Low mass (low x) Drell-Yan production at the LHC

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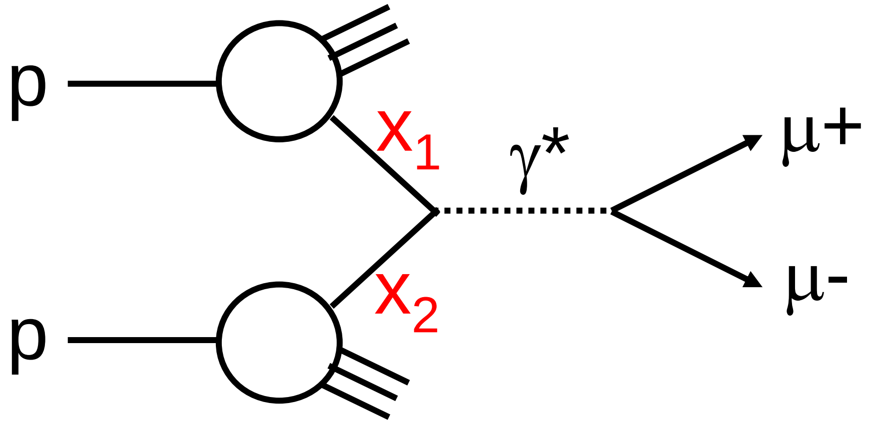


low M, high Y
very low x_2

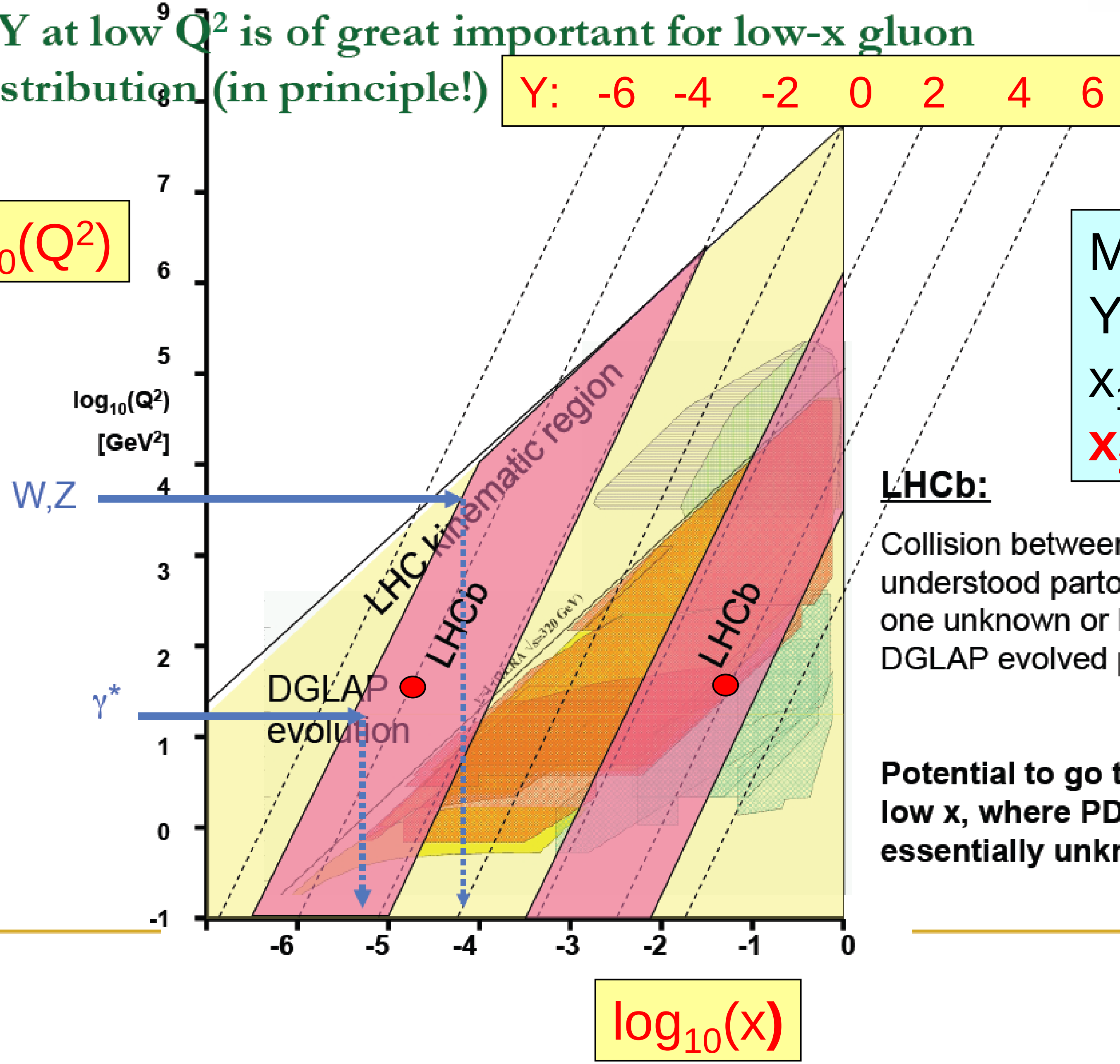
$$x_{1,2} = \frac{M}{\sqrt{s}} \exp(\pm Y)$$

Low x meeting, Paphos, Cyprus, June 2012

From Ronan McNulty (LHCb)

$$x_{1,2} = \frac{M}{\sqrt{s}} \exp(\pm Y)$$

DY at low Q^2 is of great important for low-x gluon distribution (in principle!)



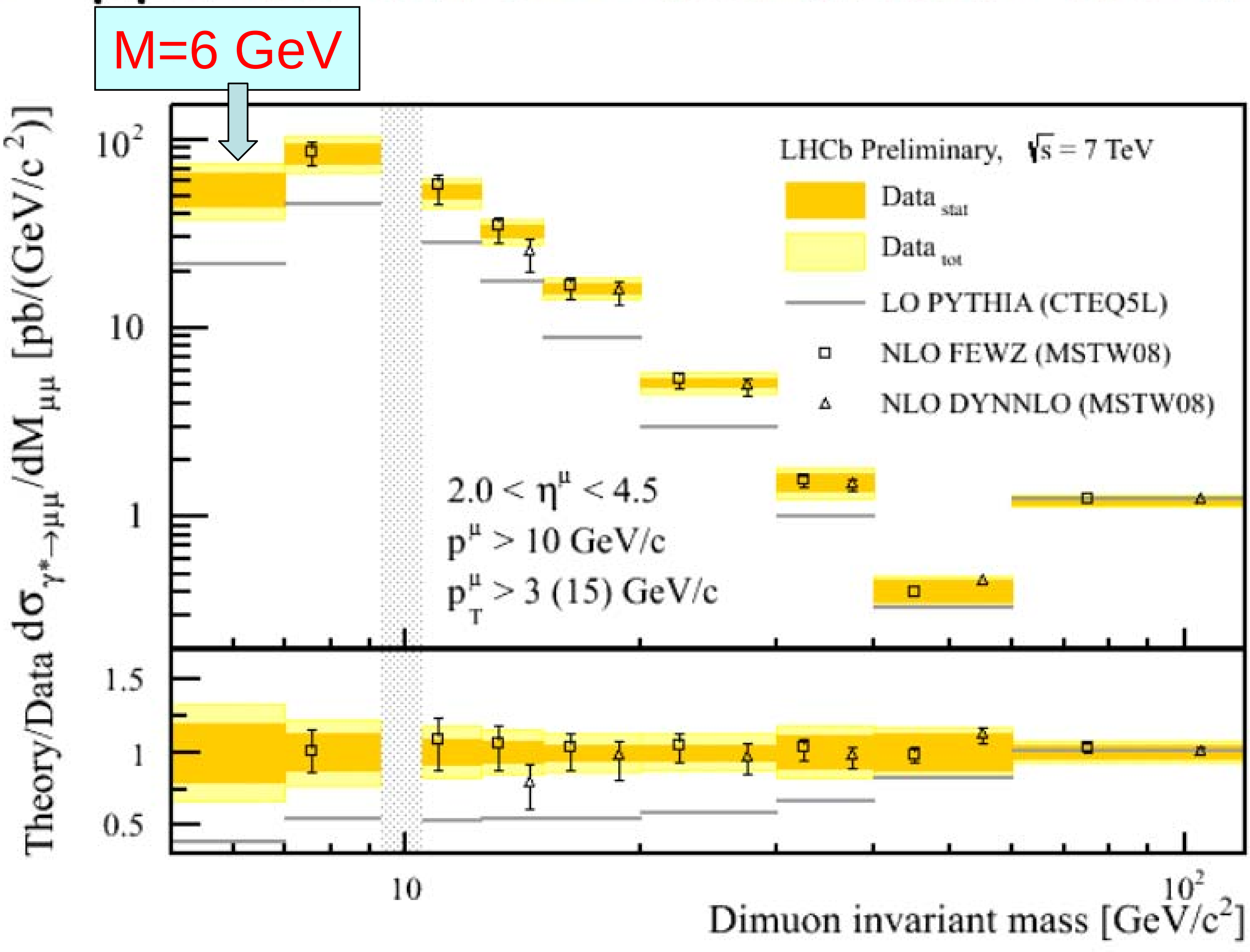
$M_{\mu+\mu-}$=6GeV
Y=4
$x_1=4.7\times10^{-2}$
$x_2=1.6\times10^{-5}$

LHCb:

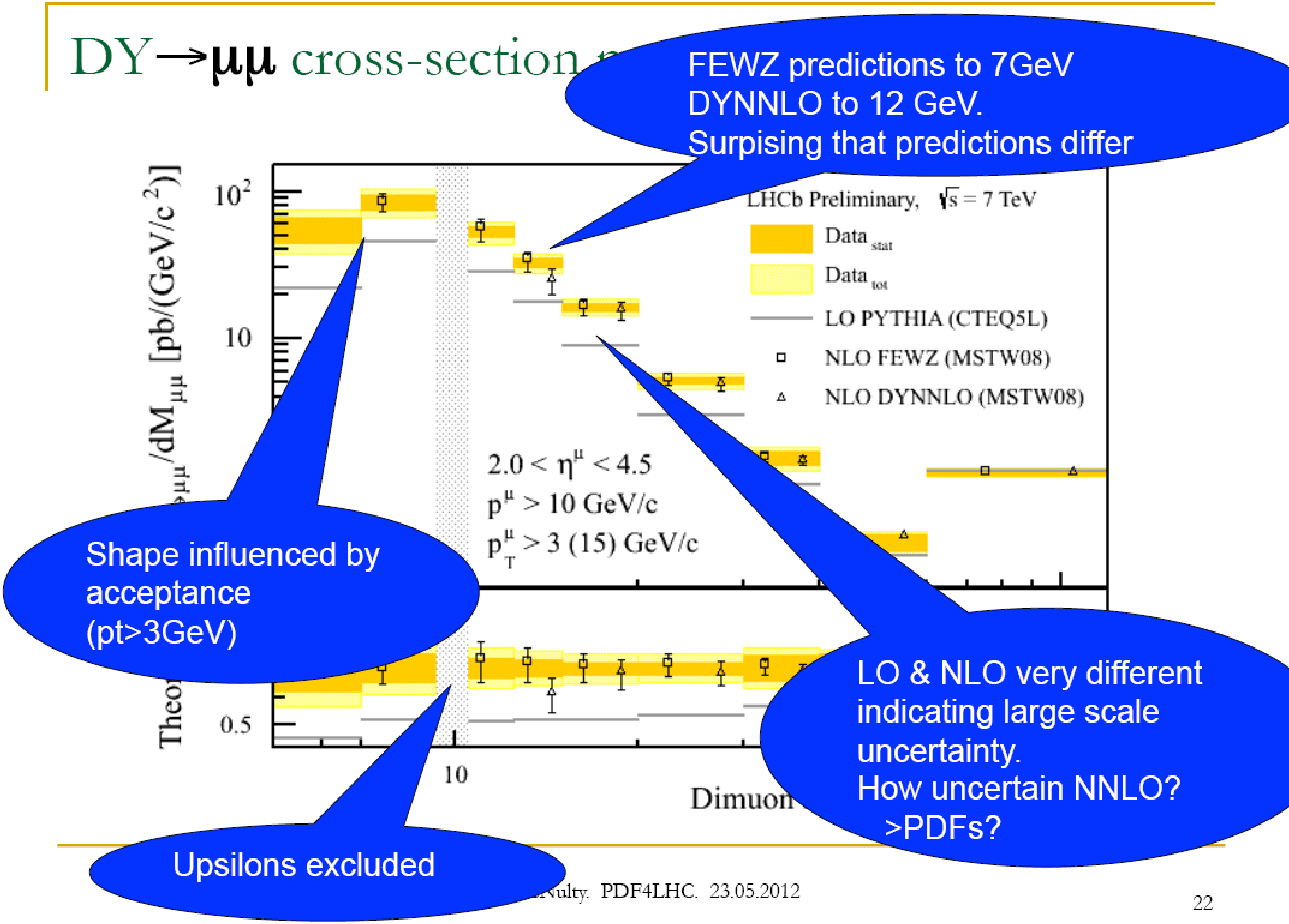
Collision between one well understood parton and one unknown or large DGLAP evolved parton.

Potential to go to very low x, where PDFs essentially unknown

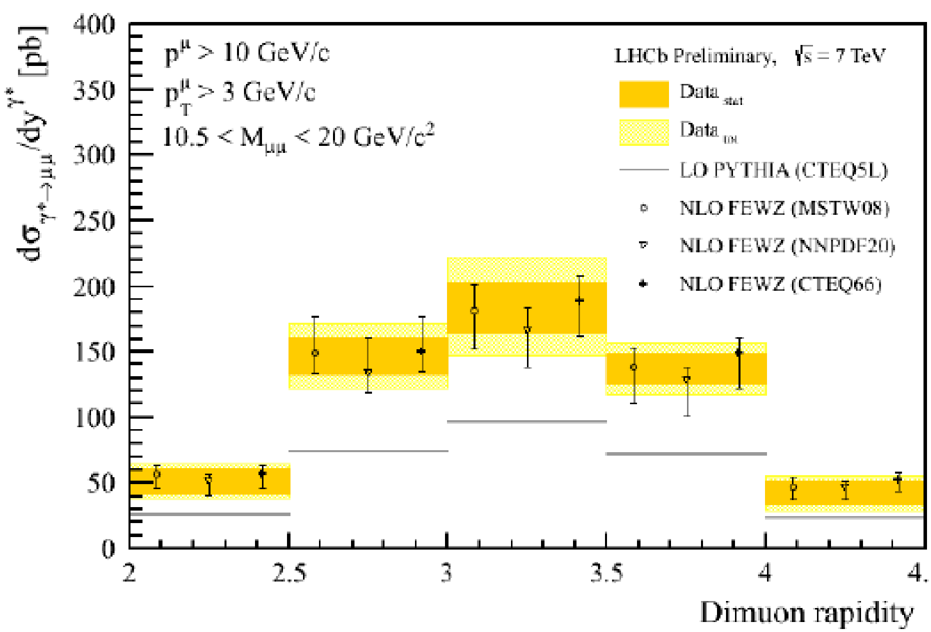
DY→μμ cross-section measured down to 5 GeV.

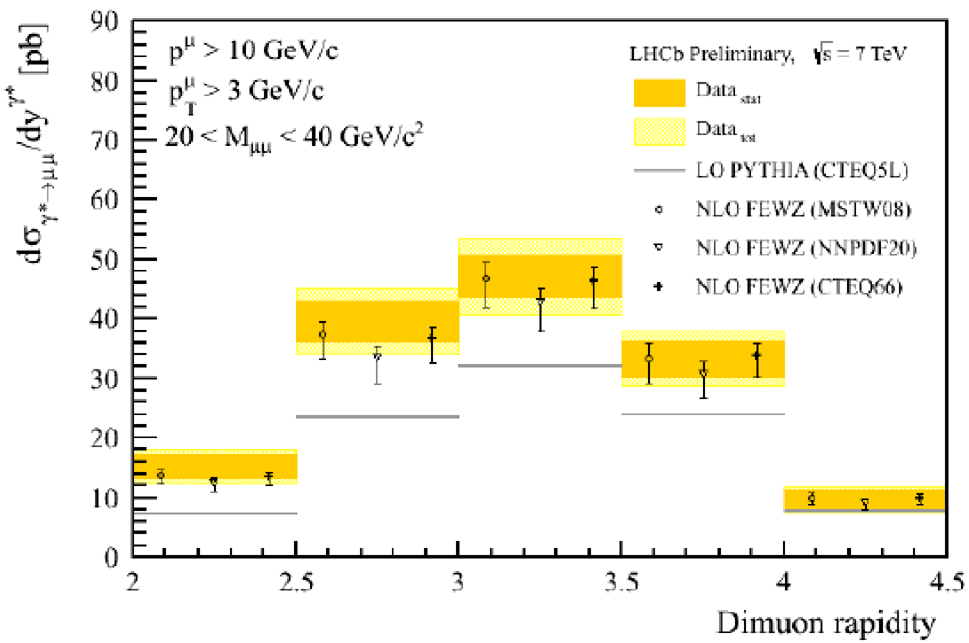


DY→μμ cross-section



Differential distributions for two mass bins.





Fully double differential distributions possible with 2011 data

The factorization scale μ_F

$$d\sigma/d^3p = \int dx_1 dx_2 \, \mathrm{PDF}(x_1, \mu_F) \, |\mathcal{M}(p; \mu_F, \mu_R)|^2 \, \mathrm{PDF}(x_2, \mu_F)$$

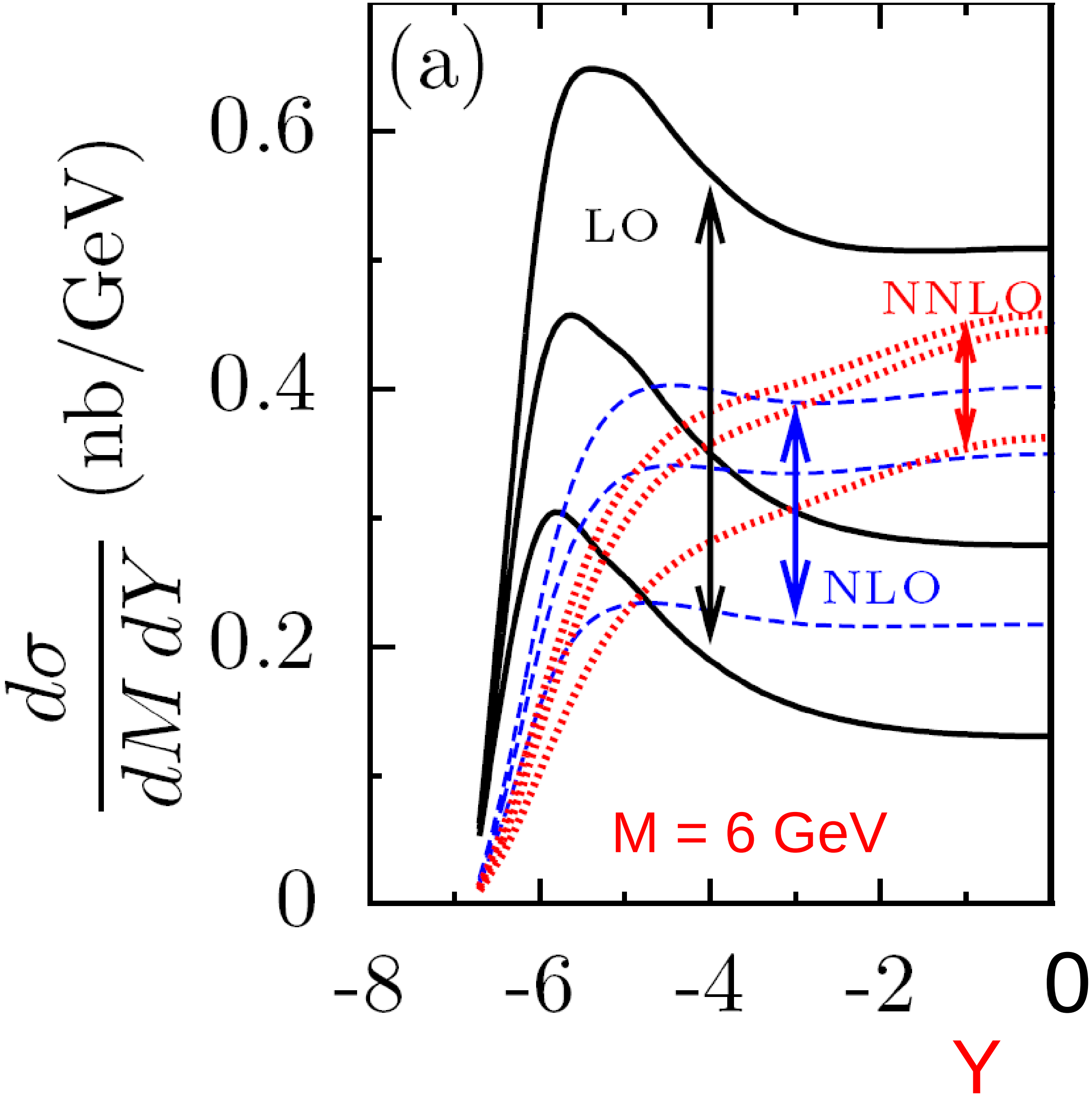
parton virtuality $q^2 < \mu_F^2$ $\quad$ $q^2 > \mu_F^2$

At low x, the PDFs strongly depend on choice of μ_F.
Worse, dominance of g at low x (i.e. low M) means
LO $q\bar{q} \rightarrow \gamma^*$ overshadowed by NLO $gq \rightarrow q\gamma^*$ subproc.

At low x, prob. to emit new parton in $\Delta\mu_F$ enhanced: mean number →

$$\langle n \rangle \simeq \frac{\alpha_s N_C}{\pi} \ln(1/x) \, \Delta \ln \mu_F^2$$

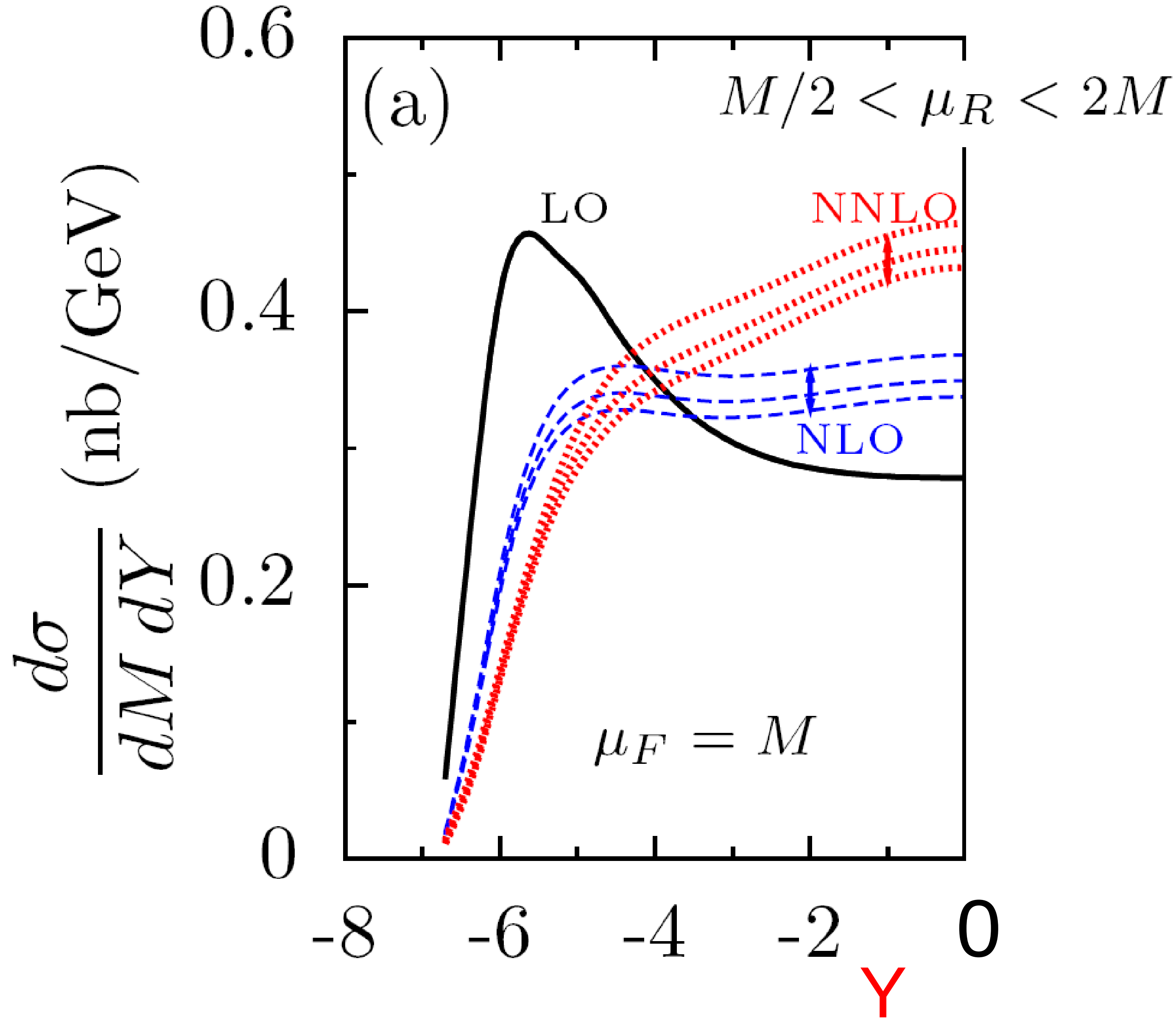
but $|\mathcal{M}^{NLO}|^2$ can emit only one → so no compensation



Large μ_F dependence

μ_F = M/2, M, 2M

Renormalization scale μ_R dependence



Idea: use NLO to fix μ_F for LO part, and to show results stable to variations of μ_F in remaining NLO part

Start with LO:

$$\sigma(\mu_F) \;=\; \mathrm{PDF}(\mu_F) \otimes C^{\mathrm{LO}} \otimes \mathrm{PDF}(\mu_F)$$

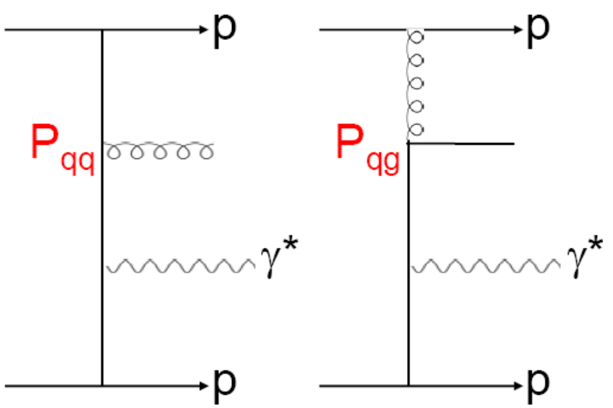
Changing scale from m to μ_F

$$\sigma(\mu_F) = \mathrm{PDF}(m) \otimes \left(C^{\mathrm{LO}} + \frac{\alpha_s}{2\pi} \ln\left(\frac{\mu_F^2}{m^2}\right) \left(P_{\mathrm{left}} C^{\mathrm{LO}} + C^{\mathrm{LO}} P_{\mathrm{right}}\right)\right) \otimes \mathrm{PDF}(m)$$

$$P_{\mathrm{left}} = P_{\bar{q}\bar{q}} + P_{\bar{q}g} \qquad\qquad P_{\mathrm{right}} = P_{qq} + P_{qg}$$

This is α_s corr^n in LO DGLAP collinear approach,
Leading Log Approx (LLA)

$$\int_{m^2}^{\mu_F^2} \frac{dk_T^2}{k_T^2} \;=\; \ln\left(\frac{\mu_F^2}{m^2}\right)$$



Now NLO expression:

$$\sigma(\mu_F) \; = \; \mathrm{PDF}(\mu_F) \otimes (C^{\mathrm{LO}} + \underline{\alpha_s C^{\mathrm{NLO}}_{\mathrm{corr}}}) \otimes \mathrm{PDF}(\mu_F)$$

C^{NLO} means $q\bar{q} \rightarrow g\gamma^*$ and $gq \rightarrow q\gamma^*$ calc better than LLA accuracy, but part already included to LLA accuracy --- subtract it off.
At this stage C^{NLO} becomes dependent on μ_F --- $C^{\mathrm{NLO}}_{\mathrm{rem}}(\mu_F)$

Changing μ_F redistributes α_s contribution between two terms

$$(\mathrm{PDF} \otimes C^{\mathrm{LO}} \otimes \mathrm{PDF}) \Longleftrightarrow (\mathrm{PDF} \otimes \alpha_s C^{\mathrm{NLO}}_{\mathrm{rem}} \otimes \mathrm{PDF})$$

The trick is to choose $\mu_F = \mu_0$ so as to minimize $C^{\mathrm{NLO}}_{\mathrm{rem}}(\mu_F)$

Choose μ_F so as much as possible of "real" NLO ladder-like form is included in LO part (where large $\alpha_s \ln(1/x)$ terms are collected in PDFs)

α_S term from LO DGLAP

main NLO subprocess

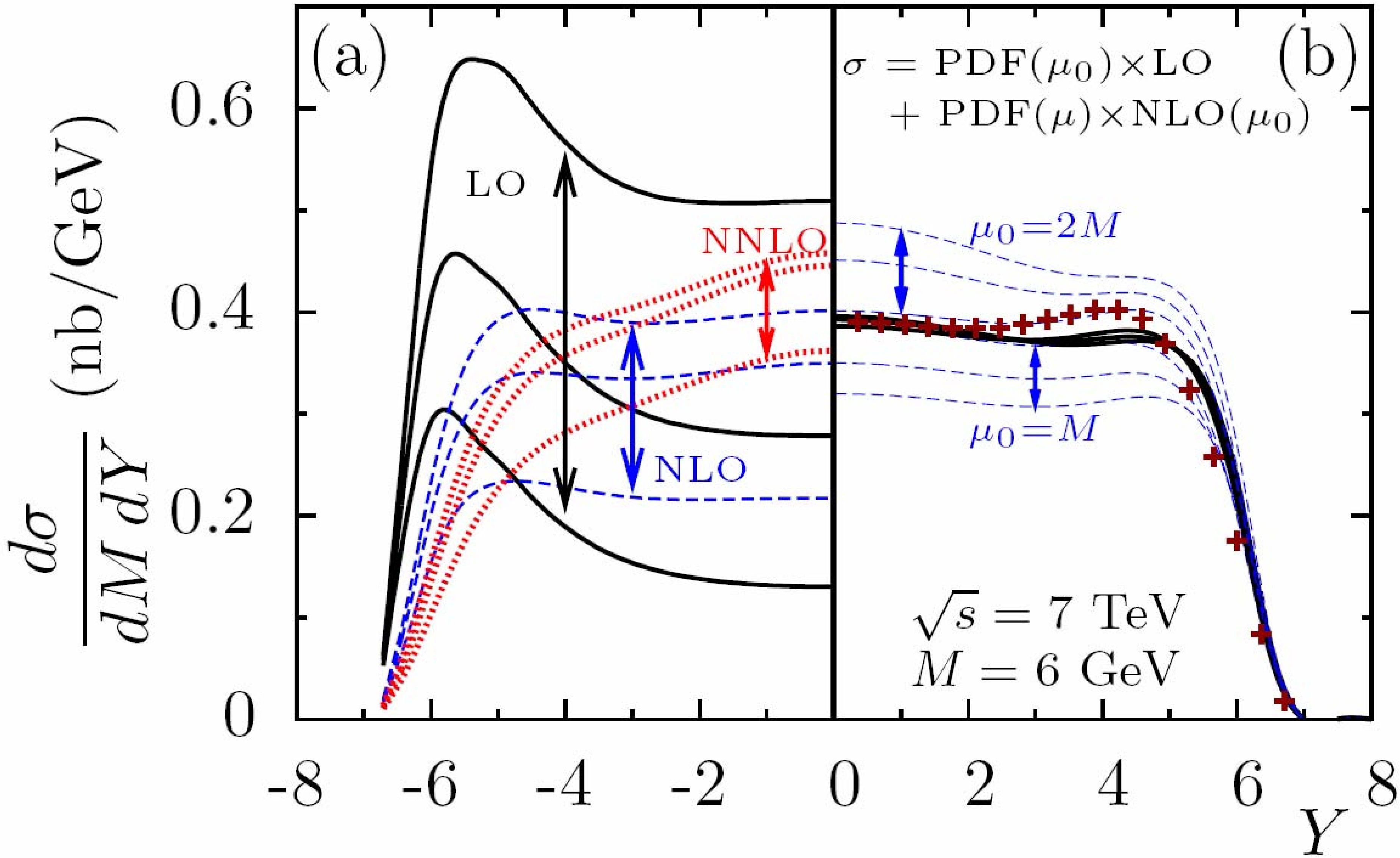
$$\int_{Q_0^2}^{\mu_F^2} dt \, \frac{d\sigma}{dt}(\alpha_S, \mathrm{LLA}) = \int_{Q_0^2}^{\mu_F^2} dt \, \frac{d\sigma}{dt}(gq \to q\gamma^*)_{\mathrm{exact}}$$

adjust μ_F until equality achieved

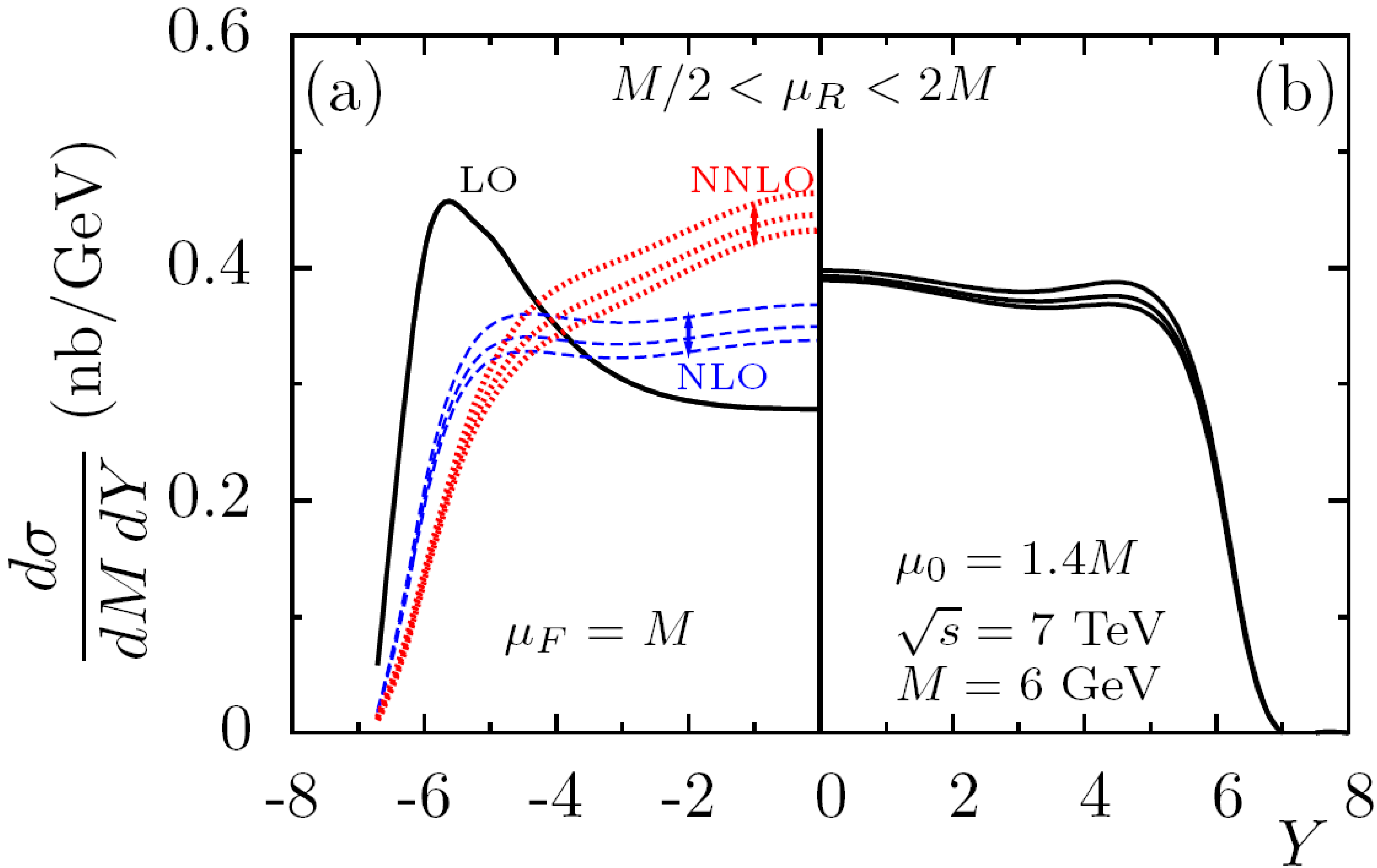
$$\mu_F \equiv \mu_0 = 1.4M$$

so (LO DGLAP $\otimes$ C^{LO}) well reproduces NLO term

minimizes $C_{\mathrm{rem}}^{\mathrm{NLO}}(\mu_F)$ for $\mu_F = 1.4M$



(a)
(b)
σ = PDF(μ_0)×LO
+ PDF(μ)×NLO(μ_0)
LO
NNLO
NLO
$\mu_0 = 2M$
$\mu_0 = M$
$\sqrt{s} = 7$ TeV
$M = 6$ GeV
$\frac{d\sigma}{dM\,dY}$ (nb/GeV)
0
0.2
0.4
0.6
-8
-6
-4
-2
0
2
4
6
8
Y



0.6
0.4
0.2
0
(a)
(b)
$M/2 < \mu_R < 2M$
LO
NNLO
NLO
$\frac{d\sigma}{dM\,dY}$ (nb/GeV)
$\mu_F = M$
$\mu_0 = 1.4M$
$\sqrt{s} = 7$ TeV
$M = 6$ GeV
-8
-6
-4
-2
0
2
4
6
8
Y

Impact on global PDFs

D-Y data make a direct measurement of q, $\bar{q}$ at μ=1.4M, with little scale ambiguity

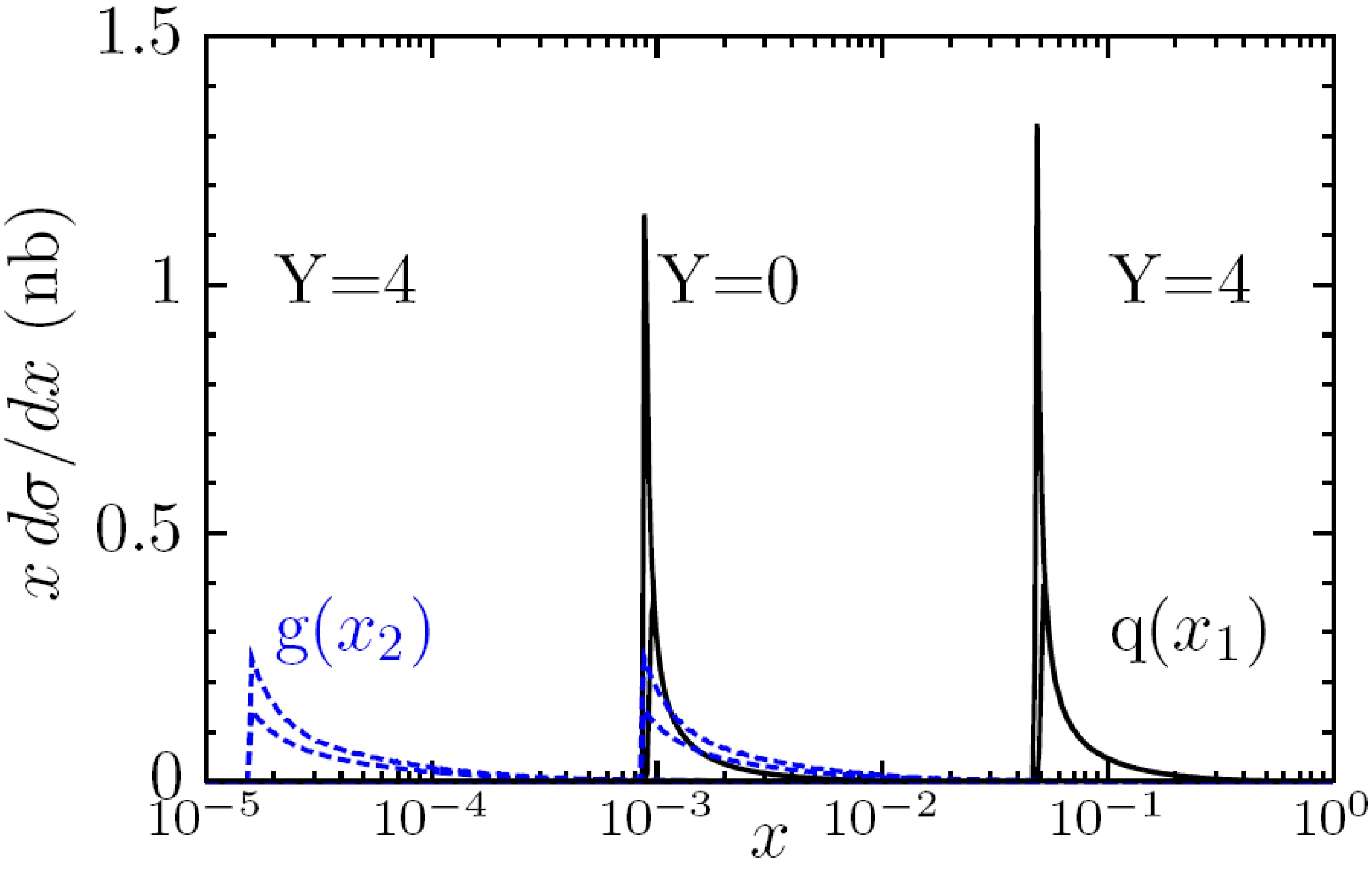
In pure DGLAP most of the q, $\bar{q}$ at low x come from $g \rightarrow q\bar{q}$ splitting

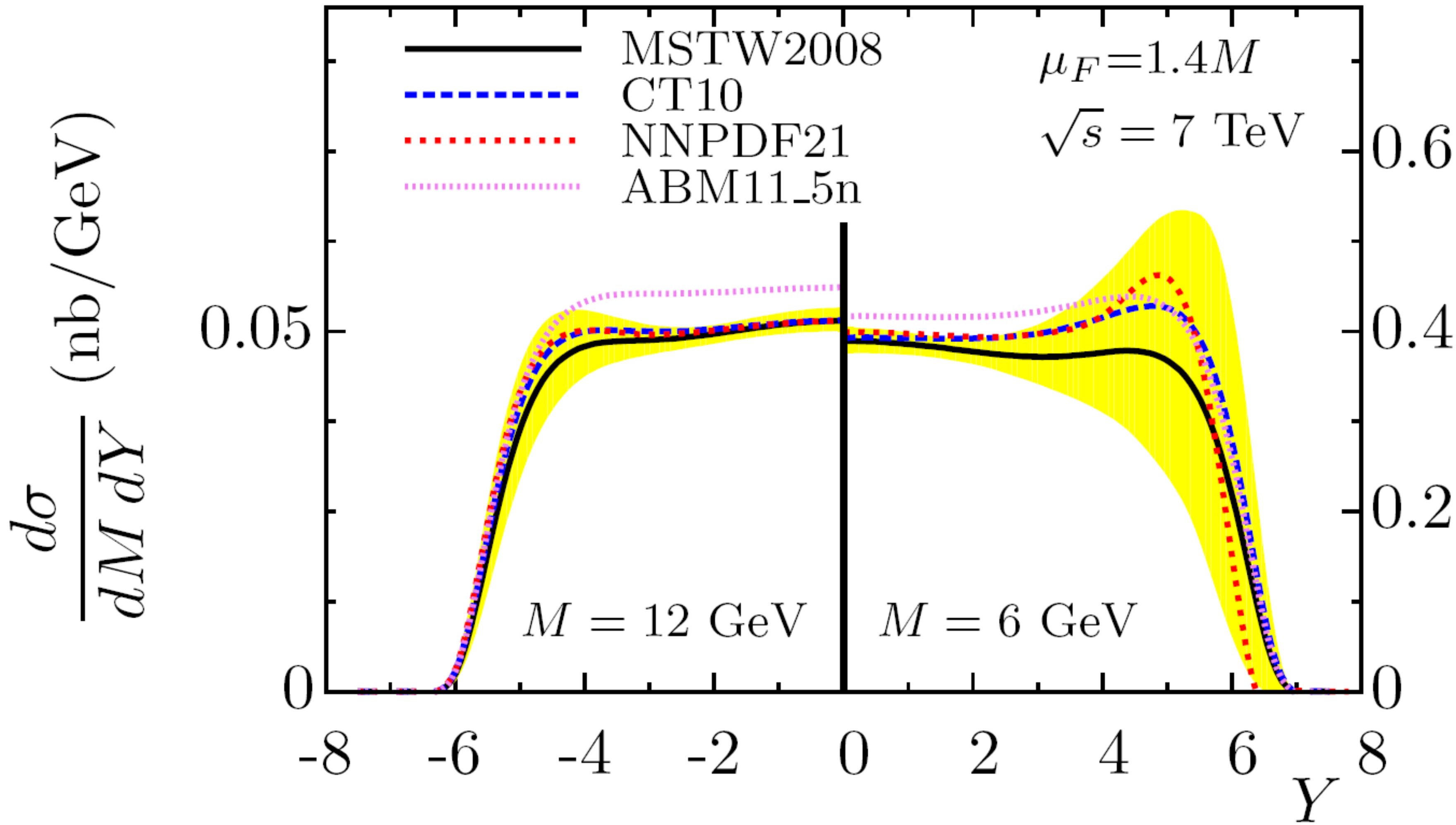
Indeed, $\sigma(gq \rightarrow q\gamma^*) > 90\% \, \sigma(DY)$

Take $g(x_2)\ q(x_1) \rightarrow q\ \gamma^*$

allows parton k_t

$$x_{1,2} = \frac{\sqrt{M^2 + k_t^2}}{\sqrt{s}} \exp(\pm Y)$$





For Y > 3, pure DGLAP PDF extrapolations become unreliable due to absence of absorptive, ln(1/x),…modifications

LHCb data provide direct measure of PDFs in this low x domain

By-product of D-Y study
arXiv:1205.6108

study revealed an inconsistency in the conventional procedure to remove infrared divergence, not only in D-Y, but in DIS….
arXiv:1206.2223

Take Drell-Yan as example:

main NLO subprocess

$$\frac{d\hat{\sigma}(gq \to q\gamma^*)}{d|t|} = \frac{\alpha^2 \alpha_s z}{9M^2} \frac{1}{|t|} \left[((1-z)^2 + z^2) + z^2 \frac{t^2}{M^4} - 2z^2 \frac{t}{M^2} \right]$$

To calculate $d\sigma/dM^2$ need to integrate over t from t=0

Starting with the gluon, subtraction of the LO DGLAP, with the $P_{qg}\ \alpha_s$ term, exactly removes infrared divergence

Consistent treatment of infrared region

$$\frac{d\hat{\sigma}^{(1)}}{dt} = \frac{d\hat{\sigma}^{\rm NLO}_{\rm rem}}{dt} + \underbrace{\frac{d\hat{\sigma}^{\rm LO}_{q\bar{q}}}{dt} \otimes \frac{\alpha_s}{2\pi} P^{\rm LO}_{\bar{q}g}}$$

$$\frac{d\hat{\sigma}^{\rm LO}}{d|t|} \otimes \frac{\alpha_s}{2\pi} P^{\rm LO}_{\bar{q}g} = \frac{\alpha^2 \alpha_s z}{9M^2} \frac{1}{|t|} [(1-z)^2 + z^2] \, \underline{\Theta(\mu_F^2 - |t|)}$$

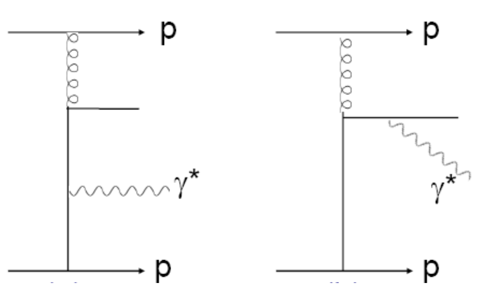
DGLAP α_s term accounts for all virtualities $|t| < \mu_F^2$, where $|t| < Q_0^2$ is hidden in input PDF

After subtraction of this LO generated term

$$\frac{d\hat{\sigma}^{\rm NLO}_{\rm rem}}{d|t|} = \frac{\alpha^2 \alpha_s z}{9M^2} \frac{1}{|t|} \left[[(1-z)^2 + z^2] \, \underline{\Theta(|t| - \mu_F^2)} + z^2 \frac{t^2}{M^4} - 2z^2 \frac{t}{M^2} \right]$$

which has no singularity as $t \to 0$.

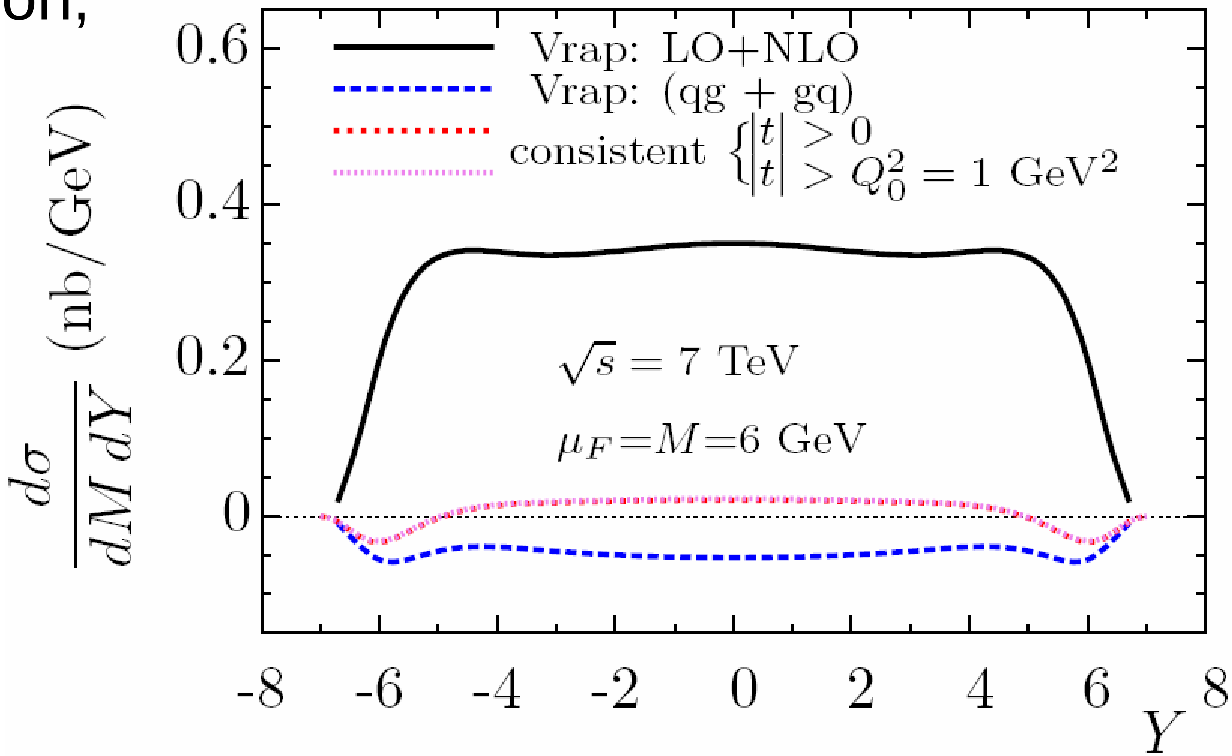
Non-singular terms vanish as Q_0^2/μ_F^2.



Conventional treatment of infrared region

Integral is regularised by working in 4+2ε dimensions. Contribution at small t gives 1/ε pole, which is absorbed in incoming PDF. Also DGLAP generated term is integrated in same 4+2ε dimensions. Dimensional regularisation makes, unnecessary and unwarranted, assumption that 1/t singular pQCD behaviour is valid below Λ_{QCD}.

After 1/ε-1/ε subtraction, it leaves a non-vanishing infrared contribution



Error is difference between red and blue curves

Second example:

Correction to the γ^*g coefficient function, C_g, in DIS

- The conventional ε-regularistion treatment of the infrared singularity gives

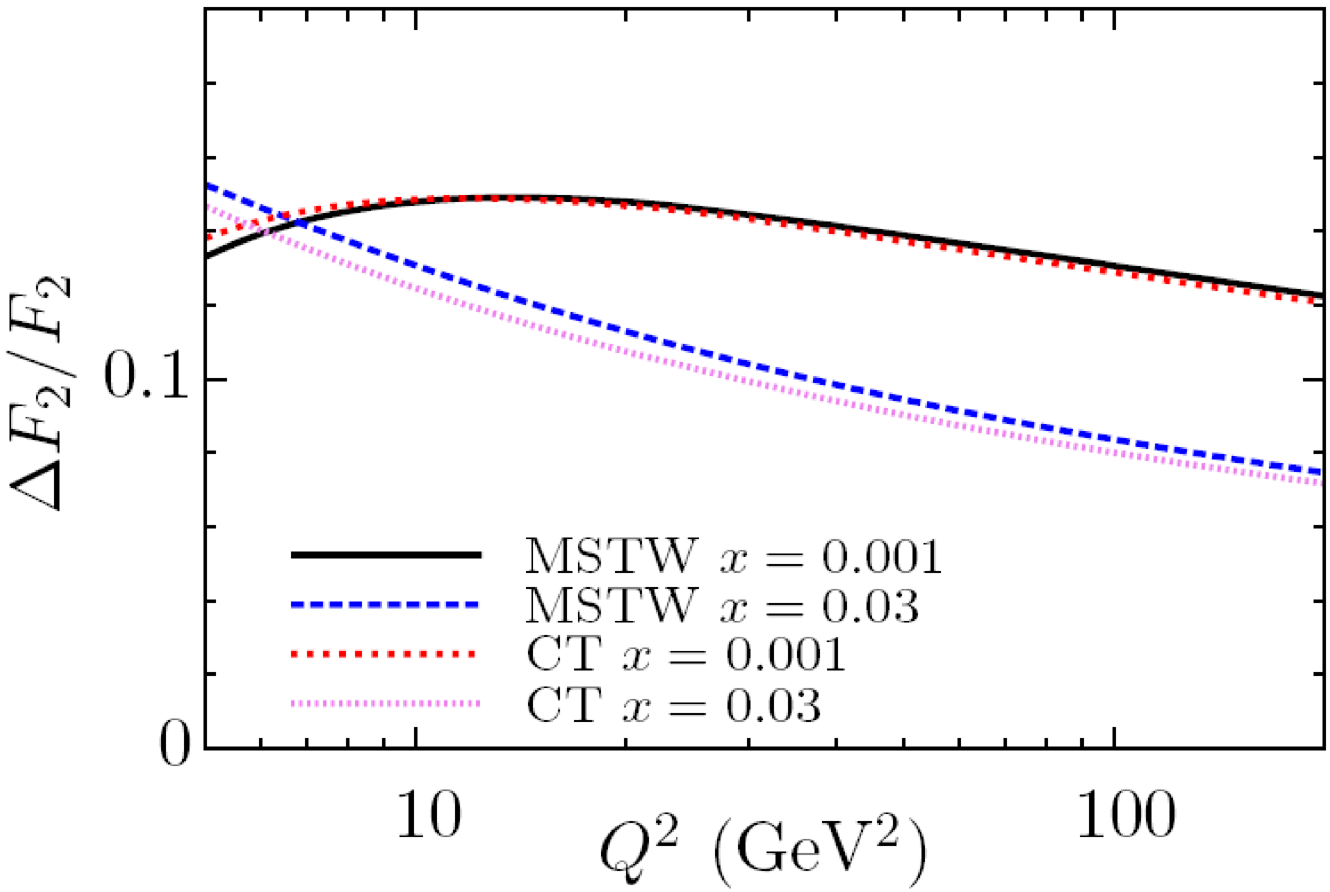
$$C_g = T_R\left([(1-z)^2+z^2]\ln\frac{1}{z} + 6z(1-z) - 1\right)$$

- Whilst the consistent explicit subtraction of the term generated by LO DGLAP evolution gives

$$C_g = T_R\left([(1-z)^2+z^2]\ln\frac{1-z}{z} + 8z(1-z) - 1\right)$$

- The correction, ΔF_2, arising from the difference →

Correction to F_2 arising from "consistent" C_g



To account for a proper treatment of the infrared region it it is necessary to perform a global analysis with a **complete set of corrected coefficient and splitting funct^s.**