A First Estimate of Triply Heavy Baryon Masses from the pNRQCD Perturbative Static Potential

Felipe J. Llanes-Estrada
in collaboration with Olga I. Pavlova and Richard Williams
(largely based on Eur.Phys.J. C72 (2012) 2019)

Universidad Complutense de Madrid
Departamento de Física Teórica I

February 5^{th} 2013, Excited QCD (Sarajevo)

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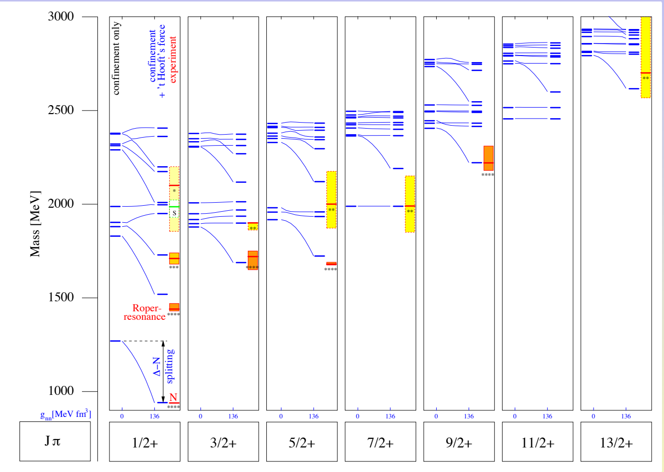
Much studied light-baryon spectrum



Löring, Metsch, Petry hep-ph/0103289

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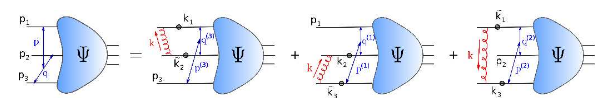
Validity of Faddeev equations?



Sanchis-Alepuz et al. hep-ph/1109.0199

QCD features Intrinsic three-body forces

Can we estimate them in the heavy-quark limit?

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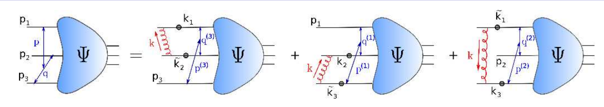

Sanchis-Alepuz et al. hep-ph/1109.0199

QCD features Intrinsic three-body forces
Can we estimate them in the heavy-quark limit?

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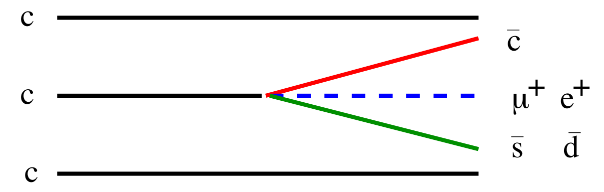
Constrain baryon number violation with heavy quarks?



- If $M(\Omega_{ccc}) > 5177$ MeV, $J/\psi D_s \mu^+$ open
- If $M(\Omega_{ccc}) > 4965$, $J/\psi D e^+$ open

Proton decay bounds + flavor universality + quark model (Gavela et al. 1981) imply unobservable rate, but nice to check in future experiments.

Except... mass unknown

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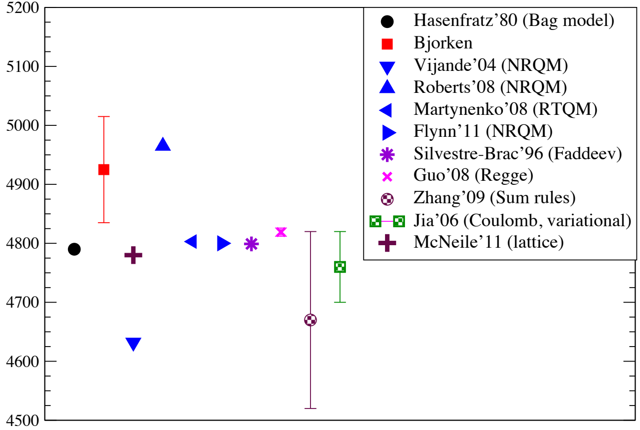
Ω_{ccc}: Existing predictions



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Last decade: pNRQCD

$$M_q \gg M_q v \gg M_q v^2, \Lambda$$

$$\begin{aligned} \mathcal{L}_{\rm pNRQCD} = & -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} \\ +\mathrm{Tr}\Big\{ \mathrm{S}^\dagger \left(i\partial_0 - V_s(r) + \dots \right) \mathrm{S} & + \mathrm{O}^\dagger \left(iD_0 - V_o(r) + \dots \right) \mathrm{O} \Big\} \\ & + g V_A(r) \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot\mathbf{E}\, \mathrm{S} + \mathrm{S}^\dagger \mathbf{r}\cdot\mathbf{E}\, \mathrm{O} \right\} \\ & + g \frac{V_B(r)}{2} \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot\mathbf{E}\, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \mathbf{r}\cdot\mathbf{E} \right\} \end{aligned}$$

Brambilla, Pineda, Soto, Vairo hep-ph/9907240

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The paradox

What happens when an irresistible force affects an unmovable object?

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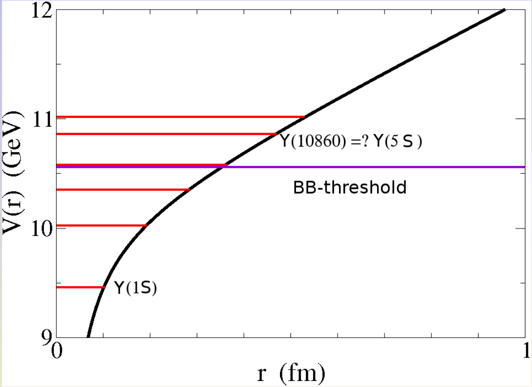
Solution in the weak sense



For matrix elements involving the ground state the "irresistible force" is not felt, simply a Coulomb potential with calculable perturbative corrections.

Ω_{ccc}: LO potential in pNRQCD

The Leading Order potential in pNRQCD is Δ-shaped
i.e. sum of two-body Coulomb interactions

$$V_{LO}^{(0)} = \frac{-2\alpha_s}{3}\left(\frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} + \frac{1}{|\mathbf{r}_2 - \mathbf{r}_3|} + \frac{1}{|\mathbf{r}_3 - \mathbf{r}_1|}\right) .$$

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Ω_{ccc}: NLO, NNLO potential in pNRQCD

NLO Still Δ shaped

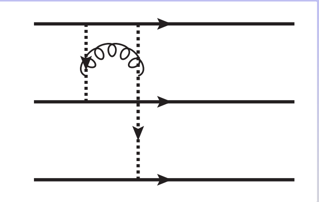
$$V^{(0)}_{LO} + V^{(0)}_{NLO} = \frac{-2}{3}\sum_i \alpha_s(|\mathbf{r}_i|^{-2})\frac{1}{|\mathbf{r}_i|}\times$$
$$\left[1+\frac{\alpha_s(|\mathbf{r}_i|^{-2})}{4\pi}\left(2\beta_0\gamma_E+a_1\right)\right]$$

NNLO: two parts, this Δ-shaped

$$V^{(0)}_{NNLO-2} = \frac{-2}{3}\sum_i \frac{\alpha_s(\mathbf{r}_i^{-2})}{|\mathbf{r}_i|}\frac{\alpha_s(\mathbf{r}_i^{-2})^2}{(4\pi)^2}\times$$
$$\left(a_2-36\pi^2+3\pi^4+\left(\frac{\pi^2}{3}+4\gamma_E^2\right)\beta_0^2+\gamma_E(4a_1\beta_0+2\beta_1)\right)$$

Brambilla, Ghiglieri, Vairo 2010

Now a three-body part is present!

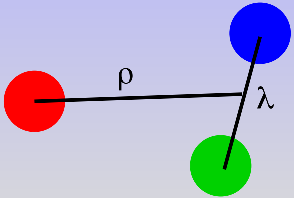


$$\hat{V}_{\text{aux}}(\mathbf{q}_2, \mathbf{q}_3) = \frac{(-i/2)(4\pi)^3 \alpha_s^3}{8|\mathbf{q}_2|^2|\mathbf{q}_3|^2} \times \tag{1}$$

$$\left[\frac{|\mathbf{q}_2 + \mathbf{q}_3|}{|\mathbf{q}_2||\mathbf{q}_3|} + \frac{\mathbf{q}_2 \cdot \mathbf{q}_3 + |\mathbf{q}_2||\mathbf{q}_3|}{|\mathbf{q}_2||\mathbf{q}_3||\mathbf{q}_2 + \mathbf{q}_3|} - \frac{1}{|\mathbf{q}_2|} - \frac{1}{|\mathbf{q}_3|} \right]$$

This is a non-Abelian QCD feature (perturbatively checked with four-jet events)

Variational computation



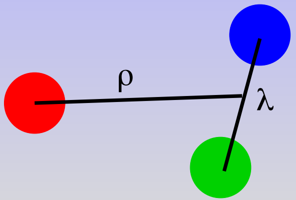
Jacobi coordinates:

$$F(\mathbf{k}_\rho, \mathbf{k}_\lambda) = \psi\left(\frac{\mathbf{k}_\rho}{\alpha_\rho}\right)\psi\left(\frac{\mathbf{k}_\lambda}{\alpha_\lambda}\right)$$

Two variational parameters α_ρ, α_λ

Minimize $\langle N|H|N\rangle(\alpha_\rho, \alpha_\lambda)$

Variational computation



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Minimize $\langle N|H|N\rangle(\alpha_\rho, \alpha_\lambda)$

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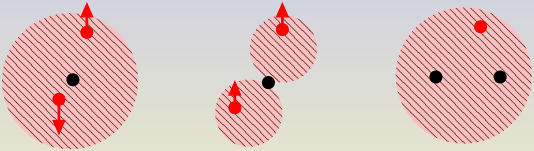
Ω_{ccc}: Estimate of the variational precision

Employ simple atomic/molecular systems!



Para–Helium **Ortho–Helium** **Dihydrogen cation**

Ω_{ccc}: Estimate of the variational precision

Helium $\tilde{k} \equiv k/(m_e \alpha_{em})$

$$\langle H \rangle_\psi = m_e \alpha_{em}^2 \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \psi^*(k_1, k_2) \times$$

$$\left[\frac{1}{2} \left(\tilde{k}_1^2 + \tilde{k}_2^2 + \tilde{k}_3^2 \frac{m_e}{m_\alpha} \right) \psi(k_1, k_2) + \right.$$

$$\left. \int \frac{4\pi d^3 q}{(2\pi)^3 q^2} \left(\psi(k_1+q, k_2-q) - 2\psi(k_1+q, k_2) - 2\psi(k_1, k_2+q) \right) \right]$$

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Ω_{ccc}: Estimate of the variational precision

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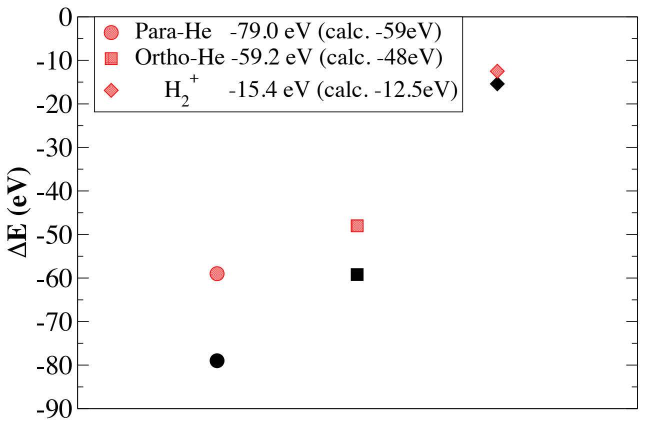
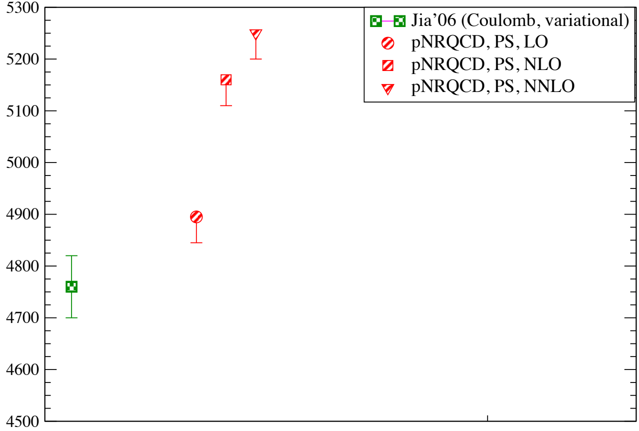


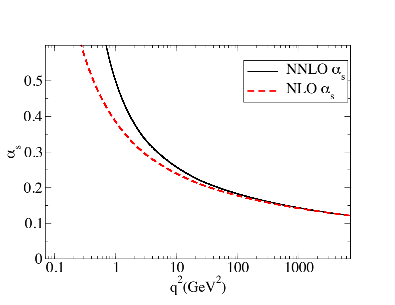
Ω_{ccc}: Variational pNRQCD predictions



Running coupling

$$\frac{\partial \alpha_s}{\partial \log q^2} = -\frac{\beta_0}{4\pi}\alpha_s^2 - \frac{\beta_1}{(4\pi)^2}\alpha_s^3$$

(Solved by Runge-Kutta to necessary order down to $\Lambda \simeq 0.4 - 0.6$ GeV)



- Pole mass: IR sensitive (renormalon). Freeze constant $\alpha_s(\mu < \Lambda) = \alpha_s(\mu = \Lambda)$
- PS scheme: Subtract IR. We set $\alpha_s(\mu < \Lambda) = 0$

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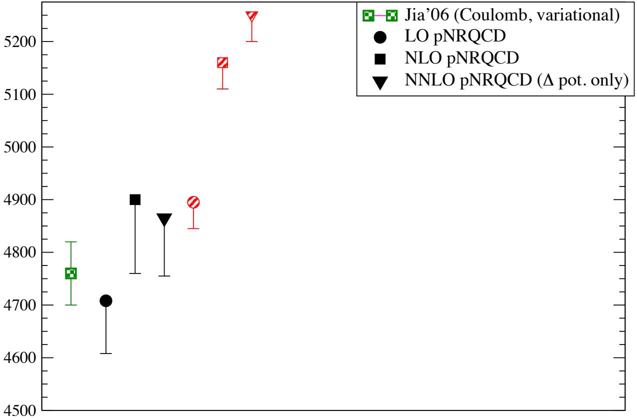
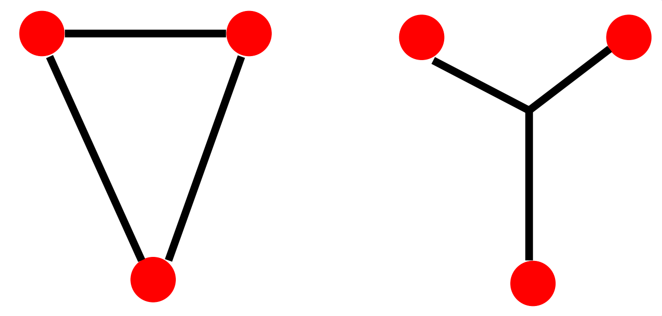
Ω_{ccc}: Potential Subtraction Scheme



(Infrared cutoff at 0.4 GeV)

Ω_{ccc}: Triangle versus Star confinement



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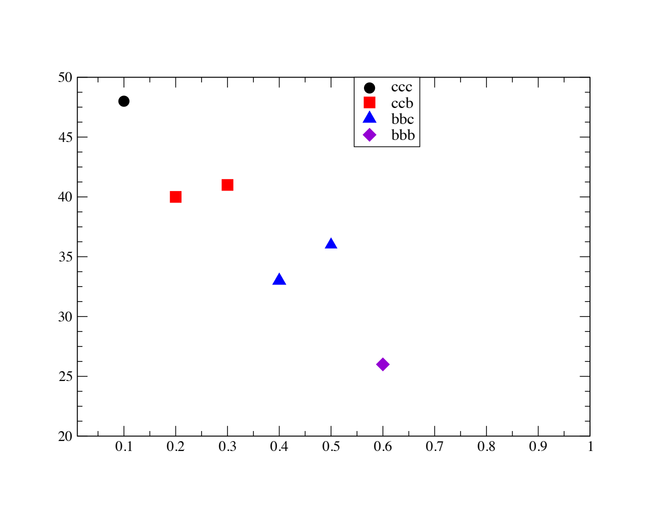
Ω_{ccc}: Triangle versus Star confinement

A new Coulomb+linear calculation by Flynn, Hernandez, Nieves 2011



$M_Y - M_\Delta$

Ω_{ccc}: Perturbative 3-body contribution

Our computation of ΔE by including/not including the three-body part:

- Scale set as $\alpha_s^3(m_c^2) \rightarrow O(1MeV)$
- Scale set as $\alpha_s(q^2)\alpha_s(q'^2)\alpha_s(qq') \rightarrow O(20MeV)$

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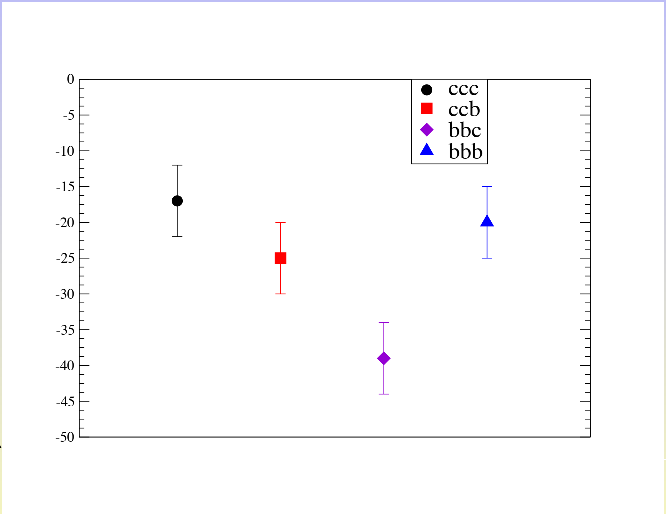
Ω_{ccc}: Perturbative 3-body contribution



$M_Y - M_\Delta$

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- Moderate value (20-40 MeV) at NNLO (good for the Faddeev formulation)
- Moderate (40 MeV) value reported by Flynn et al in a model calculation will not be enough to establish it with expt. data given the spread of errors.
- Need to develop better tests to extract at colliders

Ω_{ccc} Error estimate

- From 9-12 dimensional Montecarlo integration: 10 MeV
- Variational error: 25% of binding energy or 150 MeV (sign known)
- Perturbation theory: NLO to NNLO at most 100 MeV
- From parameters, 50 MeV
- Infrared effects: 200 *MeV*
- Our prediction from pNRQCD: 4800(250) MeV.

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Ω_{ccc}: Decay and detection

- Lightest triply-charmed state in Hilbert space $(3 \times D(1867) \simeq 5600$ MeV $> M_{\Omega_{ccc}} \in (4600-5000)$ MeV
- Long lived: weak decays

$$
\begin{aligned}
\Omega_{ccc} &\rightarrow \Xi_{ccs}\pi \\
&\rightarrow \Xi_{css}\pi\pi \\
&\rightarrow \Omega_{sss}\pi\pi\pi
\end{aligned}
$$

- 10 000-100 000 produced for 10 fb^{-1} at LHC Chen, Wu 1106.0193 (very tough)

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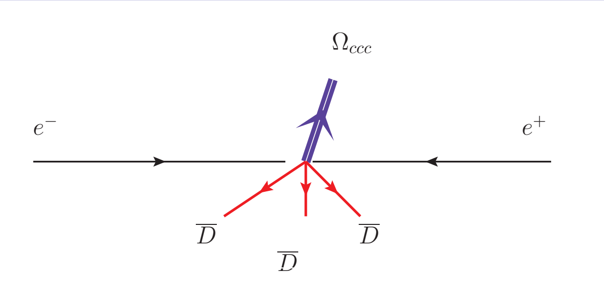
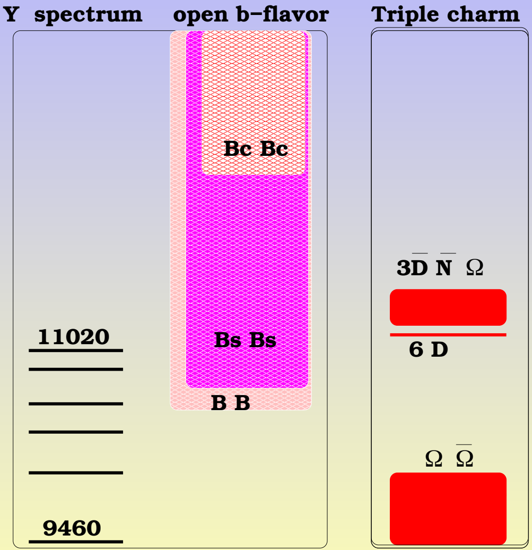
Ω_{ccc}: Decay and detection

Analyze $e^- e^+ \to \Omega_{ccc}\overline{p}\overline{DDD}$ or $\Omega_{ccc}\overline{\Lambda}_c\overline{DD}$ by recoil method (in addition to direct reconstruction) (A. Drutskoy et al. arXiv:1210.6623)

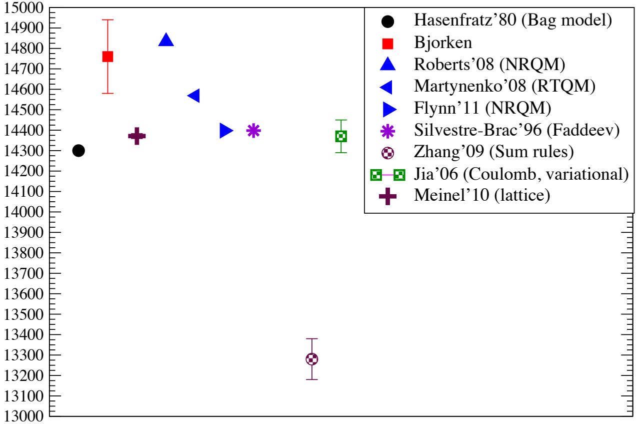


3fb at SuperB? (adapting Baranov and Slad, Phys.Atom.Nucl.67,808, 2004)

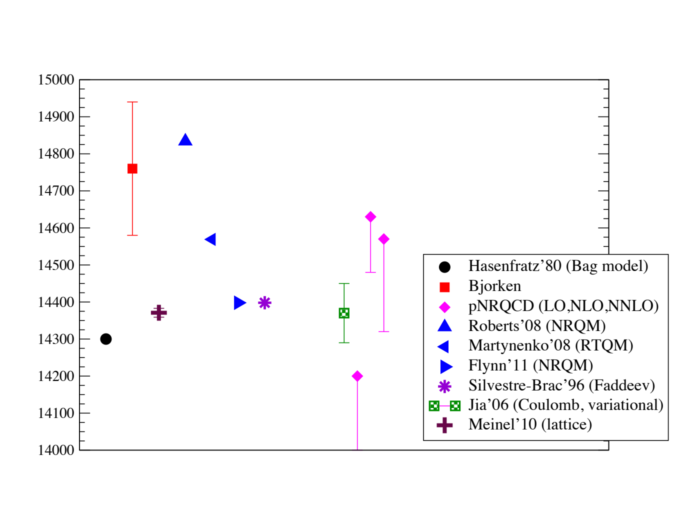
Triple charm thresholds



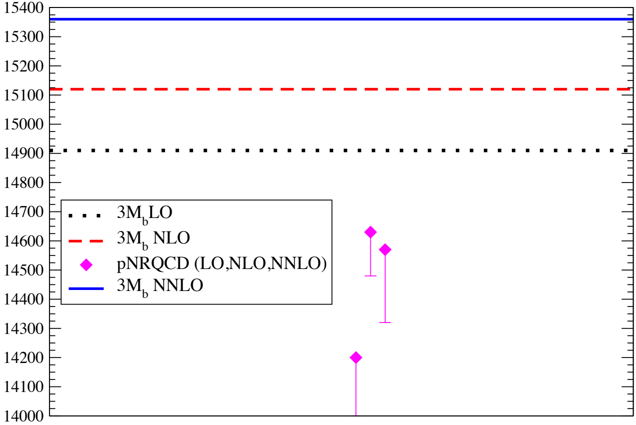
Ω_{bbb}: Existing estimates



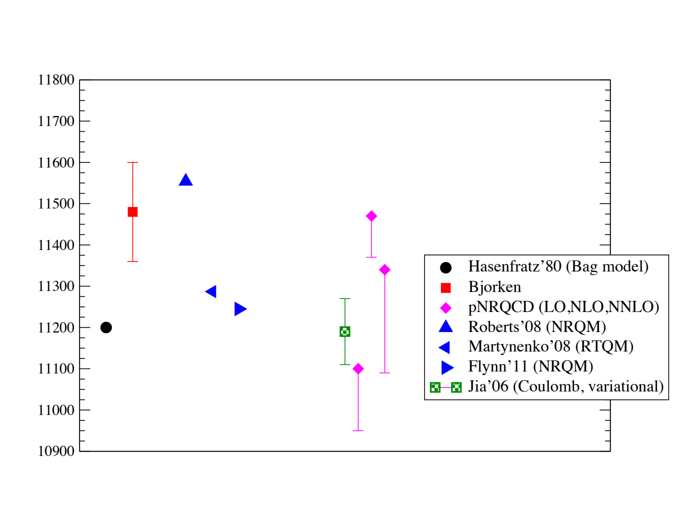
Ω_{bbb}: pNRQCD estimate



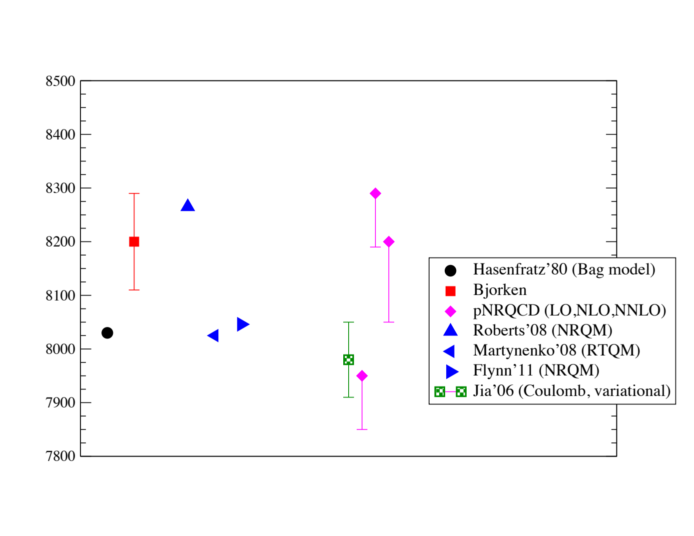
Ω_{bbb}: Binding energy



Ξ^*_{bbc}: pNRQCD estimate



Ξ^*_{ccb}: pNRQCD estimate



Summary

- Studied triply-heavy baryon systems in pNRQCD
- Predict 4900(250) MeV for Ω_{ccc}; also 8200, 11300 and 14600 MeV for ccb, bbc and bbb respectively
- Three-body force seems negligible (order 25 MeV) at NNLO

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Possible future work

1. Improve on variational calculation with systematic diagonalization
2. Adopt R-evolution for IR sensitivity

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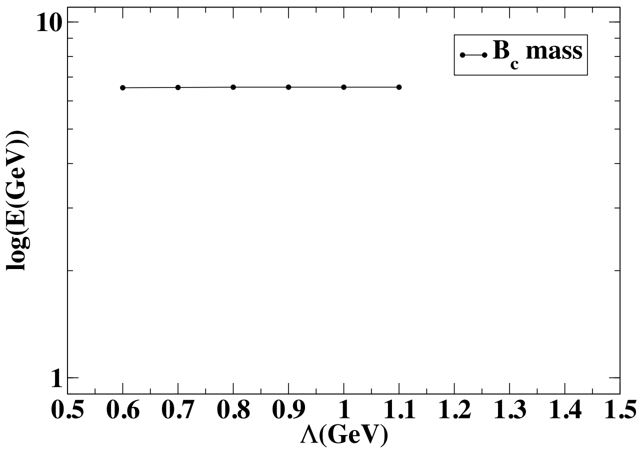
R-evolution example



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Observables

Table: Experimental data (PDG) input to MINUIT

$(M_{\eta_b} + 3M_{\Upsilon})/4$	$9443(1)\ MeV$
$(M_{\chi_{b0}} + 3M_{\chi_{b1}} + 5M_{\chi_{b2}})/9$	$9899.9(3)\ MeV$
$M_{\Upsilon(2S)}$	$10023(0.3)\ MeV$
$(M_{\eta_c} + 3M_{J/\psi})/4$	$3067.7(3)\ MeV$
$\Gamma_{J/\psi \to \eta_c \gamma}$	$1.6(4)\ keV$
$M_{B_c} + (3/4)\Delta M_{hf}$	$6317(8)\ MeV$

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$J/\psi \to \gamma\eta_c$

$$\Gamma_{J/\psi\to\eta_c\gamma} = \frac{16\alpha_{\rm em}}{3} e_c^2 \frac{k_\gamma^3}{M_{J/\psi}^2} \times \tag{2}$$

$$\left(1 + \frac{4}{3}\frac{\alpha_s((M_{J/\psi}/2)^2)}{\pi} - \frac{2}{3}\left(\frac{4}{3}\alpha_s(r^{-2})\right)^2\right)$$

(Brambilla, Jia, Vairo hep-ph/0512369)

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Quality of typical fit (many variations in our article)

Quantity	Expt.	LO	NLO	NNLO
$(M_{\eta_c}+3M_{J/\psi})/4$	3067.7(3)MeV	3068	3068	3068
$M_{B_c}+(3/4)\Delta M_{hf}$	6317(8)MeV	6112	6118	6277
$\frac{M_{\chi_{b0}}+3M_{\chi_{b1}}+5M_{\chi_{b2}}}{9}$	9899.9(3)MeV	9891	9897	9888
$(M_{\eta_b}+3M_{\Upsilon})/4$	9443(1)MeV	9542	9481	9577
$\Gamma_{J/\psi\to\eta_c\gamma}$	1.6(4)keV	2.7	3.1	2.5
$M_{\Upsilon(2S)}$	10023(0.3)MeV	9829	9800	9804

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Typical parameter set from various NNLO runs

Parameter	Fits			PDG reviews
m_c (pole, GeV)	1.67	1.83	1.76	1.47-1.83
m_b (pole, GeV)	4.96	5.12	5.08	4.71-4.98
$\alpha_s(1.3\mathrm{GeV})$	0.336	0.348	0.262	0.39

(at the Z pole $\alpha_s \in (0.105 - 0.111)$ just under world average. Tau decays for example suggest 0.1204 see for example 1107.1123 by A. Pich)

Behavior of perturbation theory

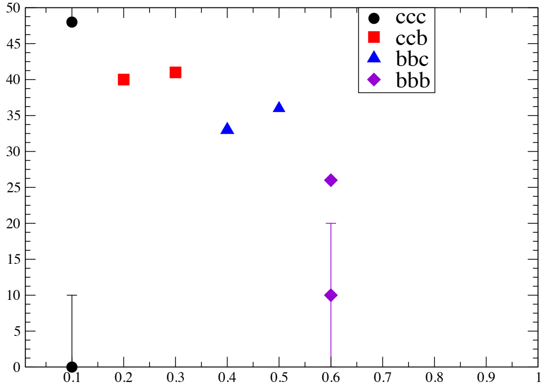
Beneke's Potential Subtraction scheme, $\lambda = 0.4\ GeV$.

Parameter	LO	NLO	NNLO
$\alpha_s(m_Z)$	0.115*	0.112	0.105
$m_c(m_c)$	1.63	1.72	1.75
$m_b(m_b)$	4.96	5.03	5.05

* ran to m_Z at NLO; at limit of allowed fit range

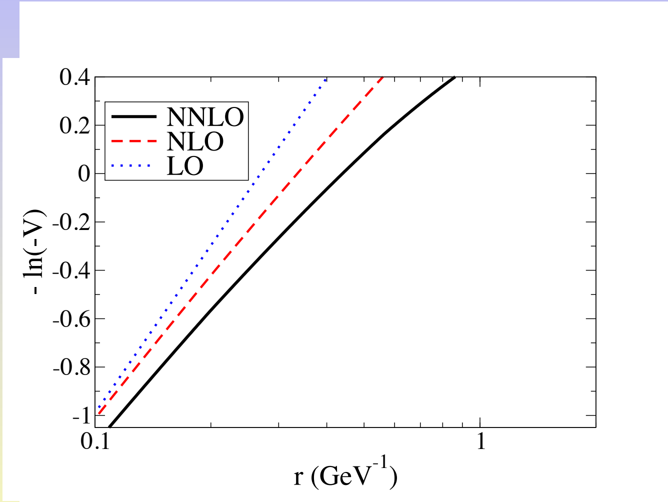
Ω_{ccc}: Perturbative energy

Superimpose ΔE by including/not including the three-body part: compatible with zero (Montecarlo error)



$M_Y - M_\Delta$

Static potential



Static potential in pNRQCD

$$V^{(0)} = V^{(0)}_{LO} + V^{(0)}_{NLO} + V^{(0)}_{NNLO} \cdots$$

The leading order potential is just the color Coulomb potential

$$V^{(0)}_{LO} = -\frac{4}{3}\frac{\alpha_s(r^{-2})}{r} \quad (3)$$

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Static potential in pNRQCD

NLO and NNLO

$$V^{(0)}_{NLO} = V^{(0)}_{LO} \times (a_1 + 2\gamma_E \beta_0)\frac{\alpha_s(r^{-2})}{4\pi}$$

$$V^{(0)}_{NNLO} = V^{(0)}_{LO} \times$$

$$\left(\gamma_E(4a_1\beta_0 + 2\beta_1) + (\frac{\pi^2}{3} + 4\gamma_E^2)\beta_0^2 + a_2\right)\frac{\alpha_s^2(r^{-2})}{(4\pi)^2}$$

$V_{1/m}$ is convention-dependent

$$V_{m^{-1}} = \frac{-\alpha_s^2(\mu)}{m_r r^2} \times \left(\frac{7}{9}\right)$$

$$-\frac{\alpha_s^3}{3\pi m_r r^2}\left\{-b_2 + \log(e^{2\gamma_E}\mu^2 r^2)\left(\frac{7\beta_0}{6} + \frac{68}{3}\right)\right\}$$

(A. Vairo 2001; N. Brambilla et al. 2000)

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