

# Lattice QCD based equation of state at finite baryon density

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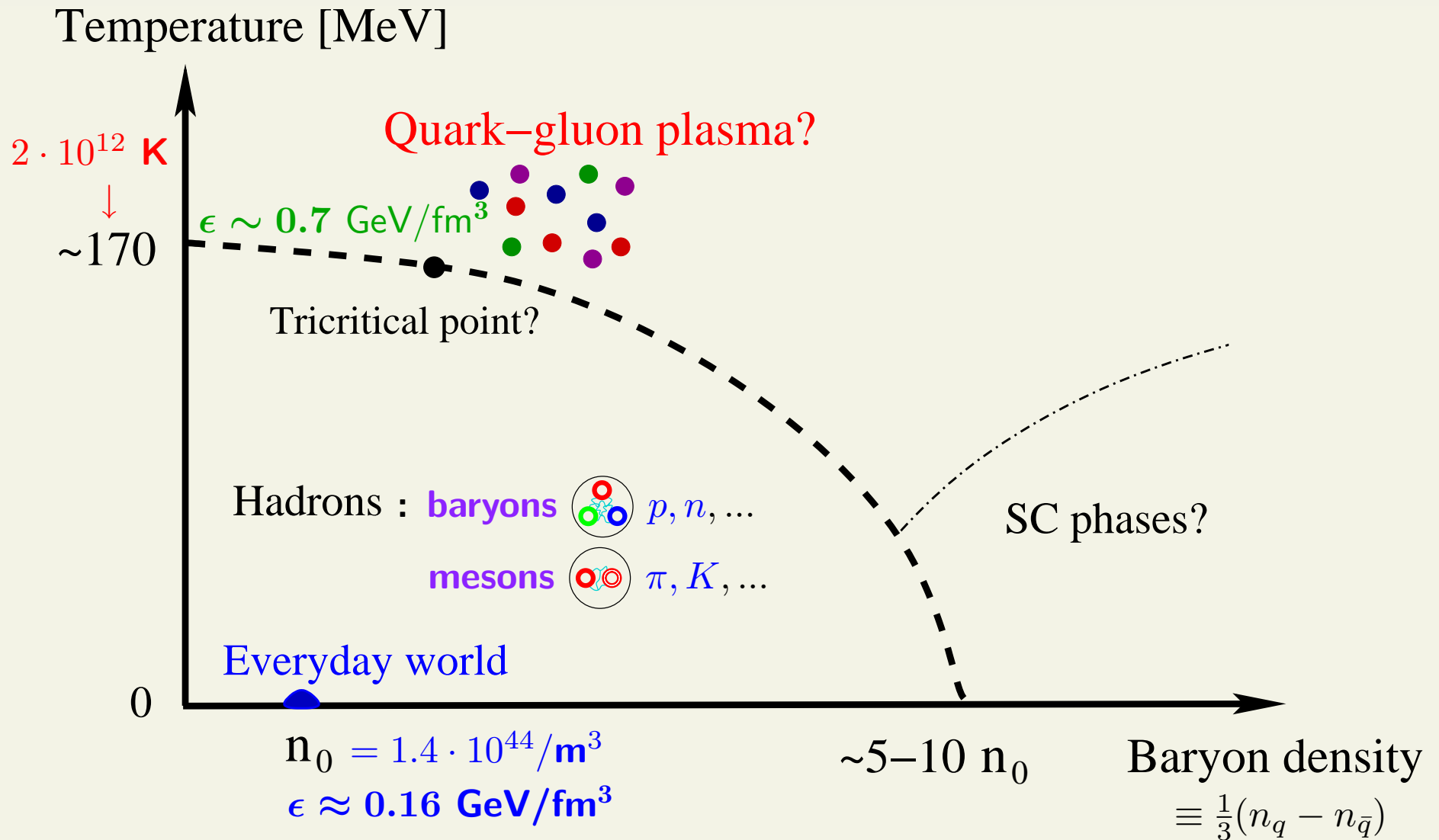
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# Nuclear phase diagram



# Taylor expansion for pressure

$$\frac{P}{T^4} = \sum_{i,j} c_{ij}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_S}{T}\right)^j,$$

where

$$c_{ij}(T) = \frac{1}{i!j!} \frac{\partial^i}{\partial(\mu_B/T)^i} \frac{\partial^j}{\partial(\mu_S/T)^j} \frac{P}{T^4},$$

i.e. moments of baryon number and strangeness **fluctuations** and **correlations**

• an EoS based on lattice calculations of these?

**But:** Most extensive study using **p4 action** with  $N_\tau = 4$

⇒ **large discretization effects?**

# Hadrons on lattice

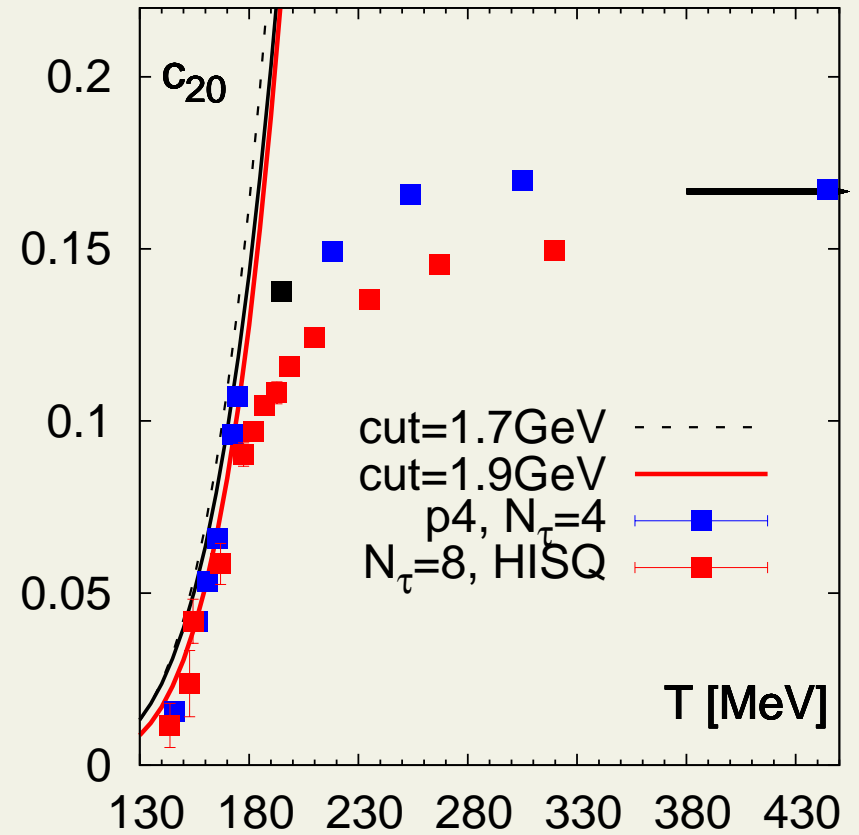
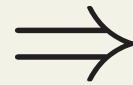
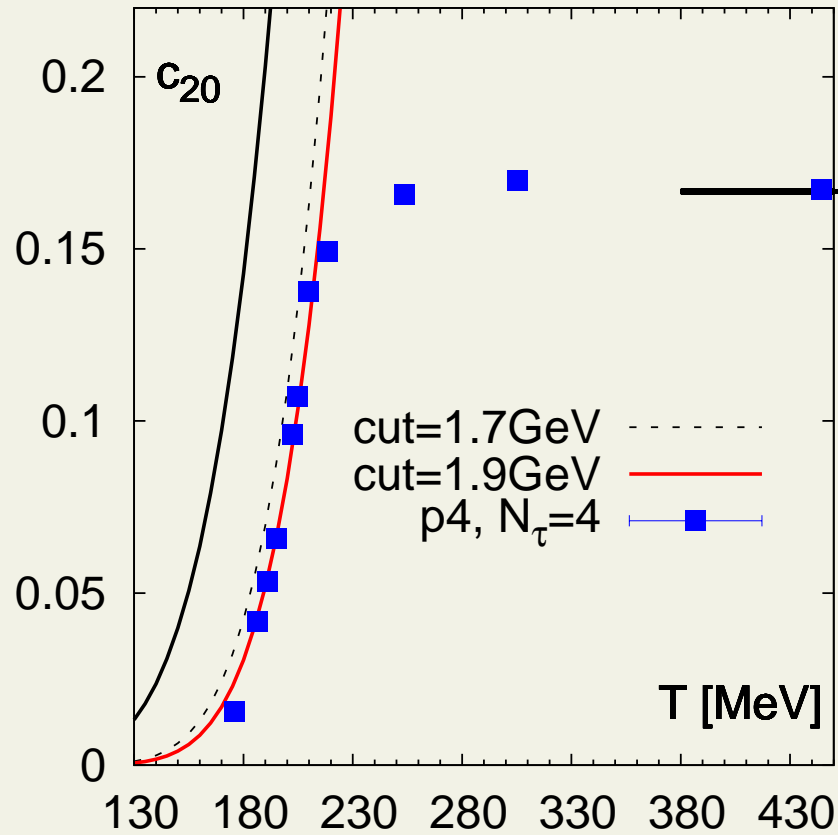
- **16** pseudoscalar mesons on lattice
- Hadron masses depend on lattice cutoff  
⇒ i.e. on **temperature**:  
E.g. for pseudoscalar mesons on asqtad calculations

$$m_{\text{ps}_i}^2 = m_{\text{ps}_0}^2 + \frac{1}{r_1^2} \frac{a_{\text{ps}}^i x + b_{\text{ps}}^i x^2}{(1 + c_{\text{ps}}^i x)^{\beta_i}}$$

$$x = (a/r_1)^2$$

$$a = \frac{1}{N_\tau T}$$

# 30 MeV shift



# Hadron Resonance Gas model

- EoS of **interacting** hadron gas well approximated by **non-interacting** gas of hadrons and resonances

$$P(T) = \sum_i \int d^3p \frac{p^2}{3E} f(p, T)$$

- valid when
  - interactions mediated by resonances
- Prakash & Venugopalan, NPA546, 718 (1992): experimental phase shifts
- Gerber & Leutwyler, NPB321, 387 (1989): chiral perturbation theory
  - ⇒ **HRG good approximation at low temperatures**
  - lattice should reproduce HRG at  $T \leq 120 - 140$  MeV
- **practical problem:** how to convert **fluid** to **particles**?
- energy conservation iff EoS is the same before and after freeze-out

# Parametrization

$$c_{ij}(T) = \frac{a_{1ij}}{\hat{T}^{n_{1ij}}} + \frac{a_{2ij}}{\hat{T}^{n_{2ij}}} + \frac{a_{3ij}}{\hat{T}^{n_{3ij}}} + \frac{a_{4ij}}{\hat{T}^{n_{4ij}}} + \frac{a_{5ij}}{\hat{T}^{n_{5ij}}} + \frac{a_{6ij}}{\hat{T}^{n_{6ij}}} + c_{ij}^{SB},$$

where  $n_{kij}$  are **integers** with  $1 < n_{kij} < 42$ ,  
and

$$\hat{T} = \frac{T - T_s}{R},$$

with  $T_s = 0.1$  or  $0$  **GeV**, and  $R = 0.05$  or  $0.15$  **GeV**.

## Constraints:

$$\begin{aligned}c_{ij}(T_{\text{sw}}) &= c_{ij}^{\text{HRG}}(T_{\text{sw}}) \\ \frac{d}{dT} c_{ij}(T_{\text{sw}}) &= \frac{d}{dT} c_{ij}^{\text{HRG}}(T_{\text{sw}}) \\ \frac{d^2}{dT^2} c_{ij}(T_{\text{sw}}) &= \frac{d^2}{dT^2} c_{ij}^{\text{HRG}}(T_{\text{sw}}) \\ \frac{d^3}{dT^3} c_{ij}(T_{\text{sw}}) &= \frac{d^3}{dT^3} c_{ij}^{\text{HRG}}(T_{\text{sw}})\end{aligned}$$

at  $T_{\text{sw}} = 155 \text{ MeV}$

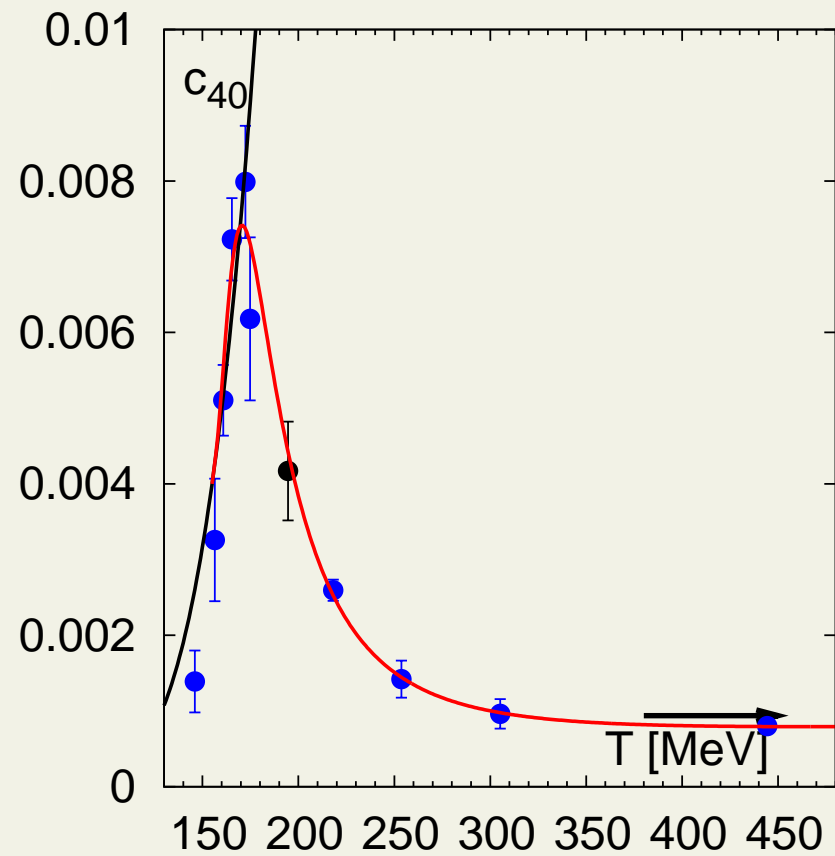
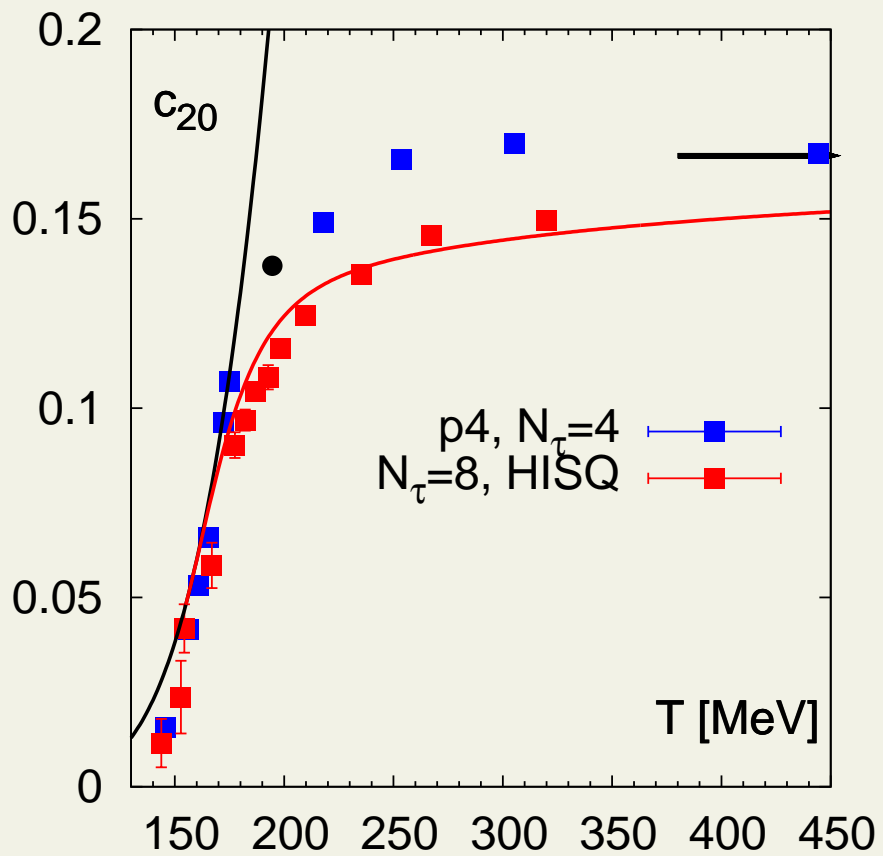
**3rd derivative** to guarantee smooth behaviour of speed of sound:

$$c_s^2 \propto \frac{d^2}{dT^2} c_{ij}$$

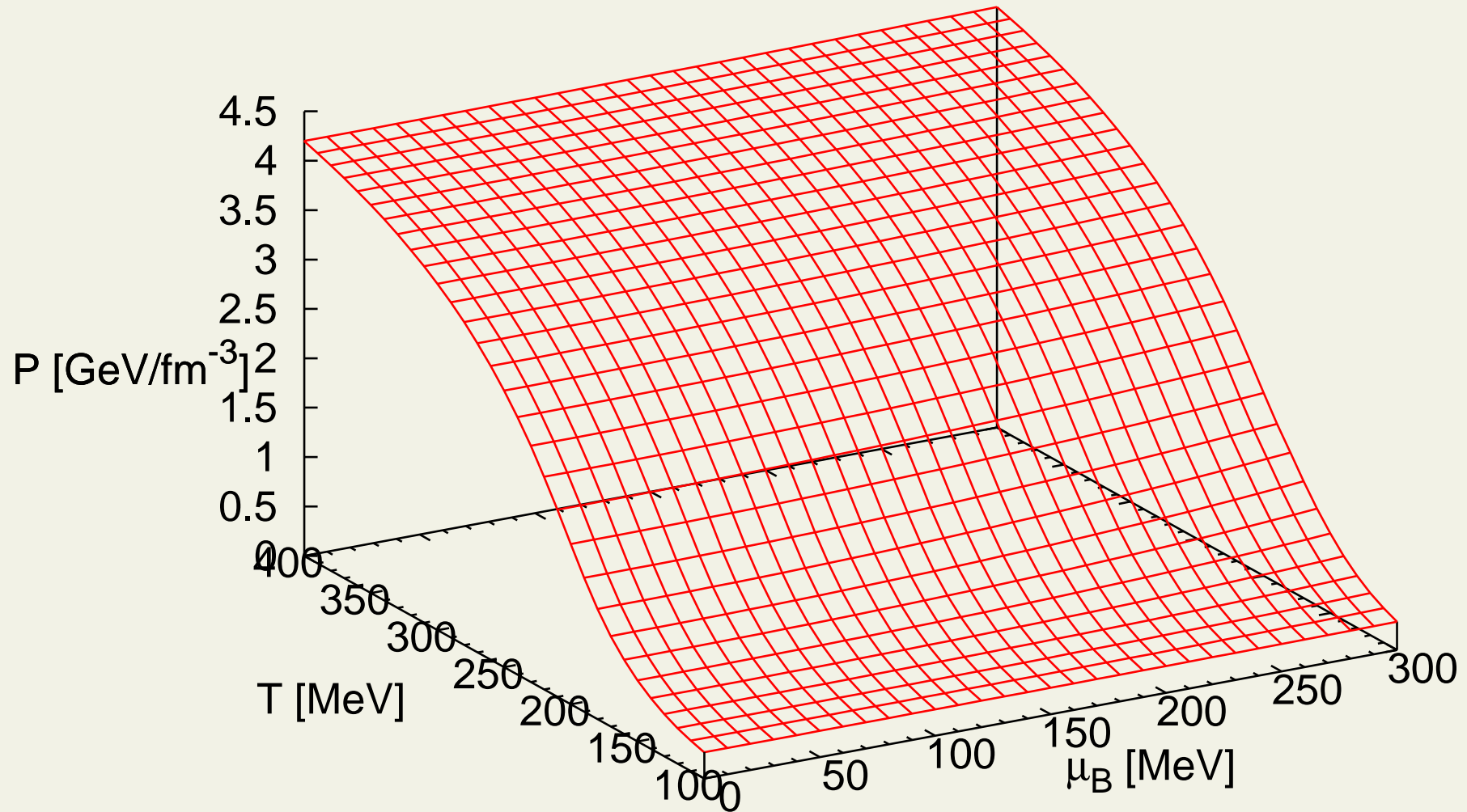
**For second order terms also**

$$c_{ij}(T = 800 \text{ MeV}) = 0.95 \cdot SB_{ij}$$

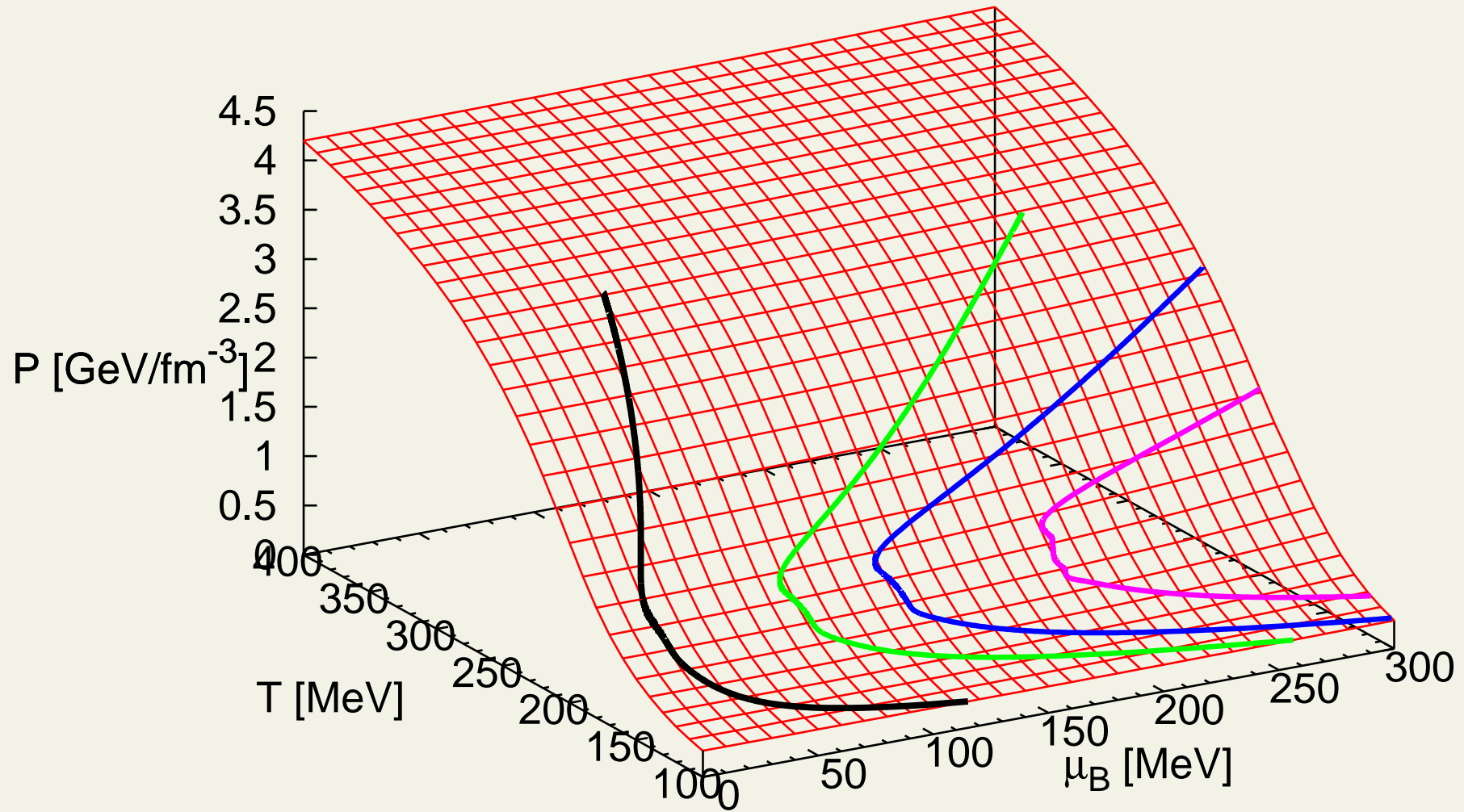




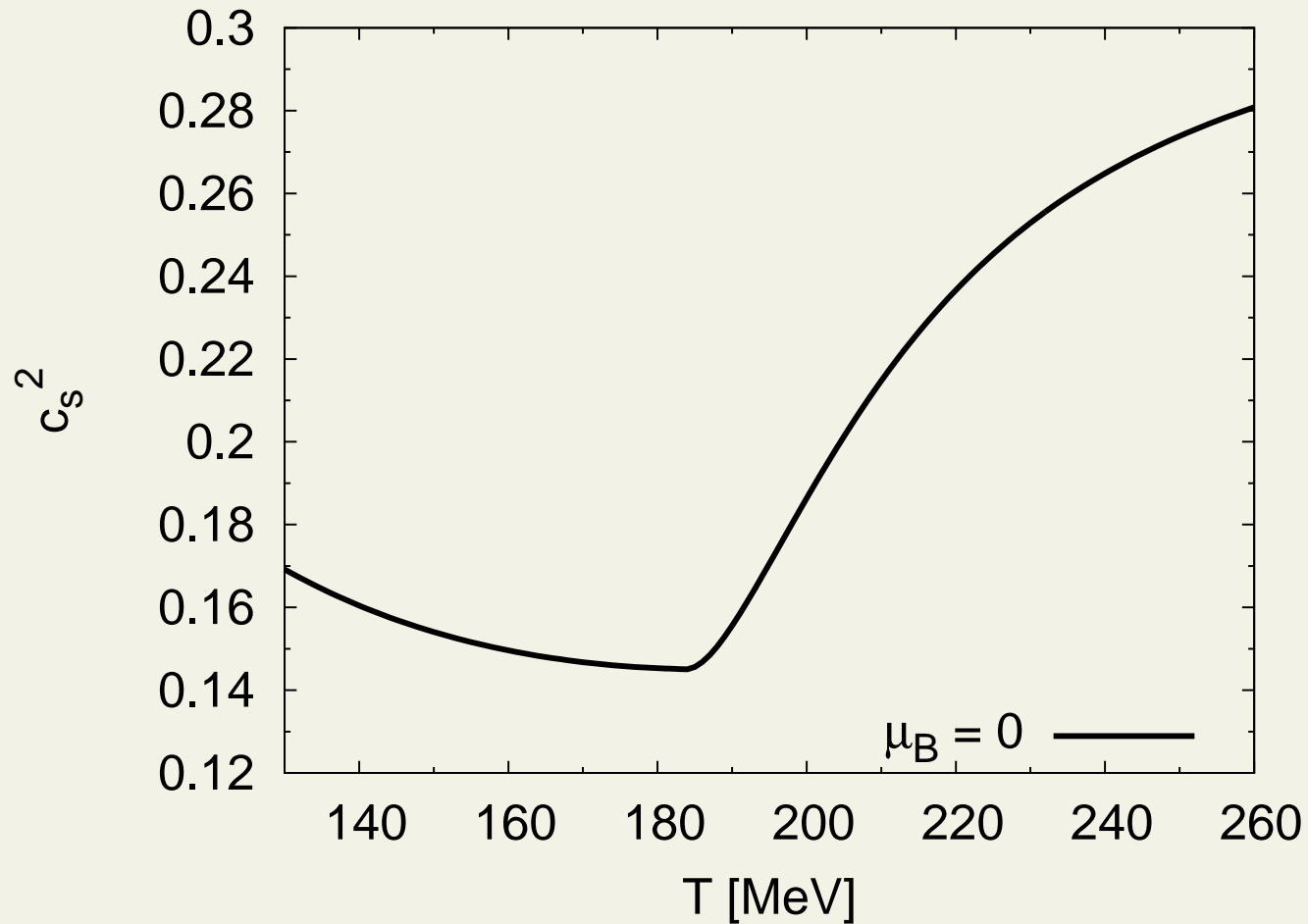
$$P/T^4$$



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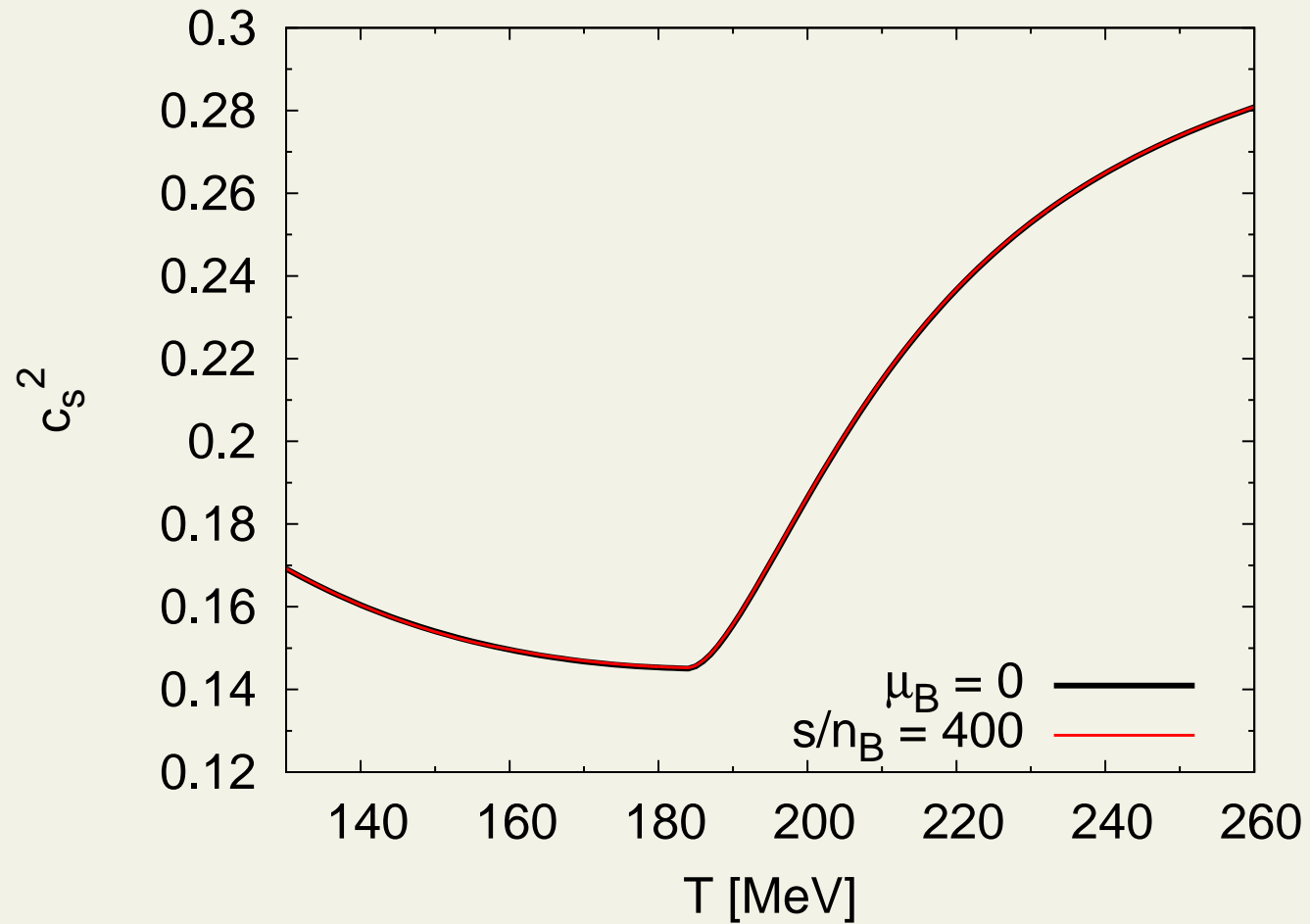


# Speed of sound

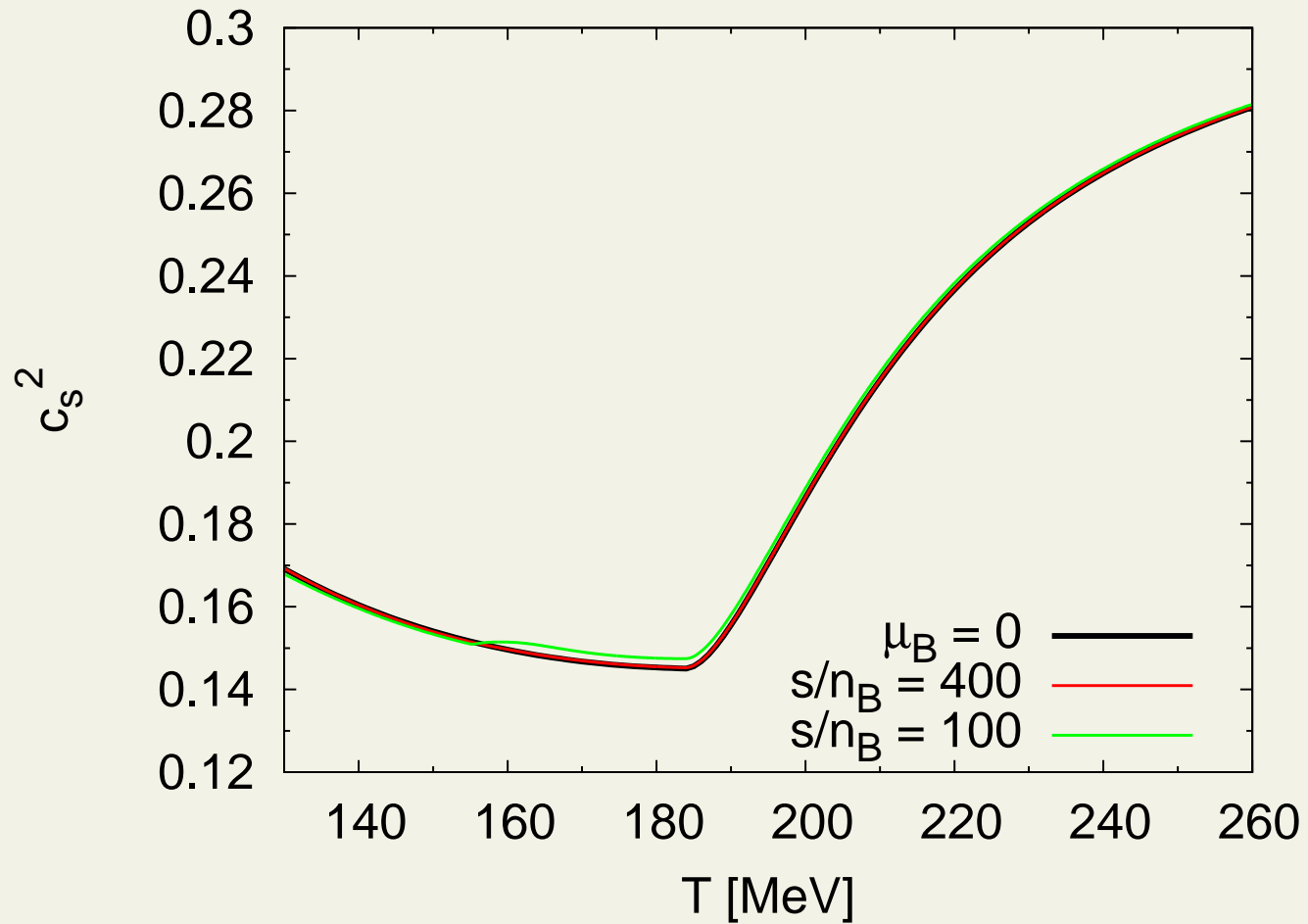


- [s95p-v1](#) parametrization by P. Petreczky and P.H.

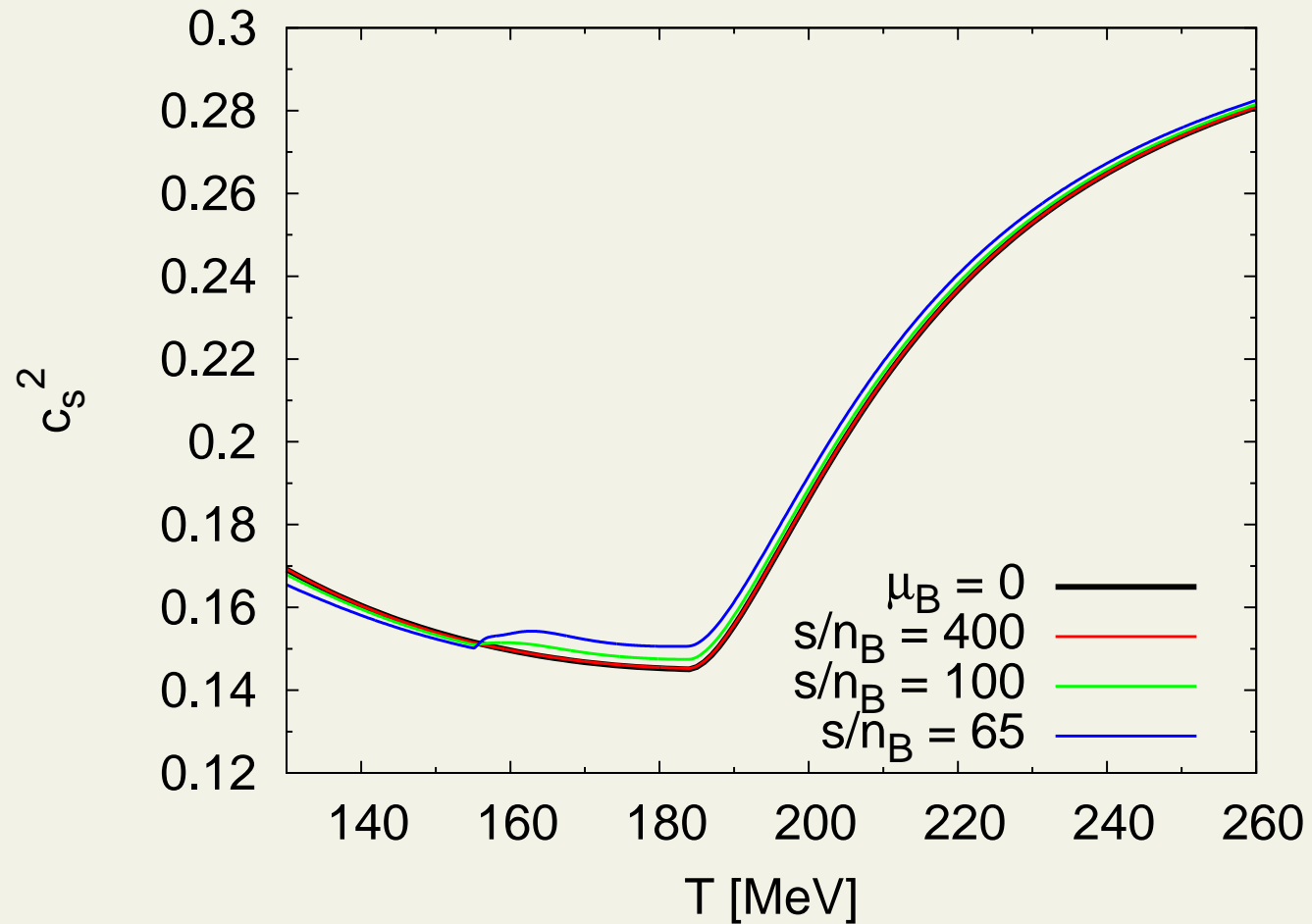
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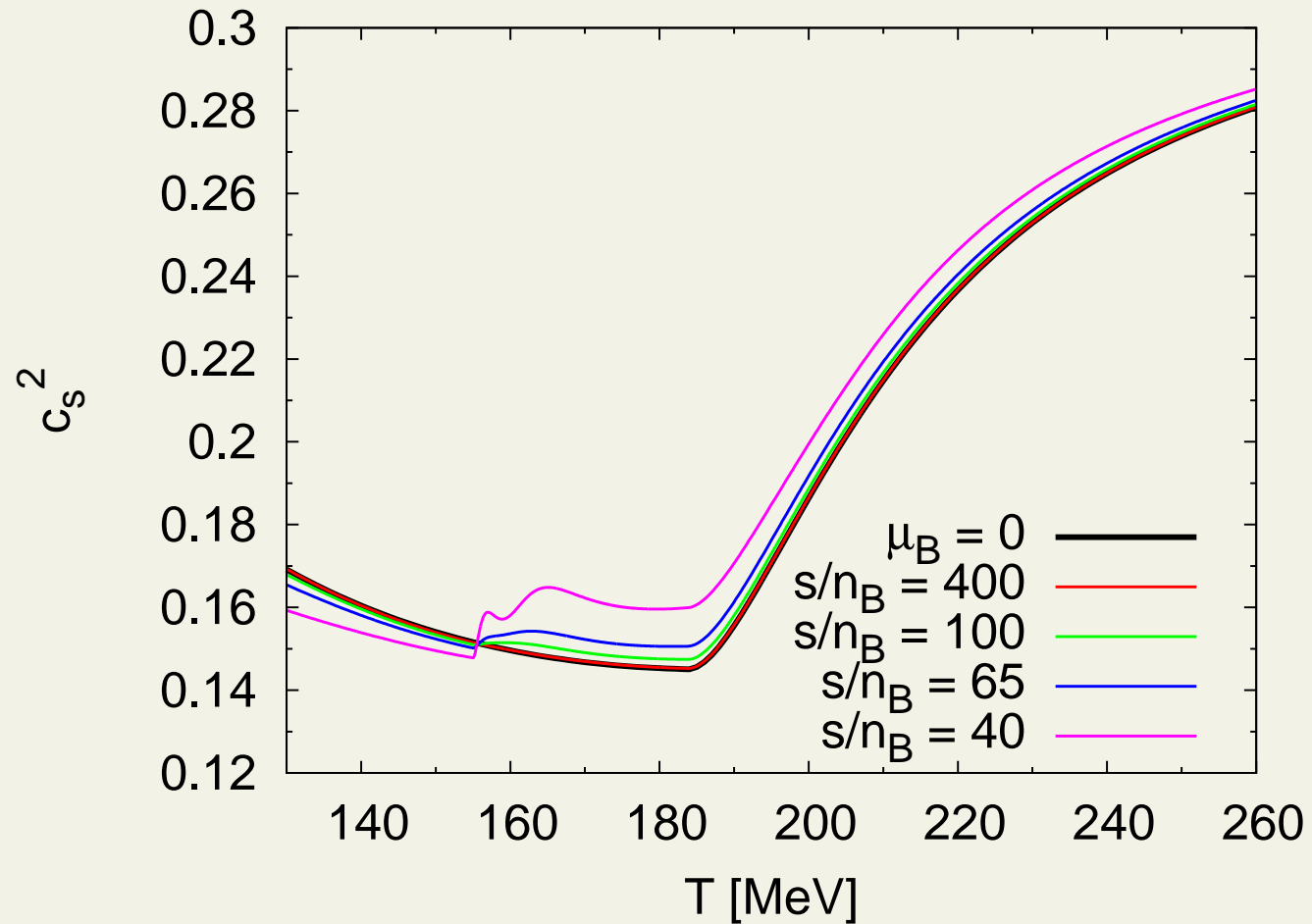
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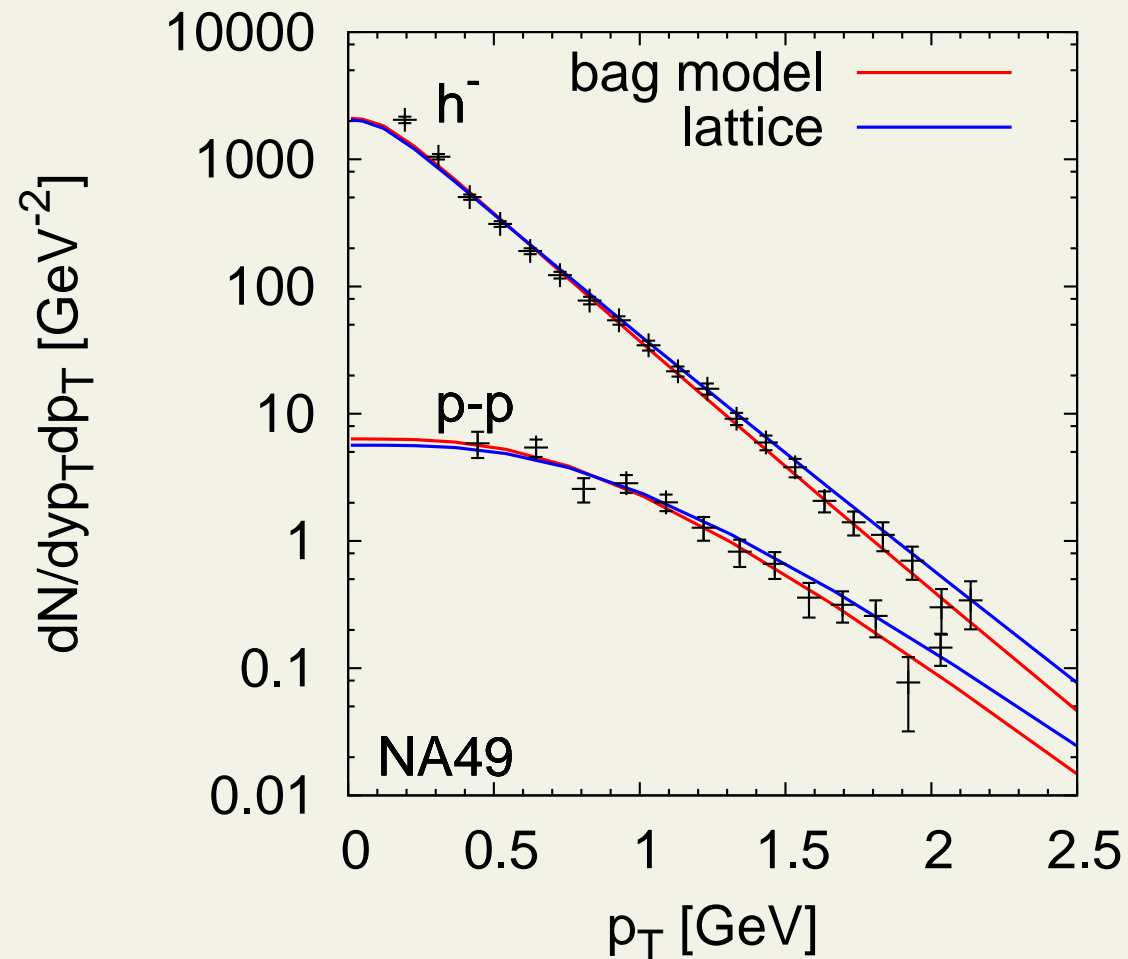


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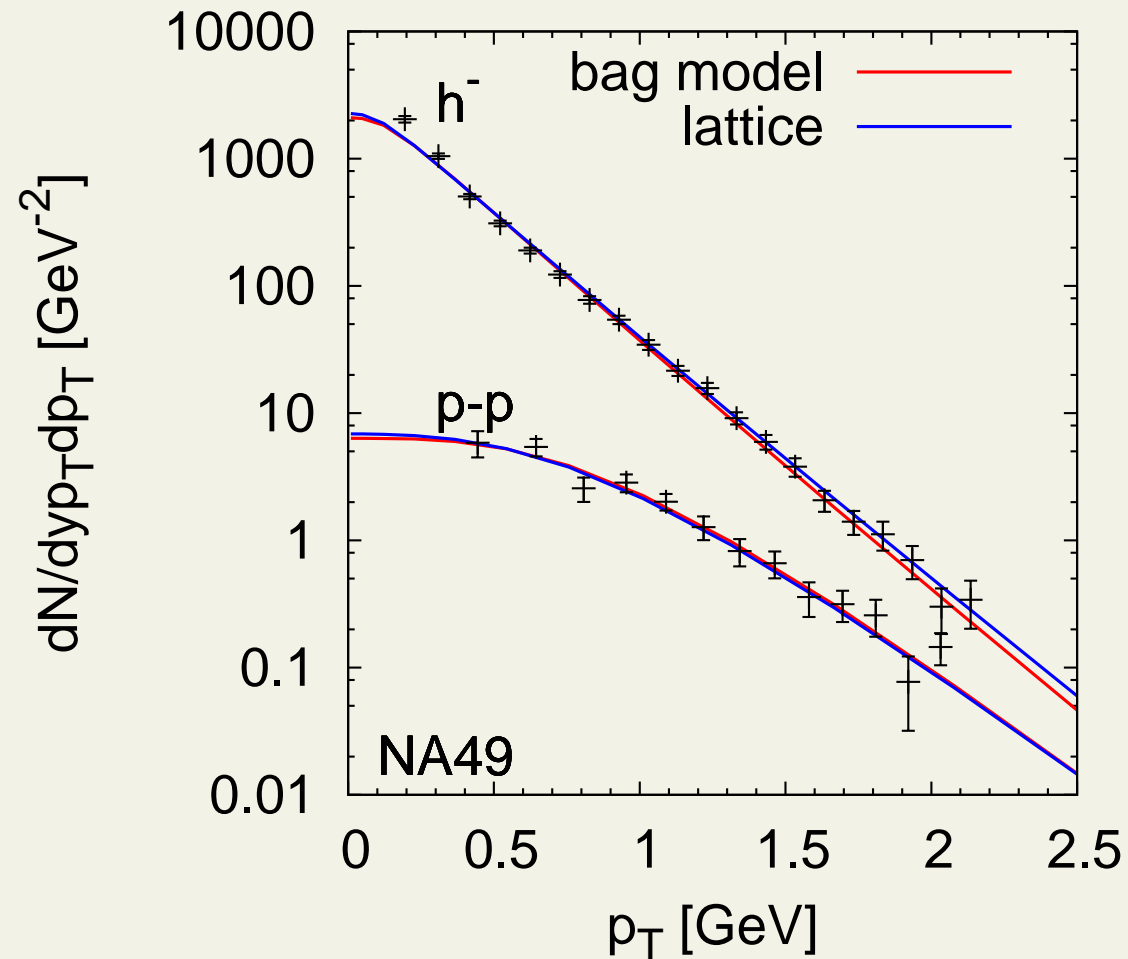


# $p_T$ -spectra at SPS



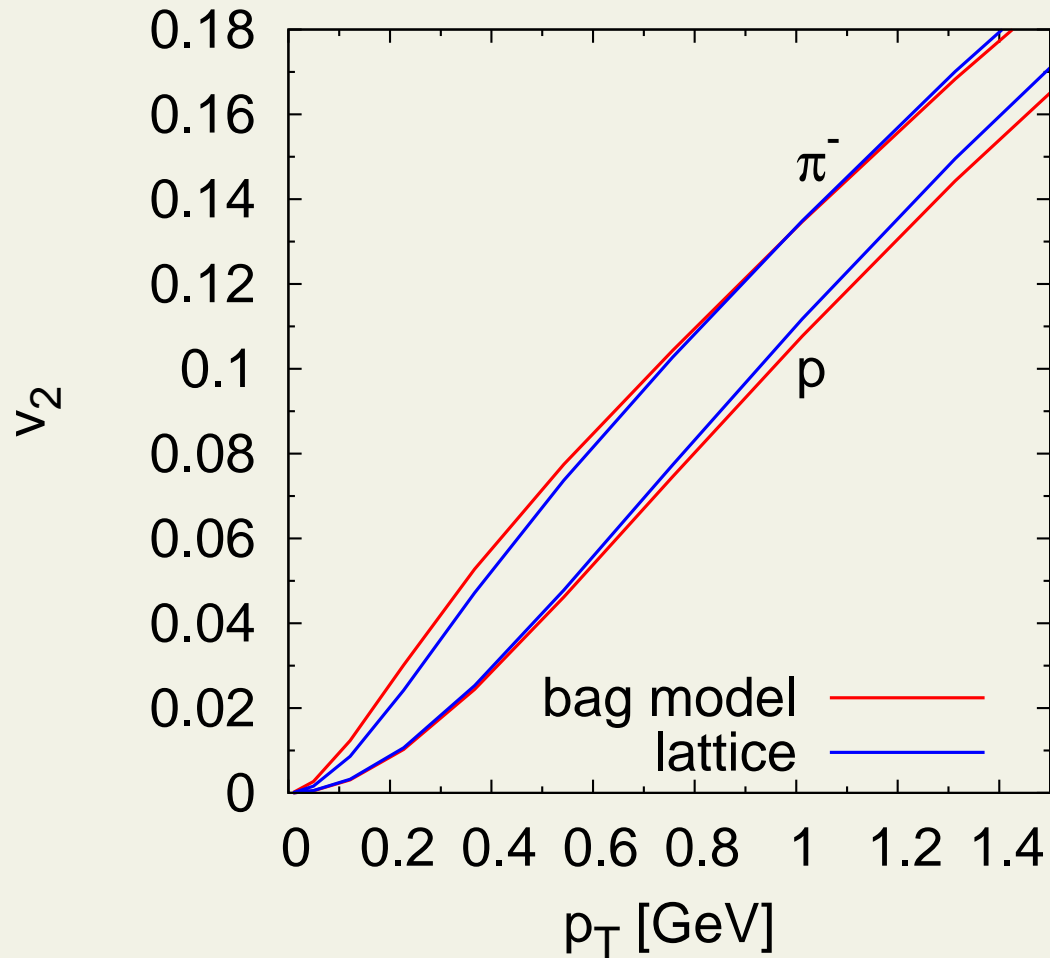
- **harder EoS, more transverse flow, flatter spectra**

# $p_T$ -spectra at SPS



- $T_{fo} \approx 120 \text{ MeV (bag)} \Rightarrow 130 \text{ MeV (lattice)}$

# $v_2$ at SPS ( $b = 7$ fm)



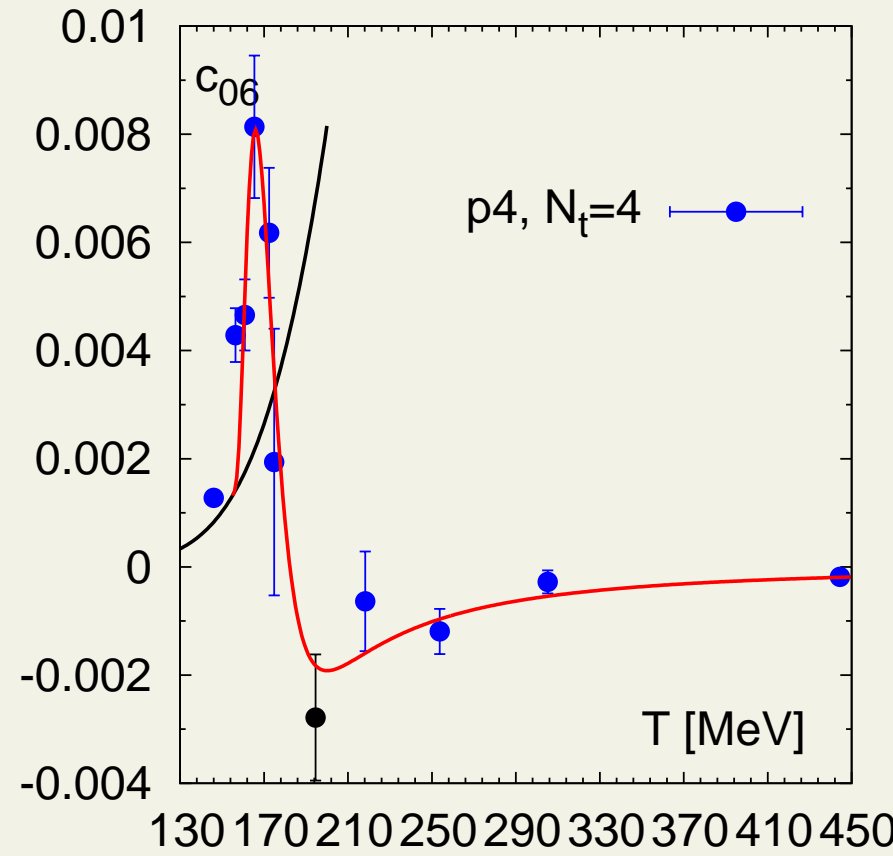
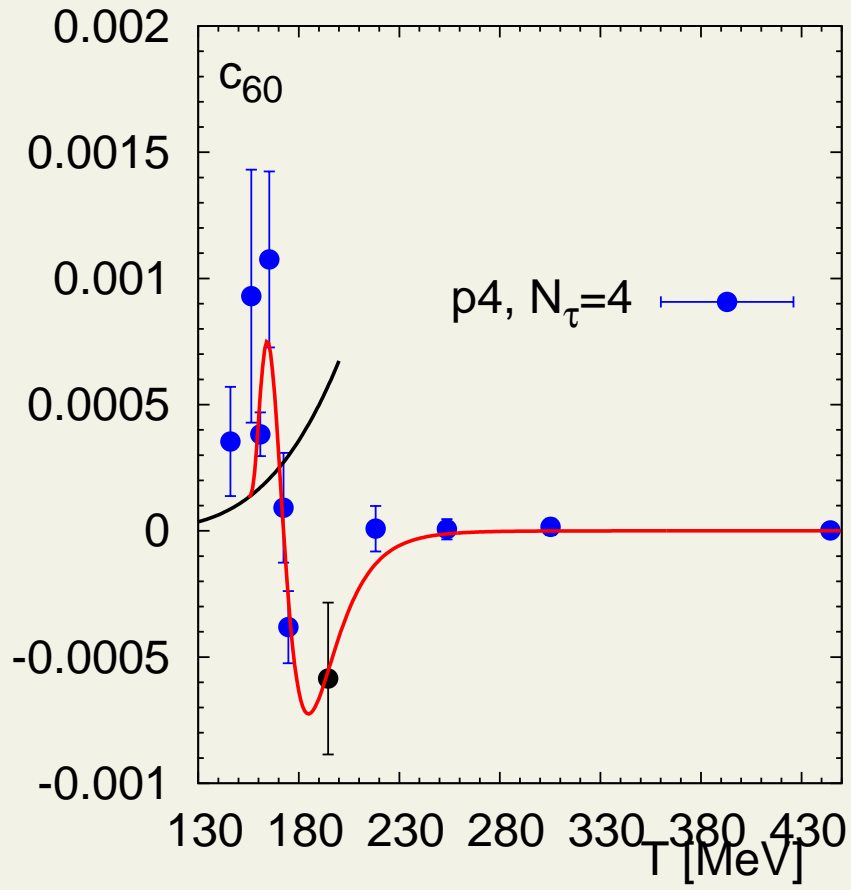
- $T_{fo} \approx 120$  MeV (bag)  $\Rightarrow$  **130** MeV (lattice)

# Conclusions

- lattice spacing dependence of hadron masses explains the difference between HRG and lattice QCD
  - **30 MeV shift** in temperature
- EoS at **finite baryon densities** based on **lattice QCD** calculations of baryon number and strangeness fluctuations and correlations
  - **~10% uncertainty** in speed of sound around the transition
- **effect on flow** when compared to bag model EoS **tiny** at SPS and (some?) RHIC low energy scan energies

# Backups

# Some fits are better than others:



# Pressure vs. Budapest-Wuppertal lattice

