D^* and D_s^* spectroscopy on the lattice

2+1+1 Setup

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Aims

- Long term project to compute spectra of mesons with strange and charm quark content using lattice QCD methods.
- Extrapolation to the physical pion mass and the continuum limit.
- Here:
  - Tuning valence s and c quark masses and computing low-lying D mesons.
  - First two points for the continuum extrapolation.

Example: Meson correlator on the lattice

- Amplitude for zero momentum pion propagation Δt:

$$\sum_{t_0}^{T=t_0+\Delta t} \langle \Omega | O_2^\dagger(T) O_1(t_0) | \Omega \rangle =$$

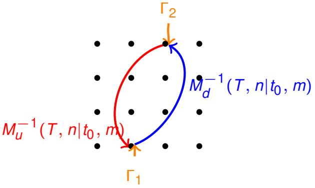
$$\sum_{n,m,t_0}^{T=t_0+\Delta t} \frac{1}{Z} \int D^{n_f}[\bar{\psi}\psi] D[U] e^{-S_F[U,\bar{\psi}^{(n_f)},\psi^{(n_f)}]-S_g[U]} O_2^\dagger(T,n) O_1(t_0,m)$$

$$O_1 = \sum_m (\bar{\psi}_u \gamma_5 \psi_d)(t_0,m), \quad O_2 = \sum_n \bar{\psi}_u \gamma_5 \psi_d)(T,n)$$

- Integration over fermionic degrees:

$$\frac{1}{Z} \sum_{n,m,t_0}^{T=t_0+\Delta t} \int DU(Det[M_u[U]]Det[M_d[U]]\ldots)e^{-S_g[U]} tr \left(M^{-1}_{u,(t_0,m|T,n)} \gamma_5 M^{-1}_{d,(T,n|t_0,m)} \gamma_5 \right)$$

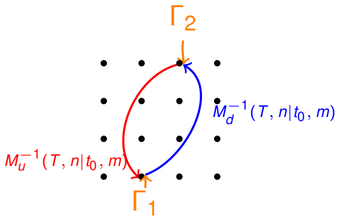
- Dimension $M : N_s^3 N_T N_c N_{dirac}$, typical: $32^3 \cdot 64 \cdot 3 \cdot 4 \sim 5 \cdot 10^7$



Example: Meson correlator on the lattice

$$\vec{R}^\dagger(0,p) M^{-1}_{(t_0,p|t_0+\Delta_t,n)} \gamma_5 M^{-1}_{(t_0+\Delta_t,n|t_0,m)} \gamma_5 \vec{R}(0,m), \quad R^*_p R_n = \delta_{p,n} + \text{noise}$$

- Method
  - Production of a large number of gauge configurations belonging to the distribution $DUDet[M[U]]e^{-S_g[U]}$ for one set of couplings $g, m_u, m_d, \ldots$
  - Calculation of $M^{-1}M^{-1}(\Delta_t)$ as good as you can on every gauge field configuration.
  - Average over all configurations $\rightarrow C_{av}(\Delta t)$.



Euclidean correlators and ground states

- Euclidean correlators projected to zero momentum

$$C_i(t) := \sum_{\vec{x},\vec{y}} \langle \Omega | O_i(\vec{x}, t) \bar{O}_i(\vec{y}, 0) | \Omega \rangle$$
$$= \sum_n \langle \Omega | O_i | n \rangle \langle n | O_i^\dagger | \Omega \rangle e^{-t m_n}$$

- large Euclidean time limit for suppression of excited states

$$\lim_{t \to \infty} C_i(t) = |\langle \Omega | O_i | n = 0 \rangle|^2 \cdot e^{-m_0 t}$$

  - m_0 lowest energy eigenstate with overlap to O_i
  - $M_{effective} = ln(\frac{C(t+1)}{C(t)})$
  - large enough to suppress excited states
  - small enough for signal without noise contribution
  - good for the calculation of the groundstate

Euclidean correlators and excited states

- Correlation matrix projected to zero momentum

$$C_{ij}(t) = \sum_{\vec{x},\vec{y}} \langle \Omega | O_i(\vec{x}, t) \bar{O}_j(\vec{y}, 0) | \Omega \rangle = \sum_n \langle \Omega | O_i | n \rangle \langle n | O_j^\dagger | \Omega \rangle e^{-t m_n}$$

- Multi exponential fit:

$$\lim_{t \to \text{large}} C_{ii}(t) = |\langle \Omega | O_i | 0 \rangle|^2 \cdot e^{-m_0 t} + |\langle \Omega | O_i | 1 \rangle|^2 \cdot e^{-m_1 t} + O(e^{-m_2})$$

- GEVP(Generalized eigenvalue problem)

$$C(t) \vec{\psi}^{(k)} = \lambda^{(k)}(t) C(t_0) \vec{\psi}^{(k)}$$

$$\lambda^{(k)}(t) \propto e^{-t m_k} + e^{-t m_k} \cdot \mathcal{O}(e^{-t \Delta m_k})$$

  - Diagonalisation
  - Δm_n := difference to the next energy level

Wilson twisted mass action

$$\begin{pmatrix} st \\ ch \end{pmatrix} = \psi_h^{phys} = e^{\frac{i}{2}\omega_h \gamma_5 \tau_1} \chi_h \quad \text{diff. masses}$$

$$\begin{pmatrix} up \\ dn \end{pmatrix} = \psi_l^{phys} = e^{\frac{i}{2}\omega_1 \gamma_5 \tau_3} \chi_l \quad \text{mass degenerated}$$

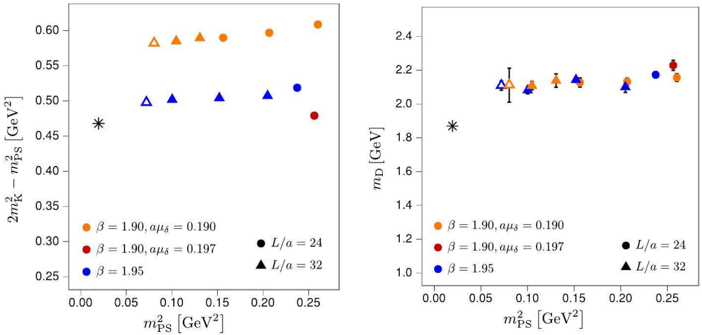
$$S_l = a^4 \sum_x \bar{\chi}_l(x) \left[D[U] + m_{0,l} + i\mu_l \gamma_5 \tau^3 \right] \chi_l(x)$$

$$S_h = a^4 \sum_x \bar{\chi}_h(x) \left[D[U] + m_{0,h} + i\mu_\sigma \gamma_5 \tau^1 + \mu_\delta \tau^3 \right] \chi_h(x)$$

$$\chi_l = \begin{pmatrix} \chi_u \\ \chi_d \end{pmatrix}, \quad \chi_h = \begin{pmatrix} \chi_s \\ \chi_c \end{pmatrix} \qquad \tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- $tan\omega_1 = \frac{\mu_l}{m_{0,l}}$
- Tuned to 'maximal' twist: $m_{0,l} = m_{0,h} \rightarrow m_{crit} \Rightarrow (\omega_1)_{ren.} = (\omega_2)_{ren.} \rightarrow \frac{\pi}{2}$
- $\Rightarrow$ automatic $\mathcal{O}(a)$ improvement for physical observables.
- TWO VERSIONS, BROKEN PARITY

ETMC $2+1+1$ ensembles

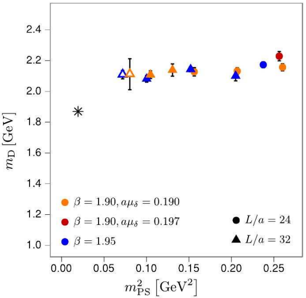


- Available $2+1+1$ configurations.
- Mismatch of strange and charm mass.
- Idea: different valence action for *s* and *c* quarks.

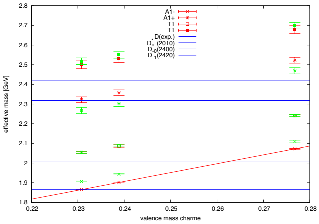
$$\chi_c := \begin{pmatrix} \chi_{c+} \\ \chi_{c-} \end{pmatrix}, \quad D_W + m_{crit} \pm i\mu_c\gamma_5$$

Introduction of strange and charm valence quarks

unitary setup



valence quark setup



- Introduction of mass degenerated doublets for charm and strange quarks(Valence sector).

$$\chi_c := \begin{pmatrix} \chi_{c+} \\ \chi_{c-} \end{pmatrix}, \quad D_W + m_{crit} \pm i\mu_c\gamma_5$$

- Calculation at different bare masses and extrapolation to the 'physical' point (c: m_D, s: $2m_K^2 - m_{PS}^2$)

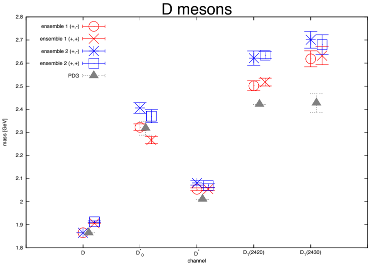
Setup

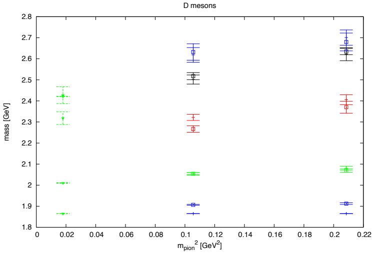
- Gauge link configurations with $2+1+1$ dynamical quark flavors (ETMC).
  - Tuning charm valence quark mass to reproduce physical m_D mass.
  - Tuning strange valence quark mass to reproduce physical value of $2m_K^2 - m_\pi^2$ mass. Weak dependence on the pion mass.
- Mixed action setup to avoid mixing of strange and charm flavor and repair mismatch in the sea.
- Gaussian distributed spin diluted timeslice sources with APE smeared gauge links.
- Parameters of the ensembles:

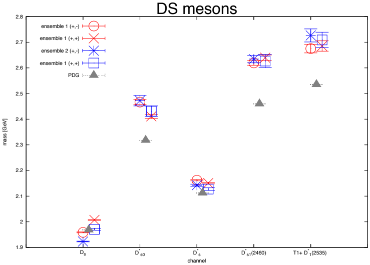
$$\beta = 1.9, \quad a = 0.0859(5)\text{fm}$$

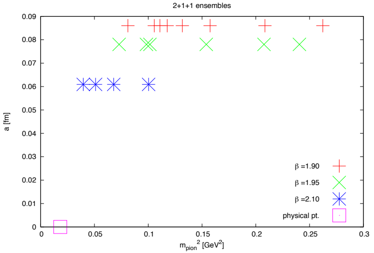
  - $(L/a)^3 \times (T/a) = 32^3 \times 64, \quad \mu = 0.004, \quad m_\pi \approx 325\text{MeV}$
  - $(L/a)^3 \times (T/a) = 24^3 \times 48, \quad \mu = 0.008, \quad m_\pi \approx 457\text{MeV}$

Desults D D_s chiral 'extrapolation'

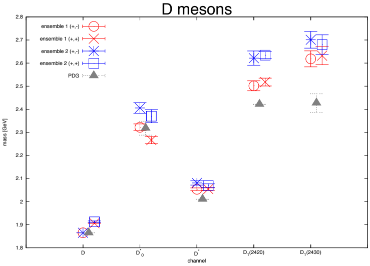


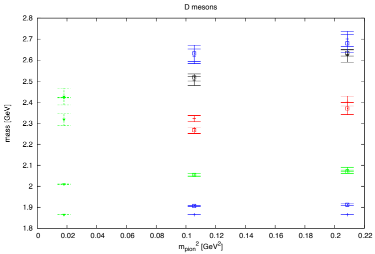


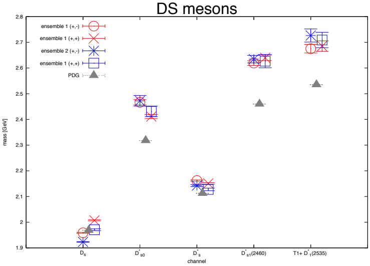


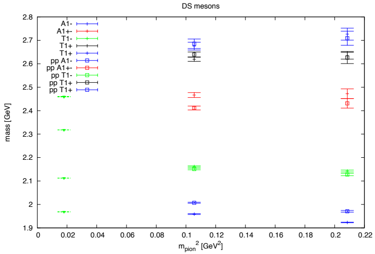


Desults D D_s chiral 'extrapolation'

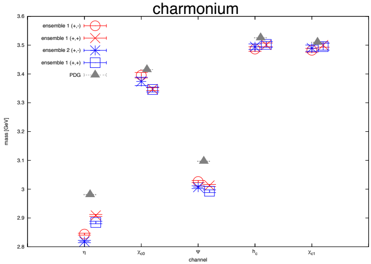


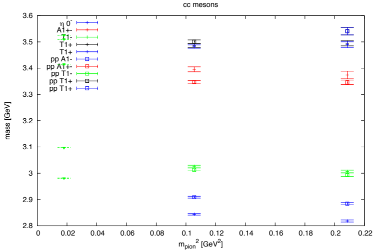




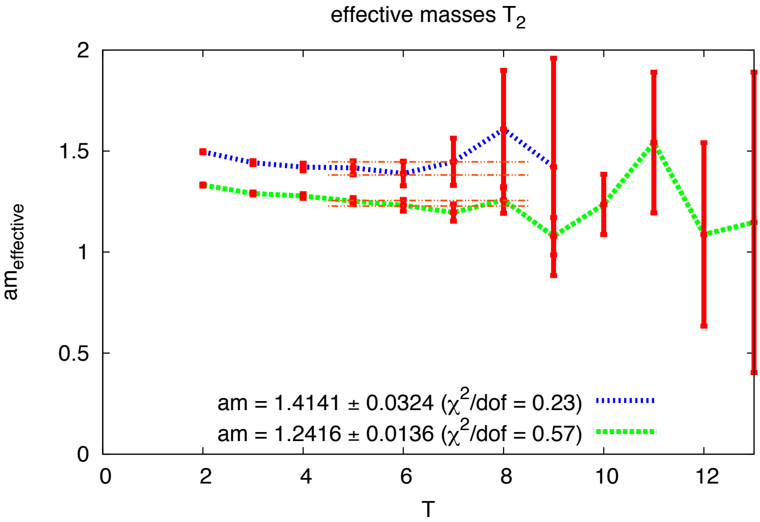


Results $\bar{c}c$ chiral 'extrapolation'





quality of the $T_2(J = 2, 3, 4, ...)$ channel



- Thank you for your attention.