Higher-order corrections for top-quark and W-boson production

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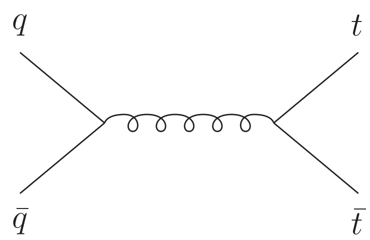
- $t\bar{t}$ **and single top production channels**
- **Higher-order two-loop corrections**
- $t\bar{t}$ **cross section at LHC and Tevatron**
- **Top** p_T **and rapidity distributions**
- t**-channel and** s**-channel production**
- tW^- **and** tH^- **production**
- W **production at large** p_T

Top production partonic processes at LO

Top-antitop pair production

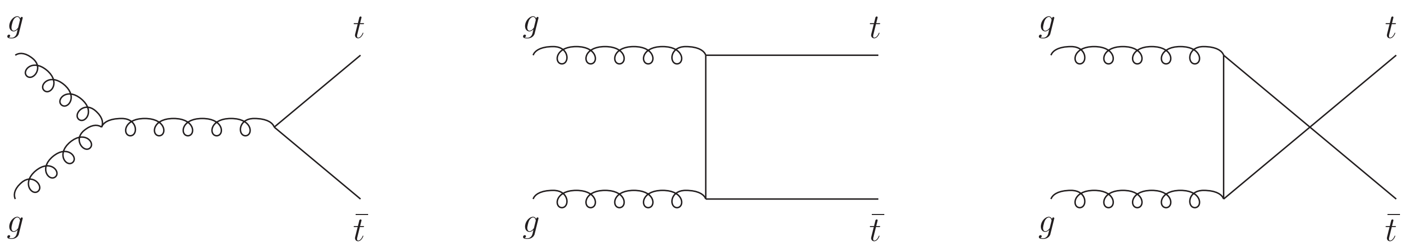
- $q\bar{q} \longrightarrow t\bar{t}$

dominant at Tevatron



- $gg \longrightarrow t\bar{t}$

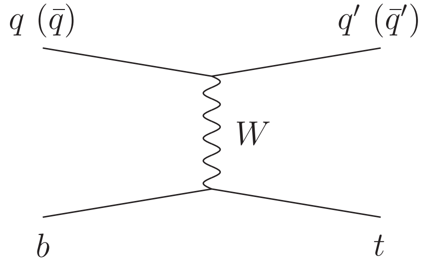
dominant at LHC



Single top quark production

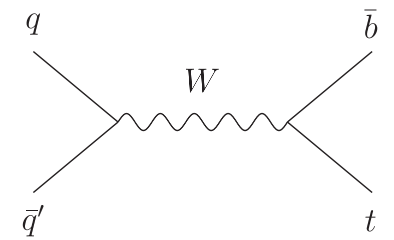
• t **channel:** $qb \rightarrow q't$ **and** $\bar{q}b \rightarrow \bar{q}'t$

dominant at Tevatron and LHC



• s **channel:** $q\bar{q}' \rightarrow \bar{b}t$

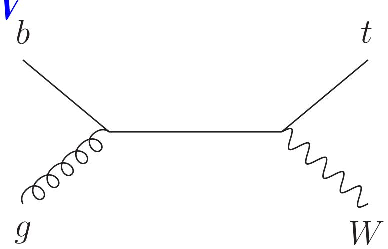
small at Tevatron and LHC

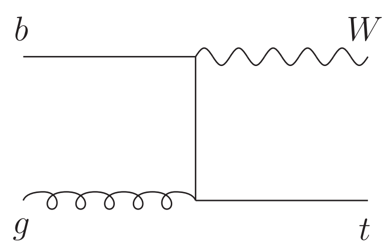


• **associated** tW **production:** $bg \rightarrow tW^-$

very small at Tevatron, significant at LHC

Related process: $bg \rightarrow tH^-$





Higher-order corrections

QCD corrections significant for top pair and single top quark production

Soft-gluon corrections from emission of soft (low-energy) gluons

Soft corrections: $\left[\frac{\ln^k(s_4/m^2)}{s_4}\right]_+$ with $k \le 2n-1$, s_4 distance from threshold

Soft-gluon corrections are dominant near threshold

Resum these soft corrections - factorization and RGE

Complete results at NNLL–two-loop soft anomalous dimension

NK, PRD 82, 114030 (2010); PRD 84, 011504 (2011) ($t\bar{t}$)

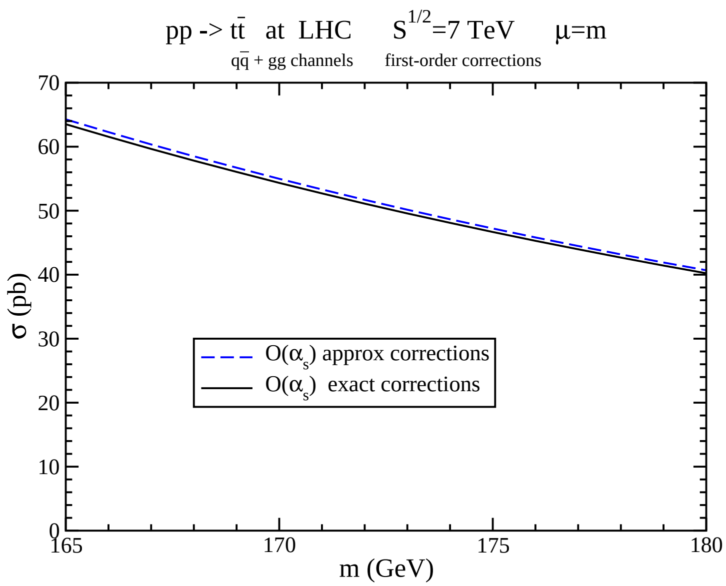
NK, PRD81, 054028 (2010); PRD 82, 054018 (2010); PRD 83,091503 (2011) (single top)

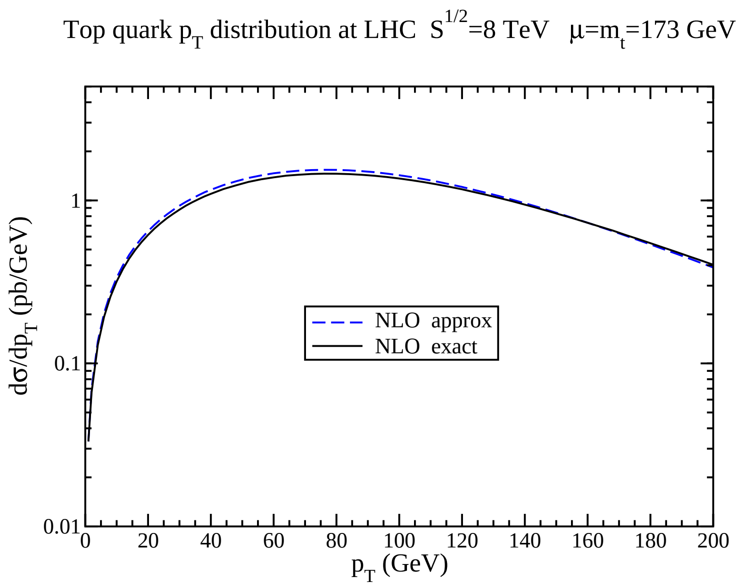
Approximate NNLO cross section from expansion of resummed cross section

This is the only calculation at the differential cross section level using the standard moment-space resummation in pQCD

Threshold approximation

Approximation works very well not only for Tevatron but also for LHC energies because partonic threshold is still important





only 1% difference between first-order approximate and exact corrections
→ less than 1% difference between NLO approximate and exact cross sections
Also true for differential distributions

For best prediction add NNLO approximate corrections to exact NLO cross section

Factorization and Resummation

Resummation follows from factorization properties of the cross section - performed in moment space

$\sigma = (\prod \psi)\, H_{IL}\, S_{LI}\, (\prod J)$ **H: hard-scattering function**

S: soft-gluon function

Use RGE to evolve soft-gluon function

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S_{LI} = -(\Gamma_S^\dagger)_{LB} S_{BI} - S_{LA}(\Gamma_S)_{AI}$$

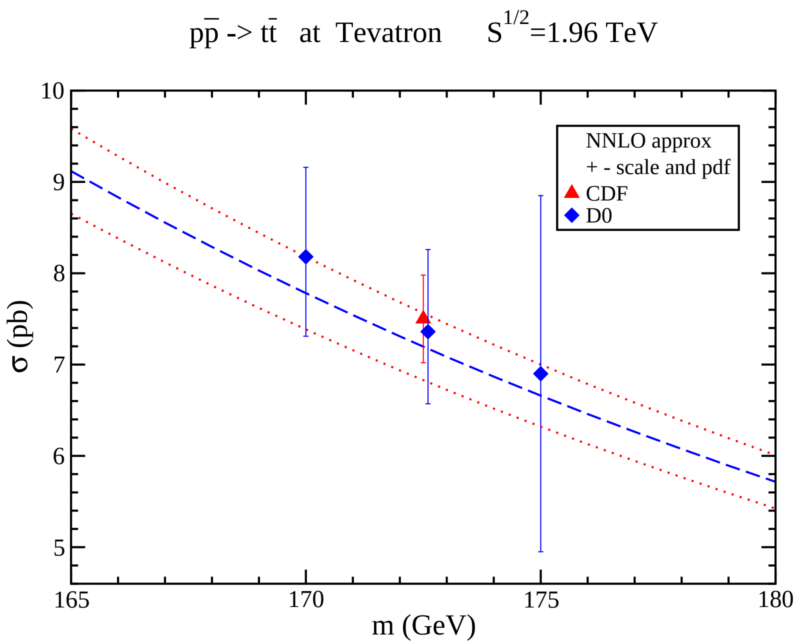
Γ_S is the soft anomalous dimension - a matrix in color space and a function of kinematical invariants s, t, u

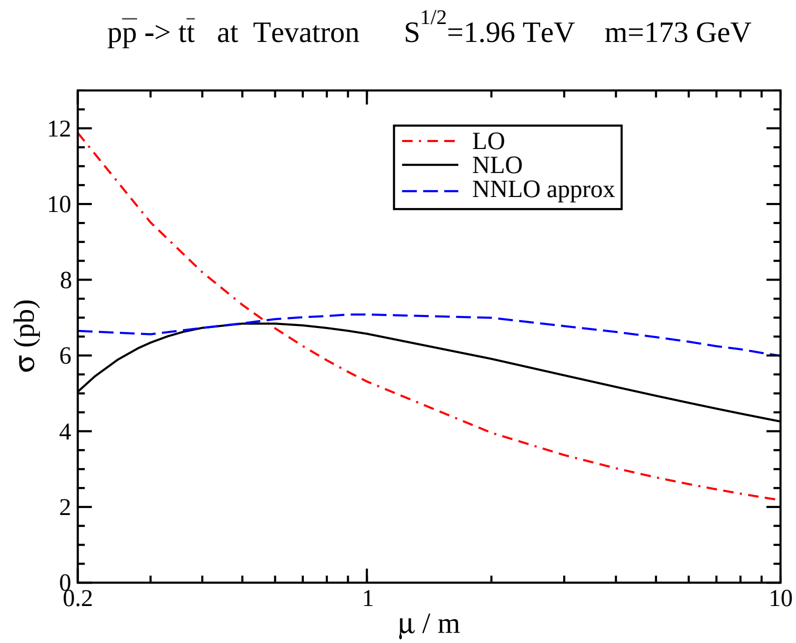
Resummed cross section

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E_i(N_i)\right] \exp\left[\sum_j E'_j(N')\right] \exp\left[\sum_{i=1,2} 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\, \gamma_{i/i}\left(\tilde{N}_i, \alpha_s(\mu)\right)\right]$$

$$\times\, \mathrm{tr}\left\{H\left(\alpha_s\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu}\, \Gamma_S^\dagger\left(\alpha_s(\mu)\right)\right] S\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}'}\right)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu}\, \Gamma_S\left(\alpha_s(\mu)\right)\right]\right\}$$

$t\bar{t}$ cross section at the Tevatron





$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 1.96\,{\rm TeV}) \quad = \quad 7.08^{+0.00}_{-0.24}{}^{+0.36}_{-0.27}\ {\rm pb}$$

scale pdf

NNLO approx: 7.8% enhancement over NLO
scale dependence greatly reduced

used MSTW 2008 NNLO pdf

Comparison of various resummation/NNLO approx approaches

Tevatron 1.96 TeV, scale uncertainty included;
pdf uncertainty not shown - same for all if same assumptions are used
use $m_t = 173$ GeV unless otherwise indicated

NLO $6.74^{+0.36}_{-0.76}$

Kidonakis, PRD 82, 114030 (2010) $7.08^{+0.20}_{-0.24}$

Aliev *et al*, **CPC 182, 1034 (2011)** $7.13^{+0.31}_{-0.39}$

Ahrens *et al*, **PLB 703, 135 (2011)** $6.65^{+0.08}_{-0.41}$

Beneke *et al*, **NPB 855, 695 (2012)** $(m_t = 173.3)$ $7.22^{+0.31}_{-0.47} \to 7.29$ **at** $m_t = 173$

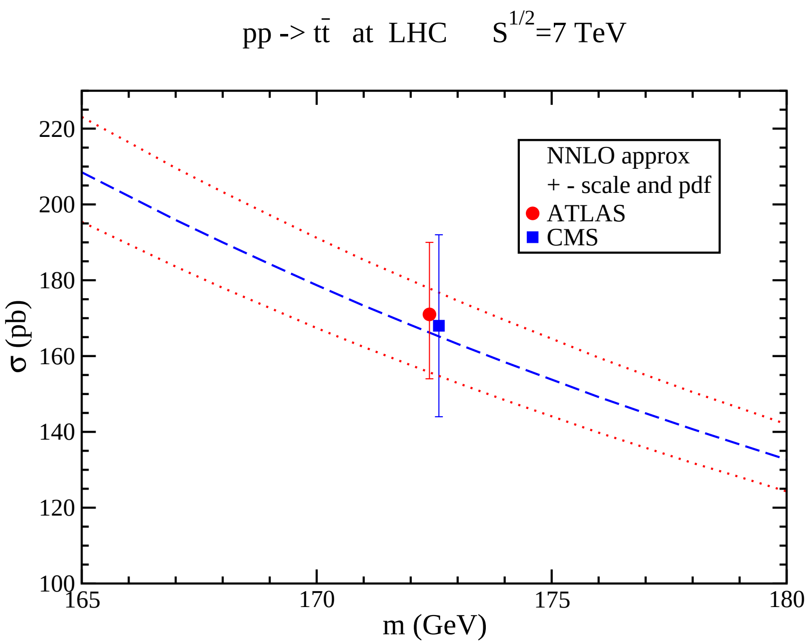
Cacciari *et al*, **PLB 710, 612 (2012)** $(m_t = 173.3)$ $6.72^{+0.24}_{-0.41} \to 6.78$ **at** $m_t = 173$

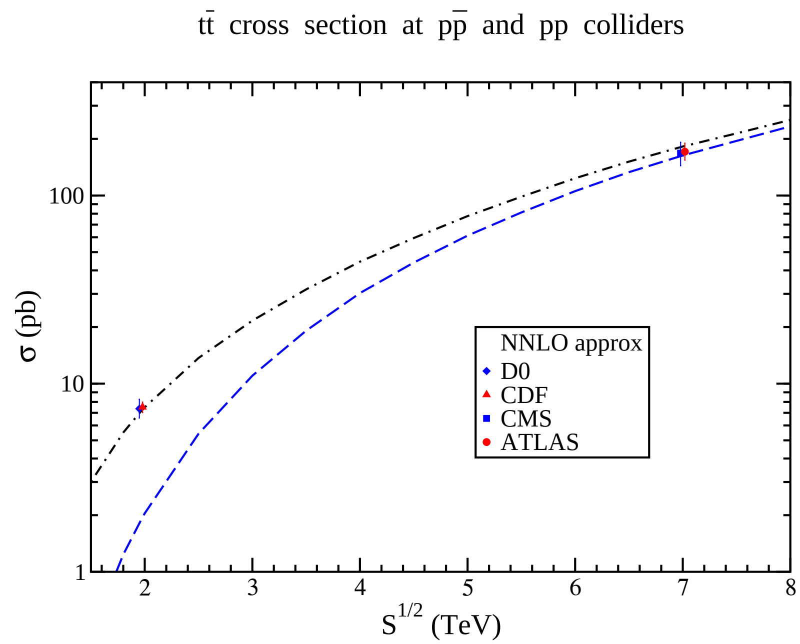
[See also
Moch *et al* **(2012)** $7.27^{+0.41}_{-0.46}$ **threshold + high-energy terms**
Brodsky & Wu (2012) $(m_t = 172.9)$ $7.626 \to 7.602$ **at** $m_t = 173$ **PMC]**

partly exact NNLO (exact for $q\bar{q}$ plus approx for gg)
Barnreuther *et al* **(2012)** $(m_t = 173.3)$ $7.005^{+0.202}_{-0.310} \to 7.07$ **at** $m_t = 173$

The PRD 82 result is by far the closest to the partly exact NNLO:
7.08 vs 7.07 with similar scale uncertainty

$t\bar{t}$ cross section at the LHC





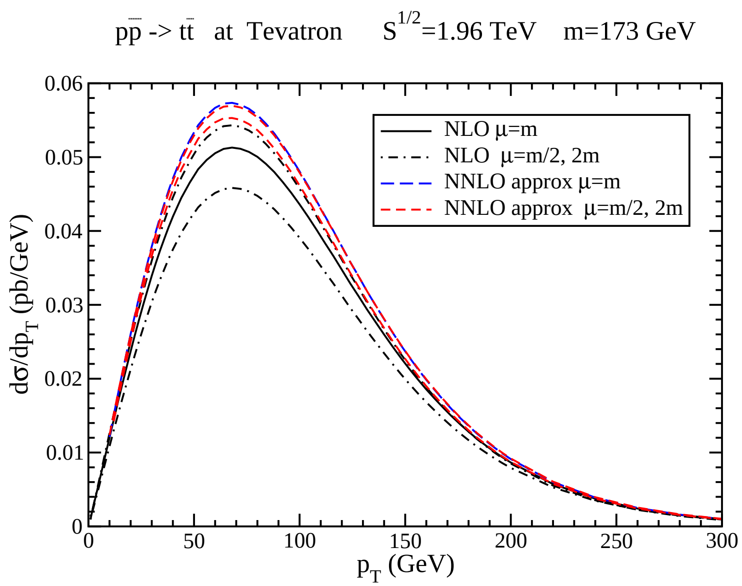
$$\sigma_{t\bar{t}}^{\mathrm{NNLOapprox}}(m_t = 173\,\mathrm{GeV},\,7\,\mathrm{TeV}) \quad = \quad 163^{+7}_{-5} \pm 9\ \mathrm{pb}$$

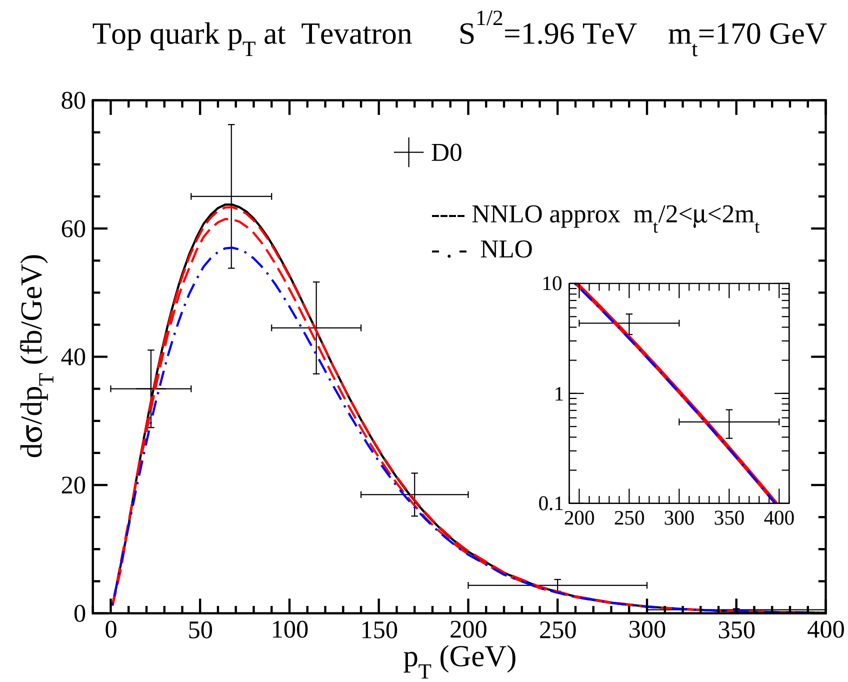
$$\sigma_{t\bar{t}}^{\mathrm{NNLOapprox}}(m_t = 173\,\mathrm{GeV},\,8\,\mathrm{TeV}) \quad = \quad 234^{+10}_{-7} \pm 12\ \mathrm{pb}$$

$$\sigma_{t\bar{t}}^{\mathrm{NNLOapprox}}(m_t = 173\,\mathrm{GeV}, 14\,\mathrm{TeV}) \quad = \quad 920^{+50}_{-39}{}^{+33}_{-35}\ \mathrm{pb}$$

NNLO approx: enhancement over NLO (same pdf) is 7.6% at 7 TeV; 7.8% at 8 TeV; 8.0% at 14 TeV

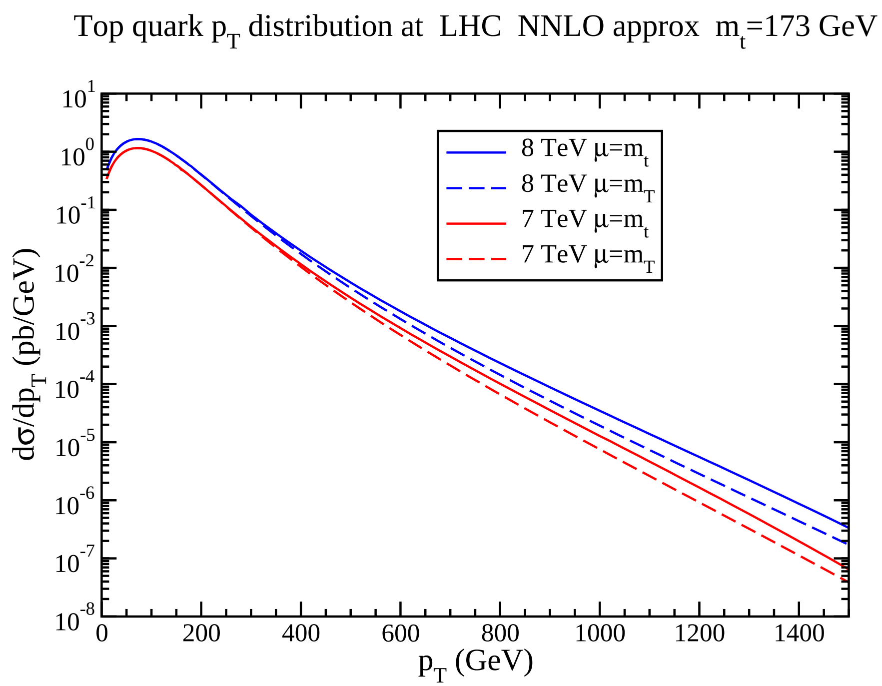
Top quark p_T distribution at Tevatron

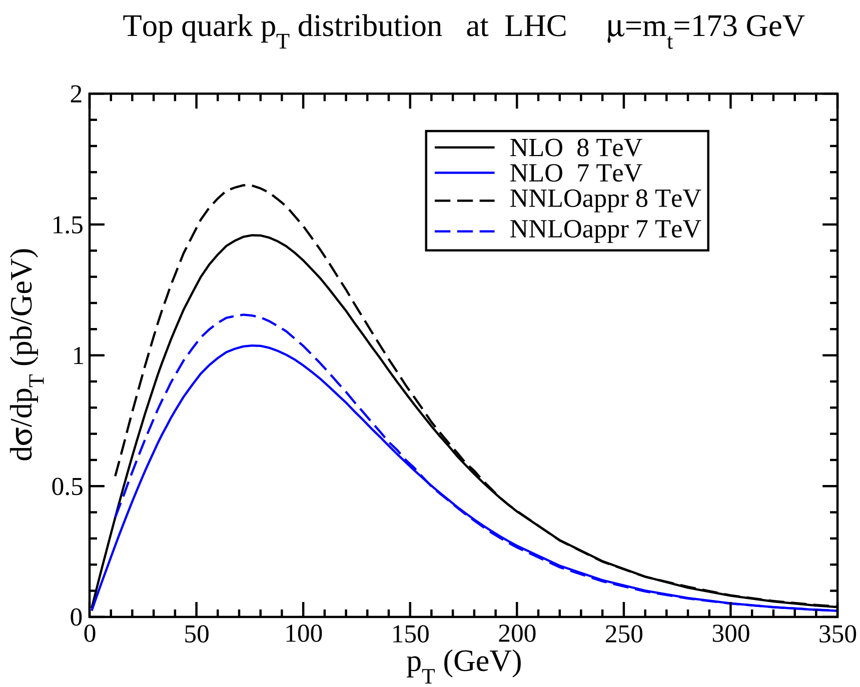




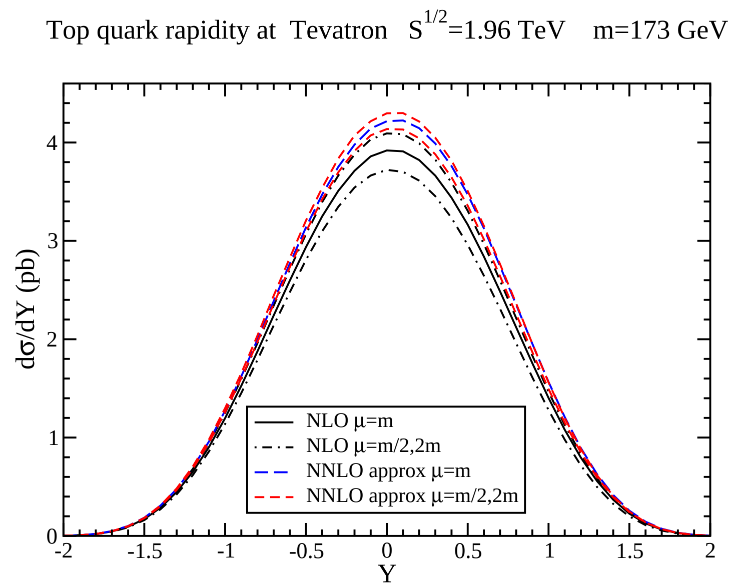
Excellent agreement of NNLO approx results with D0 data

Top quark p_T distribution at the LHC





Top quark rapidity distribution at Tevatron



Top Forward-backward asymmetry

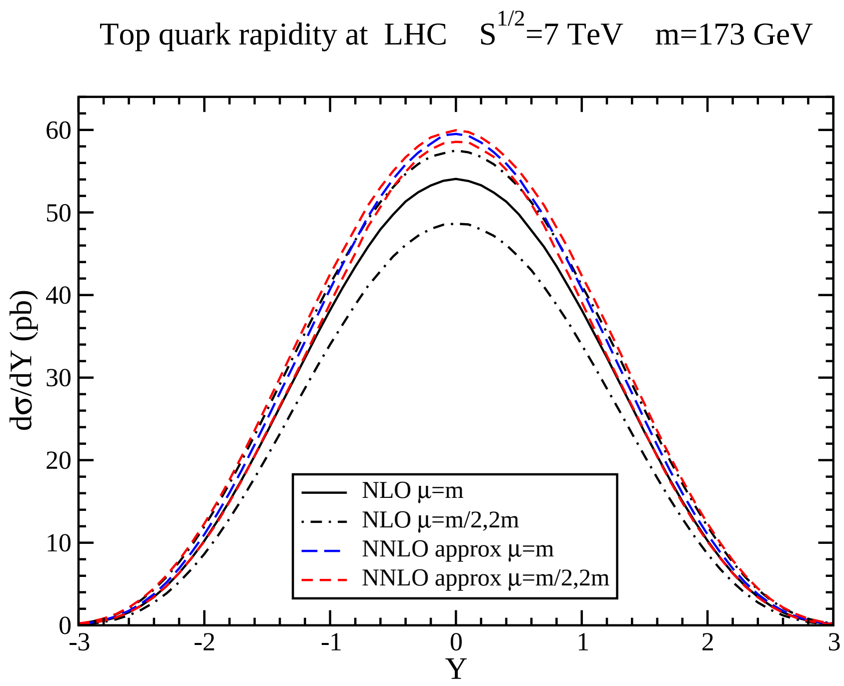
$$A_{\rm FB}=\frac{\sigma(Y>0)-\sigma(Y<0)}{\sigma(Y>0)+\sigma(Y<0)}$$

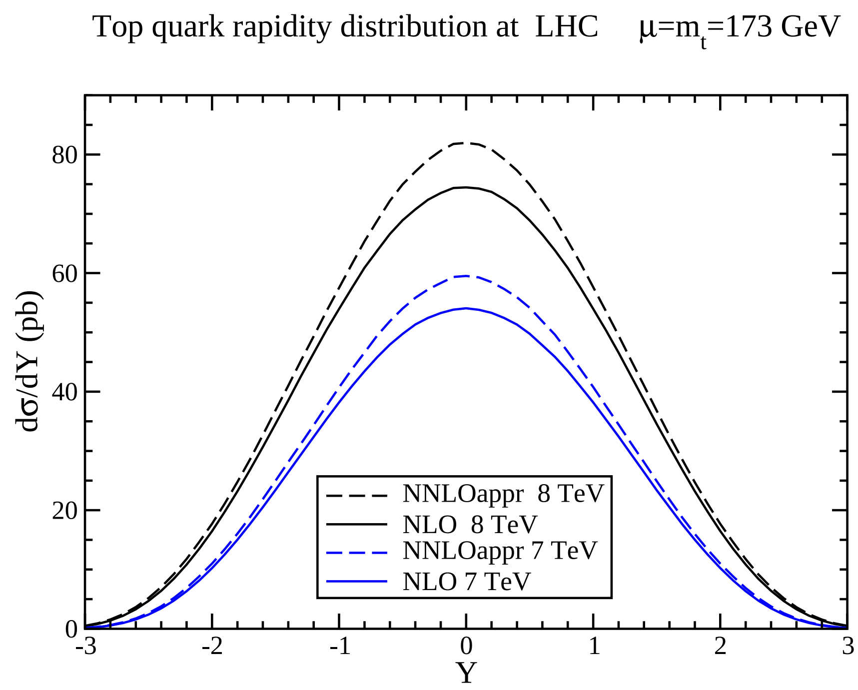
Asymmetry significant at the Tevatron

Theoretical result at Tevatron: $A_{\rm FB}=0.052^{+0.000}_{-0.006}$

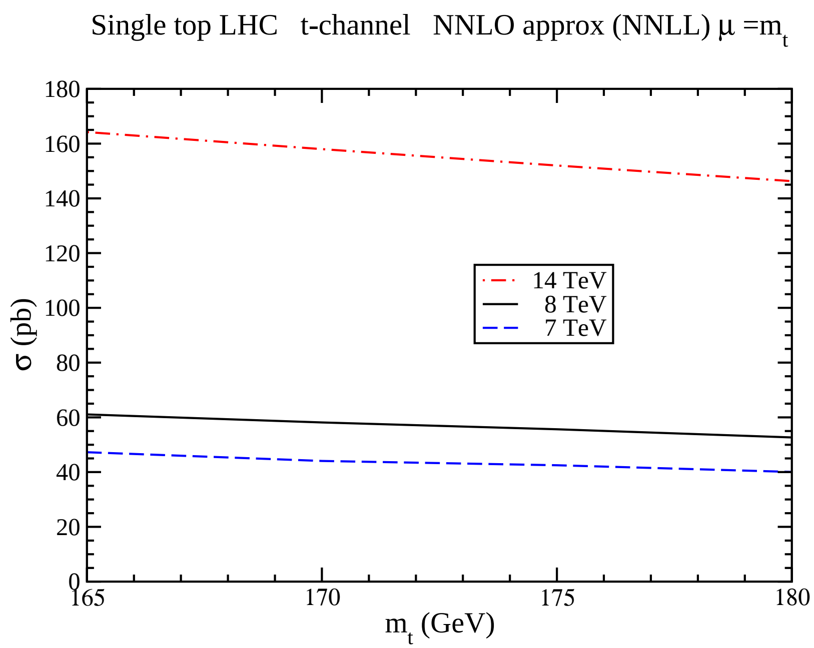
smaller than observed values

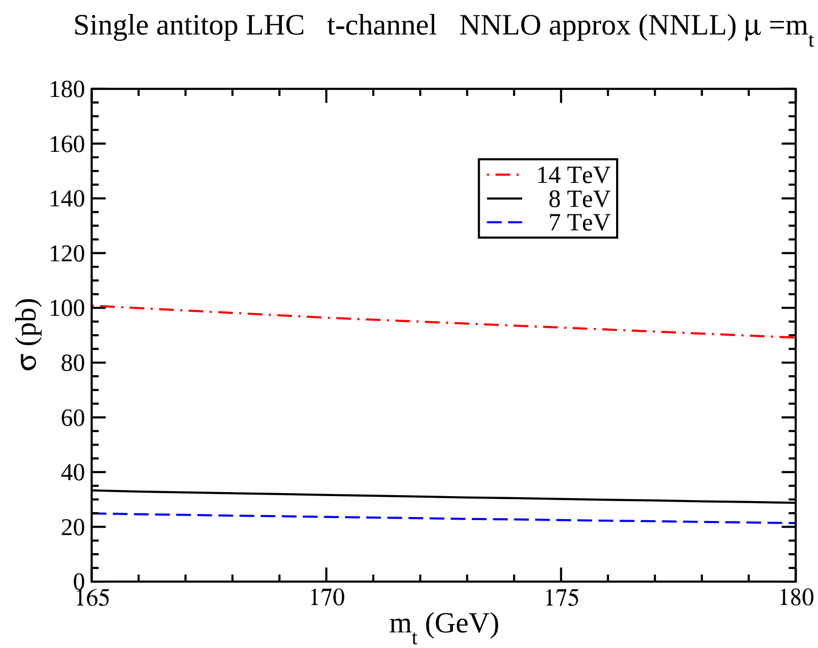
Top quark rapidity distribution at LHC





t-channel cross sections at LHC

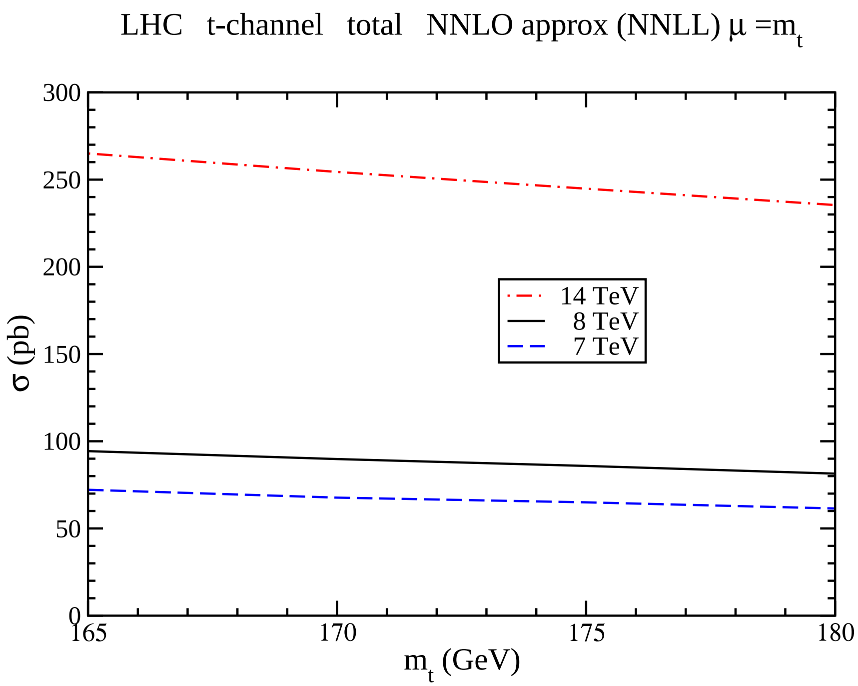


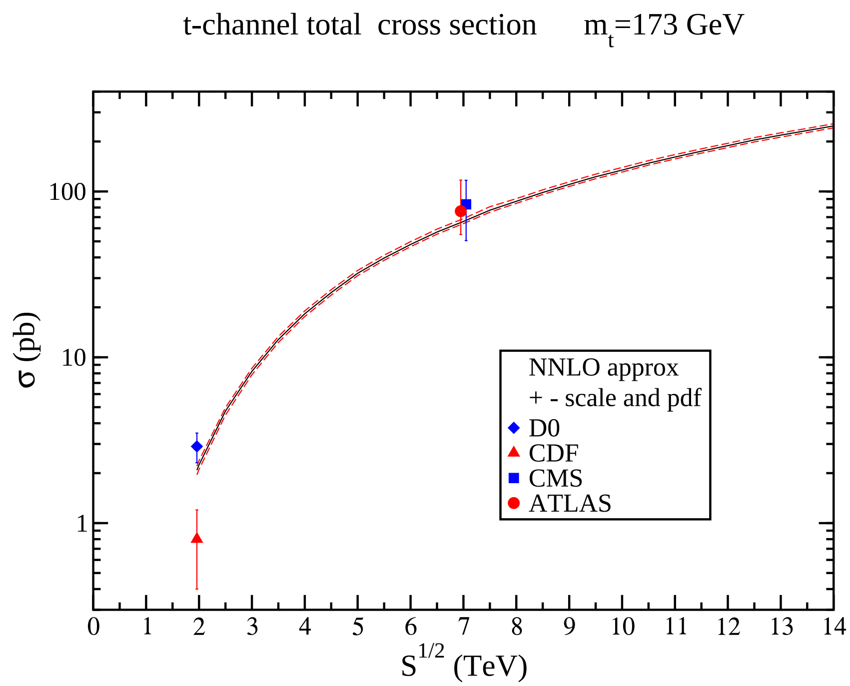


For $m_t = 173$ GeV

LHC	t	$\bar{t}$
7 TeV	$43.0^{+1.6}_{-0.2} \pm 0.8$	$22.9 \pm 0.5^{+0.7}_{-0.9}$
8 TeV	$56.4^{+2.1}_{-0.3} \pm 1.1$	$30.7 \pm 0.7^{+0.9}_{-1.1}$
14 TeV	$154^{+4}_{-1} \pm 3$	$94^{+2}_{-1}{}^{+2}_{-3}$

t-channel total cross section at LHC





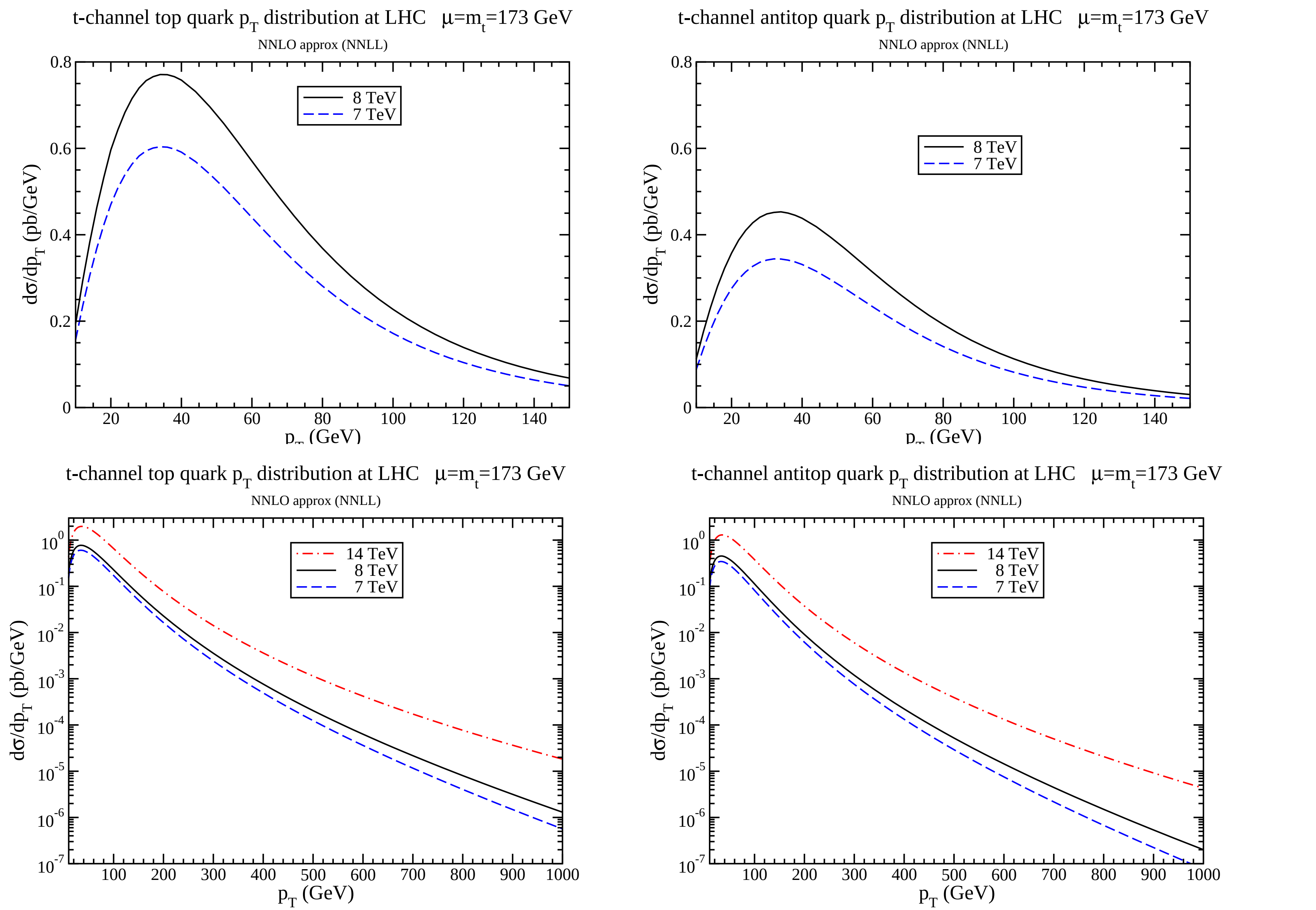
$$\sigma^{\rm NNLOapprox,\,total}_{t-\rm channel}(m_t = 173\,{\rm GeV},\,7\,{\rm TeV}) = 65.9^{+2.1\,+1.5}_{-0.7\,-1.7}\ {\rm pb}$$

$$\sigma^{\rm NNLOapprox,\,total}_{t-\rm channel}(m_t = 173\,{\rm GeV},\,8\,{\rm TeV}) = 87.2^{+2.8\,+2.0}_{-1.0\,-2.2}\ {\rm pb}$$

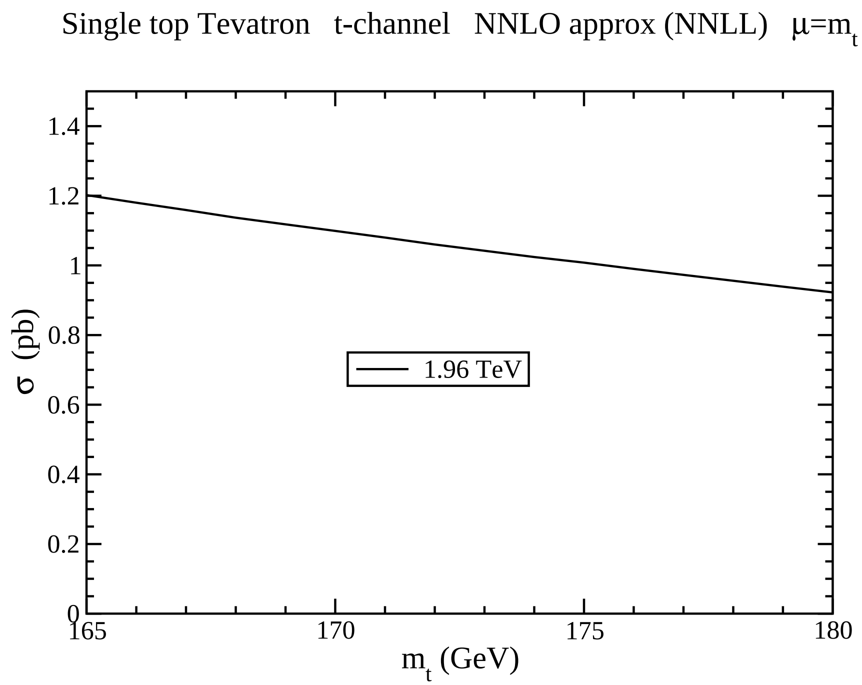
$$\sigma^{\rm NNLOapprox,\,total}_{t-\rm channel}(m_t = 173\,{\rm GeV},\,14\,{\rm TeV}) = 248^{+6\,+5}_{-2\,-6}\ {\rm pb}$$

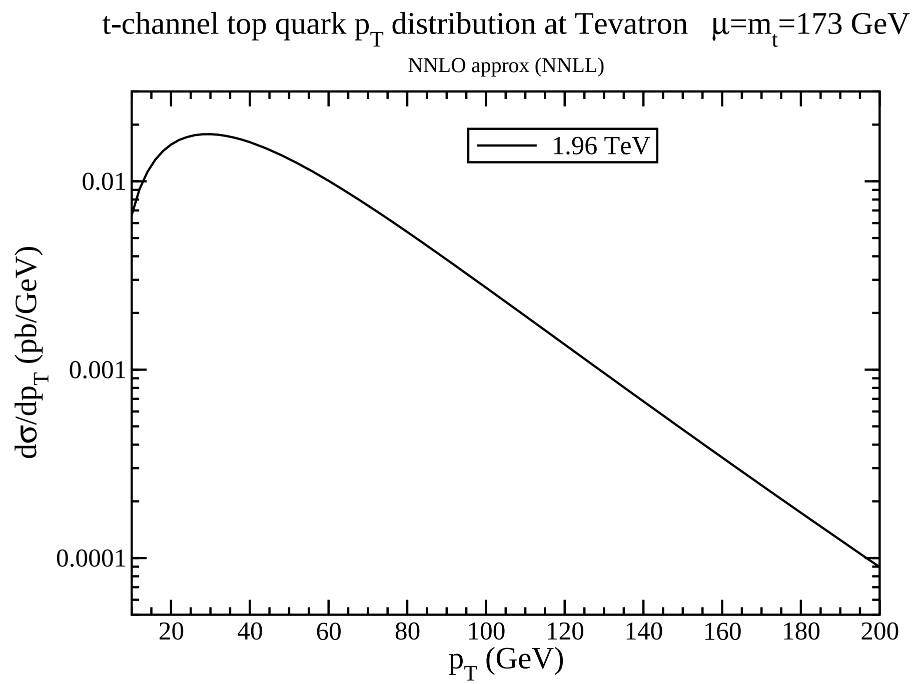
Small $\mathcal{O}(1\%)$ corrections over NLO

t-channel top and antitop p_T distributions at LHC



t-channel top quark production at Tevatron

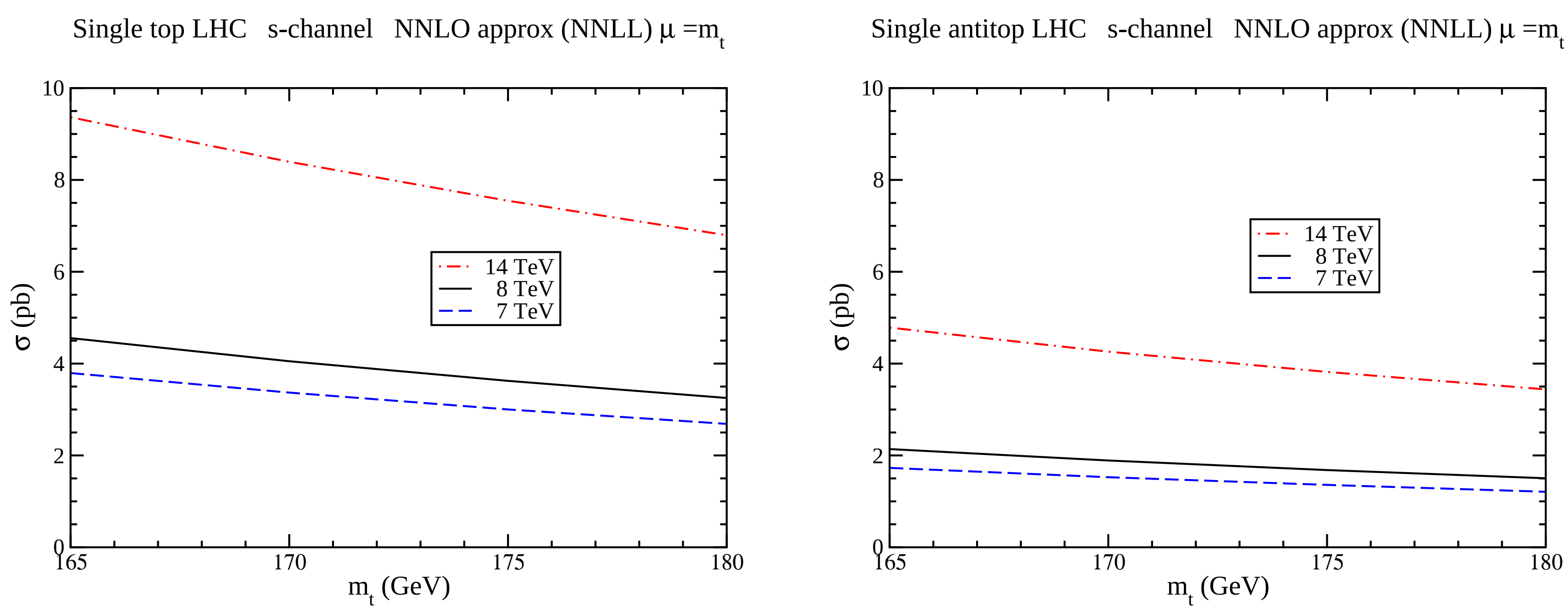




$$\sigma^{\rm NNLOapprox,\, top}_{t-{\rm channel}}(m_t = 173\,{\rm GeV},\, 1.96\,{\rm TeV}) \quad = \quad 1.04^{+0.00}_{-0.02} \pm 0.06 \;{\rm pb}$$

Cross section for antitop t-channel production at Tevatron is identical

s-channel cross sections

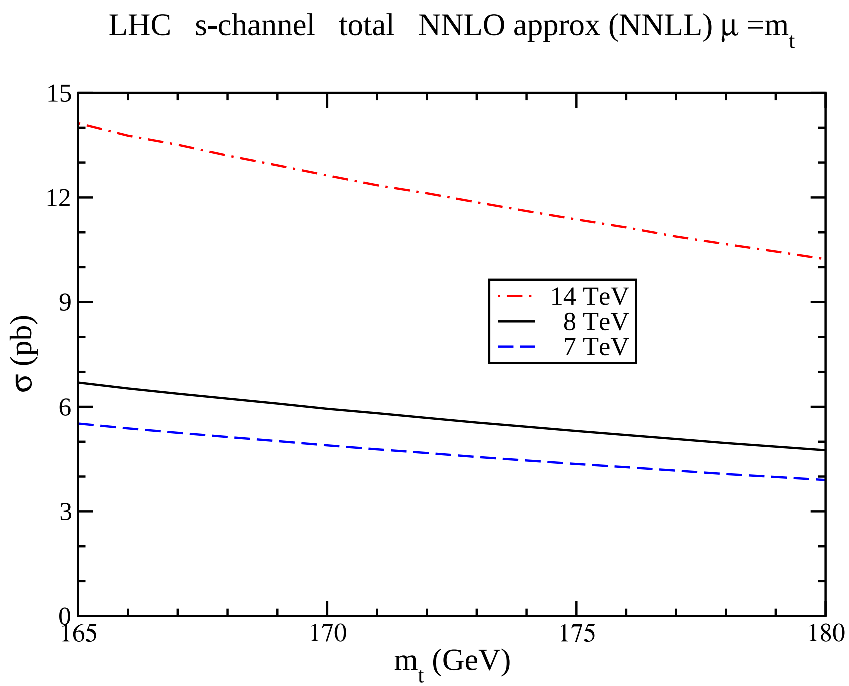


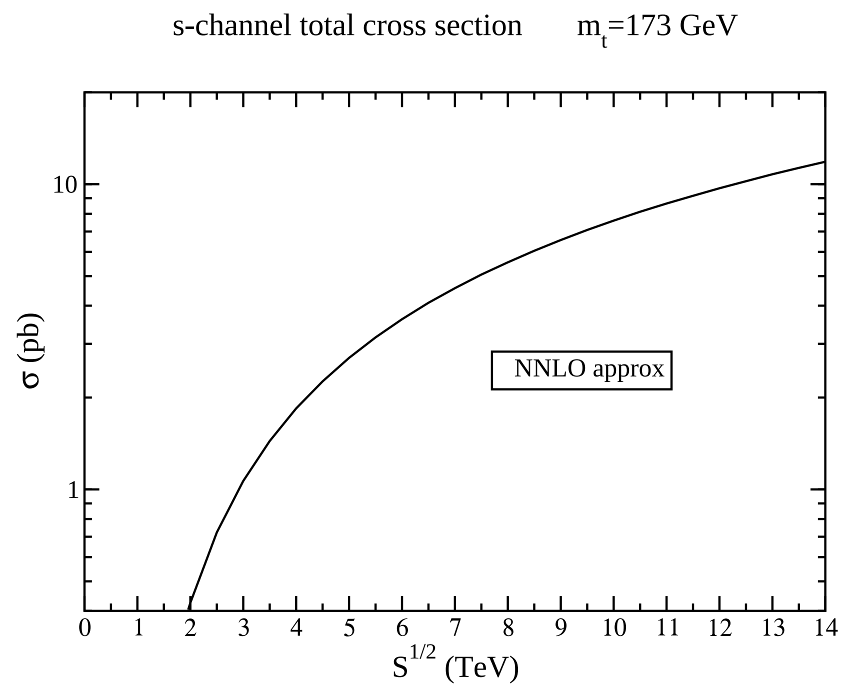
For $m_t = 173$ GeV

LHC	t	$\bar{t}$
7 TeV	$3.14 \pm 0.06^{+0.12}_{-0.10}$	$1.42 \pm 0.01^{+0.06}_{-0.07}$
8 TeV	$3.79 \pm 0.07 \pm 0.13$	$1.76 \pm 0.01 \pm 0.08$
14 TeV	$7.87 \pm 0.14^{+0.31}_{-0.28}$	$3.99 \pm 0.05^{+0.14}_{-0.21}$

At Tevatron $\sqrt{S} = 1.96$ TeV: $0.523^{+0.001\,+0.030}_{-0.005\,-0.028}$ pb for top; same for antitop

s-channel total cross section at LHC





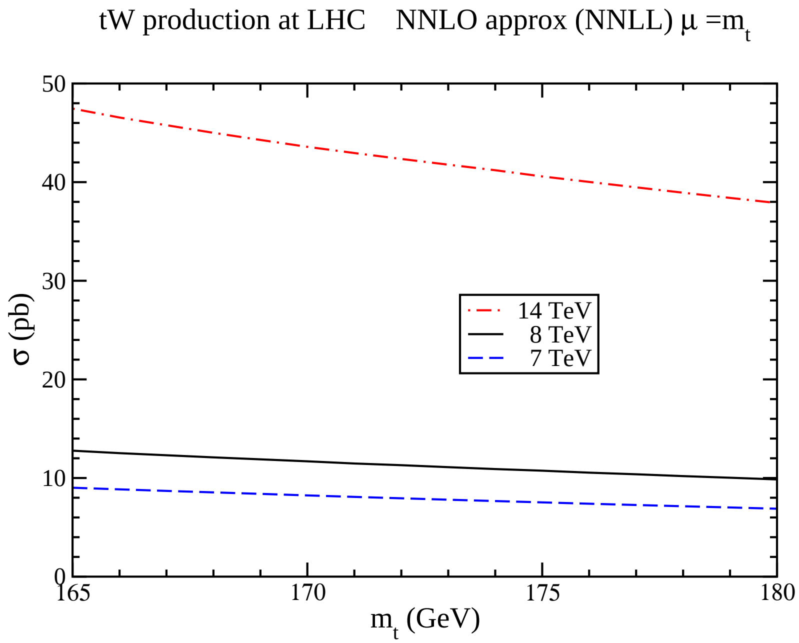
$$\sigma^{\rm NNLOapprox,\, total}_{s-{\rm channel}}(m_t = 173\,{\rm GeV},\ 7\,{\rm TeV}) \quad = \quad 4.56 \pm 0.07^{+0.18}_{-0.17}\ {\rm pb}$$

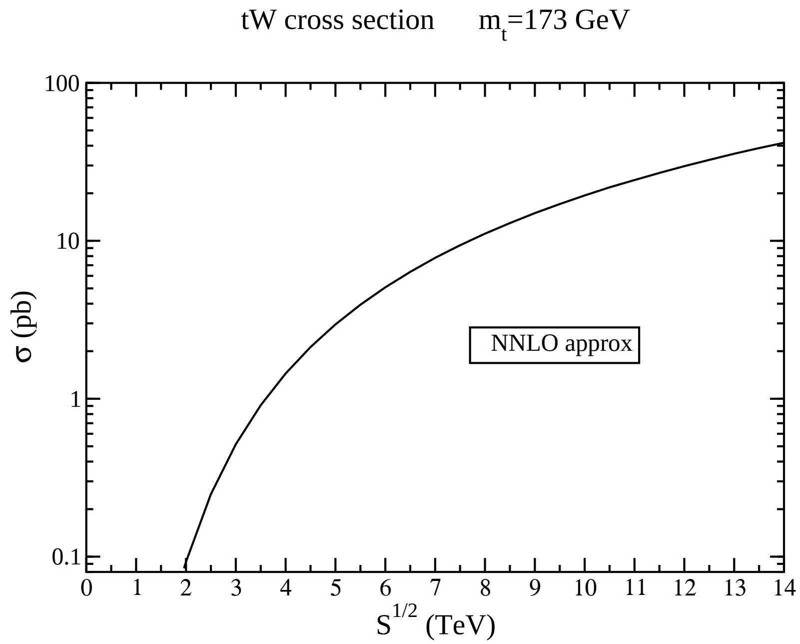
$$\sigma^{\rm NNLOapprox,\, total}_{s-{\rm channel}}(m_t = 173\,{\rm GeV},\ 8\,{\rm TeV}) \quad = \quad 5.55 \pm 0.08 \pm 0.21\ {\rm pb}$$

$$\sigma^{\rm NNLOapprox,\, total}_{s-{\rm channel}}(m_t = 173\,{\rm GeV},\ 14\,{\rm TeV}) \quad = \quad 11.86 \pm 0.19^{+0.45}_{-0.49}\ {\rm pb}$$

NNLO approx: enhancement over NLO is $\sim 10\%$

Associated tW^- production at the LHC





$$\sigma_{tW}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 7\,{\rm TeV}) = 7.8 \pm 0.2^{+0.5}_{-0.6}\ {\rm pb}$$

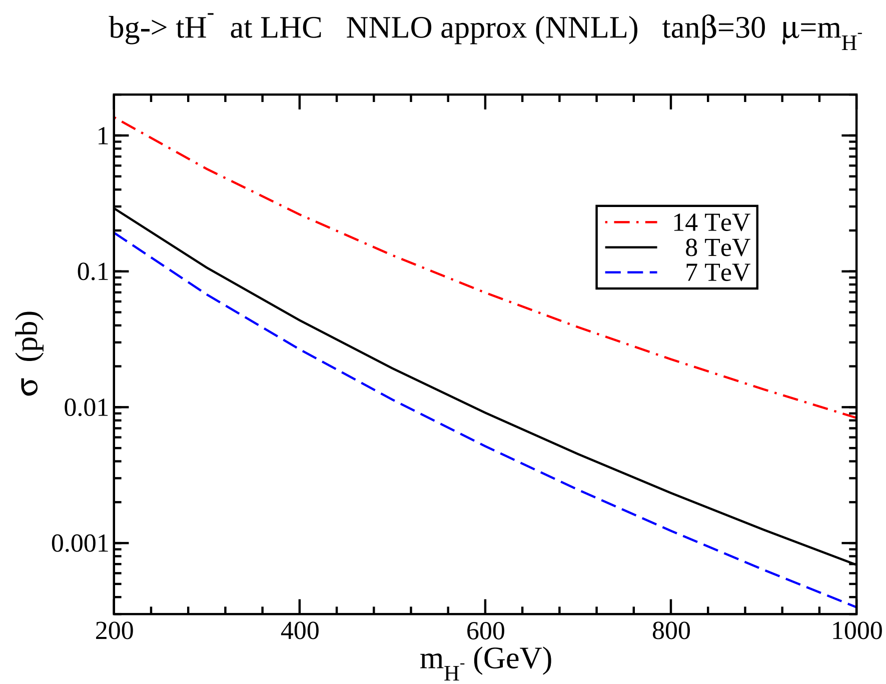
$$\sigma_{tW}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 8\,{\rm TeV}) = 11.1 \pm 0.3 \pm 0.7\ {\rm pb}$$

$$\sigma_{tW}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 14\,{\rm TeV}) = 41.8 \pm 1.0^{+1.5}_{-2.4}\ {\rm pb}$$

NNLO approx corrections increase NLO cross section by $\sim 8\%$

Cross section for $\bar{t}W^+$ production is identical

Associated production of a top quark with a charged Higgs



NNLO approx corrections increase NLO cross section by ~ 15 **to** $\sim 20\%$

W and Z production at large p_T - parton processes

W and Z hadroproduction useful in testing the SM and in estimates of backgrounds to Higgs production and new physics (new gauge bosons)

p_T distribution falls rapidly as p_T increases

Partonic channels at LO

$q(p_a) + g(p_b) \longrightarrow W(Q) + q(p_c)$

$q(p_a) + \bar{q}(p_b) \longrightarrow W(Q) + g(p_c)$

Define $s = (p_a + p_b)^2$**,** $t = (p_a - Q)^2$**,** $u = (p_b - Q)^2$ **and** $s_4 = s + t + u - Q^2$

At threshold $s_4 \to 0$

Soft corrections $\left[\frac{\ln^l(s_4/p_T^2)}{s_4}\right]_+$

Virtual corrections $\delta(s_4)$

NLO corrections

The NLO cross section can be written as

$$E_Q \frac{d\hat{\sigma}_{f_a f_b \to W(Q)+X}}{d^3Q} = \delta(s_4)\alpha_s(\mu_R^2)\left[A(s,t,u) + \alpha_s(\mu_R^2)B(s,t,u,\mu_R)\right] + \alpha_s^2(\mu_R^2)C(s,t,u,s_4,\mu_F)$$

The coefficient functions A, B, and C depend on the parton flavors

The coefficient $A(s,t,u)$ arises from the LO processes

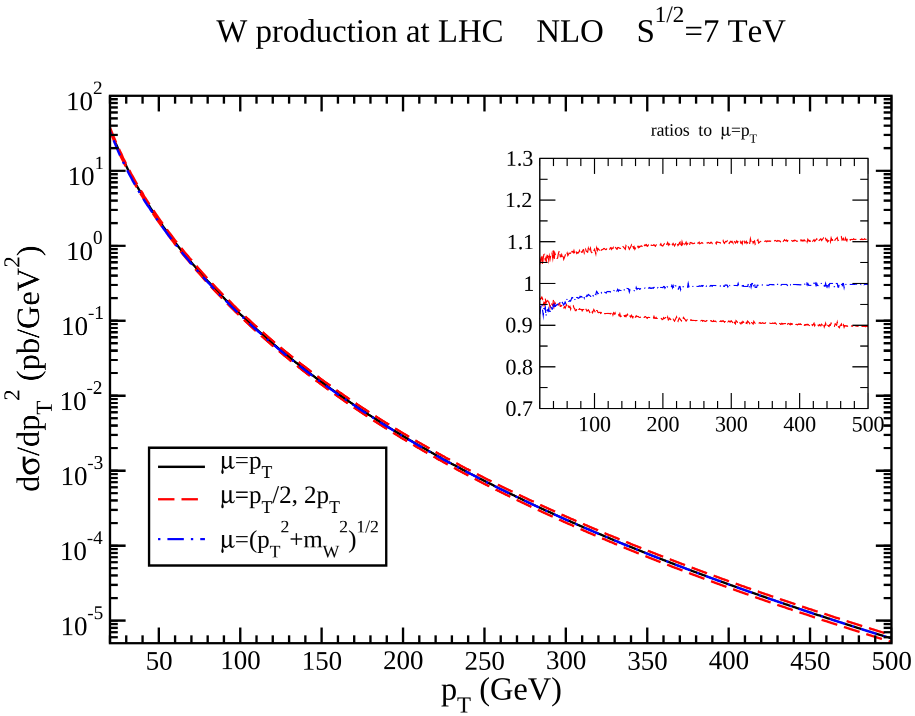
$B(s,t,u,\mu_R)$ is the sum of virtual corrections and of singular terms $\sim\delta(s_4)$ in the real radiative corrections

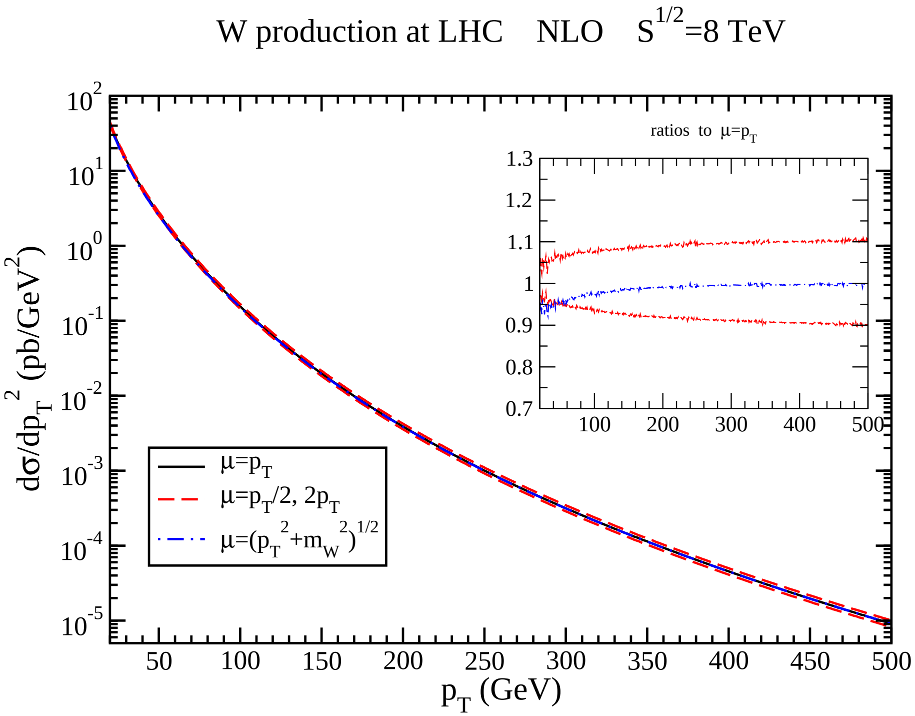
$C(s,t,u,s_4,\mu_F)$ is from real emission processes away from $s_4=0$

P.B. Arnold and M.H. Reno, Nucl. Phys. B 319, 37 (1989); (E) B 330, 284 (1990)

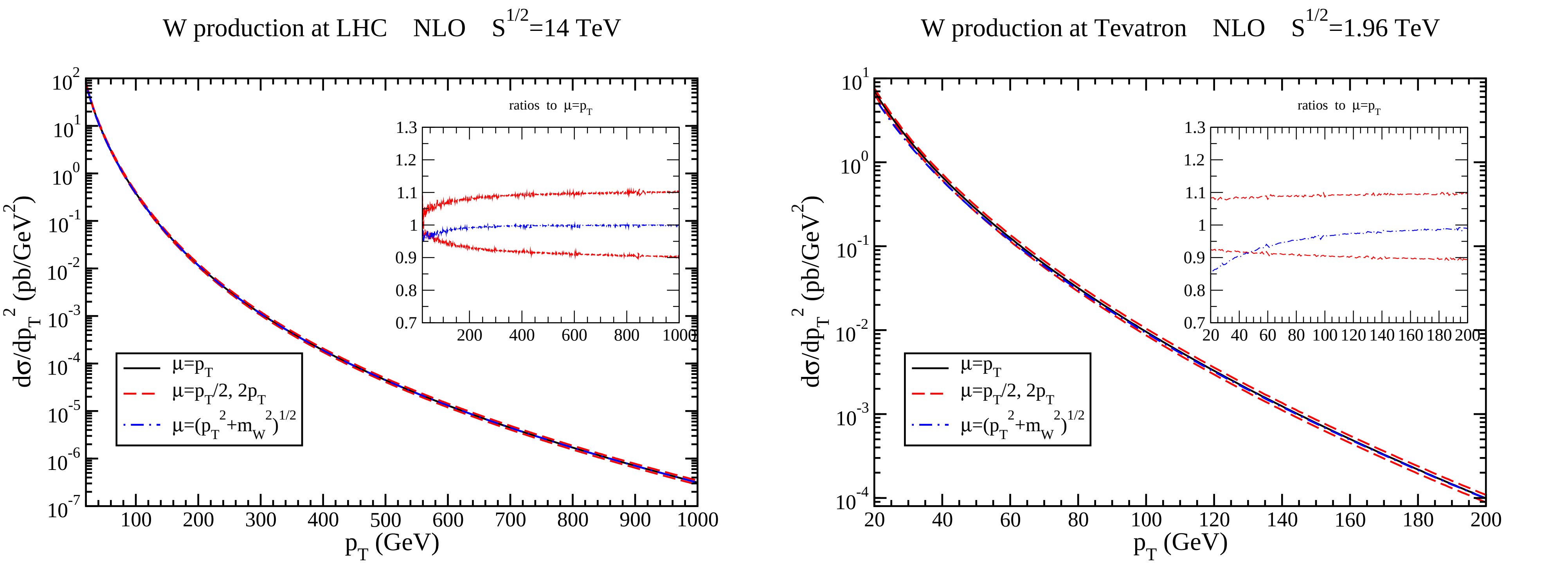
R.J. Gonsalves, J. Pawlowski, C.-F. Wai, Phys. Rev. D 40, 2245 (1989); Phys. Lett. B 252, 663 (1990)

NLO p_T distribution of the W-boson at the LHC at 7 and 8 TeV





NLO p_T distribution of the W-boson at the LHC and Tevatron



Soft-gluon corrections

$$\mathcal{D}_l(s_4) \equiv \left[\frac{\ln^l(s_4/p_T^2)}{s_4}\right]_+$$

We can formally resum these logarithms for W production at large p_T to all orders in α_s **Phys. Lett. B 480, 87 (2000)**

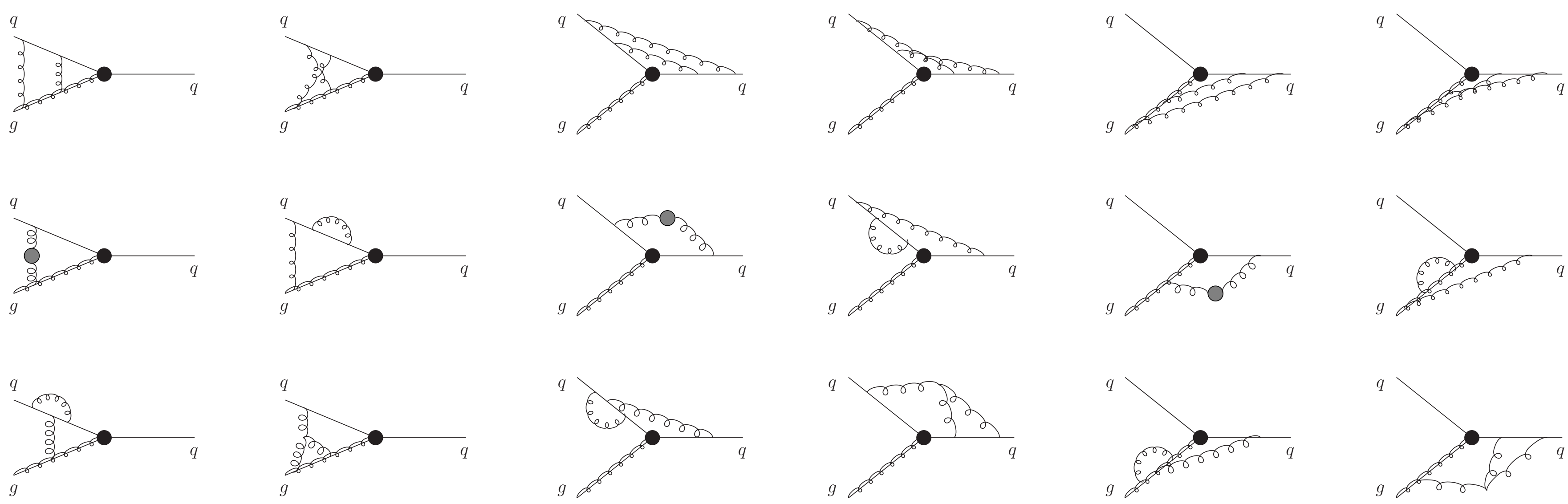
Applied to W production at the Tevatron: **JHEP 02, 027 (2004)**
and at the LHC: **Phys. Rev. Lett. 95, 222001 (2005)**

New approximate NNLO from NNLL resummation:

N. Kidonakis and R.J. Gonsalves, arXiv:1201.5265 [hep-ph]

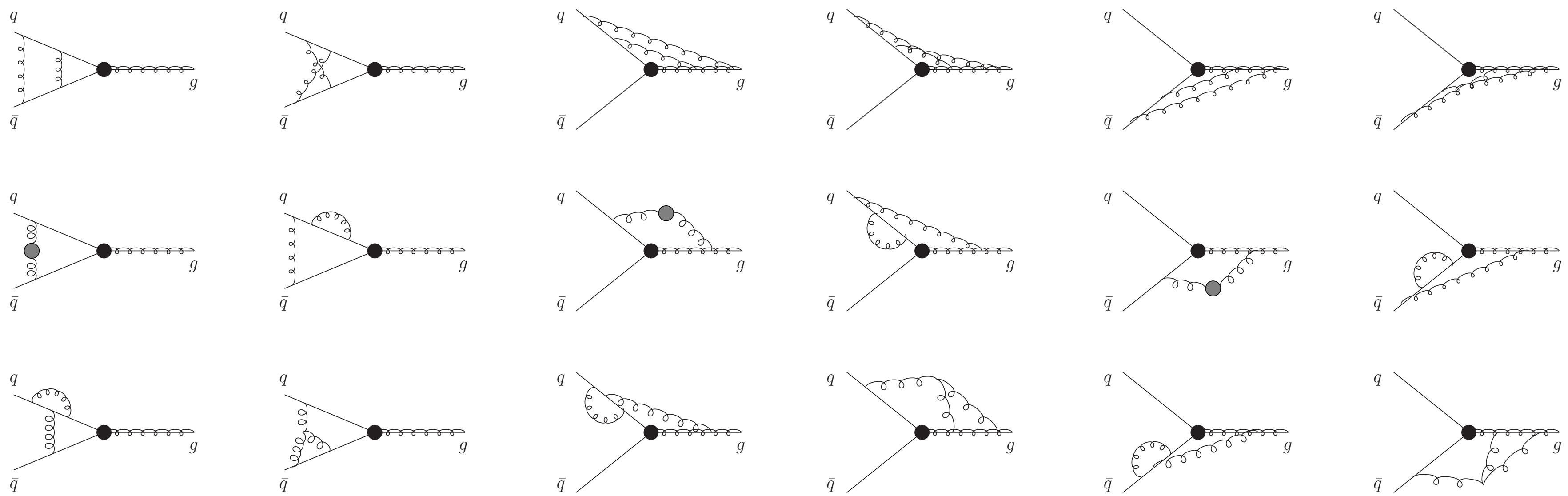
Two-loop soft anomalous dimension

Two-loop eikonal diagrams for $qg \to Wq$



Determine $\Gamma_S^{(2)}$ from UV poles of two-loop dimensionally regularized integrals

Two-loop eikonal diagrams for $q\bar{q} \to Wg$



Determine $\Gamma_S^{(2)}$ from UV poles of two-loop dimensionally regularized integrals

Two-loop soft anomalous dimension

For $qg \to Wq$ **or** $qg \to Zq$

$$\Gamma_{S,\,qg\to Wq}^{(1)} = C_F \ln\left(\frac{-u}{s}\right) + \frac{C_A}{2} \ln\left(\frac{t}{u}\right)$$

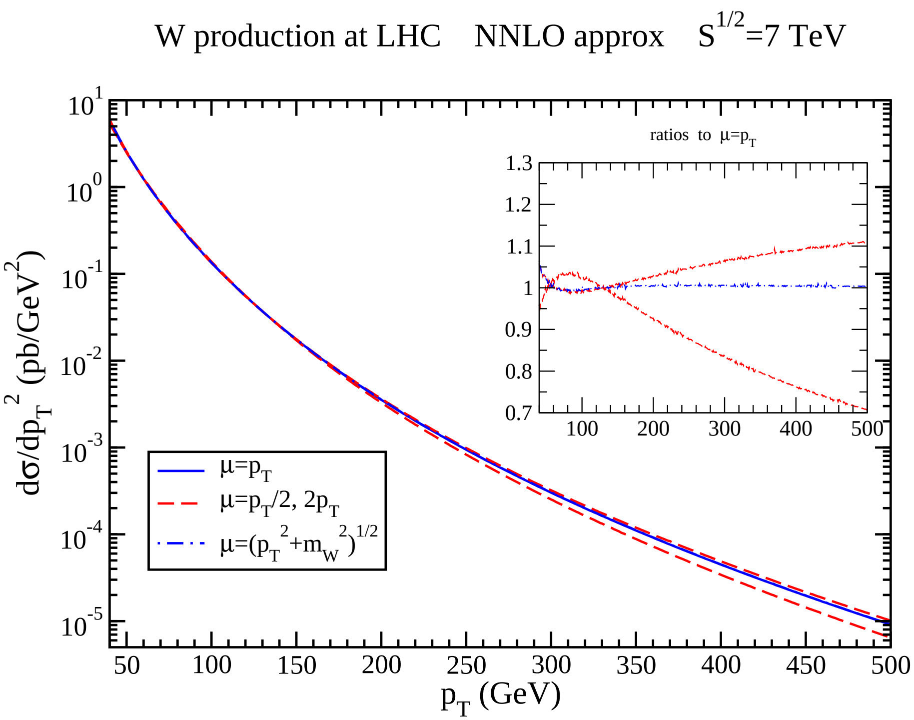
$$\Gamma_{S,\,qg\to Wq}^{(2)} = \frac{K}{2} \Gamma_{S,\,qg\to Wq}^{(1)}$$

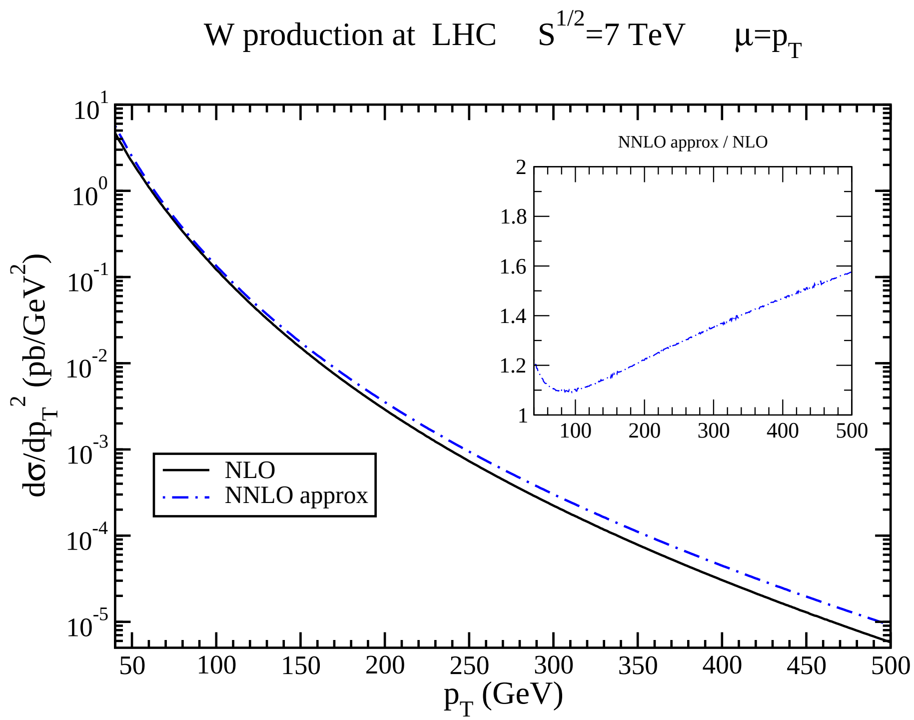
For $q\bar{q} \to Wg$ **or** $q\bar{q} \to Zg$

$$\Gamma_{S,\,q\bar{q}\to Wg}^{(1)} = \frac{C_A}{2} \ln\left(\frac{tu}{s^2}\right)$$

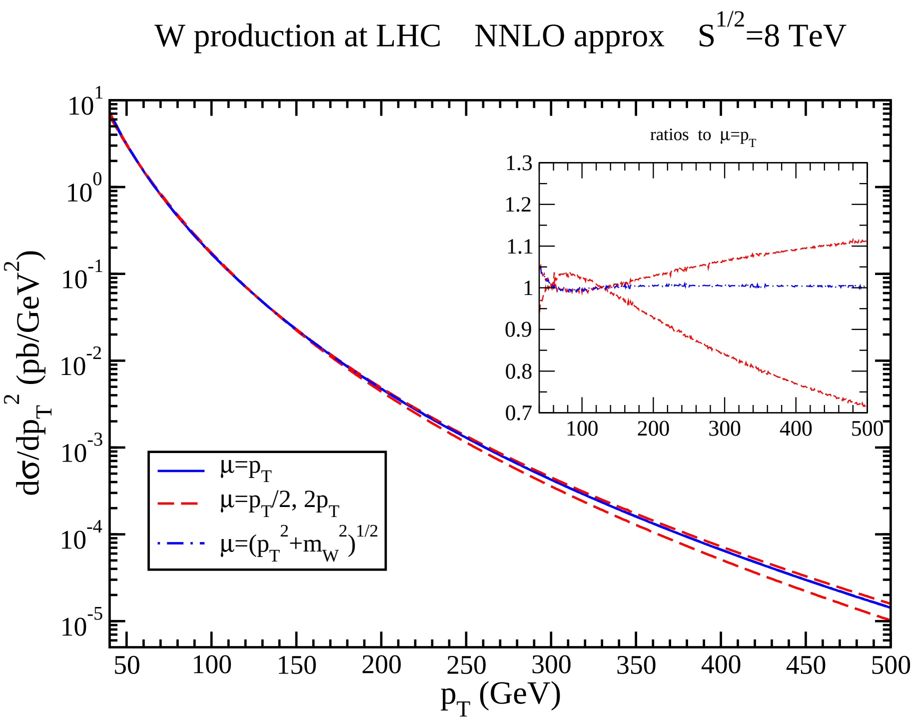
$$\Gamma_{S,\,q\bar{q}\to Wg}^{(2)} = \frac{K}{2} \Gamma_{S,\,q\bar{q}\to Wg}^{(1)}$$

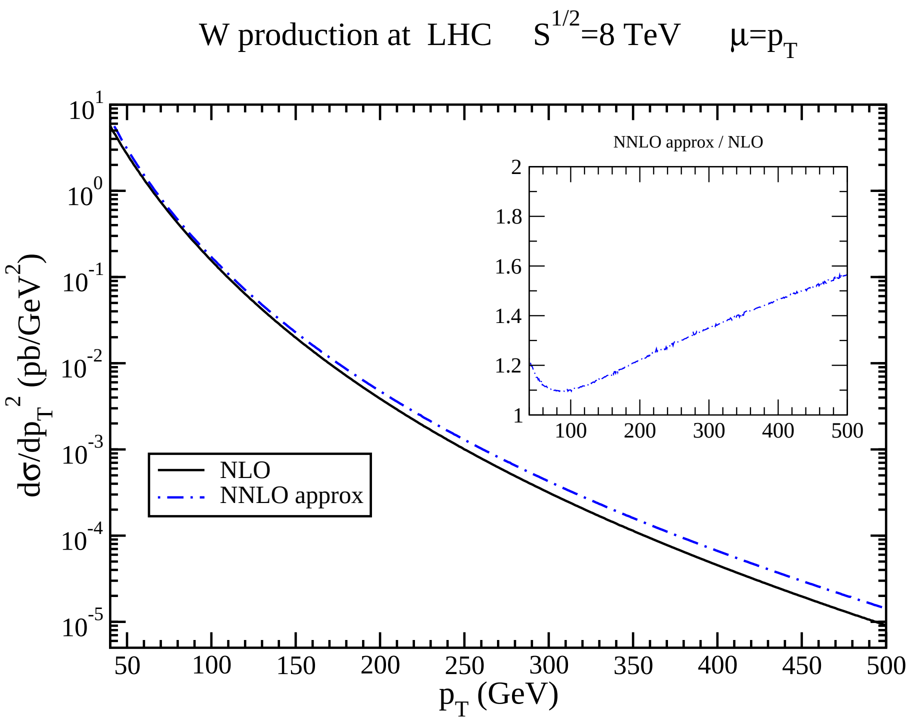
NNLO approx for W production at the LHC at 7 TeV



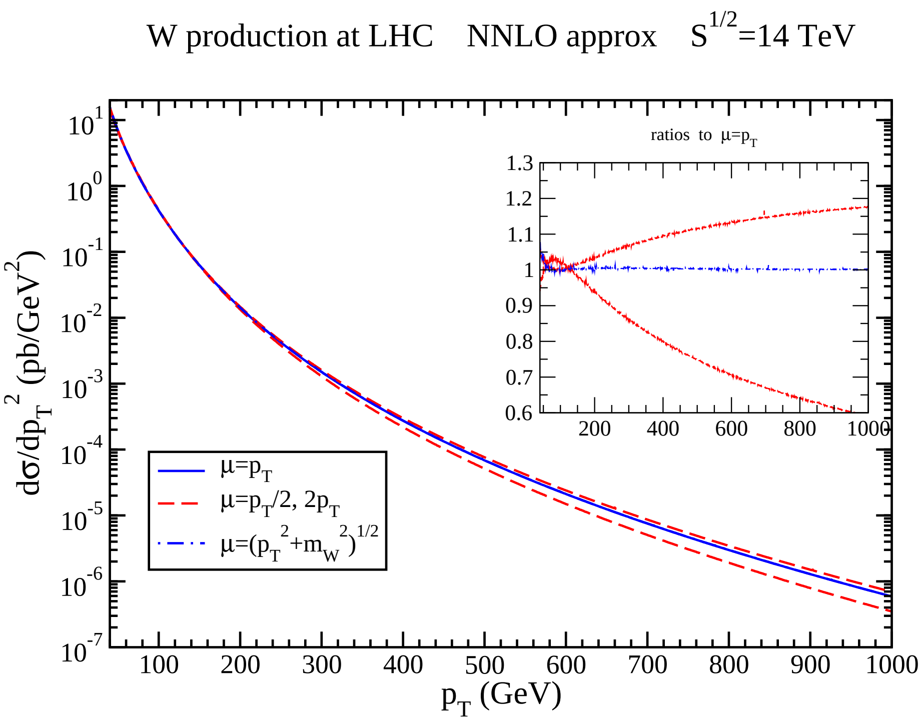


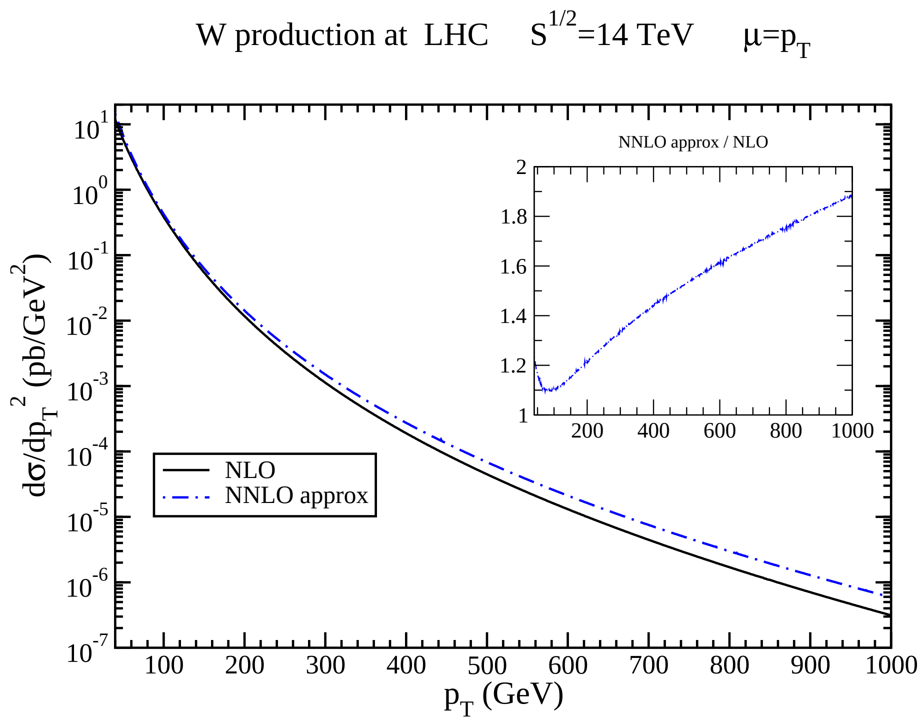
NNLO approx for W production at the LHC at 8 TeV



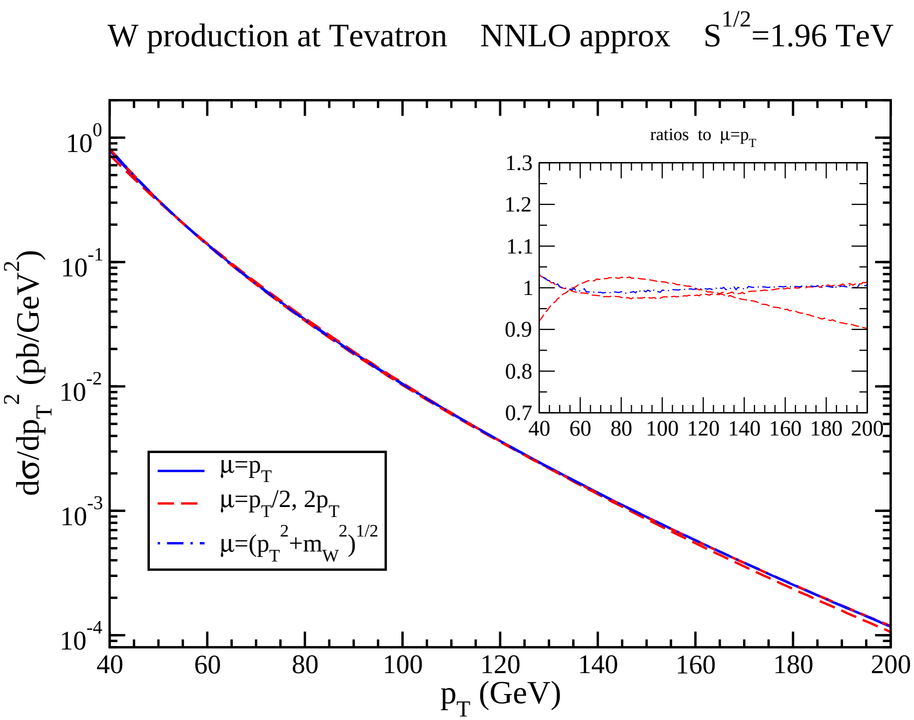


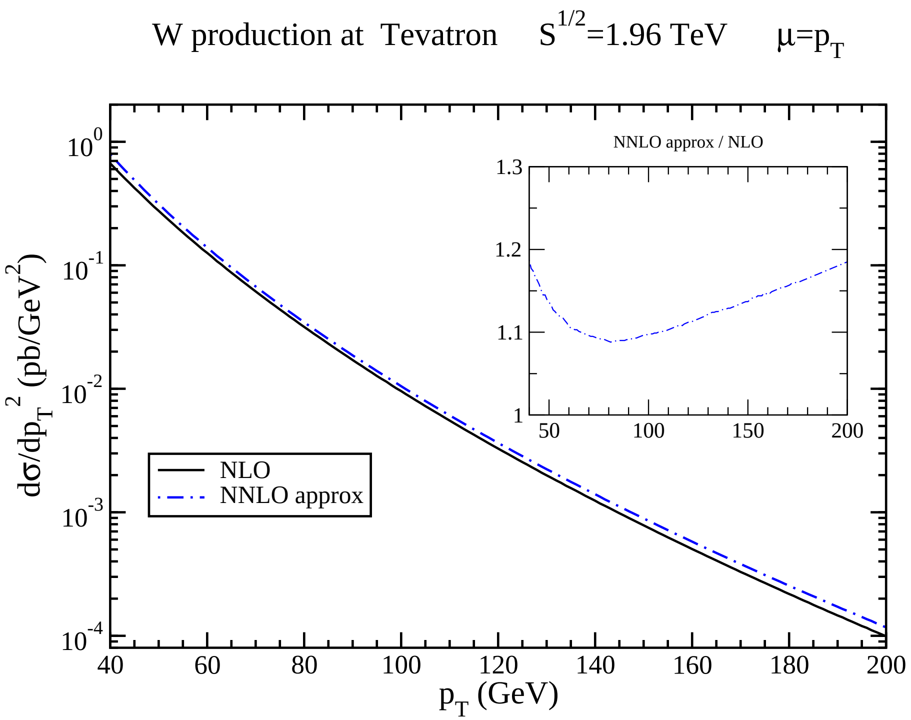
NNLO approx for W production at the LHC at 14 TeV



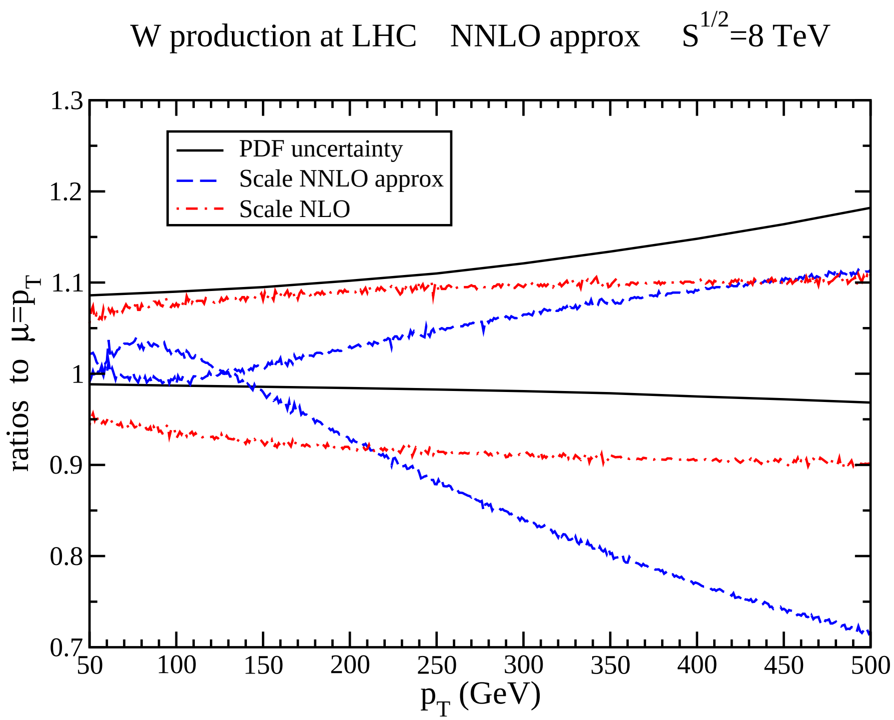


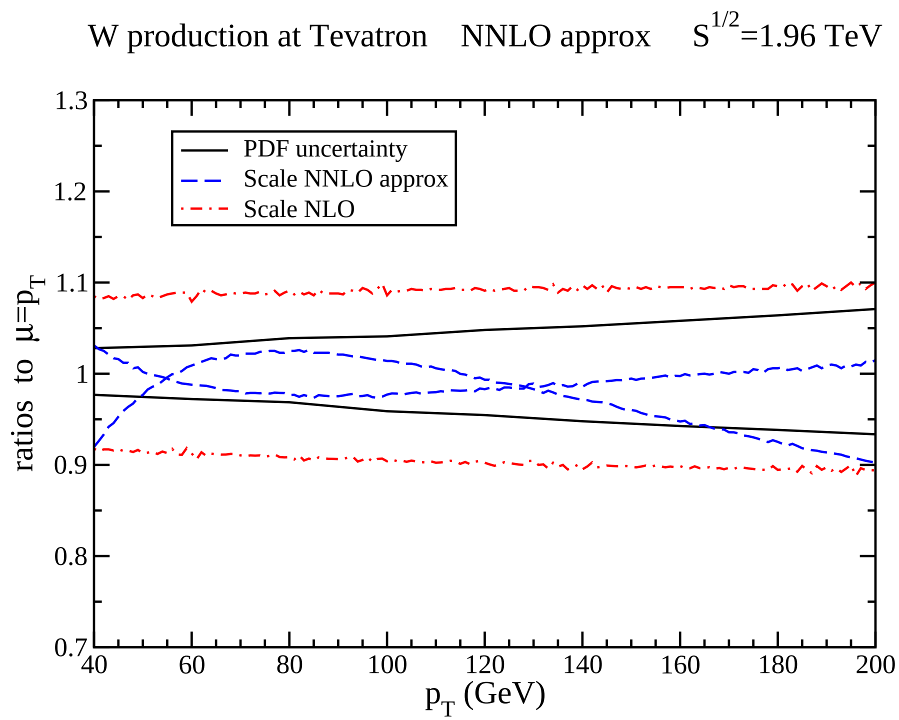
NNLO approx for W production at the Tevatron





Scale and PDF dependence





Used MSTW 2008 pdf

Summary

- **NNLL resummation for top quark pair and single top production**
- $t\bar{t}$ **production cross section**
- **top quark** p_T **and rapidity distributions**
- **single top cross sections and** p_T **distributions**
- W **production**
- **NNLO approx corrections are significant at the LHC and the Tevatron**
- **good agreement with LHC and Tevatron data**