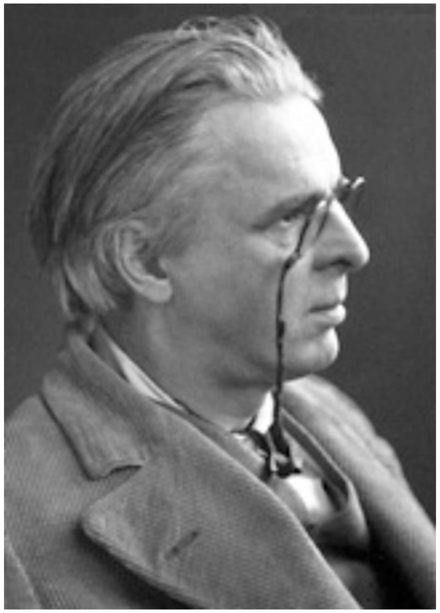
Center Symmetry and Confinement



The Irish poet, William Butler Yeats may have been prescient. In 1920 he described the Yang Mills phase transition in his apocalyptic poem *Second Coming*:

“Things fall apart,
the centre cannot hold”

TDC in preparation

An Overview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An Overview

- The talk has two purposes
  1. To explain to non-specialists (experimentalists and theorists working in other fields) what center symmetry is and the standard view of why it is important in understanding confinement.
     - Introduce some key ideas---center symmetry, Euclidean space observables, Wilson lines, Polyakov loops, n-ality, and all that
     - Motivate unbroken center symmetry as a condition for confinement (eg Yang Mills).
     - Motivate Polyakov loop as order parameter for center symmetry and confinement.
  2. To explain to specialists why what is said in part 1 is probably completely misleading.
     - Show that the Polyakov loop appears to be a good order parameter for confinement, even for theories without center symmetry.

- Examples of such theories include SU(Nc) gauge theory with Nc $\geq$5 and odd and with quarks in the 2-index symmetric or anti-symmetric representation

Purpose of the exercise is to gain insight into the nature of confinement

- Note that confinement is a very slippery subject and even 40 years after the advent of QCD it remains an elusive subject
- New insights are helpful

What is confinement in QCD?

There are two distinct notions of confinement

The real thing

All physical asymptotic states are color singlets. This notion applies directly to QCD

A cartoon

An unbroken Zn symmetry or a vanishing Polyakov loop.

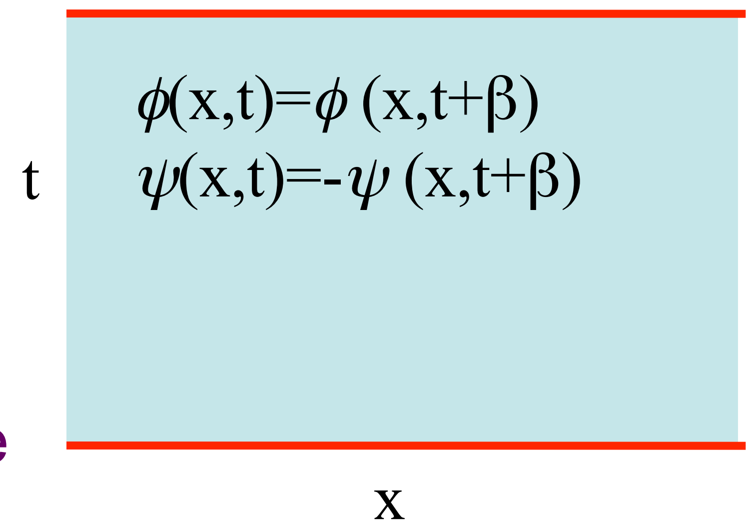
Of interest mostly to theorists looking at simplified limits or theories other than QCD!

This talk will focus on the cartoon version.

- Why
  - The basic notion of confinement as preventing asymptotic colored states is rather subtle
    - There is no order parameter for confinement in this sense. QCD has no order parameter; it is also has no deconfinement phase transition—only a cross-over
      - As a result it can be difficult for people interested in “confinement mechanisms” (assuming this is a well-posed idea) to make contact.
  - However, if one looks at some cousins of QCD such pure Yang-Mills, there is well-posed order parameter for confinement, the Polyakov loop
    - The hope is that by studying these cousin theories in which confinement is more tractable, one my get general insights into the more general issue.
    - There is no guarantee that this is true

Work in Euclidean Space

- Notion of center transformations and the Polyakov loop only make sense as a physical theory in Euclidean space.
- Center transformations require a finite time dimension and some kind of periodic b.c.
- Field theories in Euclidean space with periodic b.c. in time for the bosons and anti-periodic for fermion correspond to finite temperature theory with the extent of the time direction, β, being the inverse temperature.



$$\phi(x,t)=\phi(x,t+\beta)$$
$$\psi(x,t)=-\psi(x,t+\beta)$$

Standard field theory generating function is the partition function and all thermodynamic observables by including appropriate source and differentiating

- What is center the of a group?
  - The center of a group is the set of elements in the group which commute with all the elements of the group.
  - For SU(Nc) the center is Z_{Nc}. Namely C is an element of the center iff $C = z_i \mathbb{I}_{Nc \times Nc}$ with $z_i^{Nc} = 1$
- What is a center transformation in a gauge theory?

$$A_\mu \rightarrow A'_\mu \equiv \Omega A_\mu \Omega^+ - \Omega \partial_\mu \Omega^+$$

$$\text{with} \quad \Omega(\vec{x}, t + \beta) = C\Omega(\vec{x})$$

where C is an element of the center and W is an arbitrary function subject to these boundary conditions.

  - This looks just like a gauge transformation (and hence unphysical) but in a periodic theory, true gauge transformations are periodic whereas this is not unless C is the unit element.
  - Key thing: easy to show that if *A* is periodic so is *A*' . Thus a center trans is a legitimate trans in the theory.

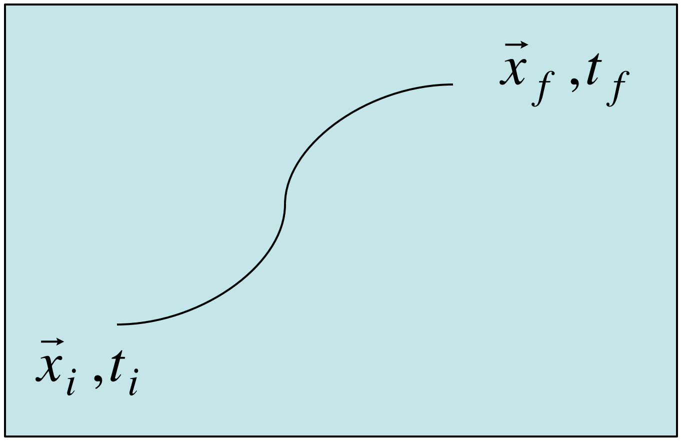
- Except for concerns about the boundary, it looks like a gauge transformation, Moreover, in a pure Yang-Mills this center transformation respects the boundary conditions: it is thus a symmetry of the action of the theory.

- Since, it seems to act so much like a gauge transformation, how do we know it is physical?
  - There exist gauge invariant (i.e. physical) observables which are NOT invariant under center transformation
  - The Polyakov loop is the simplest example.

To understand the Polyakov loop, it is useful to define the Wilson line which is defined along some path in space-time from some initial to some final point

$$w_{if} = P\exp\left(i \int_{\vec{x}_i, t_i}^{\vec{x}_f, t_f} A_\mu dx_\mu \right)$$

where P means path-ordered product



The Wilson line is a unitary matrix in color space. Under gauge transformations it has the following property

$$w_{if} \rightarrow \Omega(\vec{x}_f, t_f) w_{if} \Omega^+(\vec{x}_i, t_i)$$

Thus one can use the Wilson line to connect operators at different space-time points in a gauge invariant way.

The Polyakov loop is the trace of a Wilson line along a striaght path in the time direction from *t* to *t*+*β*.

$$\Phi(\vec{x}) = \tfrac{1}{N_c} Tr\left[T \exp\left(i \int_0^\beta d\tau \, A_0(\vec{x},\tau) \right) \right]$$

It is called a loop since one can view the periodicity as a curled up direction and it loops around that direction

The Polyakov loop is gauge invariant. This is a consequence of the trace and periodicity—a gauge transformation at *t* and *t*+*β*. are the same

$$\Phi(\vec{x}) \to \Phi'(\vec{x}) = \tfrac{1}{N_c} Tr\left[\Omega(\vec{x},\beta) T \exp\left(i \int_0^\beta d\tau \, A_0(\vec{x},\tau) \right) \Omega^+(\vec{x}_i,0) \right] = \Phi(\vec{x})$$

However, the Polyakov loop is NOT invariant under non-trivial center transformations!!

$$\Phi(\vec{x}) \rightarrow \Phi'(\vec{x}) = \tfrac{1}{N_c} Tr\left[\Omega(\vec{x},\beta) T \exp\left(i\int_0^\beta d\tau\, A_0(\vec{x},\tau)\right)\Omega^+(\vec{x}_i,0)\right] = z_i \Phi(\vec{x})$$

So center transformations have physical content since gauge-invariant objects change under it.

But, what is the physical meaning of it?

The standard physical interpretation is that it is related to the free energy of adding an external static color source in the fundamental representation (such as an infinitely massive quark ignoring the contribution from the mass itself) to the system.

$$\left|\Phi(\vec{x})\right| = \exp(-F\beta)$$

- To ask about how the free energy changes with a colored charge one needs to put in a color charge into the action in a gauge invariant way.
  - Put the color source in at $t=0$ and remove it a $t=\beta$. This will add to the action the free energy times β.
  - Need to do this in a gauge invariant way
  - The Wilson line in the time direction is the energetically cheapest way to do this
  - The absolute value is needed as the PL is complex

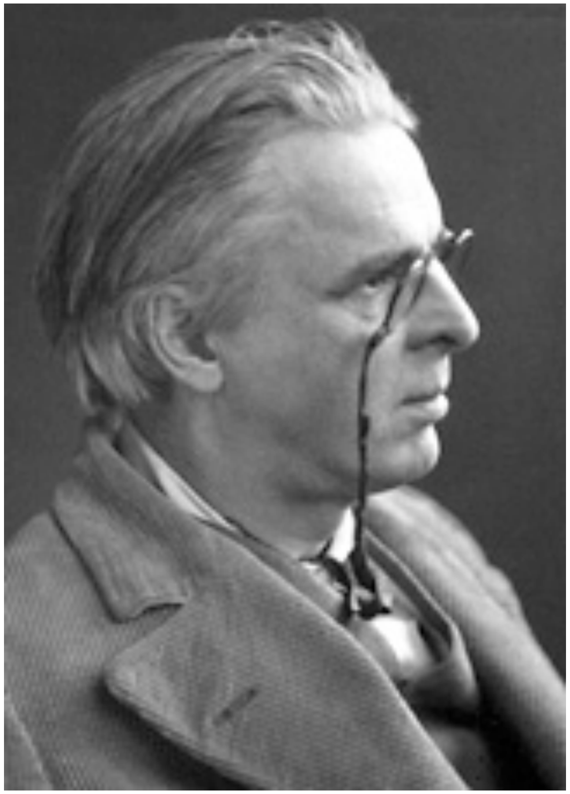
- Since $|\Phi(\vec{x})| = \exp(-F\beta)$ it follows that in a confining phase of a theory in which the free energy to add a color sources should be infinite Φ must be zero.
- **Thus the Polyakov loop really does act as an order parameter for confinement—at least for pure Yang-Mills.**

- Moreover, it is a TRUE order parameter—it is zero in one phase and nonzero in another.

- True order parameters are typically associated with symmetry*: eg. the magnetization in a Heisenberg magnetic system is zero in the high temperature phase because of rotational invariance of the Hamilitonian.*
- The symmetry is at play here is, of course, **center symmetry**.
  - The Polyakov loop must vanish for unbroken center symmetry: the action is invariant under center transformations and the Polyakov loop is not.
- Thus there is a natural association:

In a confining phase, center symmetry is unbroken and the Polyakov loop vanishes. At the deconfinement transition (which is 1st order in pure YM), center symmetry breaks spontaneously.

The deconfinement transition

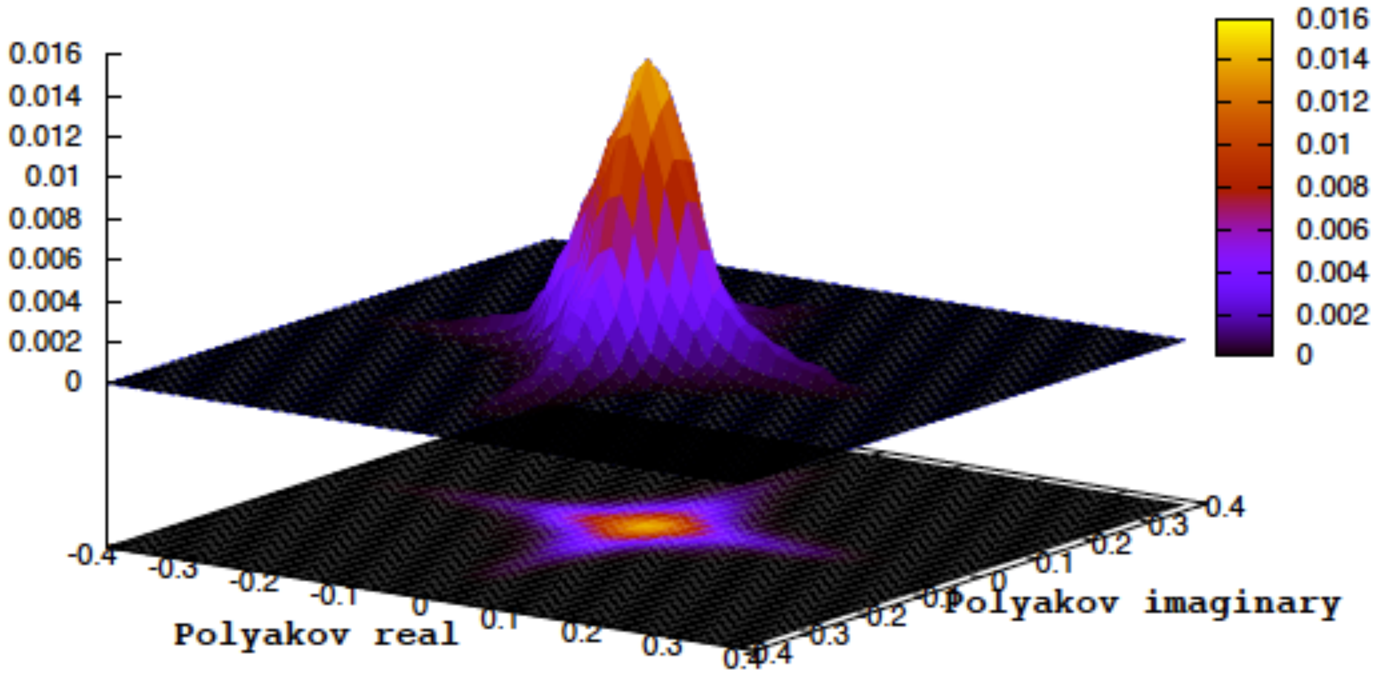


“Things fall apart, the centre cannot hold”

- Lattice results bear this out.

For example in SU(4) YM in 2+1 dimensions

(from K. Holland, M. Pepe & U.J. Weise, JHEP 0802, 041 (2008))



Probability distribution for Polyakov loop below the transition

- Φ is not the only gauge invariant operator which varies under center transformation.

  One can define analogous operators which correspond to coupling to color sources in any representation.

For example

$$\Phi_{2-index}(\vec{x}) = \tfrac{1}{2N_c}\sum_{ab}\left(w^T_{ab}(\vec{x})w^T_{ab}(\vec{x}) \pm w^T_{ab}(\vec{x})w^T_{ba}(\vec{x})\right)$$

$$w^T_{ab}(\vec{x}) \equiv \left[T\exp\left(i\int_0^\beta d\tau\, A_0(\vec{x},\tau)\right)\right]_{ab}$$

This is corresponds to coupling to color in the 2-index symmetric or antisymmetric representations.

- Under center transformations : $\Phi_{2-index}(\vec{x}) \longrightarrow z_i^2\Phi_{2-index}(\vec{x})$

- In a center symmetric (confining) phase $\Phi_{\text{2-index}}$ must vanish on mathematical grounds.
- It also must vanish on physical grounds in a confining phase: it has the interpretation of being equal to $\exp(-F_{2\text{-}index}\,\beta)$ where $F_{2\text{-}index}$ is the free energy of introducing a color charge in the 2-index representation to the system. $F_{2\text{-}index}$ should be infinite in the confining phase.
- An analogous argument applies for generalized Polyakov loops for all color sources in any representation except those with n-anility=0.
- A generalized Polyakov loop under a center transformation transforms according to

$$\Phi_n(\vec{x}) \rightarrow z_i^n \Phi_n(\vec{x})$$

$$\text{where } n \text{ is the n-ality of the representation}$$

- Thus in a center symmetric (confining) phase Polyakov loops associated with any n-ality other than zero must vanish; physically this means that the free energy of bringing in such a color charge is infinite as expected
- The n-ality of a representation is the number of fundamental representation one needs to combine to make the representation (i.e. number of boxes in the Young tableau); it is defined modulo Nc.

By construction in a Clebsch - Gordan decomposition

$R_1 \otimes R_2 = \sum_i R_i$ the n - alities follow the rule

$$N[R_i] = \mathrm{mod}[N[R_1] + N[R_2], N_c]$$

Thus adding gluons (which have n-ailty of zero) to any configuration cannot change the configurations n-aility.

- What about Polyakov loops with sources with n-ality zero? This includes color sources in the adjoint representation (i.e. the same as gluons)

$$\Phi_{\mathrm{adjoint}}(\vec{x}) = \frac{1}{2N_c}\sum_{ab}\left(\left(w_{ab}^{T}\right)^{*}(\vec{x})\,w_{ab}^{T}(\vec{x})\right)$$

- Center symmetry does not force this to vanish in center symmetric phase and thus one does not expect it to be zero.
- What about physically? Isn't the free energy of adding a static adjoint color charge infinite?

Actually no. The point is that the act of putting in an external color adjoint source induces the system to pop a gluon (or a bunch of gluons) out of the vacuum which neutralizes it. The remaining state is color neutral and of finite free energy.

- The produced gluon returns to the vacuum with the sink annihilating the external color.
- This is completely analogous to why there is no string tension between adjoints—gluons appear from vacuum and flux tubes break.
- The same thing happens with all zero n-anilty sources since any zero n-ality representation can be constructed from some number of gluons
  - Note that one can only make other zero n-anilty representations by combining gluons. So gluons cannot screen any color charge with non-zero n-ality.

- The picture of confinement for pure Yang-Mills being associated with an unbroken center symmetry holds together nicely.
  - Note that there is something odd about the order parameters for confinement: they explain only confinement for degrees of freedom which are not in the theory!!
    - They do not explain gluon confinement
    - Thy do explain quark confinement but in a theory which has no quarks.
  - What happens if I try to apply these ideas to real QCD?

- Everything Breaks!!! But it breaks in a sensible and coherent way
  - The action of the theory is not invariant under center transformations.
  - Thus there is no symmetry reason that the Polyakov loop should be zero.
  - Moreover on physical grounds one has no reason to expect that it should be.
    - Recall that the Polyakov loop told you about the free energy of bringing in an external color charge. However, the act of brining in an external fundamental color charge can induce an anti-quark to pop out of the vacuum to neutralize it.
  - An analogous thing happens for generalized Polyakov loops associated with any representation of color.

Why is the theory not center invariant?

Recall the transformation of gluons under a center transformation:

$$A_\mu \to A'_\mu \equiv \Omega A_\mu \Omega^+ - g\Omega\partial_\mu\Omega^+$$

$$\text{with} \quad \Omega(\vec{x}, t+\beta) = C\Omega(\vec{x})$$

Except for the boundary conditions this looks like a gauge transformation. This will leave the action invariant only if the quarks transform via the same "gauge transformation"

$$q(\vec{x},t) \to q'(\vec{x},t) = \Omega(\vec{x},t)q(\vec{x},t)$$

$$\text{so if} \;\; q(\vec{x},t+\beta) = -q'(\vec{x},t)$$

$$\text{then} \;\; q'(\vec{x},t+\beta) = -z_i q'(\vec{x},t)$$

But this means that q' does not satisfy the boundary conditions and the transformation is not allowed.

- Pure Yang Mills
- Theory is center symmetric. It is unbroken at small T.
- At small T, generalized Polyakov loops are zero for all non-zero n-ality.
- External color charges in all representations for all non-zero n-ality are unscreened and have infinite free energy. Zero n-ality charges are screened.

- QCD (quarks in fundamental)
- Theory explicitly breaks center symmetry. It is broken at all T.
- Generalized Polyakov loops are nonzero for all n-ality.
- External color charges in all representations for all are screened.

The Polyakov loop acts as an order parameter in the theory with center symmetry (Yang Mills) and not in the theory without it (QCD).

What about other theories?

- The same pattern holds QCD with quarks in the adjoint representation (n-ality=0) Note that with a single flavor of massless quark this is SYM.
  - This theory is invariant under center transformations so generalized Polyakov loops for nonzero n-ality vanish.
  - Color sources of nonzero n-ality are unscreened and have infinite free energy

Why is the theory center invariant?

Recall the transformation of gluons under a center transformation:

$$A_\mu \to A'_\mu \equiv \Omega A_\mu \Omega^+ - \Omega \partial_\mu \Omega^+$$

$$\text{with} \quad \Omega(\vec{x}, t + \beta) = C\Omega(\vec{x})$$

Except for the boundary conditions this looks like a gauge transformation. This will leave the action invariant only if the quarks transform via the same "gauge transformation"

$$q(\vec{x}, t) \to q'(\vec{x}, t) = \Omega(\vec{x}, t) q(\vec{x}, t) \Omega^+(\vec{x}, t)$$

$$\text{so if} \;\; q(\vec{x}, t + \beta) = -q'(\vec{x}, t)$$

$$\text{then} \; q'(\vec{x}, t + \beta) = -z_i q'(\vec{x}, t) z_i^* = -q'(\vec{x}, t)$$

But this means that q' does satisfy the boundary conditions and the transformation is allowed.

- The picture of a vanishing Polyakov loop that signals confinement and is associated with an unbroken center appears to be on a solid ground
  - This is the party line and attempts to understand confinement are often aimed at understanding a mechanism which forces center symmetry to hold.
- Is it correct? Let's consider a few more theories: What happens with QCD with quarks in **the two-index symmetric or anti-symmetric representations**? (Note these theories are of some real formal interest at large Nc as they have been shown to become equivalent to QCD with quarks in the adjoint.)

- First consider Nc=3
  - The theories are not center symmetric. This is easy to see: The two-index anti-symmetric representation is three dimensional and is the same as the (anti-) fundamental (r b – b r = $\bar{\text{g}}$). We have already seen that quarks in the fundamental break center symmetry. Quarks in the two index symmetric representation (6) behave the same way as they have the same n-ality.
  - Color sources of any n-ality can be screened so we do not expect any generalized Polyakov loops to vanish.
  - This is consistent with the pattern we have seen so far.

- Next consider Nc=4.
  - The quarks in these theories differ from fundamental quarks in n-nality. The theories are center symmetric but only for a very limited center symmetry of Z_2.

Recall that the center transformation looks like a gauge transformation. It will leave the action invariant only if

$$q_{ab}(\vec{x},t) \rightarrow q' \quad \text{with } q'_{ab}(\vec{x},t) = \Omega_{aa'}(\vec{x},t)\Omega_{bb'}(\vec{x},t)q_{ab}(\vec{x},t)$$

$$\text{so if } \; q(\vec{x},t+\beta) = -q'(\vec{x},t)$$

$$\text{then } q'(\vec{x},t+\beta) = -z_i^2 q'(\vec{x},t)$$

This means that q' satisfies the boundary conditions and the transformation is allowed, but only if $z_i = \pm 1$. But $C = -\mathbb{I}_{\mathrm{Nc}\times\mathrm{Nc}}$ is a valid nontrivial center element for SU(4)

- Nc=4 continued.
  - Recall that under center transformations generalized Polyakov loops transform according to

$$\Phi_n(\vec{x}) \rightarrow z_i^n \Phi_n(\vec{x})$$

where n is the n-ality of the representation

  - But since z_i can be ± 1, this means that all generalized Polyakov loops associated with odd n-ality source must vanish in a Z_2 symmetric phase
    - There is no constraint on the even n-ality Polyakov Loops.
  - Physically, this is consistent with the picture so far. The, odd n-ality sources cannot be screened but the two-index quarks + gluons can screen any even n-anilty source. Thus, the physical vanishing of the Polyakov loops is tied to the symmetry (albeit only Z_2) . The same argument goes through for any even Nc.

- **Nc=5 with quarks in a 2-index representation is very different.**
  - The quarks in these theories differ from fundamental quarks in n-nality. The theories are **not** center symmetric even for a limited center symmetry.

Recall that the center transformation looks like a gauge transformation. It will leave the action invariant only if

$$q_{ab}(\vec{x},t) \to q' \quad \text{with } q'_{ab}(\vec{x},t) = \Omega_{aa'}(\vec{x},t)\Omega_{bb'}(\vec{x},t)q_{ab}(\vec{x},t)$$

$$\text{so if } \; q(\vec{x},t+\beta) = -q'(\vec{x},t)$$

$$\text{then } q'(\vec{x},t+\beta) = -z_i^2 q'(\vec{x},t)$$

q' satisfies the boundary conditions and the transformation is allowed only if $z_i=\pm 1$. But $C=-\mathbb{I}_{Nc\times Nc}$ is **not** a center element for SU(5) since -1 is not a 5th root of 1.

- Nc=5 with quarks in a 2-index representation
  - Since there is no center symmetry there is no symmetry reason for the generalized Polyakov loop to vanish for any n-anilty.
  - But, on physical grounds one expects that the standard Polyakov loop (and generalized Polyakov loops for odd n-ality) to vanish in a confined phase.
    - The, odd n-ality sources cannot be screened by any number of two-index quarks + gluons .
    - One expects the same behavior not just for Nc=5 for any odd Nc>3
- Apparently the Polyakov loop can act like an order parameter in the absence of center symmetry

What does this mean?

- **There seem to be two possibilities:**

1. **Despite the physical argument the Polyakov loop does not vanish at low temperatures for these theories.**

2. **A vanishing Polyakov loop does not require center symmetry.**

1. **Despite the physical argument the Polyakov loop does not vanish for these theories.**

- **This possibility seems unlikely. However, if true, it is important since it implies the physical interpretation relating the Polyakov loop to the free energy is wrong.**
  - This possibility can be ruled out or confirmed by lattice calculation.
  - Note that if it is true then the Polyakov loops must approach 0 as Nc➔∞: It is known that these theories become equivalent (in a common sector) to QCD adjoint which is center symmetric. There is "an emergent center symmetry" (Shiffman & Unsal 2007)

2. A vanishing Polyakov loop does not require center symmetry.

- **This possibility is interesting: it contradicts the standard view that center symmetry is essential for the Polyakov loop to act as an order parameter.**
- **It is also interesting in that this appears to be a case of a true order parameter which is unconnected to a symmetry**
  - **unless some new and interesting symmetry is discovered.**
- **It also has interesting implications for attempts to understand confinement by focusing on theories with the Polyakov loop a good order parameter**

– **The key point is that whatever "confinement mechanism" is at play at making the Polyakov loop zero it either a) depends on center symmetry for pure Yang-Mills but not for theories with quarks in the A or S reps or b) does not depend on center symmetry.**

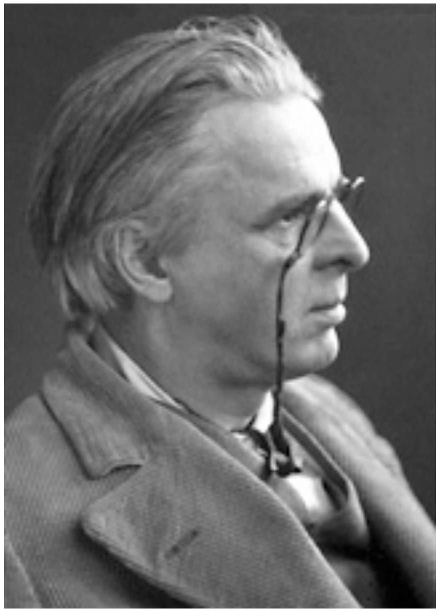
a) **If this is true, than the confinement mechanism is not generic. In this case it is hard to see why the mechanism in Yang Mills case would give any insight into real QCD.**

b) **This is interesting in itself as it suggests the connection between confinement and center symmetry is weaker than generally appreciated. Rather than being a key feature of confinement, it is essentially an accident for YM.**

A note for the cognoscenti

- There has been at lot of recent interest in YM theory based on G2 which has no center but does have a 1st order phase transition which often described as a “deconfining transition”. Aren’t the examples given here essentially the same?
- No!!! In the case of G2, the Polyakov loop is nonzero in both the high and low temperature phases (although small in the low temperature phase). It is not obvious that one should describe the transition as “deconfining”.

Summary



- An unbroken center symmetry is often taken as a the condition for a confining phase; its breaking signals deconfinement.
- The Polyakov loop is often taken as an order parameter for center symmetry and thus confinement; this is consistent with physical interpretation.
- However, theories exist which lack center symmetry for which it seems likely that the Polyakov loop and some of its generalizations appear to be order parameters for confinement.