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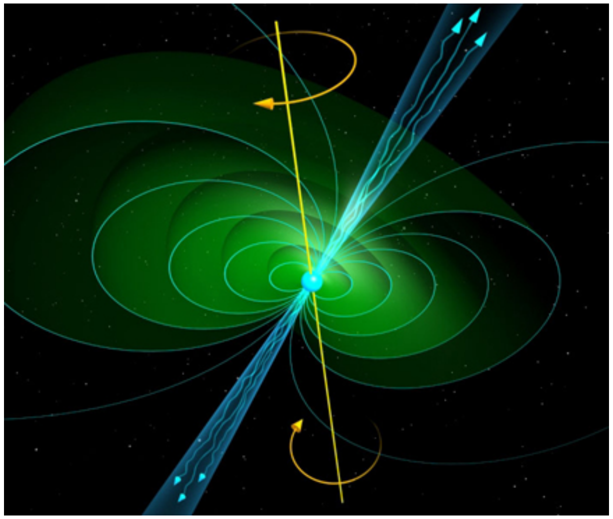
QCD in strong magnetic fields

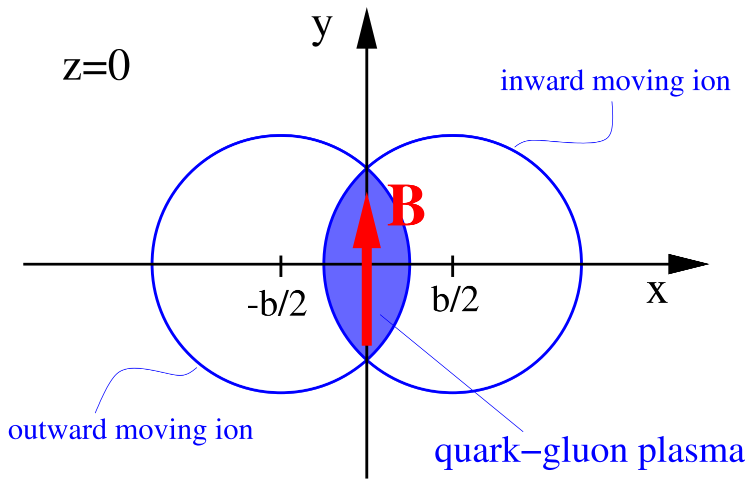
D. Kharzeev, K. Landsteiner, A. Schmitt, H.-U. Yee (Eds.), Lect. Notes Phys., to appear
[arXiv:1211.6245 [hep-ph]]

- chiral magnetic effect
  and anomaly-induced transport
- phase structure in a magnetic field
  - magnetic catalysis at $T = \mu = 0$
  - surprises at finite T, μ
    ("Inverse magnetic catalysis")

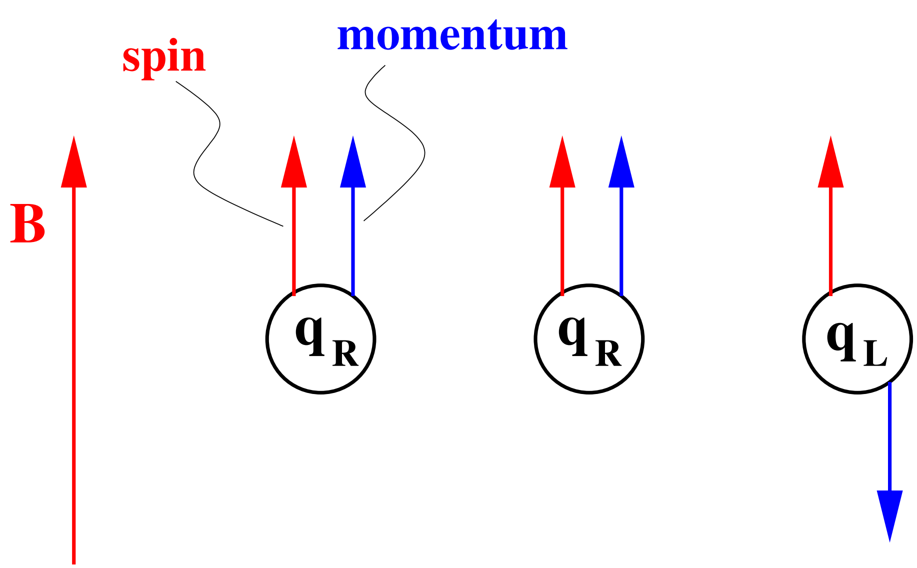


• QCD matter in strong magnetic fields (page 1/2)

Non-central heavy-ion collisions



- Chiral magnetic effect
  D.E. Kharzeev, L.D. McLerran, H.J. Warringa
  NPA 803, 227 (2008)

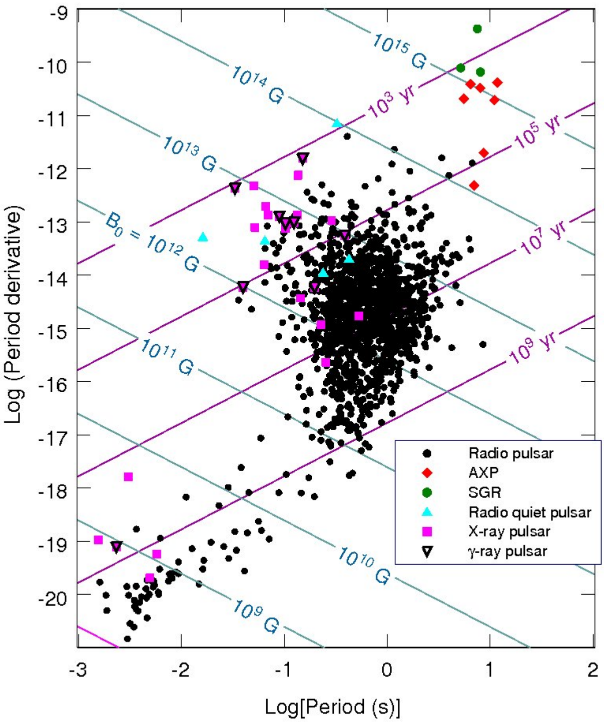


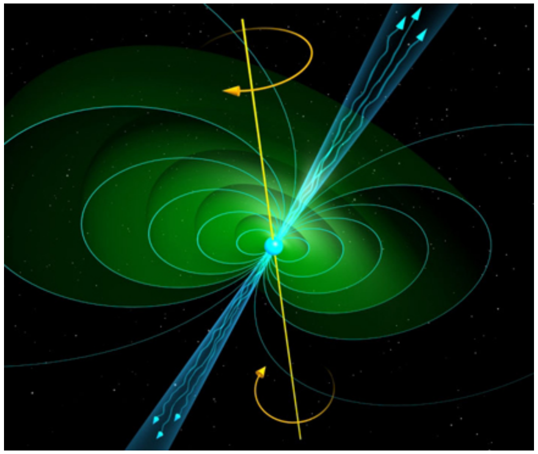
$$J = \frac{e^2 N_c}{2\pi^2}\mu_5 B$$

⇒ electric current parallel to B

• QCD matter in strong magnetic fields (page 2/2)

Compact stars
("Magnetars")





- magnetic fields from star's progenitor, strongly enhanced (flux conserved)
- surface magnetic field measured via

$$B \propto (P\dot{P})^{1/2}$$

(magn. dipole radiation)

A. K. Harding, D. Lai, Rept. Prog. Phys. 69, 2631 (2006)

• QCD at nonzero T, μ, and B

- heavy-ion collisions:

  temporarily $B \lesssim 10^{19}\,\mathrm{G}$

  Skokov, Illarionov, Toneev,
  Int. J. Mod. Phys. A 24, 5925 (2009)

  (*compare:*
  earth's magn. field: $B \simeq 0.6\,\mathrm{G}$
  LHC supercond. magnets: $B \simeq 10^5\,\mathrm{G}$)

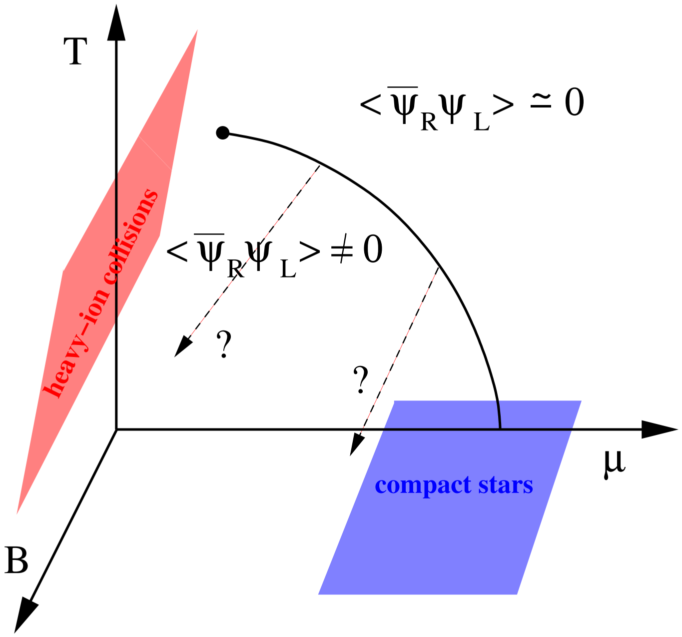
- magnetars:

  at surface $B \lesssim 10^{15}\,\mathrm{G}$

  Duncan, Thompson, Astrophys.J. 392, L9 (1992)

  larger in the interior,
  $B \sim 10^{18-20}\,\mathrm{G}$?

  Lai, Shapiro, Astrophys.J. 383, 745 (1991)
  E. J. Ferrer *et al.*, PRC 82, 065802 (2010)



effect on QCD phase transitions?

$$\Lambda^2_{\mathrm{QCD}} \sim (200\,\mathrm{MeV})^2 \sim 2\times 10^{18}\,\mathrm{G}$$

$(1\,\mathrm{eV}^2 \simeq 51.189\,\mathrm{G})$

• Magnetic catalysis (page 1/2)

K. G. Klimenko, Theor. Math. Phys. 89, 1161-1168 (1992)

V. P. Gusynin, V. A. Miransky, I. A. Shovkovy, PLB 349, 477-483 (1995)

- (massless) fermions in **Nambu-Jona-Lasinio (NJL)** model

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}\, i\gamma^\mu \partial_\mu \psi + G \left[(\bar{\psi}\psi)^2 + (\bar{\psi}\, i\gamma^5 \psi)^2 \right]$$

- mean-field approximation: $\bar{\psi}\psi = \langle\bar{\psi}\psi\rangle +$ fluctuations

Zero magnetic field:
dynamical fermion mass

$$M \propto \langle \bar{\psi}\psi \rangle \neq 0$$

for coupling $g > g_c = 1$

dimensionless coupling $g \equiv G\Lambda^2/\pi^2$

• Magnetic catalysis (page 1/2)

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Nonzero magnetic field:

$M \neq 0$ for *arbitrarily small* g,

$$M \propto \sqrt{eB}\, e^{-{\rm const.}/eBg}$$

at weak coupling $g \ll 1$

dimensionless coupling $g \equiv G\Lambda^2/\pi^2$

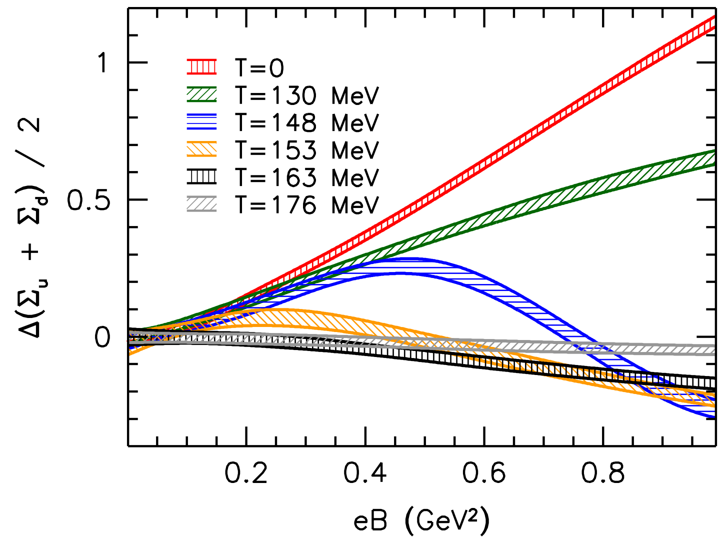
- **Magnetic catalysis (page 2/2)**

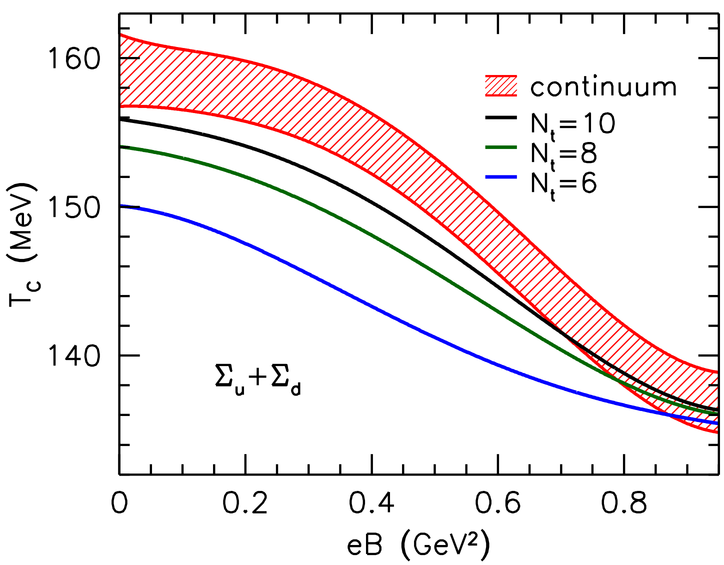
Analogy to BCS Cooper pairing:

BCS superconductor	**Magnetic catalysis**
Cooper pair condensate $\langle\psi\psi\rangle$	chiral condensate $\langle\bar{\psi}\psi\rangle$
$\Delta \propto \mu\, e^{-\text{const.}/G\nu_F}$ (ν_F: d.o.s. at $E=\mu$ Fermi surface)	$M \propto \sqrt{eB}\, e^{-\text{const.}/G\nu_0}$ (ν_0: d.o.s. at $E=0$ surface)
pairing dynamics effectively (1+1)-dimensional because of Fermi surface	effectively (1+1)-dimensional in lowest Landau level (LLL) because of magn. field
gap equation $\Delta = \frac{\mu^2 G}{2\pi^2}\int_0^\infty dk\, \frac{\Delta}{\sqrt{(k-\mu)^2+\Delta^2}}$	gap equation (LLL) $M = \frac{\lvert q\rvert BG}{2\pi^2}\int_{-\infty}^\infty dk_z\, \frac{M}{\sqrt{k_z^2+M^2}}$

- **Magnetic catalysis in QCD? ($\mu = 0$)**
  - Yes, but only for small T!
  - condensate *decreases* with B for large T
  - critical temperature *decreases* with B

  G. S. Bali *et al.*, JHEP 1202, 044 (2012)





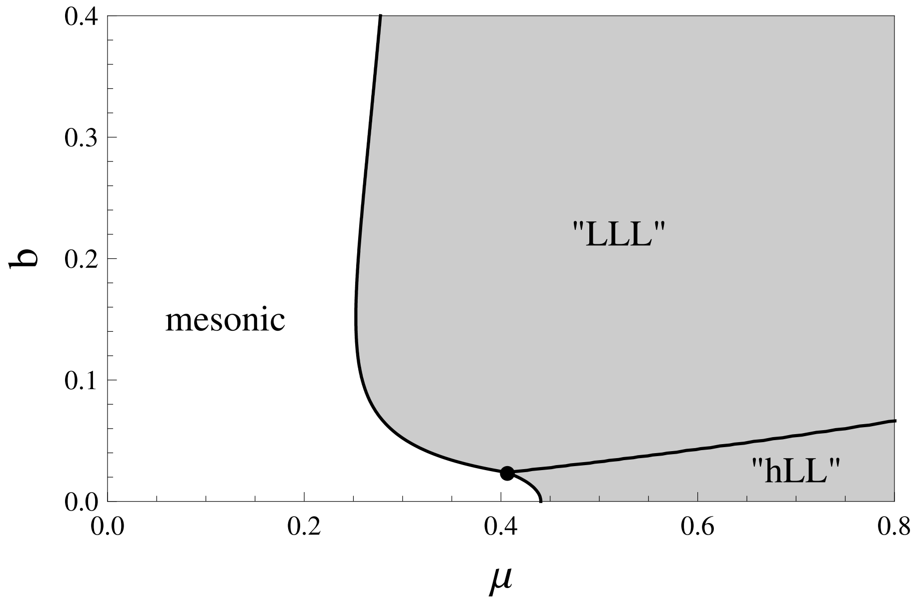
- in contrast to all model studies
- for possible explanations, see

  K. Fukushima, Y. Hidaka, PRL 110, 031601 (2013)
  T. Kojo, N. Su, arXiv:1211.7318 [hep-ph]

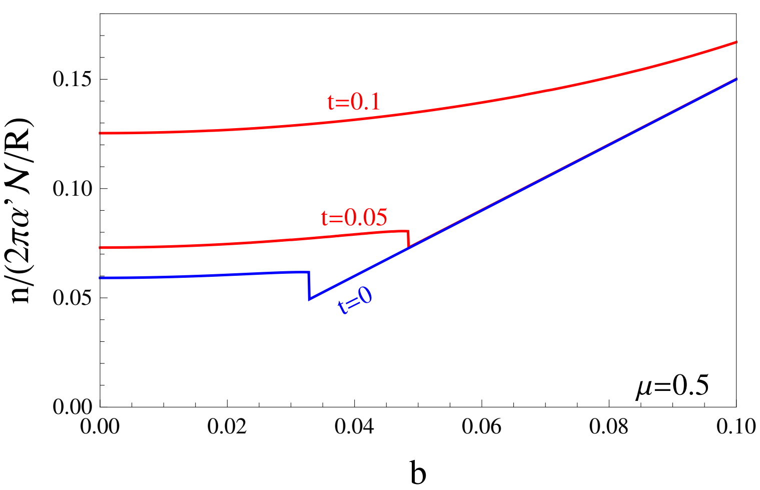
• Effect of nonzero μ (holographic model)

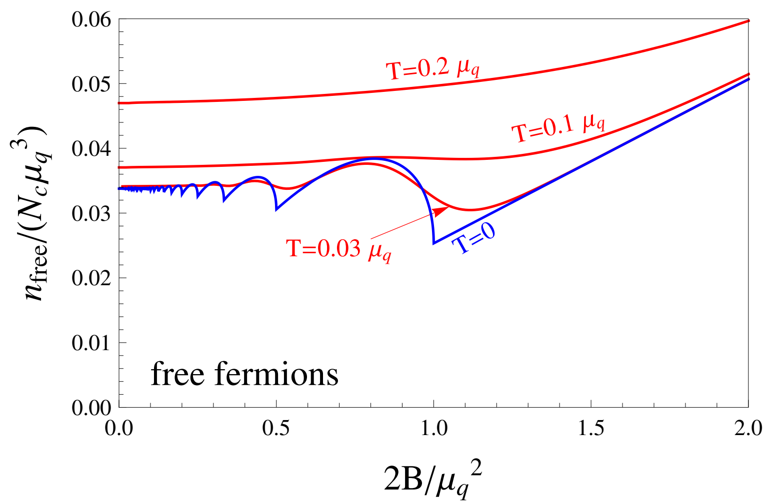
- $T = 0$ phase diagram in the Sakai-Sugimoto model (deconfined phase)

  F. Preis, A. Rebhan, A. Schmitt, JHEP 1103, 033 (2011)



Apparent Landau level transition G. Lifschytz, M. Lippert, PRD 80, 066007 (2009):





- **Inverse magnetic catalysis at nonzero μ**

Why does B *restore* chiral symmetry for certain μ?

- free energy *gain* from $\bar\psi - \psi$ pairing increases with B (magnetic catalysis)
- μ induces free energy *cost* for pairing; this cost depends on B!

NJL (weak coupling):

E. V. Gorbar *et al.*, PRC 80, 032801 (2009)

$$\Delta\Omega \propto B[\mu^2 - M(B)^2/2]$$

just like Clogston limit $\delta\mu = \frac{\Delta}{\sqrt{2}}$ in superconductivity

A. Clogston, PRL 9, 266 (1962)

B. Chandrasekhar, APL 1, 7 (1962)

$\rightarrow$ **no inverse catalysis**

Sakai-Sugimoto:

large B:

$$\Delta\Omega \propto B[\mu^2 - 0.12\, M(B)^2]$$

$\rightarrow$ **no inverse catalysis**

small B:

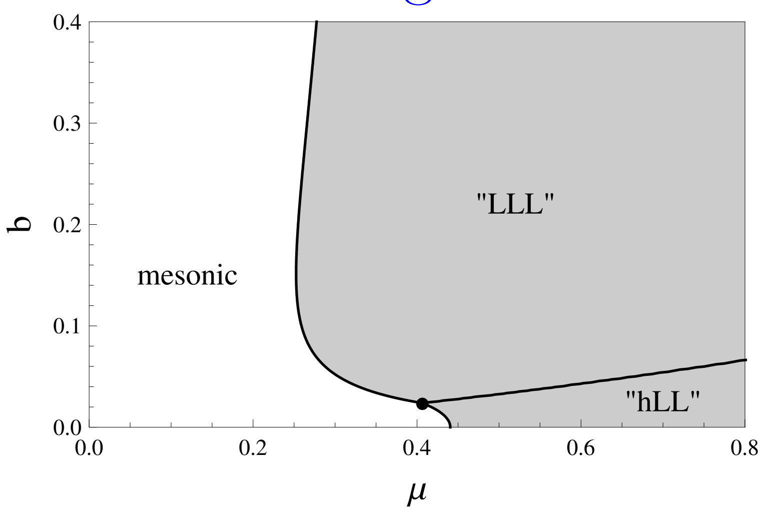
$$\Delta\Omega \propto \mu^2 B - \mathrm{const} \times M(B)^{7/2}$$

$\rightarrow$ **inverse catalysis possible**

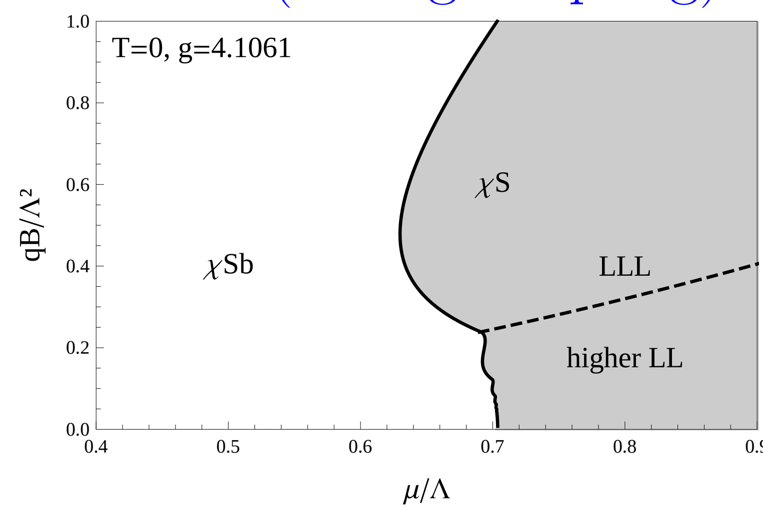
• Comparison with NJL calculation ($T = 0$)

F. Preis, A. Rebhan and A. Schmitt, arXiv:1208.0536 [hep-ph]

Sakai-Sugimoto



NJL (strong coupling)



- NJL at large g: inverse magnetic catalysis like in Sakai-Sugimoto!
- inverse magnetic catalysis in NJL and related models:

D. Ebert, K. G. Klimenko, M. A. Vdovichenko and A. S. Vshivtsev, PRD 61, 025005 (2000)

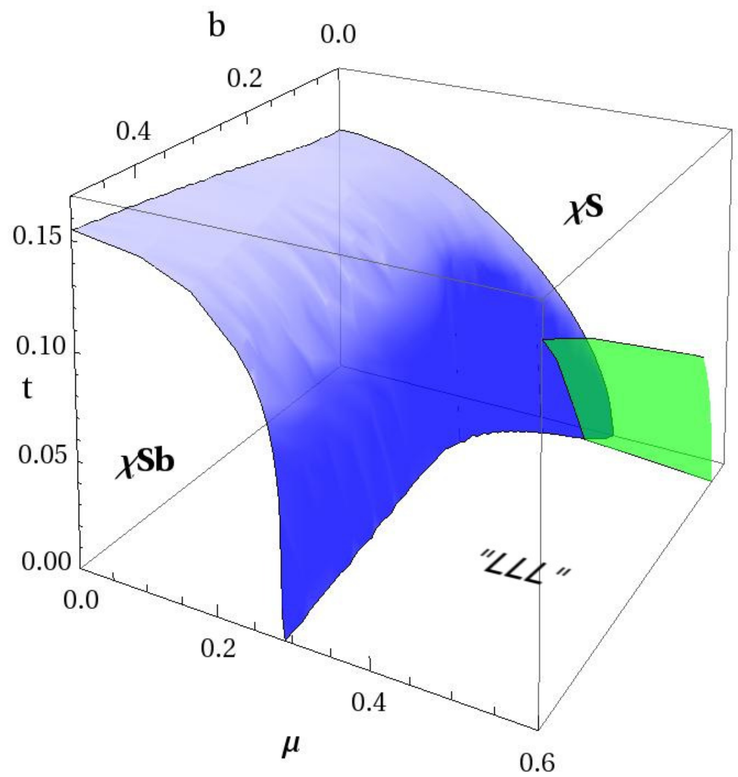
T. Inagaki, D. Kimura and T. Murata, Prog. Theor. Phys. 111, 371 (2004)

B. Chatterjee, H. Mishra and A. Mishra, PRD 84, 014016 (2011)

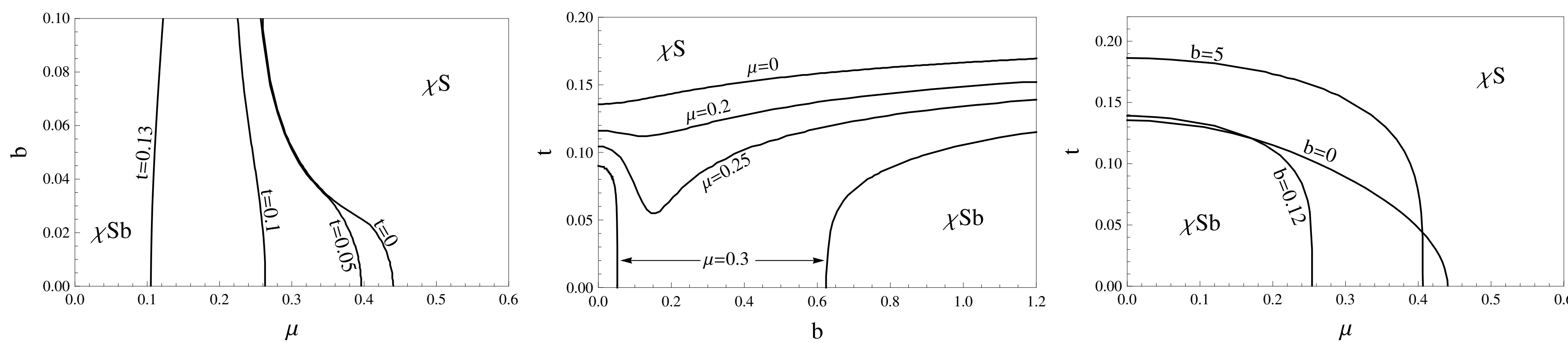
S. S. Avancini, D. P. Menezes, M. B. Pinto and C. Providencia, PRD 85, 091901 (2012)

J. O. Andersen and A. Tranberg, JHEP 1208, 002 (2012)

• Phase structure at nonzero temperature

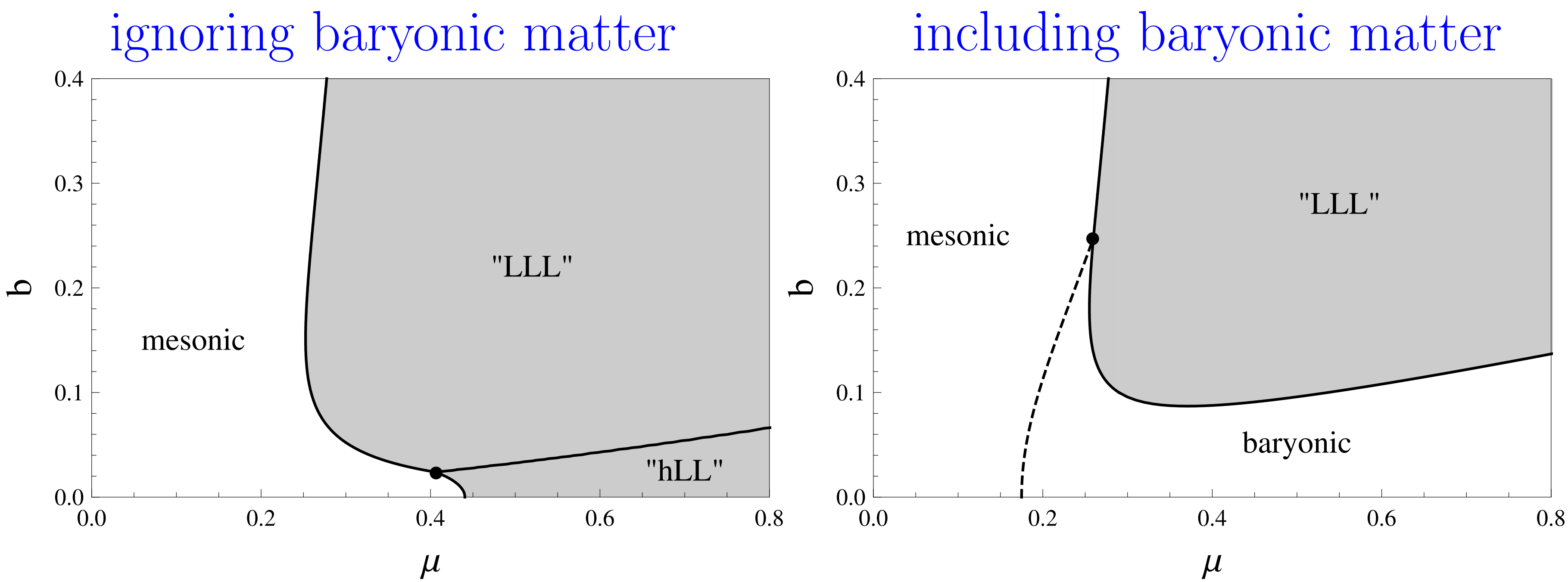


blue: chiral phase transition
green: "LLL" transition



• Including baryonic matter

F. Preis, A. Rebhan, A. Schmitt, JPG 39, 054006 (2012)



- small b: baryonic matter prevents the system from restoring chiral symmetry
- baryon onset line intersects chiral phase transition line $\rightarrow$ large b: mesonic matter superseded by quark matter
- with baryonic matter, IMC plays an even more prominent role in the phase diagram

- **Summary**

- QCD matter in strong magnetic fields ...

  ... is of phenomenological relevance
  (heavy-ion collisions & compact stars)

  ... yields rich physics which may teach us more about QCD

- "typically", a magnetic field favors chiral symmetry breaking
  (non-confining models at $\mu = 0$)

- surprises (= opposite effect) have been found

  – in lattice QCD ($\mu = 0$)

  – at nonzero μ (models)

- for instance: critical μ reduced from $\mu \simeq 400\,\mathrm{MeV}$ ($B = 0$) to $\mu \simeq 230\,\mathrm{MeV}$ at $B \simeq 1.0 \times 10^{19}\,\mathrm{G}$
  $\rightarrow$ quark matter favored in compact stars?

• Open questions

- Is the observed charge separation at RHIC due to the chiral magnetic effect?
- What is the physics behind decreasing $T_c(B)$ in QCD?
- Is there inverse magnetic catalysis at nonzero μ in QCD (as the models suggest)?
- How does the "chiral shift" affect inverse magnetic catalysis?
- Can we describe realistic baryonic matter with holographic methods?