Calculating the analytic structure of QFT Green's functions (on GPUs)

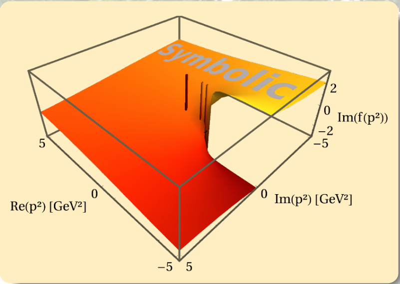
A. Windisch, M. Q. Huber and R. Alkofer

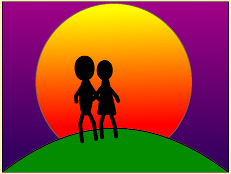
Excited QCD 2013

3-9 February 2013 *Bjelasnica Mountain, Sarajevo*

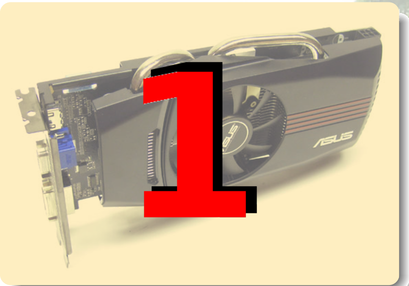
Europe/Sarajevo timezone

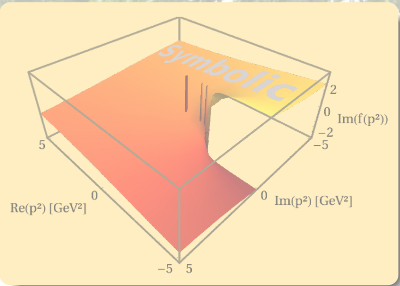
Outline

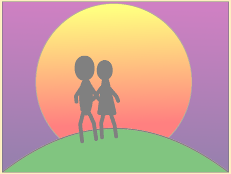




Outline







NVIDIA

GPU Computing

CUDA

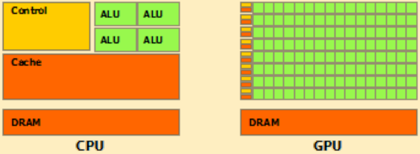
Graphics Processing Unit

NVIDIA

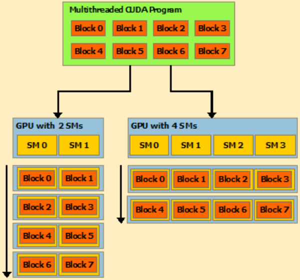
Compute Unified Device Architecture

CUDA C	CUDA Fortran
NVIDIA	The Portland Group

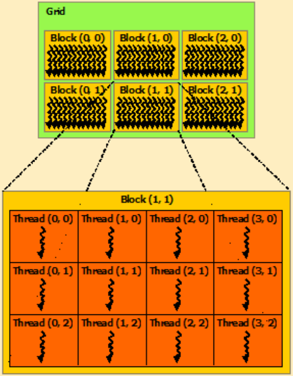
CPU vs. GPU [nvidia.com]



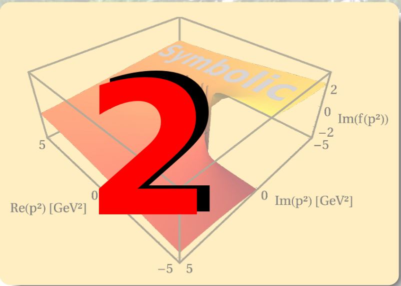
Grids and Blocks [nvidia.com]

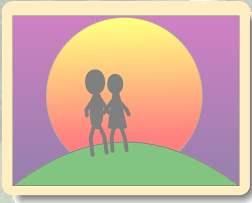


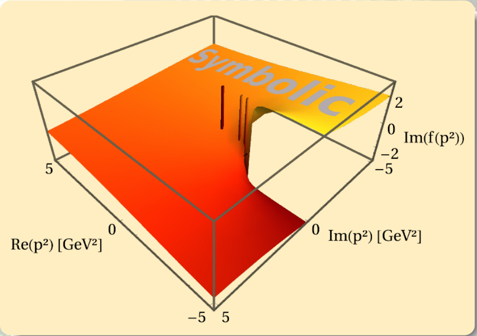
Threads [nvidia.com]



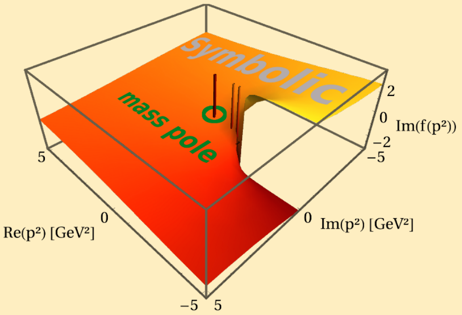
Outline



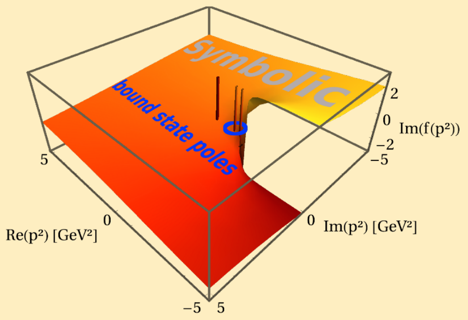




Analytic structure

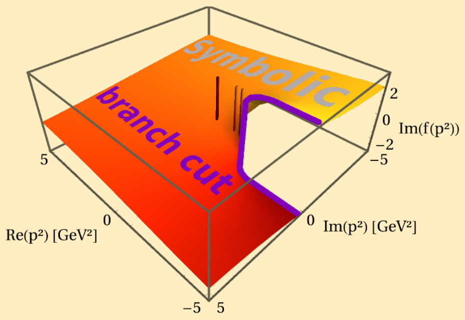


mass



mass

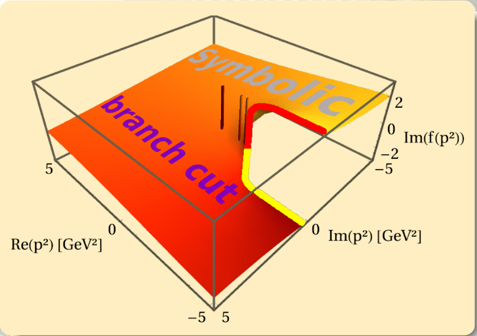
bound states



mass

bound states

spectral density

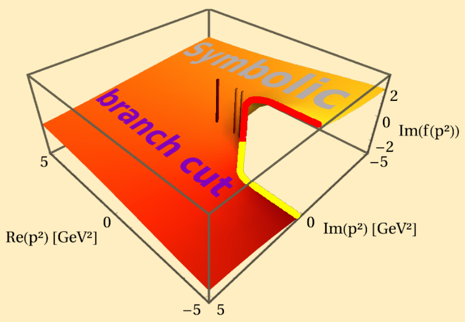


mass

bound states

spectral density

$\rho(p^2) = \frac{1}{2\pi i}\,[\,\text{■} - \text{■}\,]$



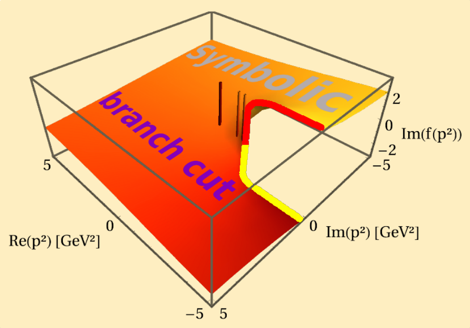
mass

bound states

spectral density

$$\rho(p^2) = \frac{1}{2\pi i}\,[\blacksquare - \blacksquare]$$

positivity



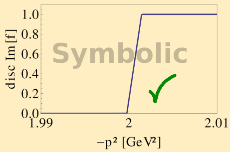
mass

bound states

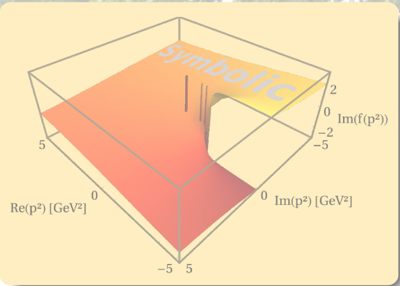
spectral density

$$\rho(p^2) = \frac{1}{2\pi i}\left[\,\blacksquare - \blacksquare\,\right]$$

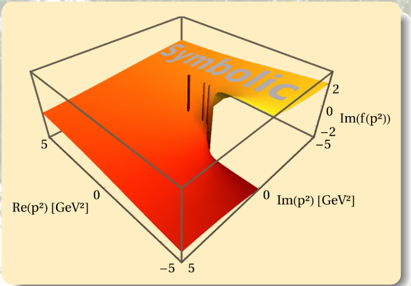
positivity

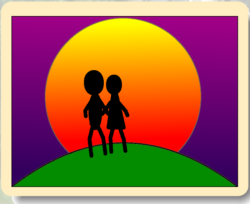


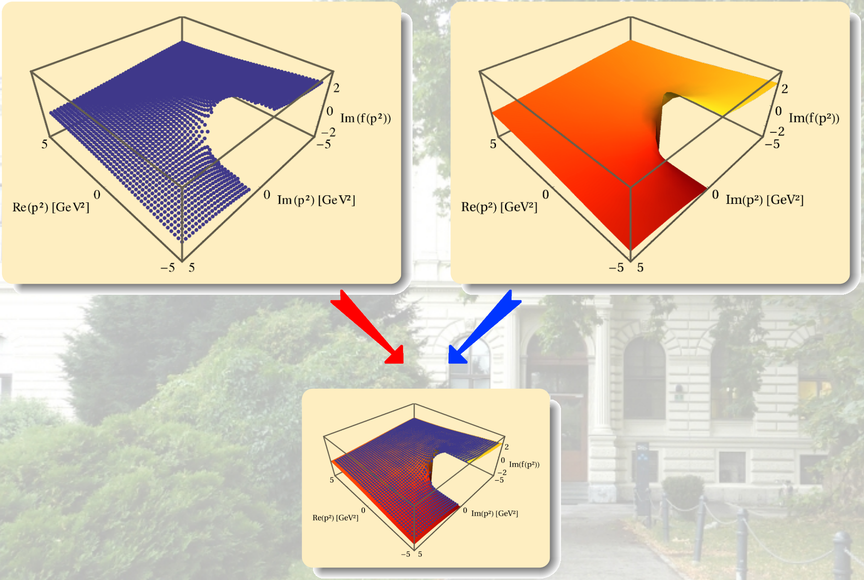
Outline




ASUS


Symbolic
2
0
Im(f(p²))
−2
−5
5
Re(p²) [GeV²]
0
0
Im(p²) [GeV²]
−5
5




2
0
Im(f(p²))
−2
−5
5
Re(p²) [GeV²]
0
Im(p²) [GeV²]
−5
5

Method

Application

Outlook

Complex integration: AW, R. Alkofer, G. Haase and M. Liebmann, *Comput.Phys.Commmun.* **184** (2013), arXiv:1205.0752

L. Baulieu et al., Phys. Rev. D **82**, 025021 (2010), [0912.5153]

$$\mathcal{O}_{4d}(p^2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(p-k)^2 - i\sqrt{2}\theta^2} \frac{1}{k^2 + i\sqrt{2}\theta^2}$$

L. Baulieu et al., Phys. Rev. D **82**, 025021 (2010), [0912.5153]

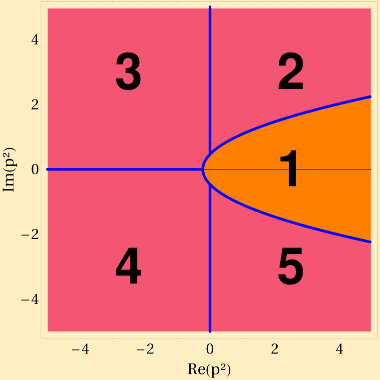
$$\mathcal{O}_{4d}^{sub,resc.}(x) = \left(1 - \frac{\pi}{2x} + \frac{\sqrt{1-x^2}}{x} \arccos\{x\}\right)$$

Subtracted and rescaled, $x = p^2, \; y = k^2, \; 2\sqrt{2}\theta^2 \equiv 1$

$$\mathcal{O}_{4d}^{sub,resc.}(x) = \frac{2}{\pi}\int_0^{\xi^2} dy y \int_{-1}^{1} dz \sqrt{1-z^2} \frac{-x+2\sqrt{x}\sqrt{y}z}{(x+y-2\sqrt{x}\sqrt{y}z-\frac{i}{2})(y^2+\frac{1}{4})}$$

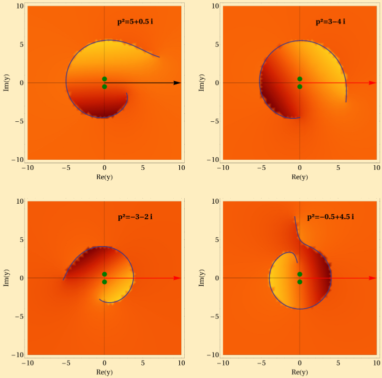
Subtracted and rescaled, $x = p^2,\ y = k^2,\ 2\sqrt{2}\theta^2 \equiv 1$

$$\mathcal{O}_{4d}^{sub,resc.}(x) = \frac{2}{\pi}\int_0^{\xi^2} dy\, y \int_{-1}^{1} dz \sqrt{1-z^2}\, \frac{-x+2\sqrt{x}\sqrt{y}z}{(x+y-2\sqrt{x}\sqrt{y}z-\frac{i}{2})(y^2+\frac{1}{4})}$$



Generic structure

$$\int_0^{\xi^2} dy\, y \int_{-1}^{1} dz \sqrt{1-z^2}\, f(x,y,z)$$



Left: y-plane after angular integration.
Right: p^2-plane.

Left: y-plane after angular integration.
Right: p^2-plane.

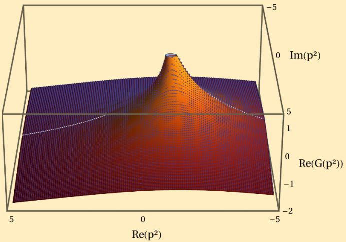
Escaping for $\Re(p^2) > 0$

Escaping for $\Re(p^2) <= 0,\ \Im(p^2) <= 0$

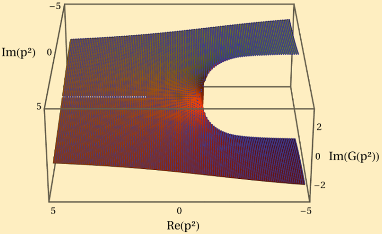
Subtracted and rescaled, $x = p^2$, $y = k^2$, $2\sqrt{2}\theta^2 \equiv 1$, **Dots**

$$\mathcal{O}_{4d}^{sub,resc.}(x) = \frac{2}{\pi}\int_0^{\xi^2} dy\, y \int_{-1}^{1} dz \sqrt{1-z^2}\, \frac{-x+2\sqrt{x}\sqrt{y}z}{(x+y-2\sqrt{x}\sqrt{y}z-\frac{i}{2})(y^2+\frac{1}{4})}$$

Solution, $\mathfrak{Re}$



Solution, $\mathfrak{Im}$



L. Baulieu et al., Phys. Rev. D **82**, 025021 (2010), **Solid**

$$\mathcal{O}_{4d}^{sub,resc.}(x) = \left(1 - \frac{\pi}{2x} + \frac{\sqrt{1-x^2}}{x}\arccos\{x\}\right)$$

CPU	GPU		
Intel® Xeon® Processor X5650, 1 core	NVIDIA GeForce® GTX® 550 Ti	NVIDIA GeForce® GTX® 480	NVIDIA GeForce® Tesla® C2070

`real 252m1.869s`
`user 252m0.982s`
`sys 0m0.102s`
	`real 6m21.590s`
	`user 6m21.564s`
	`sys 0m0.068s`
		`real 4m21.111s`
		`user 4m20.804s`
		`sys 0m0.280s`
			`real 2m38.290s`
			`user 0m0.021s`
			`sys 0m0.052s`

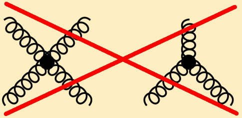
1	≈ 39.6	≈ 57.9	≈ 95.5

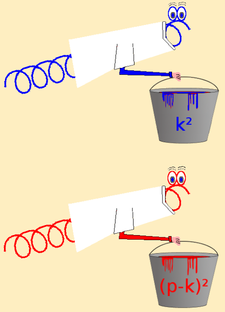
Method

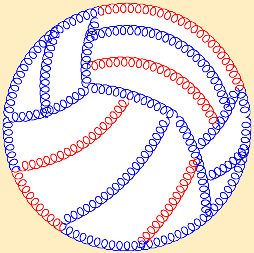
Application

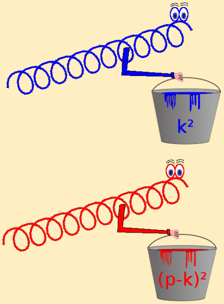
Outlook

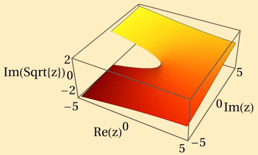
$$\langle F^2(x)F^2(0)\rangle$$



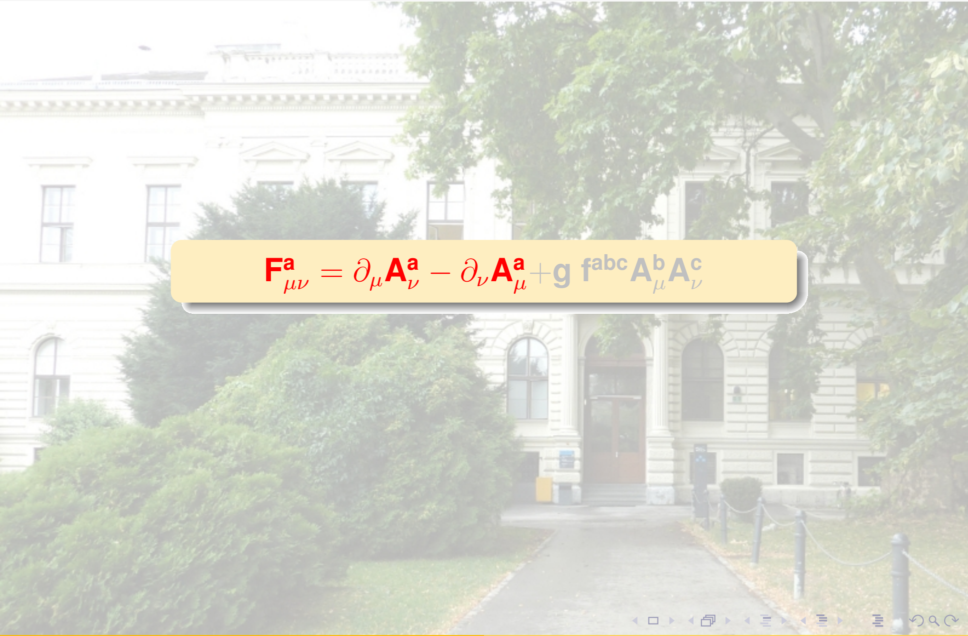




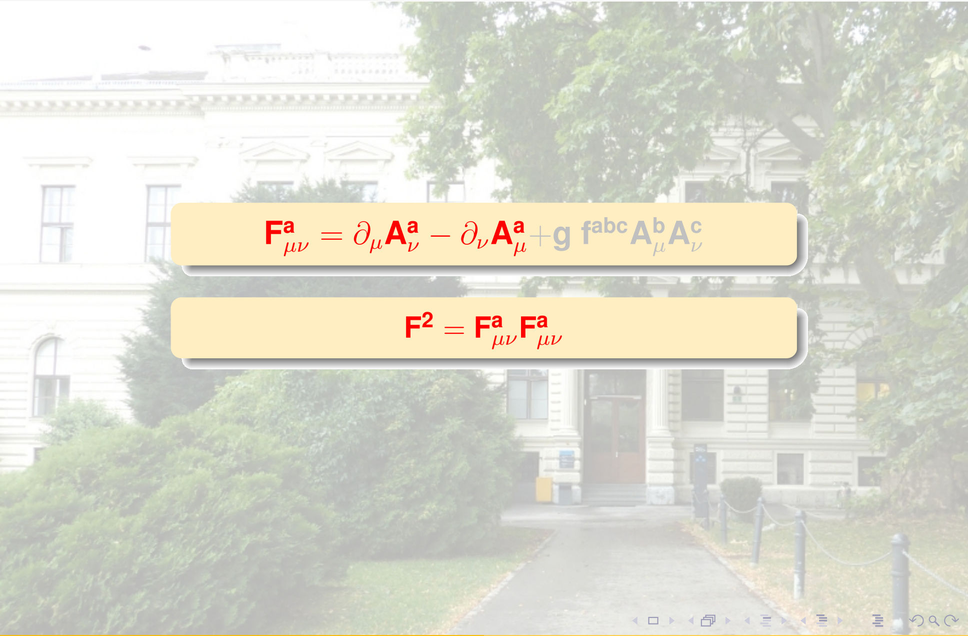




$$\mathbf{F}^{\mathbf{a}}_{\mu\nu} = \partial_\mu \mathbf{A}^{\mathbf{a}}_\nu - \partial_\nu \mathbf{A}^{\mathbf{a}}_\mu + \mathbf{g}\ \mathbf{f}^{\mathbf{abc}} \mathbf{A}^{\mathbf{b}}_\mu \mathbf{A}^{\mathbf{c}}_\nu$$



$$\mathbf{F}^{\mathbf{a}}_{\mu\nu} = \partial_\mu \mathbf{A}^{\mathbf{a}}_\nu - \partial_\nu \mathbf{A}^{\mathbf{a}}_\mu + \mathbf{g}\ \mathbf{f}^{\mathbf{abc}} \mathbf{A}^{\mathbf{b}}_\mu \mathbf{A}^{\mathbf{c}}_\nu$$



$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu A^c_\nu$$

$$F^2 = F^a_{\mu\nu} F^a_{\mu\nu}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu A^c_\nu$$

$$F^2 = F^a_{\mu\nu} F^a_{\mu\nu}$$

$$\langle F^2(x) F^2(0) \rangle$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu A^c_\nu$$

$$F^2 = F^a_{\mu\nu} F^a_{\mu\nu}$$

$$\langle F^2(x) F^2(0) \rangle$$

$$\langle F^2(x) F^2(0) \rangle = \int \frac{d^D k}{(2\pi)^D} \exp\{ik \cdot x\} \mathscr{O}(k)$$

$$\mathbf{F}^{\mathbf{a}}_{\mu\nu} = \partial_\mu \mathbf{A}^{\mathbf{a}}_\nu - \partial_\nu \mathbf{A}^{\mathbf{a}}_\mu + \mathbf{g}\,\mathbf{f}^{\mathbf{abc}} \mathbf{A}^{\mathbf{b}}_\mu \mathbf{A}^{\mathbf{c}}_\nu$$

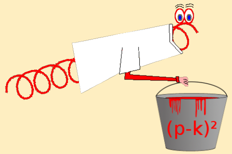
$$\mathbf{F}^2 = \mathbf{F}^{\mathbf{a}}_{\mu\nu} \mathbf{F}^{\mathbf{a}}_{\mu\nu}$$

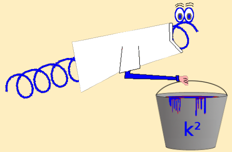
$$\langle \mathbf{F}^2(\mathbf{x}) \mathbf{F}^2(\mathbf{0}) \rangle$$

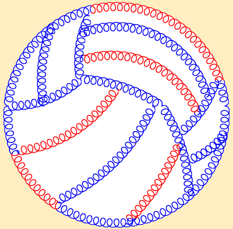
$$\langle \mathbf{F}^2(\mathbf{x}) \mathbf{F}^2(\mathbf{0}) \rangle = \int \frac{\mathbf{d}^{\mathbf{D}}\mathbf{k}}{(\mathbf{2}\pi)^{\mathbf{D}}} \exp\{\mathbf{ik} \cdot \mathbf{x}\} \mathscr{O}(\mathbf{k})$$

$\mathscr{O}(k)$: **The "F^2 *correlator*"**

$$\mathcal{O}_d(p^2) = 8C \int \frac{d^d k}{(2\pi)^d} \left[\mathcal{G}\left((p-k)^2\right) \mathcal{G}\left(k^2\right) (p-k)^2 k^2 + (d-2)\left((p-k)\cdot k\right)^2 \right]$$

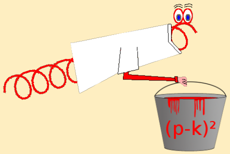




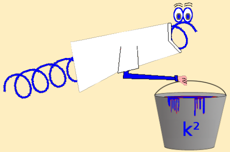


Glueball correlators at the Born level: AW, M.Q. Huber and R. Alkofer, **[1212.2175], [1301.3525]**

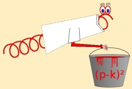
$$\mathcal{O}_d(p^2) = 8C \int \frac{d^d k}{(2\pi)^d} \left[\mathcal{G}\left((p-k)^2\right) \mathcal{G}\left(k^2\right) (p-k)^2 k^2 + (d-2)\left((p-k) \cdot k\right)^2 \right]$$



$$D_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{l_\mu l_\nu}{l^2} \right) \mathcal{G}\left(l^2\right), \quad l = (p-k)$$



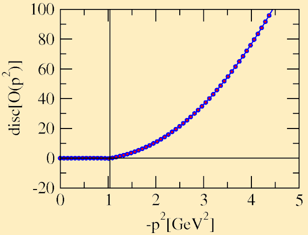
$$D_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \mathcal{G}\left(k^2\right)$$



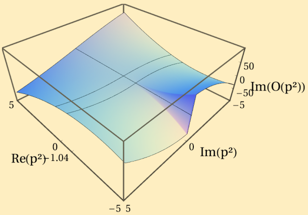
Alkofer et al., PRD70 (2004) 014014, hep-ph/0309077

$$\mathscr{G}(p^2) = w\frac{1}{p^2}\left(\frac{p^2}{p^2+\Lambda^2}\right)^{2\kappa}$$

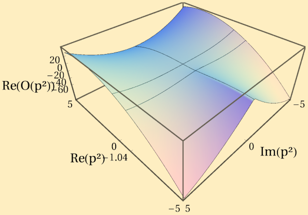
Discontinuity

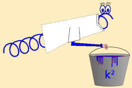


$\Im(\mathscr{O}_{4d})$



$\Re(\mathscr{O}_{4d})$

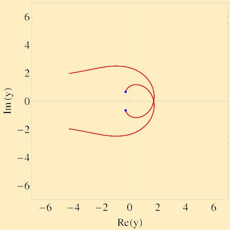




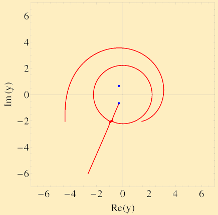
Cucchieri et al., PRD85 (2012) 094513, [1111.2327]

$$\mathscr{G}(p^2) = C\frac{p^2+s}{p^4+u^2p^2+t^2}$$

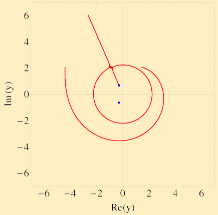
$x = -2.03\ GeV^2$

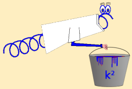


$x = -1.18 - 2.63i\ GeV^2$



$x = -1.18 + 2.63i\ GeV^2$

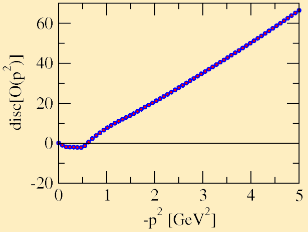




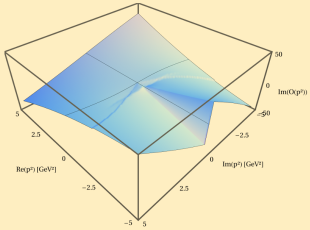
Cucchieri et al., PRD85 (2012) 094513, [1111.2327]

$$\mathcal{G}(p^2) = C\frac{p^2+s}{p^4+u^2p^2+t^2}$$

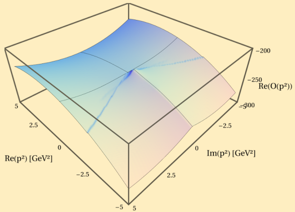
Discontinuity



$\Im(\mathcal{O}_{4d})$



$\Re(\mathcal{O}_{4d})$

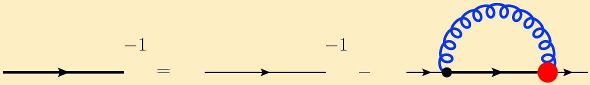


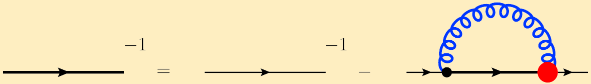
Method

Application

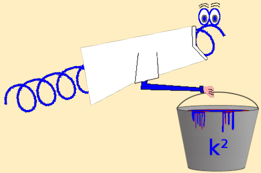
Outlook

quark propagator, $x \in \mathbb{C}$



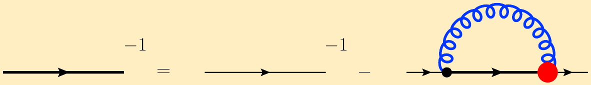


Gluon

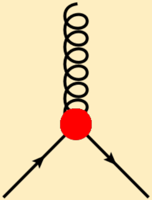


Possible input

- ***S. Strauss, Ch. Fischer*** **and** ***Ch. Kellermann***, *Phys.Rev.Lett.* **109** (2012) 252001, [1208.6239]
- ***Scaling–*** **and** ***decoupling fits***



Quark-Gluon vertex

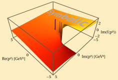


Possible input

- bare
- models
- *Ch. Fischer* and *R. Williams*, *Phys.Rev.Lett.* **103** (2009) 122001, [0905.2291]
- full study in $\mathbb{R}_0^+$, (*M. Hopfer*)

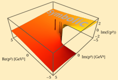
Summary

Task.

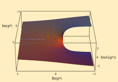


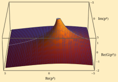
Summary

Task.



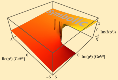
Method. **AW, R. Alkofer, G. Haase and M. Liebmann [1205.0752]**



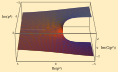


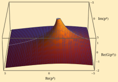
Summary

Task.

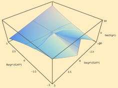


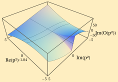
Method. **AW, R. Alkofer, G. Haase and M. Liebmann [1205.0752]**





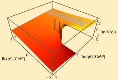
Application. **AW, M.Q. Huber and R. Alkofer [1212.2175], [1301.3525]**



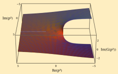


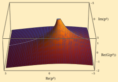
Summary

Task.

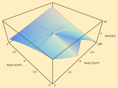


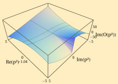
Method. AW, R. Alkofer, G. Haase and M. Liebmann [1205.0752]



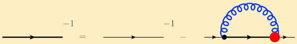


Application. AW, M.Q. Huber and R. Alkofer [1212.2175], [1301.3525]





Future.




Thank You for Your Attention!