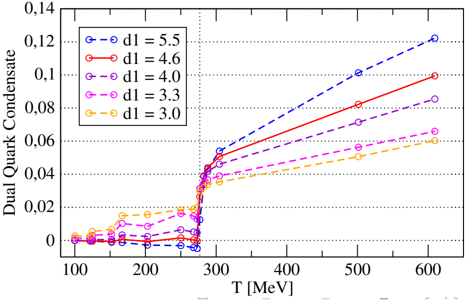
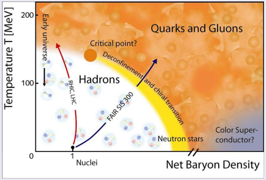


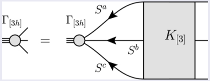


(Future) Applications

- **exploring the QCD phase diagram using DSEs**
  - study critical endpoint
    $\rightsquigarrow$ cf. e.g. Luecker, Fischer, ...
  - investigate CFL/CSC phases
    $\rightsquigarrow$ cf. e.g. Mueller, [Nickel], Buballa, ...



- **bound-state equations**
  - Bethe-Salpeter/Fadeev equations
    cf. e.g. Bhagwat, Eichmann, Maris, Nicmorus, Swanson, Tandy, Vujinovic, Williams, ...



Prerequisites for these Applications

- **more detailed understanding of the relevant vertex tensor structure**
  - **non-perturbative contributions**
  - **vertex dressing functions obtain a non-zero value due to $D\chi SB$**
  - **how are these effects accounted for in models?**

The Quark-Gluon Vertex - Earlier Investigations (2008)

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The quark–gluon vertex in Landau gauge QCD: Its role in dynamical chiral symmetry breaking and quark confinement

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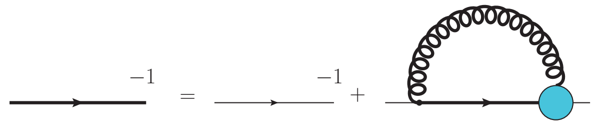
ABSTRACT

The infrared behavior of the quark–gluon vertex of quenched Landau gauge QCD is studied by analyzing its Dyson–Schwinger equation. Building on previously obtained results for Green functions in the Yang–Mills sector, we analytically derive the existence of power-law infrared singularities for this vertex. We establish that dynamical

Dyson-Schwinger Equation for the Quark Propagator



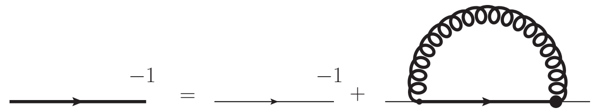
$$S^{-1}(p) = Z_2 S_0^{-1}(p) + g^2 Z_{1F} C_{N_c} \int \text{đ}k \; \gamma^\mu \; S(k) \; \boldsymbol{\Gamma}^\nu(\boldsymbol{p},\boldsymbol{k}) \; D_\gamma^{\mu\nu}(p-k)$$

Some Comments

- **Quark-Gluon Vertex**: rainbow truncation, BC/CP-type vertex constructions, ...
  - $D\chi SB$ $\Leftrightarrow$ effective interaction strength
  - put effective interaction e.g. into gluon propagator
  - and/or use sophisticated vertex models
- **Goal**: **full self-consistent solution**
  - at vanishing temperature $\rightsquigarrow$ 12 tensors

Consider $T = \mu = 0$

Dyson-Schwinger Equation for the Quark Propagator



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Dyson-Schwinger Equations

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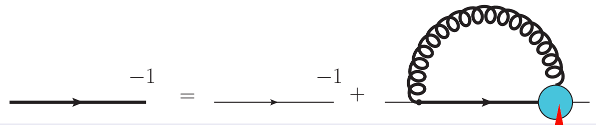

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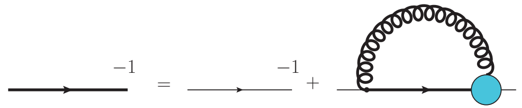
Basis (if $T = \mu = 0$)

$$\Gamma^\nu \rightsquigarrow \begin{Bmatrix} \mathbb{1} \\ \not{k} \\ \not{p} \\ \not{k}\not{p} \end{Bmatrix} \otimes \begin{Bmatrix} \gamma^\nu \\ k^\nu \\ p^\nu \end{Bmatrix}$$

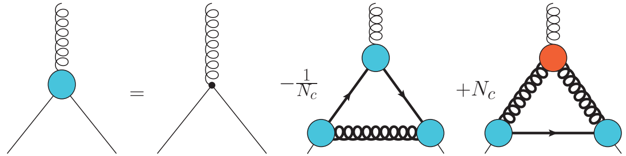
i.e. $\Gamma^\nu \propto \sum_{i=1}^{12} \lambda_i \Gamma_i$

The Coupled System

Dyson-Schwinger Equation for the Quark Propagator



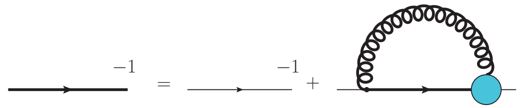
Dyson-Schwinger Equation for the Quark-Gluon Vertex



Remarks/Ingredients

- **dress all vertices**, [Alkofer, Fischer, Llanes-Estrada, Schwenzer, Annals Phys. 324 (2009)]
  - correspondence to DSE-like equation in 3PI formalism, [Berges, PRD70 (2004)]
- **gluon propagator** from lattice/DSE calculations
- **3-gluon vertex** $\rightarrow$ only models available

Dyson-Schwinger Equation for the Quark Propagator



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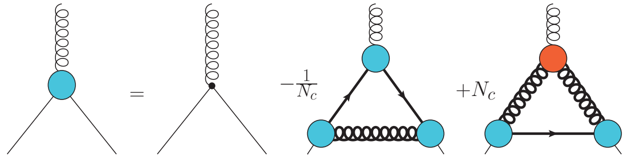

Simplifications/Tools

- **investigate scalar theory** ⇔ fundamentally charged scalars
  - no Dirac structure → simplified playground
- **calculations performed on GPUs**

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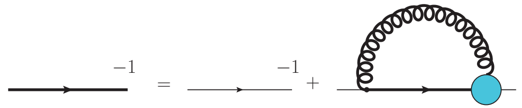
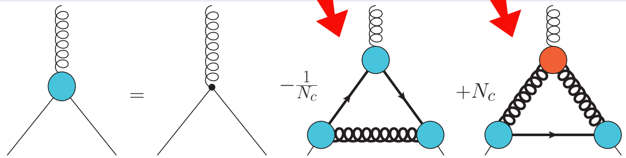


Abelian Diagram non-Abelian Diagram

Dyson-Schwinger Equation for the Quark-Gluon Vertex



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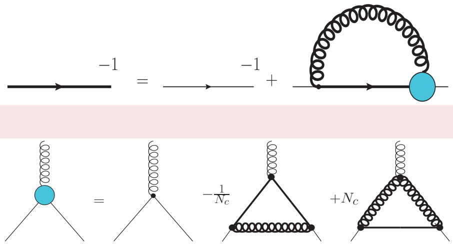
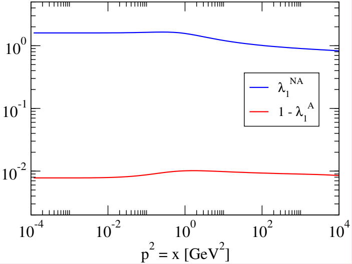
The Coupled System in a Leading Order Skeleton Expansion

Contribution of the Abelian/non-Abelian Diagram

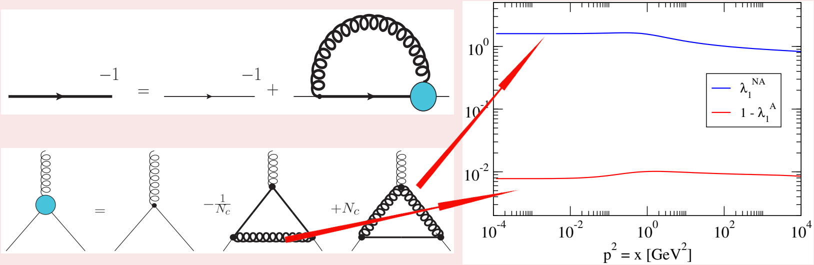




Leading Order Skeleton Expansion

- pick out an **arbitrary vertex dressing function**
  - here: λ_1 ($\Leftrightarrow \gamma^\mu$) at $p_1^2 = p_2^2 = p^2 \ \wedge \ p_1.p_2 = 0$
- Abelian diagram is sub-leading - non-Abelian diagram is important
- *Leading Order Skeleton Expansion generates no dynamical mass !!*

Contribution of the Abelian/non-Abelian Diagram



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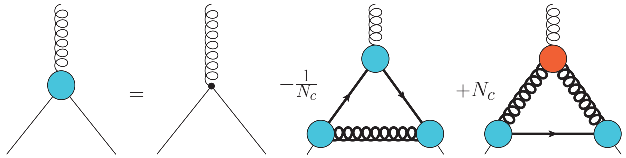
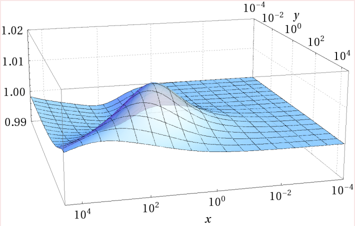
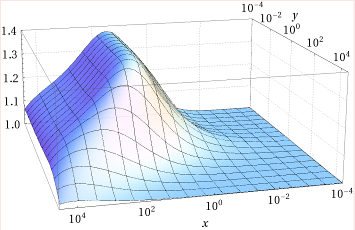
Investigating a Scalar Theory

Scalar QCD ⇝ Fundamentally Charged Scalars ⇝ No Dirac Structure



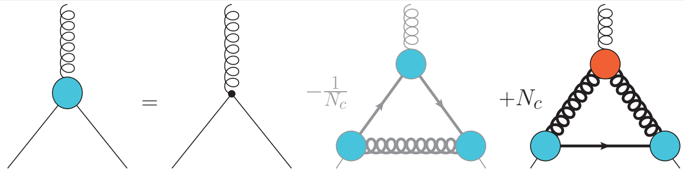
Abelian/non-Abelian Diagram in a Scalar Theory, [MH, diploma thesis, 2011]





non-Abelian Diagram

The Quark-Gluon Vertex - Step-by-Step



Solution Strategy - Step 1

- include the **non-Abelian diagram**
- include the **Abelian diagram (suppressed!?)**

non-Abelian Diagram

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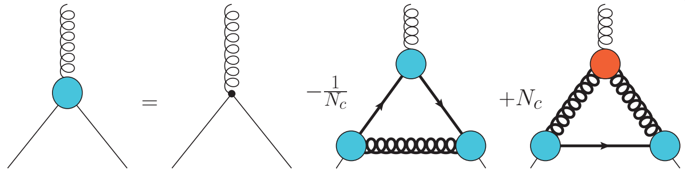

Solution Strategy - Step 2

- include the **non-Abelian diagram** ✓
- include the **Abelian diagram (suppressed!?)**

Numerics

Numerics

GPU Programing

- calculations on **GPUs**
- **Mephisto GPU Cluster**
  - Research Core Area 'Modeling and Simulation'

Code Development

- CUDA-C++/FORTRAN
- OpenACC
- GPU- and CPU Cluster

Solving the Ghost-Gluon System on GPUs

- include the gluon propagator directly from a DSE calculation
- can be done on GPUs, [MH, Alkofer, Haase, arXiv:1206.1779]
- advantages: more flexibility, include recent findings

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Step 1 - Discretization

$$\Gamma^\mu(p_1^2, p_2^2, p_1 \cdot p_2) \to \Gamma^\mu(x, y, z) \to \Gamma^\mu[x_i][y_n][z_j]$$

Step 2 - Load Distribution

$$\begin{array}{lcl} \Gamma^\mu[x_1][y_n][z_j] & \leftarrow & \mathbf{GPU_1}\ (\text{processID} = 0) \\ \Gamma^\mu[x_2][y_n][z_j] & \leftarrow & \mathbf{GPU_2}\ (\text{processID} = 1) \\ \vdots & & \vdots \\ \Gamma^\mu[x_N][y_n][z_j] & \leftarrow & \mathbf{GPU_N}\ (\text{processID} = \text{N-1}) \end{array}$$

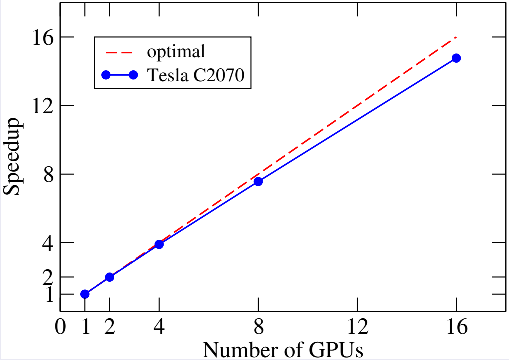
Step 3 - Global Synchronization

collect GPU data and repeat until convergence

Some Technical Details

- coarse graining with **openMPI** / fine graining with **CUDA**
  - in principle **arbitrary number of GPUs** possible
  - **minimal communication costs** / **optimal load balancing**
  - very **good scaling behaviour** of the code

Scaling of the CUDA Code



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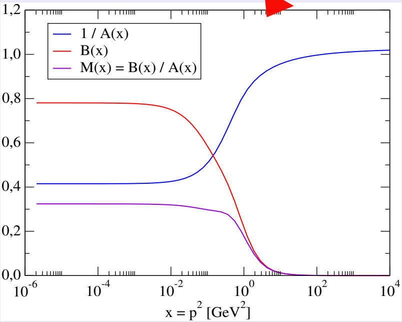
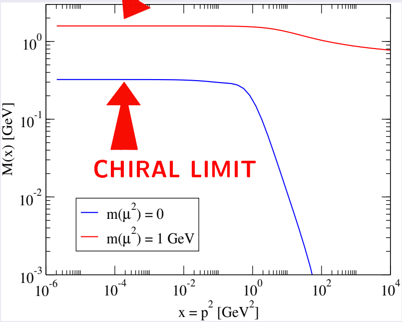
Some Results

CHIRAL LIMIT MASSIVE CASE

Quark Propagator





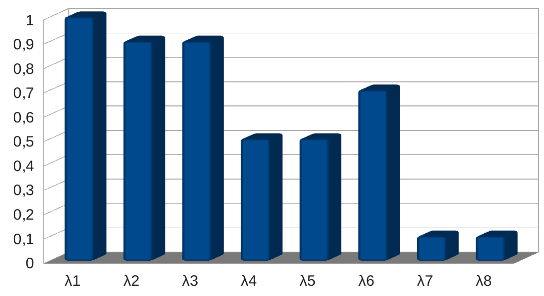
Quark Propagator Dressing Functions

- $S^{-1}(p) = i\not{p}\,A(p^2) + \mathbb{1}B(p^2)$
- non-trivial observation: dynamical mass generation
  - only observed in self-consistent treatment of the system

Relevance of Different Tensor Structures - The Idea

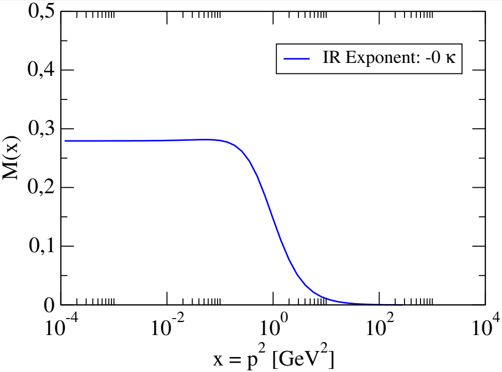
- isolate important tensor structures: $\Gamma^{\mu} \propto \sum_{i=1}^{8} \lambda_i \Gamma_i^{\mu}$
- λ_i . . . dressing function
- Γ_i . . . corresponding basis element (tensor component)

Contribution of Tensor Structures (sketched)



- **problem: some λ's may be crucial for numerical stability**

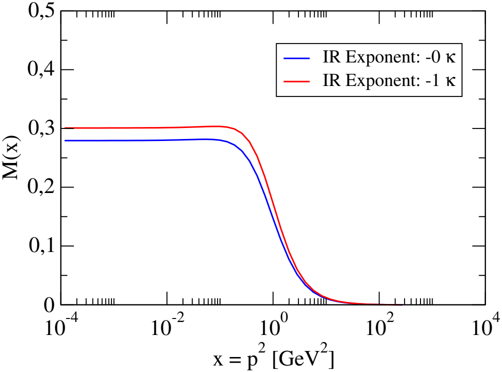
3-Gluon Vertex Dependence



3-Gluon-Vertex $\rightarrow$ bare tensor structure $\times$ model

- $\Gamma^{\mu\nu\sigma}_{3g,old} = \left(\frac{x}{x+d_1}\right)^{-0\kappa} \left(d_3 \frac{d_1}{d_1+x} + d_2 \log\left[\frac{x}{d_1}+1\right]\right)^{17/44} \Gamma^{\mu\nu\sigma}_{3g,0}$
- $x = p_1^2 + p_2^2 + p_3^2$; $d_1, d_2, d_3 \ldots$ free parameters

3-Gluon Vertex Dependence



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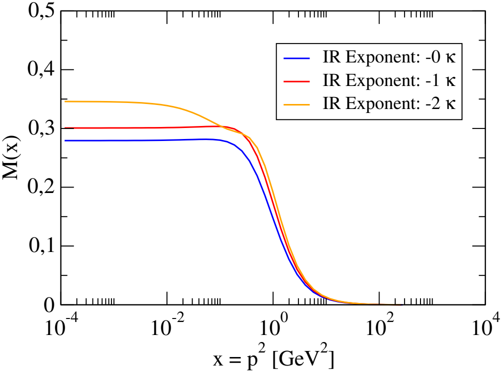

3-Gluon-Vertex → bare tensor structure × model

- $$\Gamma^{\mu\nu\sigma}_{3g,old} = \left(\frac{x}{x+d_1}\right)^{-2\kappa}\left(d_3\frac{d_1}{d_1+x}+d_2\log\left[\frac{x}{d_1}+1\right]\right)^{17/44}\Gamma^{\mu\nu\sigma}_{3g,0}$$
- $x = p_1^2 + p_2^2 + p_3^2$; $d_1, d_2, d_3 \ldots$ free parameters

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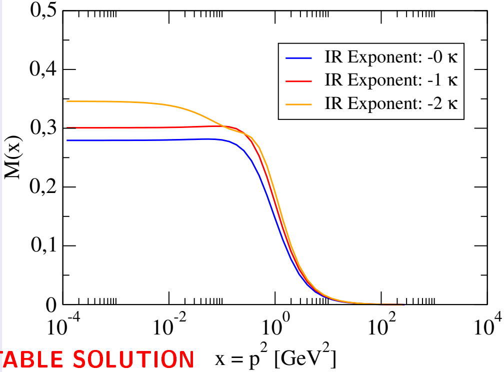

NO STABLE SOLUTION

3-Gluon-Vertex $\rightarrow$ bare tensor structure $\times$ model

- $$\Gamma^{\mu\nu\sigma}_{3g,old} = \left(\frac{x}{x+d_1}\right)^{-3\kappa} \left(d_3 \frac{d_1}{d_1+x} + d_2 \log\left[\frac{x}{d_1}+1\right]\right)^{17/44} \Gamma^{\mu\nu\sigma}_{3g,0}$$
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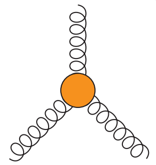
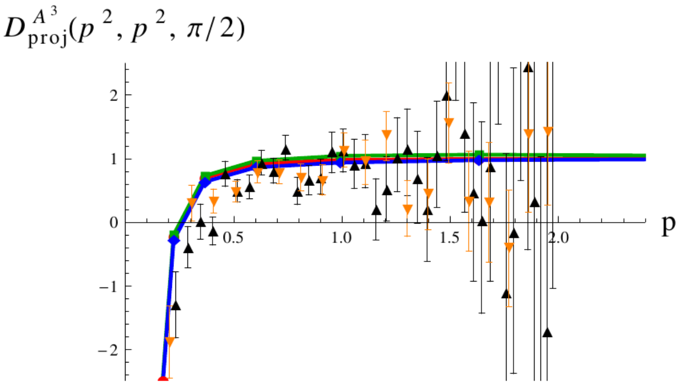
Taking a Closer Look at the 3-Gluon Vertex



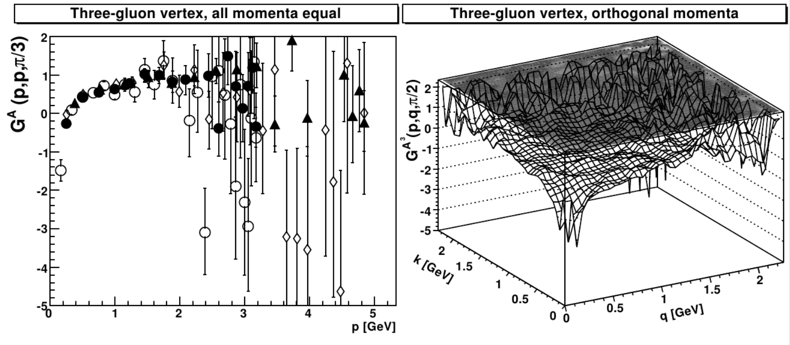
3-Gluon Vertex - Sign Flip in 2D



Lattice Data for **2 Dimensions**, [Huber, Maas and Smekal, JHEP 1211 (2012)]

- **lattice data indicates sign flip in 2D**

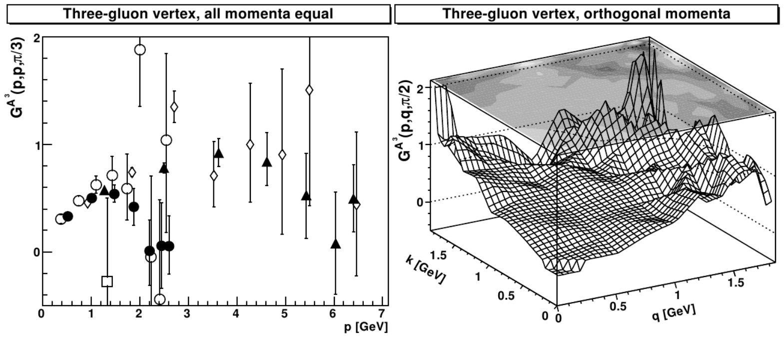
3-Gluon Vertex - Sign Flip in 3D



Lattice Data for **3 Dimensions**, [Cucchieri, Maas, Mendes, PRD77 (2008)]

- **lattice data indicates sign flip in 3D**

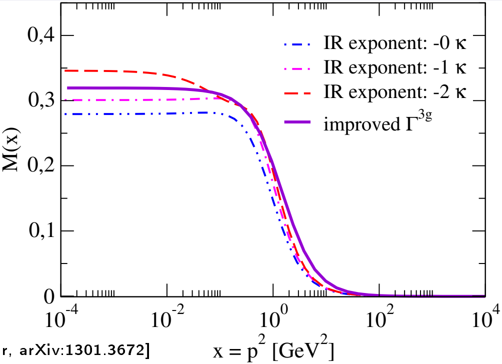
3-Gluon Vertex - Sign Flip in 4D



Lattice Data for **4 Dimensions**, [Cucchieri, Maas, Mendes, PRD77 (2008)]

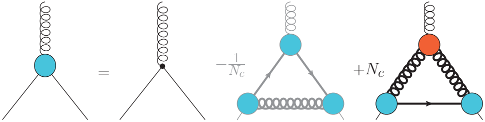
- **lattice data indicates sign flip in 4D**, cf. also [Huber and Smekal, arXiv:1211.6092]
- **try to implement sign flip also in the quark-gluon vertex calculations**
- **stabilizes the system considerably** ☺

Full Calculation with non-Abelian Diagram

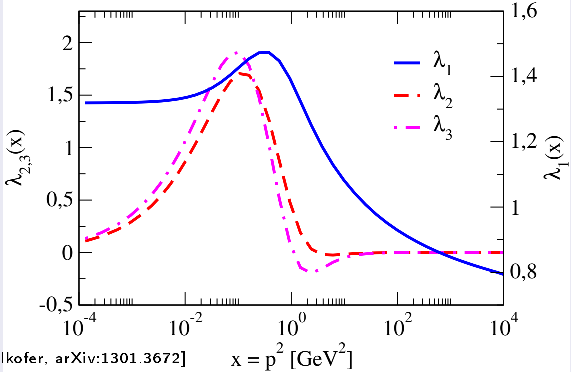


[MH, Windisch, Alkofer, arXiv:1301.3672]

- **full calculation taking non-Abelian diagram into account**

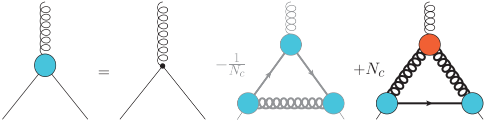


Some Selected Quark-Gluon Vertex Dressing Functions



[MH, Windisch, Alkofer, arXiv:1301.3672]

- full calculation taking non-Abelian diagram into account



Summary

- quark confinement $\leftrightsquigarrow$ $D_\chi SB$
- quark-gluon vertex $\Leftrightarrow$ quark/gluon coupling
- non-Abelian part of the vertex DSE included
- isolate important tensor structure relevant for DχSB
  - crucial for $T \neq 0$ and $\mu \neq 0$ calculations
  - useful for phenomenological model building

Outlook

- calculate with Abelian diagram, i.e. the full system
- extend towards $\mu - T$ $\rightarrow$ study critical endpoint

Thank You For Your Attention!

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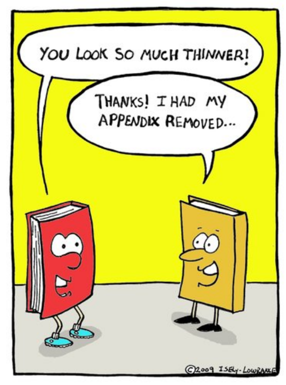
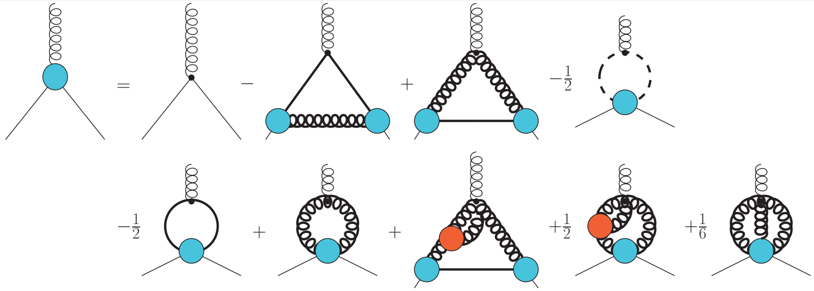
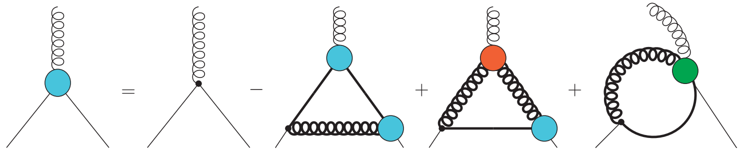
Appendix


YOU LOOK SO MUCH THINNER!
THANKS! I HAD MY APPENDIX REMOVED...
©2009 ISELY-LOWRANCE

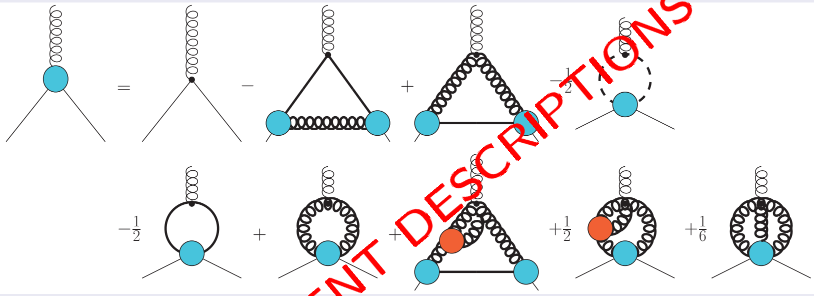
Version 1



Version 2

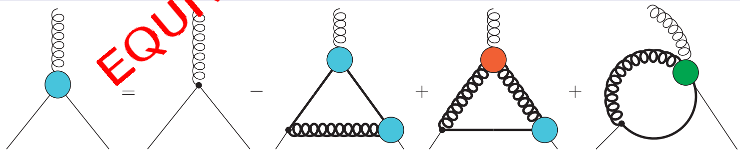


Version 1



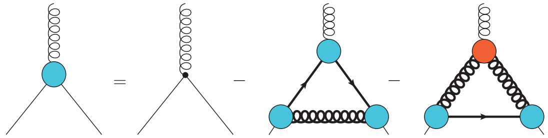
EQUIVALENT DESCRIPTIONS

Version 2

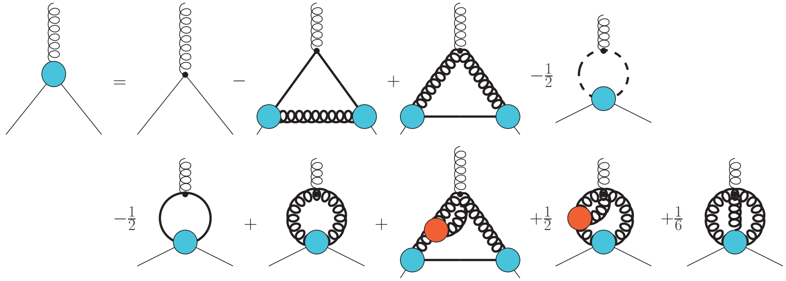


Dyson-Schwinger Equation for the Quark-Gluon Vertex II

Analogy to 3PI Formalism [Berges, PRD70 (2004)]



The Resulting "Simplified" System



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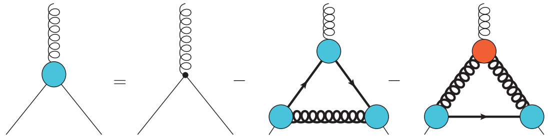

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