Study of tetraquark system from meson-meson scattering with a color flip-flop model

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Motivation

- The existence of tetraquarks is still debated
- Various experimental tetraquark candidates
- Theoretically is a four-body problem, with a four body force
- The system of two mesons is the simplest system with more than one color singlet
Static Potential I

- Static potential from the lattice: \( V_{FF} = \min(V_I, V_{II}, V_T) \)

- This kind of potentials were postulated to prevent the difficulties arising from Casimir potentials \( V_C = \sum_{i<j} C_{ij} V(r_{ij}) \)
In more detail: $V_I$ and $V_{II}$ are the two-meson potentials

$$V_I = V_M(r_{13}) + V_M(r_{24})$$
$$V_{II} = V_M(r_{14}) + V_M(r_{23})$$

$V_M = K - \frac{\gamma}{r} + \sigma r$

$V_T$ is the tetraquark potential:

$$V_T = 2K - \gamma \sum_{i<j} \frac{C_{ij}}{r_{ij}} + \sigma L_{min}(x_1, x_2, x_3, x_4)$$

Variational method calculations with this potential indicate that this could bind a tetraquark \footnote{Vijande et al, Phys. Rev. D76 114013}
The potential was studied in the lattice using a Wilson Loop operator. This can be improved by the use of a variational basis.
We used this method to calculate\(^2\) the color fields for a static \(QQQ\bar{Q}\bar{Q}\) system

Mesons to mesons transition (anti-parallel geometry)

\(\text{Phys. Rev. D84 054508 and Phys. Rev. D86 014503}\)
Static color fields III

- Tetraquark to mesons transition (parallel geometry)
Also results for the excited states in both cases

Parallel:

Anti-parallel:
Flip-flop toy model I

To have some qualitative insight, we solved an unitarized model with a simplified Flip-flop potential

\[ V_{FF} = \sigma \min(2r, \sqrt{3}\rho + r) \]
We can determine the model’s bound states by using finite differences.

We find a bound state for $l_r = 3$.

This does not work for scattering with more than one channel.
Flip-flop toy model III

- Project in the meson states: \( \psi_i(\rho) = \int d^3r \, \phi_i(r)^* \Psi(r, \rho) \) with
  \[- \frac{\hbar^2}{2m} \nabla^2_r \phi_i + 2\sigma r \phi_i = \epsilon_i \phi_i \]

- Coupled channels equation:
  \[- \frac{\hbar^2}{2m} \nabla^2_\rho \psi_i + V_{ij} \psi_j = (E - \epsilon_i) \psi_i \]

- Use asymptotic behavior (scattering \( i \to j \)):
  \[
  \begin{align*}
  \psi_j &\to \delta_{ij} \hat{j}_\rho(k_i \rho) + f_{ij}^l \frac{e^{ik_j \rho}}{\rho} \quad \text{if channel is open} \\
  \psi_j &\to 0 \quad \text{otherwise}.
  \end{align*}
  \]

- This way we can determine the phase shifts
We find resonances with high angular momentum

This was in agreement with some authors\textsuperscript{4} who postulated that the centrifugal barrier would prevent the tetraquark from decaying

\textsuperscript{4} Karliner and Lipkin, Phys. Lett. B 575, 249
However this potential neglects some important effects

- Only one transition is considered
- Color is omitted

We have two linearly independent color singlets

The potential of the system is a $2 \times 2$ matrix

Lattice results give $v_0 = \min(V_I, V_{II}, V_T)$

We have to find the whole matrix!
Two meson states:

- $|C_I\rangle = \frac{1}{3}|Q_i Q_j \bar{Q}_i \bar{Q}_j\rangle$
- $|C_{II}\rangle = \frac{1}{3}|Q_i Q_j \bar{Q}_j \bar{Q}_i\rangle$

Antisymmetric and symmetric color states:

- $|A\rangle = \frac{\sqrt{3}}{2}(|C_I\rangle - |C_{II}\rangle)$
- $|S\rangle = \sqrt{\frac{3}{8}}(|C_I\rangle + |C_{II}\rangle)$

Note that $\langle C_I | C_{II} \rangle = \frac{1}{3}$

Ground state can be $|C_I\rangle$, $|C_{II}\rangle$ or $|A\rangle$
Potential II

- The corresponding eigenvector should be $|C_I\rangle$, $|C_{II}\rangle$ or $|A\rangle$
  - $|C_I\rangle$ when $v_0 = V_I$
  - $|C_{II}\rangle$ when $v_0 = V_{II}$
  - $|A\rangle$ when $v_0 = V_T$

- Since the potential is hermitian, the second eigenvector must be orthogonal to the first one:
  - $|\overline{C}_I\rangle$ when $v_0 = V_I$, with $\langle \overline{C}_I | C_I \rangle = 0$
  - $|\overline{C}_{II}\rangle$ when $v_0 = V_{II}$, with $\langle \overline{C}_{II} | C_{II} \rangle = 0$
  - $|S\rangle$ when $v_0 = V_T$
Potential III

- For the transition to be as smooth as possible, we impose
  - $v_1 = \min(V_{II}, V_T)$ when $v_0 = V_I$
  - $v_1 = \min(V_I, V_T)$ when $v_0 = V_{II}$
  - $v_1 = \min(V_I, V_{III})$ when $v_0 = V_T$

- Lattice results seem to agree with this in the transition region \(^5\)
- With this we can construct a color-dependent flip-flop potential

\(^5\)Bornyakov et al., hep-lat/0508006
Meson-meson scattering I

- Wave-function is expanded as $\Psi = \Psi^A |C_A\rangle$
- Contra-variant color states $|C^A\rangle$ with $\langle C^A | C_B \rangle = \delta^A_B$
- Kinetic Energy operator
  $$\hat{T}_S = (\hat{T} + \hat{V}_I) |C_I\rangle \langle C^I| + (\hat{T} + \hat{V}_{II}) |C_{II}\rangle \langle C^{II}|$$
- Scattering potential
  $$\hat{V}_S = \hat{V} - \hat{V}_I |C_I\rangle \langle C^I| - \hat{V}_{II} |C_{II}\rangle \langle C^{II}|$$
- Schrödinger equation:
  $$T^A_A \Psi^A + \sum_B V^A_B \Psi^B = E \Psi^A$$
Each color component is expanded as

$$\Psi^A = \sum_k \phi^1_k(\rho_1) \phi^2_k(\rho_2) \psi^A_k(\mathbf{r}_A)$$

The $\phi^i_k(\rho_i)$ are eigenfunctions of the mesonic hamiltonian.

Integrate intra-mesonic degrees of freedom:

$$-\frac{\hbar^2}{2\mu_\alpha} \nabla^2 \psi^\alpha(\mathbf{r}) + \int d^3\mathbf{r}' \ v^\alpha_\beta(\mathbf{r}, \mathbf{r}') \psi^\beta(\mathbf{r}') = (E - \epsilon_\alpha) \psi^\alpha(\mathbf{r})$$

$v^\alpha_\beta$ is calculated using Monte Carlo (Metropolis algorithm).

By looking at the asymptotic behavior we can calculate the $T$ matrix.
For now we do the following approximations:

- Consider only heavy quarks
- Neglect spin effects
- Neglect dynamical quark effects
- Non-relativistic kinematics
Exotic channels

- We study the exotic $QQq\bar{q}$ channels
- In this case the Hamiltonian matrix has the form:
  \[
  \hat{H} = \begin{bmatrix}
  \hat{D} & \hat{A} \\
  \hat{A} & \hat{D}
  \end{bmatrix}
  \]
- Eigenfunctions have the form $\Psi_\xi = \begin{bmatrix} u \\ \xi u \end{bmatrix}$, with $\xi = \pm 1$
Results I

- Results for $m_Q = m_b$ and $m_q = m_c$
- We use $\sigma = 0.19 \text{GeV}^2$ and $\gamma = 0.4$
- Phase shifts, for $L = 0$

\[
\begin{align*}
L = 0, \xi = +1 & \quad \text{(preliminary results)} \\
L = 0, \xi = -1 & \quad \text{(preliminary results)}
\end{align*}
\]

- Bound state (Binding energy < 1 MeV)
Results II

- Phase shifts, for $L = 1$:

  \[ L = 1, \xi = +1 \]

  \[ L = 1, \xi = -1 \]

  (preliminary results)

- Resonance mass and width can be determined using Newton’s method
An unitarized method to compute the meson-meson scattering was developed.

In the heavy mass limit we find a bound states and resonances for $L = 0$.

For the $L = 1$ case we find a resonance.

Refinements should be easy to include:
- Spin-spin interactions, etc.
- Other potential models.

In principle, poles can be find directly using Newton’s method or similar.