From a complex scalar field to the hydrodynamics of dense quark matter

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arXiv:1212.0670 [hep-ph]

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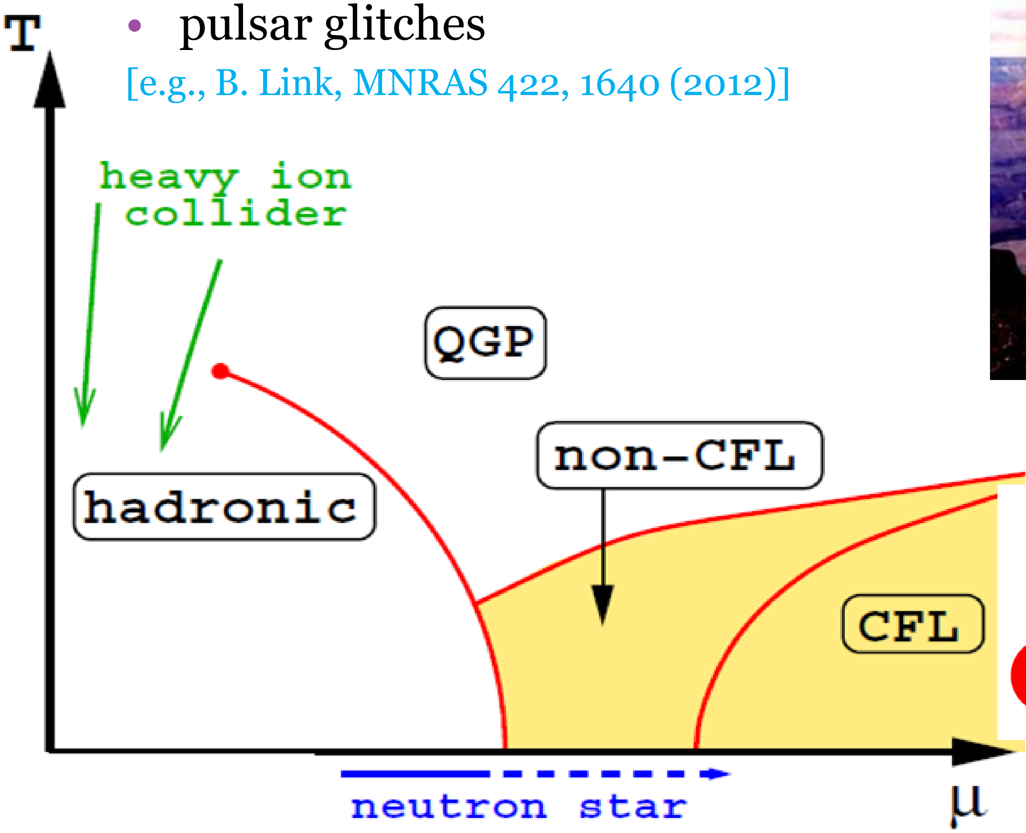
dense matter in the QCD phase diagram

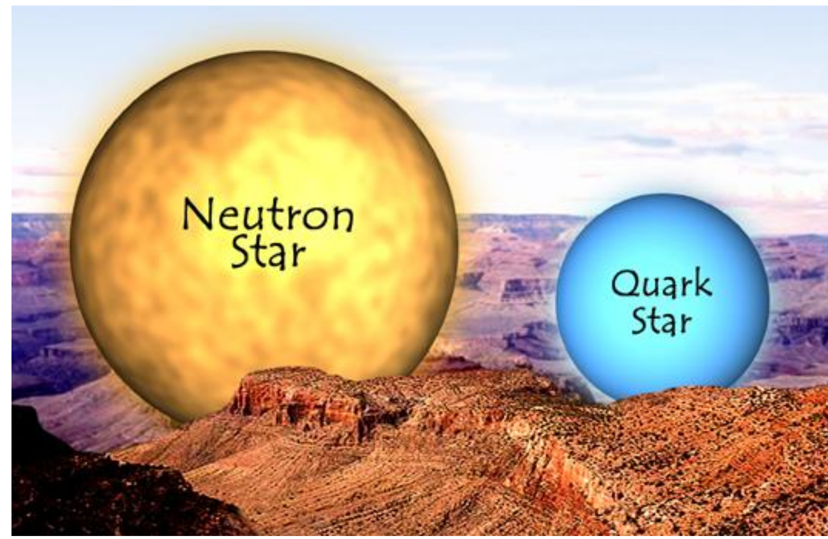
- r mode instability

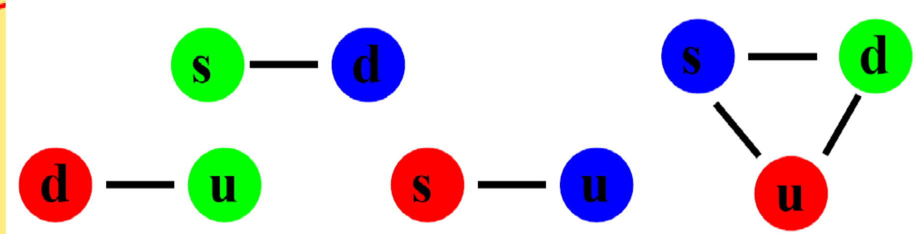
[e.g., N. Andersson, Astrophys. J. 502, 708 (1998)]

- pulsar glitches

[e.g., B. Link, MNRAS 422, 1640 (2012)]







Superfluids in dense quark matter

- CFL breaks chiral symmetry and Baryon conservation:

  → octet of (pseudo) goldstone modes

  $$\boxed{SU(3)_C \otimes SU(3)_L \otimes SU(3)_R \otimes U(1)_B \to SU(3)_{L+R+C} \otimes Z(2)}$$

  → CFL is a superfluid! - (exact) Goldstone mode due to $U(1)_B$

- Kaon condensation in CFL at high (but not asymptotically high) densities:

  → going down in density: m_s becomes non negligible

  → CFL reacts on stress on pairing pattern by developing a kaon condensate

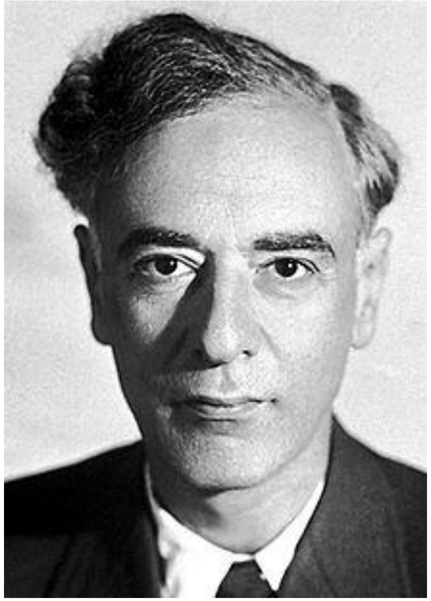
  [(P. F. Bedaque, T. Schäfer, NPA 697, 802,2002)]

  $$\boxed{U(1)_s \to 1}$$

  → on top of CFL breaking pattern: (not exact) Goldstone mode due to $U(1)_S$

- how many superfluid components are to expect in CFL with kaon condensation?

Landau´s model of superfluidity



Comparison: Landau vs microscopic model

Landau model

- Phenomenological model based on hydrodynamic equations such as:

$$\boxed{\vec{g} = \rho_s \vec{v}_s + \rho_n \vec{v}_n}$$

$$\boxed{\epsilon = \epsilon_n + \epsilon_s + \frac{\rho_s v_s^2}{2} + \frac{\rho_n v_n^2}{2}}$$

- Fluid formally divided into superfluid and normal density (very successful describing superfluid He4)

$$T = \begin{cases} 0 & \rho = \rho_s \\ > T_C & \rho = \rho_n \end{cases}$$

- Variables : $\rho, \ \vec{v}_s, \ T$, ...

microscopic model

- QFT model based on SSB and the existence of Goldstone modes
- Condensate related to superfluid, elementary excitations to the normal fluid part
- Variables:

  - gradients of Goldstone fields:

    $\partial_0 \psi, \ \vec{\nabla}\psi, \ T$,...

SSB in a φ^4 model

$$\mathcal{L} = \partial_\mu \varphi \partial^\mu \varphi^* - m^2 |\varphi|^2 - \lambda |\varphi|^4$$

- from effective theory for mesons: $\lambda_{eff} = \frac{4\mu_{K^0}^2 - m_{K^0}^2}{6 f_\pi^2}$ $\quad \mu_{K^0} = \frac{m_s^2 - m_d^2}{2\mu}$
- ansatz for the condensate: $\varphi(x) \rightarrow \frac{\rho(x)}{\sqrt{2}} e^{i\psi(x)} + fluctuations$

$$\mathcal{L} = -U + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \mathcal{L}^{(4)}$$

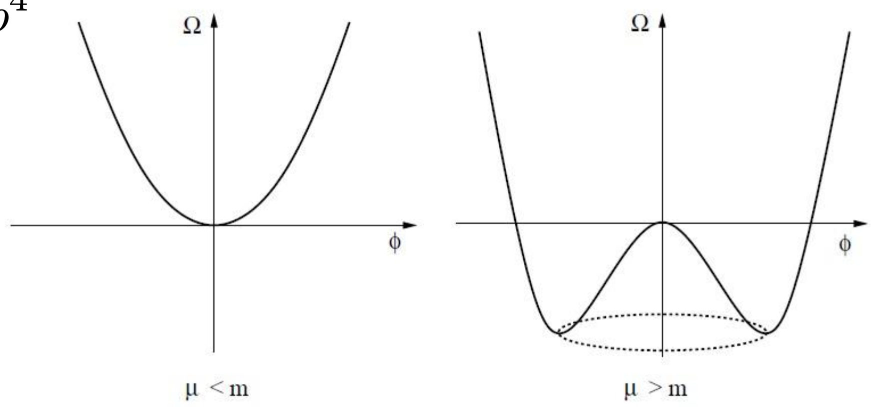
$$U = -\frac{1}{2}\partial_\mu \rho \partial^\mu \rho - \frac{\rho^2}{2}\left(\sigma^2 - m^2\right) + \frac{\lambda}{4}\rho^4$$

$$\sigma^2 = \partial_\mu \psi \partial^\mu \psi$$

- assumption:

$$\boxed{\rho,\ \partial_\mu \psi\ = cons}$$

homogeneous superflow



Comparison: Landau vs microscopic model

hydrodynamics

- super current

  $j^\mu = n_s v^\mu$
- stress-energy tensor

  $T^{\mu\nu} = (\epsilon_s + P_s) v^\mu v^\nu - g^{\mu\nu} P_s$
- superfluid density

  $n_s = \sqrt{j^\mu j_\mu} = v^\mu j_\mu$
- superfluid velocity

  $v^\mu = \gamma\left(1, \vec{v}_s\right)$
- energy density

  $\epsilon_s = v_\mu v_\nu T^{\mu\nu}$
- pressure

  $P_s = -\frac{1}{3}(g_{\mu\nu} - v_\mu v_\nu) T^{\mu\nu}$

microscopic model

- Noether current

  $j^\mu = \partial\mathcal{L}/\partial\left(\partial_\mu\psi\right) = \partial^\mu\psi(\sigma^2 - m^2)/\lambda$
- stress-energy tensor

  $T^{\mu\nu} = \partial^\mu\psi\partial^\nu\psi(\sigma^2 - m^2)/\lambda - g^{\mu\nu}\mathcal{L}$
- superfluid density

  $n_s = \sigma(\sigma^2 - m^2)/\lambda$
- superfluid velocity

  $v^\mu = \partial^\mu\psi/\sigma$
- energy density

  $\epsilon_s = v_\mu \partial^\mu\psi\, n_s - \mathcal{L}$
- pressure:

  $P_s = \mathcal{L} + (v_\mu\partial^\mu\psi - \sigma) n_s$

Superfluidity from φ⁴ model (T=0)

- connection to thermodynamics:

$$\epsilon_s + P_s = \mu_s n_s \;\rightarrow\; \mu_s = \sigma = v^\mu \partial_\mu \psi$$

→ chemical potential and flow velocity of the superfluid are both determined in terms of the phase of the condensate:

-rotations of the phase around the U(1) circle generate the chem. pot.
-number of rotations/unit length gives rise to the superflow velocity

→ Lorentz factor in σ: $\sigma = \sqrt{(\partial_0\psi)^2 - \left(\vec{\nabla}\psi\right)^2} = \partial_0\psi\sqrt{1-\left(\frac{\vec{\nabla}\psi}{\partial_0\psi}\right)^2} = \mu\sqrt{1-v^2}$

finite temperature

- use pressure as effective action:

$$\Gamma_{eff}[\Phi, S] = -U(\Phi) - \frac{1}{2} Tr\, ln\, S^{-1} \qquad S^{-1}(k) = \begin{pmatrix} -k^2 + 2(\sigma^2 - m^2) & 2ik \cdot \partial\psi \\ -2ik \cdot \partial\psi & -k^2 \end{pmatrix}$$

- Anisotropic dispersions (Goldstone + massive):

$$\epsilon_{1,\mathbf{k}} = \sqrt{\frac{\sigma^2 - m^2}{3\sigma^2 - m^2}}\, \zeta(\cos\theta)\, |\mathbf{k}| + \mathcal{O}(|\mathbf{k}|^3) \qquad \epsilon_{2,\mathbf{k}} = \sqrt{2}\sqrt{3\sigma^2 - m^2 + 2(\nabla\psi)^2} + \mathcal{O}(|\mathbf{k}|)$$

$$\zeta(\theta) = \frac{\sqrt{1 - v_s^2}\sqrt{1 - \frac{v_s^2}{3}(1 + 2cos^2\theta)} + \frac{2|v_s|}{\sqrt{3}} cos\theta}{1 - \frac{v_s^2}{3}}$$

- Anisotropic pressure: $P\delta_{ij} \to \{P_\perp, P_{||}\}$

- low T approximation: use linear and cubic terms in k in the dispersions.
- to go (numerically) up to T_c : use 2 particle irreducible formalism (CJT)

  [J.M. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. D10, 2428 (1974)]

  [M.G. Alford, M. Braby, A. Schmitt: J.Phys.G35:025002 (2008)]

finite temperature

- two major difficulties to be resolved:

→ How to define thermodynamics in a relativistic two fluid system ?

[I. M. Khalatnikov and V.V. Lebedev, Phys. Lett. 91A, 70 (1982)]

[V.V. Lebedev and I. M. Khalatnikov, Sov. Phys. JETP 56, 923 (1982)]

[B. Carter, in Relativistic Fluid Dynamics (Noto 1987), edited by A. Anile and M. Choquet-Bruhat (Springer-Verlag, Berlin, 1989)]

[B. Carter and I. M. Khalatnikov, Phys. Rev. D 45, 4536 (1992)]

[N. Andersson and G. Comer, Living Rev. Relativity 10, 1 (2005)
[...]

→ In which frame of reference are the microscopic calculations performed?

Carter`s canonical two fluid formalism

[B. Carter and I. M. Khalatnikov, PRD 45, 4536 (1992)]

- generalization of TD relation (generalized Pressure and Energy):

$$\epsilon + P = \mu n + T s \quad \rightarrow \quad \Lambda + \Psi = \partial\psi \cdot j + \theta \cdot s$$

$$d\Lambda = \partial_\mu \psi \, dj^\mu + \Theta_\mu ds^\mu \qquad d\Psi = j_\mu d(\partial^\mu \psi) + s_\mu d\Theta^\mu \qquad \partial_\mu j^\mu = 0$$

$$\Lambda = \Lambda[j^2, s^2, s \cdot j] \qquad \Psi = \Psi[(\partial\psi)^2, \theta^2, \partial\psi \cdot \theta] \qquad \partial_\mu s^\mu = 0$$

→hydro description either in terms of conserved currents or their conjugated momenta!
(this set of variables differs from $\partial_\mu j_s^\mu \neq 0$ and $\partial_\mu j_n^\mu \neq 0$)

- definition of coefficients ($\mathcal{A}$,$\mathcal{B}$,$\mathcal{C}$), $\mathcal{A}$ "entrainment coefficient"

$$\partial^\mu \psi = \frac{\partial \Lambda}{\partial j_\mu} = \mathcal{B} j^\mu + \mathcal{A} s^\mu \,, \Theta^\mu = \frac{\partial \Lambda}{\partial s_\mu} = \mathcal{A} j^\mu + \mathcal{C} s^\mu$$

- stress energy tensor:

$$T^{\mu\nu} = -g^{\mu\nu}\Psi + j^\mu \partial^\nu \psi + s^\mu \Theta^\nu \qquad \partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = -g^{\mu\nu}\Psi + \mathcal{B} j^\mu j^\nu + \mathcal{C} s^\mu s^\nu + \mathcal{A}\left(j^\mu s^\nu + s^\mu j^\nu\right)$$

relativistic two fluid formalism

- relation to formalism of Landau\Khalatnikov:
  [e.g., D. T. Son, Int. J. Mod. Phys. A 16S1C, 1284 (2001)]

$$T^{\mu\nu} = (\epsilon_n + P_n)u_n^\mu u_n^\mu + P_n g^{\mu\nu} + (\epsilon_s + P_s)u_s^\mu u_s^\mu + P_s g^{\mu\nu}$$

$$j^\mu = n_n u^\mu + n_s \frac{\partial^\mu \psi}{\sigma} \qquad s^\mu = s u^\mu$$

- expressible through coefficients:

$$\mathcal{A} = -\frac{\sigma n_n}{s n_s}, \qquad \mathcal{B} = \frac{\sigma}{n_s}, \qquad \mathcal{C} = \frac{\sigma n_n^2}{s^2 n_s} + \frac{\mu n_n + sT}{s^2}$$

- the microscopic calculations are performed in the normal fluid restframe (restframe of the heat bath) , in this frame, we can identify:

$$\Psi = \frac{1}{3}\left(g^{\mu\nu} - u^\mu u^\nu\right)\left(j_\mu \partial_\nu \psi - T_{\mu\nu}\right) \rightarrow \boxed{\frac{T}{V}\Gamma = \Psi = T_\perp} \qquad u^\mu = (1, \vec{0})$$

$$\Lambda + \Psi = \partial\psi \cdot j + \theta \cdot s \qquad s^0 = \frac{\partial \Psi}{\partial T} \qquad \rightarrow \qquad \boxed{\theta^0 = T}$$

relation to microscopic theory

- coefficients:

$$\mathcal{A} = -\frac{\partial^0\psi}{s^0\vec{\nabla}\psi\cdot\vec{j}}\left(\vec{v}_s^2 j^0\partial^0\psi - \vec{j}\cdot\vec{\nabla}\psi\right) \qquad \mathcal{B} = -\frac{\left(\vec{\nabla}\psi\right)^2}{\vec{\nabla}\psi\cdot\vec{j}}$$

$$\mathcal{C} = \frac{j^0\partial^0\psi\left(\vec{v}_s^2 j^0\partial^0\psi - \vec{j}\cdot\vec{\nabla}\psi\right) + s^0\theta^0\left(\vec{j}\cdot\vec{\nabla}\psi\right)}{\left(s^0\right)^2\left(\vec{j}\cdot\vec{\nabla}\psi\right)}$$

- some exemplary results:

$$\frac{T}{V}\Gamma_{eff} \simeq \frac{\mu^4}{4\lambda}\left(1-v_s^2\right)^2 + \frac{\pi^2T^4}{10\sqrt{3}}\frac{\left(1-v_s^2\right)^2}{(1-3v_s^2)^2} - \frac{4\pi^4T^6}{105\sqrt{3}}\frac{\left(1-v_s^2\right)^2}{(1-3v_s^2)^5}\left(5+30v_s^2+9v_s^4\right)$$

$$n_s \simeq \frac{\mu^3}{\lambda}(1-v_s^2) - \frac{4\pi^2T^4}{5\sqrt{3}\mu}\frac{1-v_s^2}{(1-3v_s^2)^3} + \frac{8\pi^4T^6}{105\sqrt{3}}\frac{1-v_2^2}{(1-3v_s^2)^6}\left(95+243v_s^2-135v_s^4-27v_s^6\right)$$

$$n_n \simeq \frac{4\pi^2T^4}{5\sqrt{3}\mu}\frac{\left(1-v_s^2\right)^2}{(1-3v_s^2)^3} - \frac{16\pi^4T^6}{35\sqrt{3}\mu^3}\frac{\left(1-v_s^2\right)^2}{(1-3v_s^2)^6}\left(15+38v_s^2-9v_s^4\right)$$

relation to analog gravity (linear dispersion)

[Carter, Langlois; Phys Rev. D51, 5855 (1995)] relation of sonic metric and superfluidity in the cold limit
[Barcelo, Liberati , Visser; Living Rev.Rel.8:12,2005] fluids as laboratory for black hole physics
[Manarelli, Manuel; Phys.Rev.D77:103014,2008] transport theory from analog gravity

- sonic metric: $G^{\mu\nu} = g^{\mu\nu} + \left(\frac{1}{u_s^2} - 1\right) v_s^\mu v_s^\nu$
- dispersions: $G^{\mu\nu} k_\mu k_\nu = 0$

→ eliminate v_s , T, μ: $\sigma^2 = \mu^2\left(1 - v_s^2\right)$ $\partial\psi \cdot \theta = \mu T - \frac{\mathcal{A}}{\mathcal{B}}\left(\nabla\psi\right)^2 = \frac{1 - v_s^2}{1 - 3v_s^2}\mu T + \mathcal{O}(T^3)$

$$\theta^2 = T^2 - \frac{\mathcal{A}^2}{\mathcal{B}^2}\left(\nabla\psi\right)^2 = \frac{(1 - v_s^2)(1 - 9v_s^2)}{(1 - 3v_s^2)^2} T^2 + \mathcal{O}(T^4)$$

→ $\Psi(\partial\psi^2, \theta^2, \partial\psi\cdot\theta) = \frac{\sigma^4}{4\lambda} + \frac{\pi^2}{90\sqrt{3}}\left[\theta^2 + 2\frac{(\partial\psi\cdot\theta)^2}{\sigma^2}\right]^2 = \frac{\sigma^4}{4\lambda} + \frac{\pi^2}{90\sqrt{3}}\left[G^{\mu\nu}\theta_\mu\theta_\nu\right]^2$

first and second sound

- two fluid system allows for two sound modes:

→first sound:
normal and super fluid densities
oscillate in phase , pressure wave,
weak temperature dependence

→second sound:
normal and super fluid densities
oscillate out of phase , temperature wave

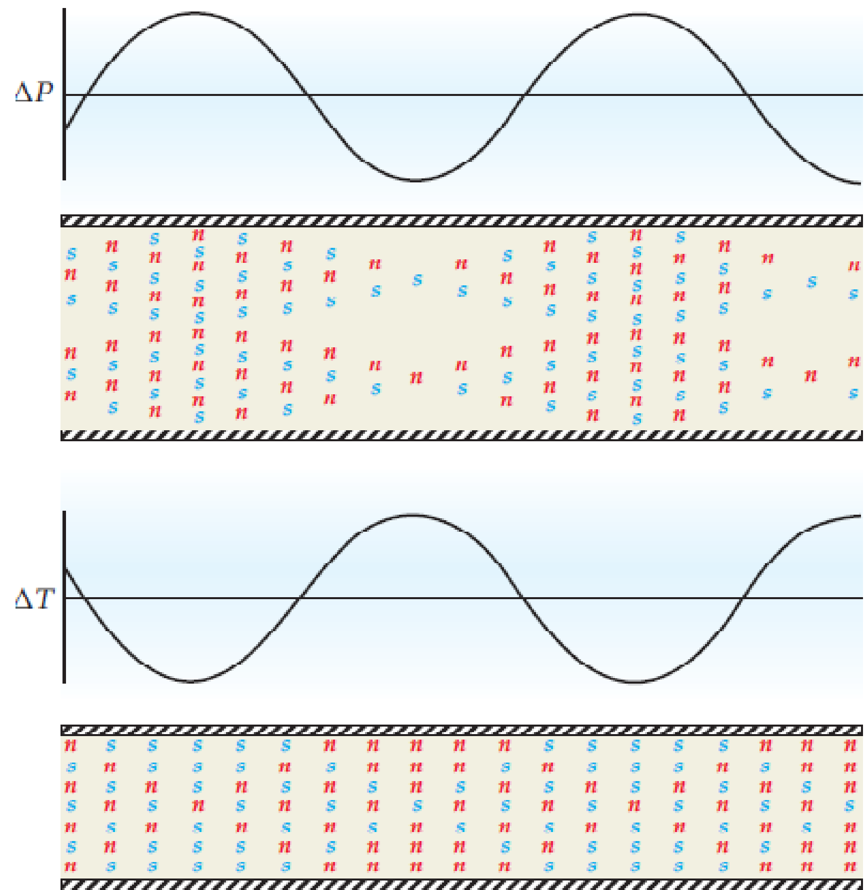


figure: [R.J. Donnelly, Physics Today 62, 10 (2009)]

first and second sound- limit cases

- the general form of the 2 equations which determine the sound velocities is complicated, discussion of limit cases:

→ T=0:

$$0 \simeq (g^{\mu\nu} + 2v_s^\mu v_s^\nu)\,\partial_\mu\partial_\nu\delta\mu(\vec{x},t) = G^{\mu\nu}k_\mu k_\nu\delta\mu(\vec{x},t)$$

we recover the term linear in k of the dispersion relation:

$$\epsilon_k = u_1 k + \mathcal{O}(k^3)$$

→ v_s =0:

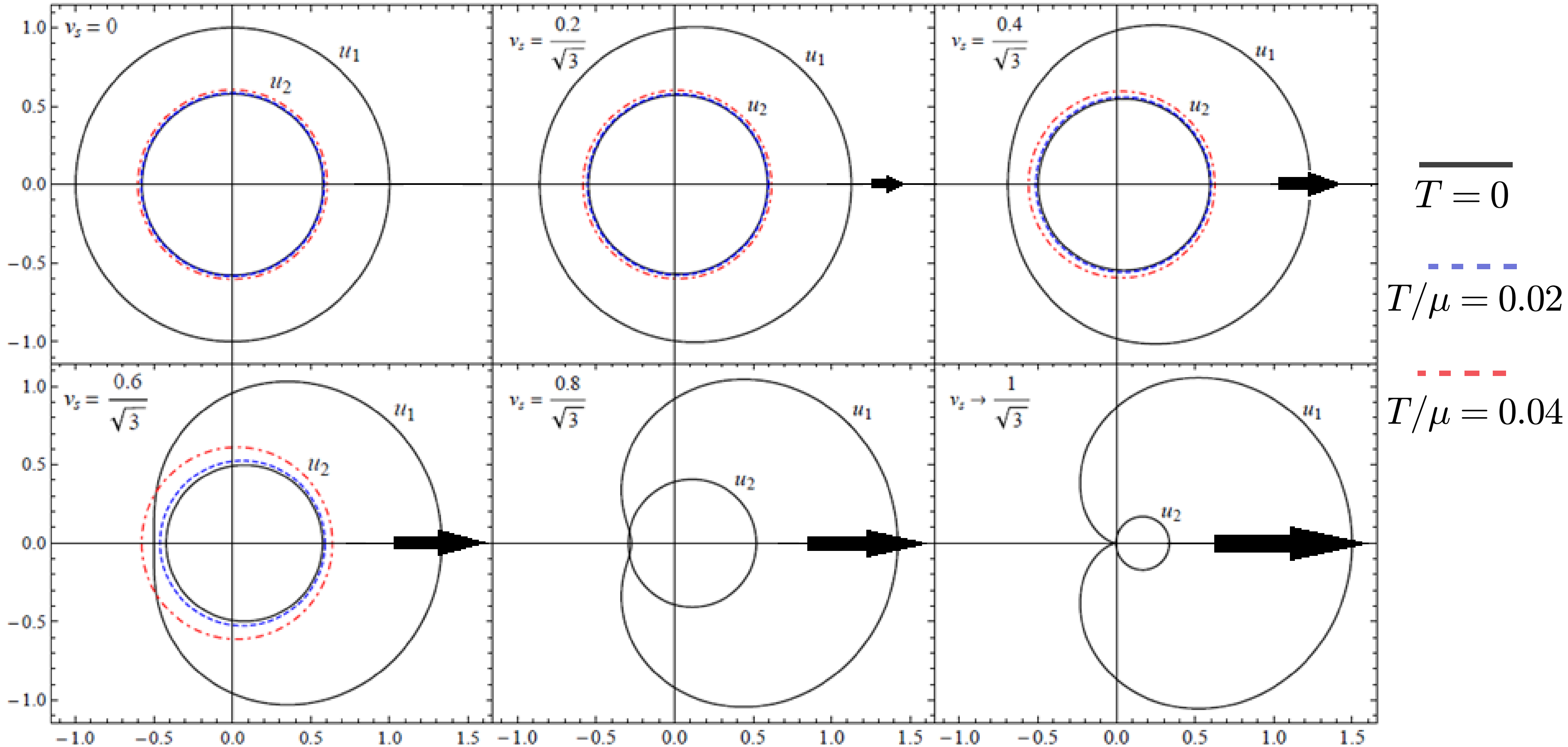
$$0 = w\left(\frac{\partial s}{\partial\mu}\frac{\partial n}{\partial T} - \frac{\partial n}{\partial\mu}\frac{\partial s}{\partial T}\right)\omega^2 - n_s s^2\vec{k}^4$$

$$+\left[s^2\mu\frac{\partial n}{\partial\mu} + \left(\mu n_n^2 + w n_s\right)\frac{\partial s}{\partial T} - 2\mu s n_n\frac{\partial s}{\partial\mu}\right]\omega^2\vec{k}^2$$

$$w = \mu n_n + Ts$$

we recover the (velocity independent) results for first and second sound given for example in: [Herzog, Kovtun, Son; Phys.Rev. D79, 066002 (2009)]

first and second sound: full results



what we haved discussed:

→The microscopic physics of CFL and kaon condensation can be translated into a multifluid hydrodynamic system

- we provide a translation in between existing descriptions of superfluid hydrodynamics
- we show, how the hydro parameters emerge from an underlying microscopic field theory
- we study the (anisotropic) first and second sound modes

what is left to do:

→ towards a complete hydro description of CFL and kaon condensation:

- use the CJT formalism to numerically go up to T_c

  [work in progress; Mark G. Alford, S. Kumar Mallavarapu, Andreas Schmitt , Stephan Stetina]

- include effect of weak interactions (include a $U(1)_s$ breaking term into the effective Lagrangian)

  [work in prgress; Denis Parganlija, Andreas Schmitt]

- repeat the analysis for a fermionic system of Cooper pairs

Thank you!