Study of tetraquark system from meson-meson scattering with a color flip-flop model

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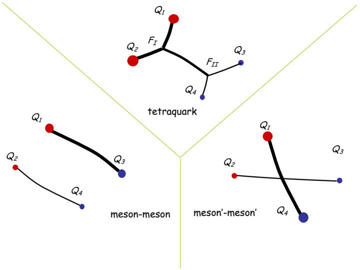
Excited QCD 2013

Motivation

- The existence of tetraquarks is still debated
- Various experimental tetraquark candidates
- Theoretically is a four-body problem, with a four body force
- The system of two mesons is the simplest system with more than one color singlet

Static Potential I

- Static potential from the lattice: $V_{FF} = \min(V_I, V_{II}, V_T)$



- This kind of potentials were postulated to prevent the difficulties arising from Casimir potentials $V_C = \sum_{i<j} C_{ij} V(r_{ij})$

Static Potential II

- In more detail: V_I and V_{II} are the two-meson potentials

$$
\begin{aligned}
V_I &= V_M(r_{13}) + V_M(r_{24}) \\
V_{II} &= V_M(r_{14}) + V_M(r_{23})
\end{aligned}
$$

- $V_M = K - \frac{\gamma}{r} + \sigma\, r$
- V_T is the tetraquark potential:

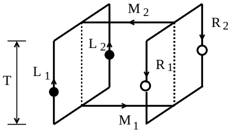
$$
V_T = 2K - \gamma \sum_{i<j} \frac{C_{ij}}{r_{ij}} + \sigma\, L_{min}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4)
$$

- Variational method calculations with this potential indicate that this could bind a tetraquark [1]

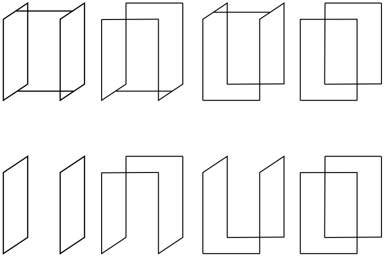
[1] Vijande et al, Phys. Rev. D76 114013

Static color fields I

- The potential was studied in the lattice using a Wilson Loop operator

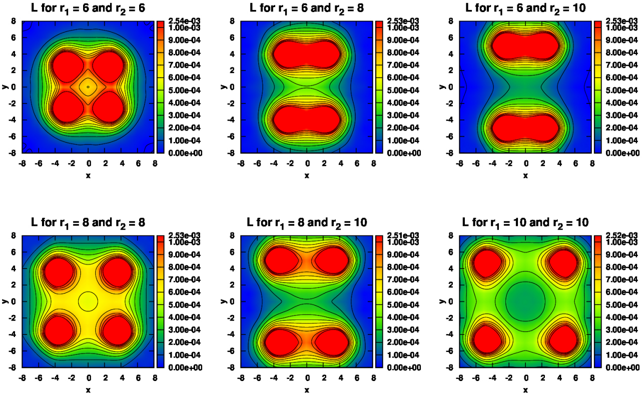


- This can be improved by the use of a variational basis:



Static color fields II

- We used this method to calculate[2] the color fields for a static $QQ\bar{Q}\bar{Q}$ system
- Mesons to mesons transition (anti-parallel geometry)



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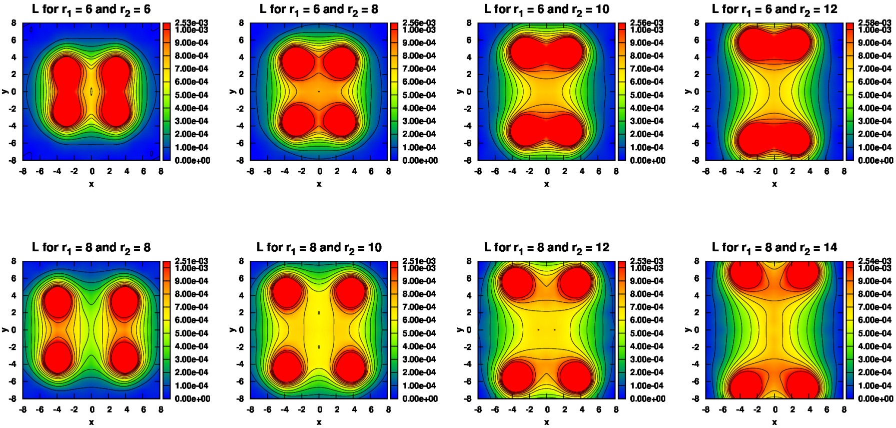
[2] Phys. Rev. D84 054508 and Phys.Rev. D86 014503

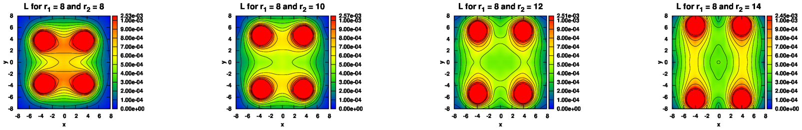
Static color fields III

- Tetraquark to mesons transition (parallel geometry)

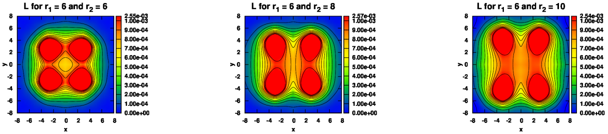


Static color fields IV

- Also results for the excited states in both cases
  - Parallel:



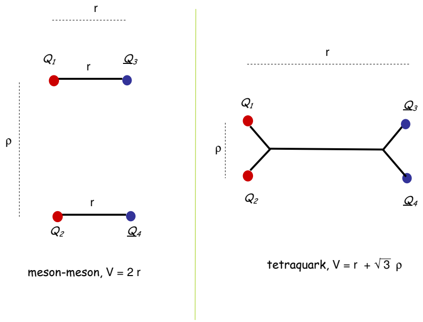
  - Anti-parallel:



Flip-flop toy model I

- To have some qualitative insight, we solved[3] an unitarized model with a simplified Flip-flop potential

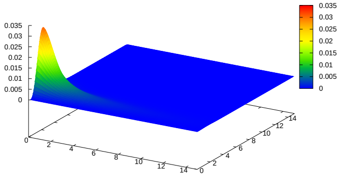
$$V_{FF} = \sigma \min(2r, \sqrt{3}\rho + r)$$



[3] PhysRevD.83.094010

Flip-flop toy model II

- We can determine the model's bound states by using finite differences
- We find a bound state for $l_r = 3$



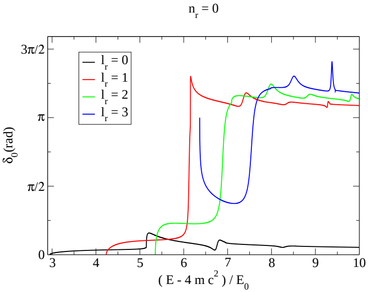
- This does not work for scattering with more than one channel

Flip-flop toy model III

- Project in the meson states: $\psi_i(\rho) = \int d^3r\ \phi_i(r)^*\Psi(r,\rho)$ with $-\frac{\hbar^2}{2m}\nabla_r^2\phi_i + 2\sigma r\phi_i = \epsilon_i\phi_i$
- Coupled channels equation: $-\frac{\hbar^2}{2m}\nabla_\rho^2\psi_i + V_{ij}\psi_j = (E-\epsilon_i)\psi_i$
- Use asymptotic behavior (scattering $i \to j$):
  - $\psi_j \to \delta_{ij} j_{l_\rho}(k_i\rho) + f_{ij}^{l\rho}\frac{e^{ik_j\rho}}{\rho}$ if channel is open
  - $\psi_j \to 0$ otherwise.
- This way we can determinate the phase shifts

Flip-flop toy model IV

- We find resonances with high angular momentum



- This was in agreement with some authors [4] who postulated that the centrifugal barrier would prevent the tetraquark from decaying

[4] Karliner and Lipkin, Phys. Lett. B 575, 249

Potential I

- However this potential neglects some important effects
  - Only one transition is considered
  - Color is omitted
- We have two linearly independent color singlets
- The potential of the system is a 2×2 matrix
- Lattice results give $v_0 = \min(V_I, V_{II}, V_T)$
- We have to find the whole matrix!

Color states

- Two meson states:
  - $|\mathcal{C}_I\rangle = \frac{1}{3}|Q_i Q_j \overline{Q}_i \overline{Q}_j\rangle$
  - $|\mathcal{C}_{II}\rangle = \frac{1}{3}|Q_i Q_j \overline{Q}_j \overline{Q}_i\rangle$
- Antisymmetric and symmetric color states:
  - $|\mathcal{A}\rangle = \frac{\sqrt{3}}{2}\big(|\mathcal{C}_I\rangle - |\mathcal{C}_{II}\rangle\big)$
  - $|\mathcal{S}\rangle = \sqrt{\frac{3}{8}}\big(|\mathcal{C}_I\rangle + |\mathcal{C}_{II}\rangle\big)$
- Note that $\langle\mathcal{C}_I|\mathcal{C}_{II}\rangle = \frac{1}{3}$
- Ground state can be $|\mathcal{C}_I\rangle$, $|\mathcal{C}_{II}\rangle$ or $|\mathcal{A}\rangle$

Potential II

- The corresponding eigenvector should be $|\mathcal{C}_I\rangle$, $|\mathcal{C}_{II}\rangle$ or $|\mathcal{A}\rangle$
  - $|\mathcal{C}_I\rangle$ when $v_0 = V_I$
  - $|\mathcal{C}_{II}\rangle$ when $v_0 = V_{II}$
  - $|\mathcal{A}\rangle$ when $v_0 = V_T$
- Since the potential is hermitian, the second eigenvector must be orthogonal to the first one:
  - $|\overline{\mathcal{C}_I}\rangle$ when $v_0 = V_I$, with $\langle\overline{\mathcal{C}_I}|\mathcal{C}_I\rangle = 0$
  - $|\overline{\mathcal{C}_{II}}\rangle$ when $v_0 = V_{II}$, with $\langle\overline{\mathcal{C}_{II}}|\mathcal{C}_{II}\rangle = 0$
  - $|\mathcal{S}\rangle$ when $v_0 = V_T$

Potential III

- For the transition to be as smooth as possible, we impose
  - $v_1 = \min(V_{II}, V_T)$ when $v_0 = V_I$
  - $v_1 = \min(V_I, V_T)$ when $v_0 = V_{II}$
  - $v_1 = \min(V_I, V_{II})$ when $v_0 = V_T$
- Lattice results seem to agree with this in the transition region [5]
- With this we can construct a color-dependent flip-flop potential

[5] Bornyakov et al., hep-lat/0508006

Meson-meson scattering I

- Wave-function is expanded as $\Psi = \Psi^A|\mathcal{C}_A\rangle$
- Contra-variant color states $|\mathcal{C}^A\rangle$ with$\langle\mathcal{C}^A|\mathcal{C}_B\rangle = \delta^A_B$
- Kinetic Energy operator

$$\hat{\mathrm{T}}_S = (\hat{\mathrm{T}} + \hat{\mathrm{V}}_I)|\mathcal{C}_I\rangle\langle\mathcal{C}^I| + (\hat{\mathrm{T}} + \hat{\mathrm{V}}_{II})|\mathcal{C}_{II}\rangle\langle\mathcal{C}^{II}|$$

- Scattering potential

$$\hat{\mathrm{V}}_S = \hat{\mathrm{V}} - \hat{\mathrm{V}}_I|\mathcal{C}_I\rangle\langle\mathcal{C}^I| - \hat{\mathrm{V}}_{II}|\mathcal{C}_{II}\rangle\langle\mathcal{C}^{II}|$$

- Schrödinger equation:

$$T_s{}^A{}_A\Psi^A + \sum_B V_S{}^A{}_B\Psi^B = E\Psi^A$$

Meson-meson scattering II

- Each color component is expanded as

$$\Psi^A = \sum_k \phi_k^1(\boldsymbol{\rho}_1)\phi_k^2(\boldsymbol{\rho}_2)\psi_k^A(\mathbf{r}_A)$$

- The $\phi_k^i(\boldsymbol{\rho}_i)$ are eigenfunctions of the mesonic hamiltonian
- Integrate intra-mesonic degrees of freedom:

$$-\frac{\hbar^2}{2\mu_\alpha}\nabla^2\psi^\alpha(\mathbf{r}) + \int d^3\mathbf{r}'\ v^\alpha_\beta(\mathbf{r},\mathbf{r}')\psi^\beta(\mathbf{r}') = (E-\epsilon_\alpha)\psi^\alpha(\mathbf{r})$$

- v^α_β is calculated using Monte Carlo (Metropolis algorithm)
- By looking at the asymptotic behavior we can calculate the T matrix

Meson-meson scattering III

- For now we do the following approximations:
  - Consider only heavy quarks
  - Neglect spin effects
  - Neglect dynamical quark effects
  - Non-relativistic kinematics

Exotic channels

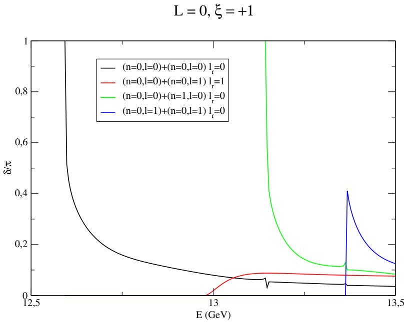
- We study the exotic $QQ\bar{q}\bar{q}$ channels
- In this case the Hamiltonian matrix has the form:

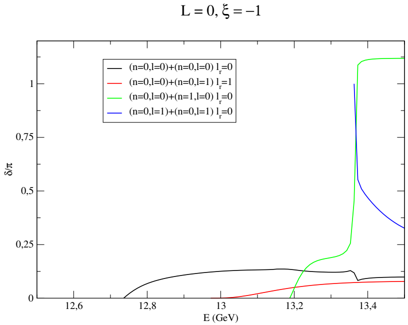
$$\hat{\mathrm{H}} = \left[\begin{array}{cc} \hat{\mathrm{D}} & \hat{\mathrm{A}} \\ \hat{\mathrm{A}} & \hat{\mathrm{D}} \end{array} \right]$$

- Eigenfunctions have the form $\Psi_\xi = \left[\begin{array}{c} u \\ \xi u \end{array} \right]$, with $\xi = \pm 1$

Results I

- Results for $m_Q = m_b$ and $m_q = m_c$
- We use $\sigma = 0.19\,\mathrm{GeV}^2$ and $\gamma = 0.4$
- Phase shifts, for $L = 0$:



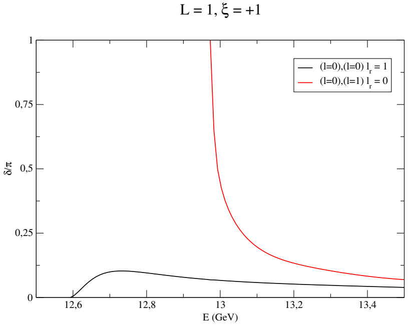


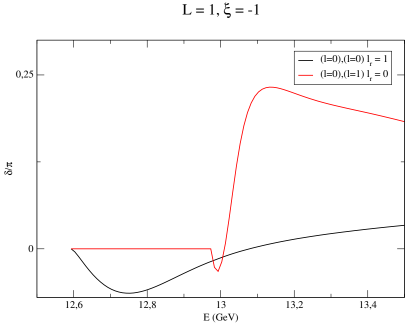
(preliminary results)

- Bound state (Binding energy$< 1\,\mathrm{MeV}$)

Results II

- Phase shifts, for $L = 1$:





(preliminary results)

- Resonance mass and width can be determined using Newton's method

Discussion/Conclusion

- An unitarized method to compute the meson-meson scattering was developed
- In the heavy mass limit we find a bound states and resonances for $L = 0$
- For the $L = 1$ case we find a resonance
- Refinements should be easy to include
  - Spin-spin interactions, etc
  - Other potential models
- In principle, poles can be find directly using Newton's method or similar