QCD with two light dynamical Chirally Improved quarks

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channel	valence quark content
N^+	uud
Δ^{++}	uuu
Λ^0	uds
Σ^+	uus
Ξ^0	uss
Ω^-	sss

Example: Few baryon channels

Aim of the presented work:
Ab-initio determination of the (low-lying) excited baryon and meson spectrum from QCD.

Chosen approach:
Lattice regularization.

"particle"	spin	parity	E [MeV]	exp. quality
proton	1/2	+	938	****
neutron	1/2	+	940	****
nucleon	1/2	+	1440	****
nucleon	3/2	-	1520	****
nucleon	1/2	-	1535	****
nucleon	1/2	-	1650	****
nucleon	5/2	-	1675	****
nucleon	5/2	+	1680	****
nucleon	3/2	-	1700	***
nucleon	1/2	+	1710	***
nucleon	3/2	+	1720	****
nucleon	3/2	+	1900	**
nucleon	5/2	+	1900	***
nucleon	7/2	+	1990	**
nucleon	3/2	-	2080	**
nucleon	1/2	-	2090	*
nucleon	1/2	+	2100	*
nucleon	7/2	-	2190	****
nucleon	5/2	-	2200	**
nucleon	9/2	+	2220	****
nucleon	9/2	+	2250	****
nucleon	11/2	-	2600	***
nucleon	13/2	+	2700	**

Example: Lowest (exp.) states for the nucleon channel

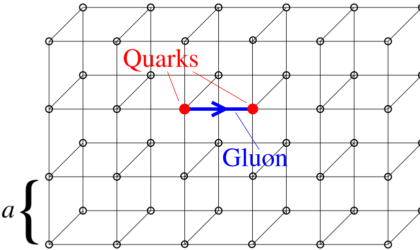
Outline

- Setup of the simulation
- Results on the excited meson and baryon spectrum
- Exploring the content of states
- Discussion of finite volume effects
- Conclusion

Setup of the simulation

Discretization:

- $\int dx \;\to \Sigma_x$
- $\partial_\mu \quad \to \frac{1}{2a}(\delta^m_{n+\hat{\mu}} - \delta^m_{n-\hat{\mu}}) + \mathcal{O}(a^2)$



Motivation:

- Gauge-covariant regularization
- Non-perturbative quantization
- Ab-initio numerical calculations

Numerical evaluation: Limits a posteriori to be performed

- Continuum limit: $a \quad \to 0$
- Infinite volume limit: $V \quad \to \infty$
- $(m_\pi \to m_{\pi,\text{phys}}$ or $m_\pi \to 0)$
- Mapping to continuum physics (e.g. spin).

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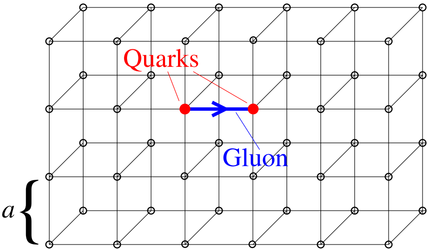

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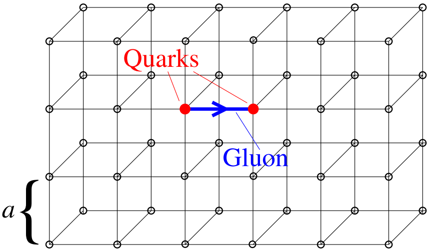

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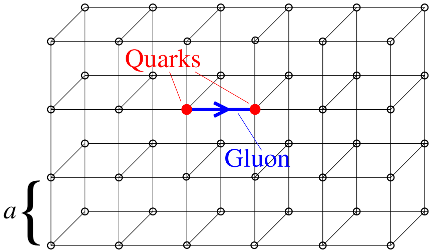

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- Energy levels are extracted from euclidean hadron correlators:

$$\langle O(t) O^\dagger(0) \rangle = \sum_n A_n e^{-tE_n}$$

- E.g. a pion correlator is given by ($O = u\gamma_5 \bar{d}$):

$$\begin{aligned} \langle O(t) O^\dagger(0) \rangle &= \frac{1}{Z} \int \mathcal{D}[U] \mathcal{D}^{n_f} [\bar{\psi}, \psi] e^{-S_G[U]} e^{-S_F[\psi, \bar{\psi}, U]} O(t) \bar{O}(0) \\ &= \frac{1}{Z} \int \mathcal{D}[U] e^{-S_G[U]} [\det(D)]^{n_f} \ \mathrm{tr} \left[D^{-1}(0,t) \gamma_5 D^{-1}(t,0) \gamma_5 \right] \end{aligned}$$

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  - Importance sampling for the gauge action and the fermion determinant to generate the gauge configurations.
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- Exponential fit to the correlator yields energy levels.

- **Variational method**: Construct several interpolators, compute all cross-correlations, obtaining the **correlation matrix**:

$$C_{ij}(t) = \langle 0|O_i(t)O_j^\dagger(0)|0\rangle$$

- Solve its generalized eigenvalue problem:

$$C(t)\,\vec{v}_k = \lambda_k(t)\,C(t_0)\,\vec{v}_k$$

- Each eigenvalue is related only to a single energy level at large time separations.

$$\boxed{\lambda_k(t,t_0) \propto \mathrm{e}^{-t\,E_k}\left(1+\mathcal{O}(\mathrm{e}^{-t\,\Delta E})\right)}$$

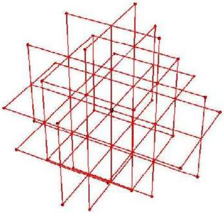
- Extraction of excited states possible.
- Eigenvectors tell about the content of the states.
- Michael NPB259(1985)58; Lüscher/Wolff NPB339(1990)222; Blossier et al. JHEP(2009)0904:094

- Lattice breaks chiral symmetry explicitly
- Recovery:
  - Continuum limit or
  - Particular discretizations of the Dirac operator obeying:

$$\boxed{\mathbf{D}\,\gamma_5 + \gamma_5\,\mathbf{D} = \mathbf{a}\,\mathbf{D}\,\gamma_5\,\mathbf{D}}$$ "**Ginsparg-Wilson** equation"

- Chirally Improved Dirac operator:
  General ansatz for bilinear fermion action ($\bar{\psi}_n D_{nm} \psi_m$):
  (Gattringer, PRD63(2001)114501)

$$D_{nm} = \sum_{\alpha=1}^{16} \Gamma_\alpha \sum_{p \in \mathcal{P}^\alpha_{m,n}} c_p^\alpha \prod_{l \in p} U_l \, \delta_{n,m+p}$$



- Plug in GW-equation, calculate coefficients
- Improved theory: reduced discretization errors

- 2 light dynamical Chirally Improved quarks.
- Lüscher-Weisz improved gauge action.

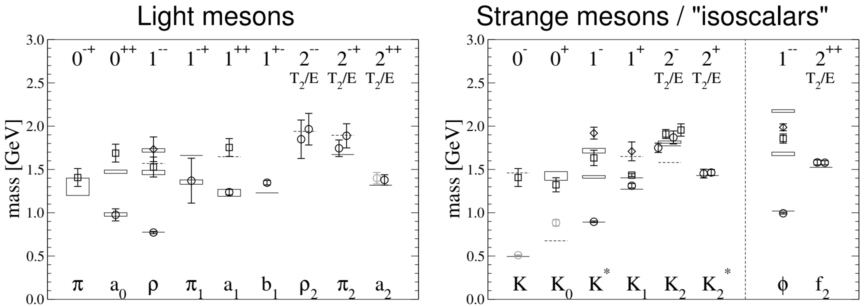
- 7 ensembles for $\mathbf{16^3} \times \mathbf{32}$:

set	$\mathbf{m}_\pi$[MeV]	**a**[fm]	β_{LW}	m_0	$m_\pi L$	configs
A50	**596(7)**	**0.1324(11)**	4.70	-0.050	6.40	200
A66	**255(7)**	**0.1324(11)**	4.70	-0.066	2.72	200
B60	**516(6)**	**0.1366(15)**	4.65	-0.060	5.72	300
B70	**305(6)**	**0.1366(15)**	4.65	-0.070	3.38	200
C64	**588(6)**	**0.1398(14)**	4.58	-0.064	6.67	200
C72	**451(5)**	**0.1398(14)**	4.58	-0.072	5.11	200
C77	**330(5)**	**0.1398(14)**	4.58	-0.077	3.74	300

- Lattice spacing **a** from Sommer parameter of $r_{0,exp} = 0.48\,\mathrm{fm}$ at the physical point.
- Coarse lattices possible due to improved action.
- Gattringer et al., PRD79(2009)054501, GPE et al., PRD85(2012)034508

Results

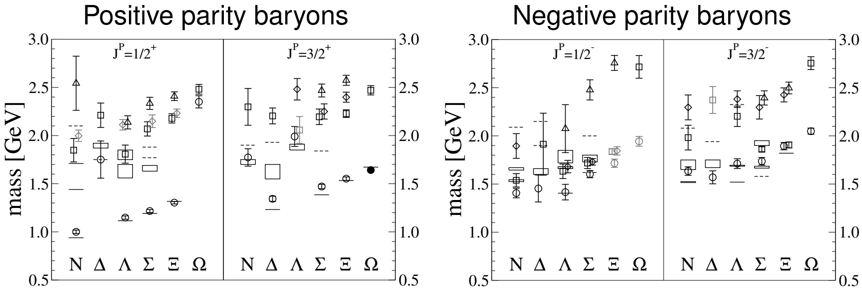
Finite volume meson spectrum (LO χPT extrapolated, $L \approx 2.2$ fm)



- In general good agreement with experiment.
- Most ground states established, some excitations still unclear.

(GPE et al., PRD85(2012)034508, arXiv:1112.1601)

Finite volume baryon spectrum (LO χPT extrapolated, $L \approx 2.2$ fm)



- Ground state masses close to experiment, excitations often too high.
- Finite size effects probably non-negligible, to be discussed.

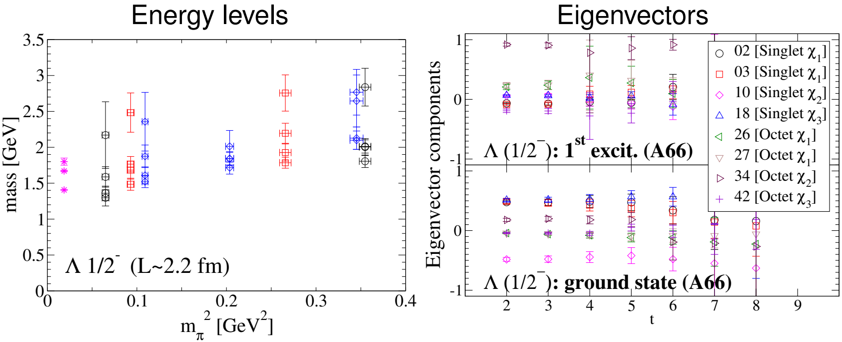
(GPE et al., arXiv:1212.2032, arXiv:1301.4318, to appear in PRD)

For exact $SU(3)_V$ (here: heavy m_π):

- Λ: singlet/octet are orthogonal.
- Σ, Ξ: octet/decuplet are orthogonal.
- Mesons: C-parity $= \pm$ are orthogonal.

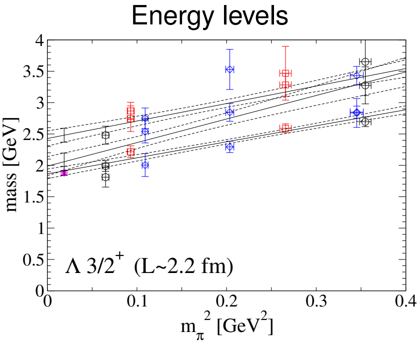
From $SU(3)_V$ towards physical m_π:

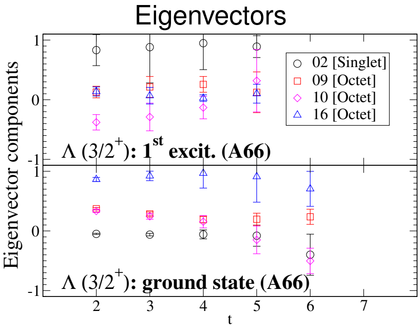
- Mixing is expected to increase.
- Contribution to the physical states can be analyzed with the variational method.



- Extrapolations compatible with $\Lambda(1405)$, $\Lambda(1670)$ and $\Lambda(1800)$.
- Ground state dominated by singlet (in the finite basis used),
- 1^{st} excitation dominated by octet,
- 2^{nd} excitation dominated by octet,
- 3^{rd} excitation dominated by singlet.
- Singlet/octet mixing increases towards the physical point.

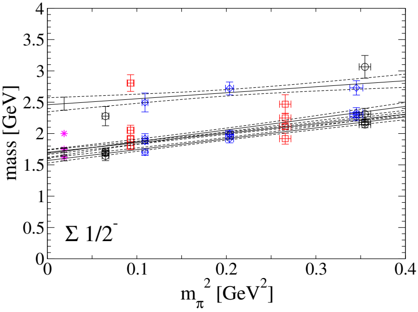
Singlet/octet analysis: Λ, $I(J^P) = 0(3/2^+)$



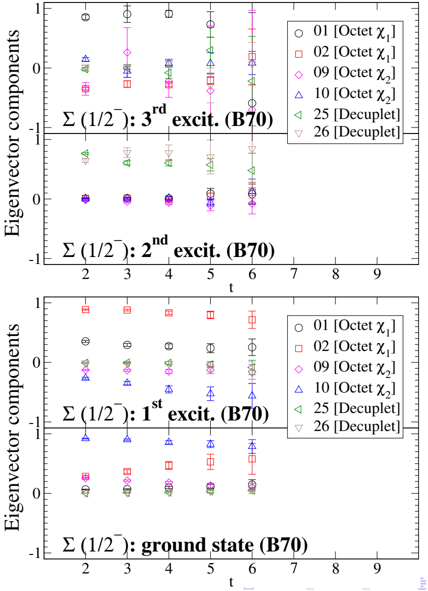


- Problem: Pointlike Λ singlet spin $3/2$ vanishes due to Fierz identities.
- Solution: Different quark smearing widths.
- 1^{st} excitation is dominated by singlet.

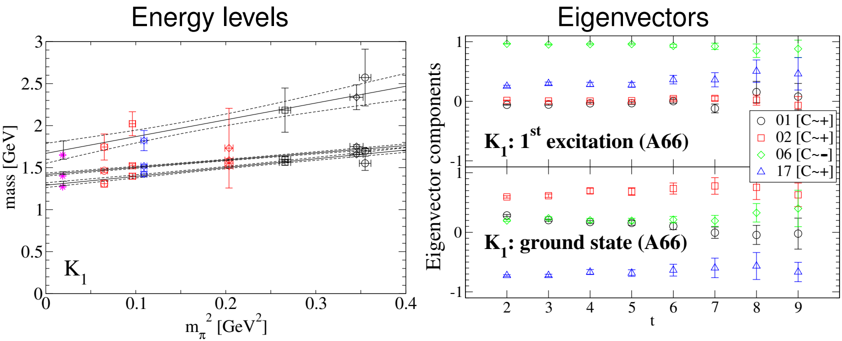
Energy levels



Eigenvectors

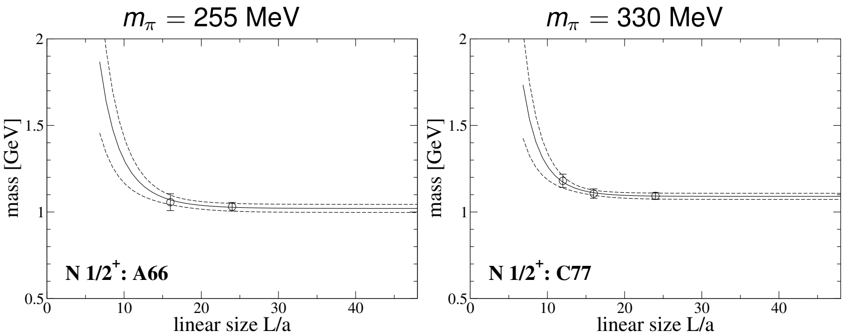


- 3 low states (PDG: 1+1).
- Ground state: octet,
- 1^{st} excitation: octet,
- 2^{nd} excitation: decuplet,
- 3^{rd} excitation: octet.
- Octet/decuplet mixing negligible (Ξ similar).



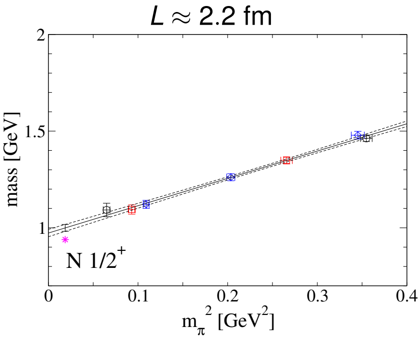
- Extrapolations agree with $K_1(1270)$, $K_1(1400)$ and $K_1(1650)$ (lhs).
- Ground state dominated by $C \approx +$,
- 1^{st} excitation dominated by $C \approx -$,
- 2^{nd} excitation dominated by $C \approx +$.
- Mixing of C-parity increases towards smaller m_π.

Finite volume analysis: Nucleon



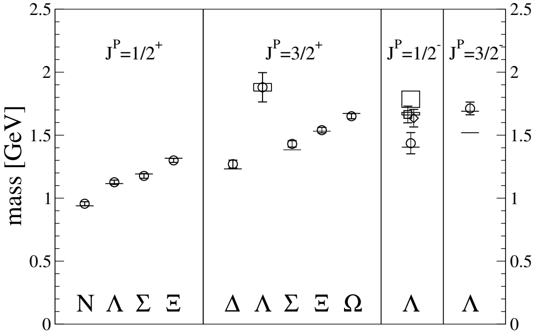
- Data for different volumes at two different pion masses:
  - A66 ($m_\pi = 255$ MeV): $L \approx 3.2, 2.2$ fm
  - C77 ($m_\pi = 330$ MeV): $L \approx 3.2, 2.2, 1.6$ fm
- Combined fit for the two sets.

Finite volume analysis: Nucleon extrapolation



- Finite volume effects significant at small quark masses.
- Infinite volume limit agrees with experimental data within statistical errors.

Summary of infinite volume limit



- Infinite volume extrapolations of low-lying levels agree well with experiment.
- Further systematics may play a role for remaining deviations ($n_f = 2$, multi-particle interpolators, discretization effects etc.).

- Hadron spectrum from 2 dynamical Chirally Improved quarks.
- 7 (10) ensembles with 250 MeV $< m_\pi <$ 600 MeV.
- Overall good agreement of energy levels with experiment.
- $SU(3)_f$ breaking effects investigated considering:
  - Singlet/octet content of Λ baryons.
  - Octet/decuplet content of Σ and Ξ baryons.
  - C-parity mixing of strange mesons.
- Fierz identities sidestepped, resulting interpolators important.
- Discussion of finite volume effects.
- Infinite volume limits in very good agreement with experiment.