SOFT SCATTERING AT EXCEEDINGLY HIGH ENERGIES

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INTRODUCTION

The subject of soft scattering cross sections at exceedingly high energies and the approach to saturation was re-kindled recently by Block and Halzen (BH), following their analysis of σ_{inel} at AUGER 57 TeV.

The theoretical interest in this issue has been kept alive over the last 50 years since the publication of Froissart bound. However, the road toward a systematic phenomenological study of saturation has been blocked by the lack of data. Very partial information is obtained from Cosmic Rays data on p-Air cross sections, available, with large errors, up to $10^4\,TeV.$

In this presentation I wish to promote the non conventional idea that there is a reasonable probability that saturation in p-p collisions IS NOT ATTAINED up to the Planck scale.

Having no relevant data above AUGER57, I shall investigate the outputs of BH, GLM and KMR models, aiming to identify model independent features.

KMR predictions go up to 100 TeV. BH and GLM go up to the Planck scale.

Recall that, model predictions at exceedingly high energies are based on the reproduction of ISR-LHC7 data. Regardless, they may have diverging predictions at higher energies.

The features of BH are relatively simple. It is a single channel eikonal model in which only σ_{tot}, σ_{el} (and σ_{inel}) are calculated. The model is based on a logarithmic parametrization supported by Finite Energy Sum Rules.

GLM and KMR are updated $I\!P$ models, conceptually similar, but different in the details of their assumptions and modellings. The models have a multi-layered architecture which incorporates a super critical $I\!P$ trajectory. $\alpha_{I\!P}(t) = 1 + \Delta_{I\!P} + \alpha'_{I\!P} t$, where, $\Delta_{I\!P} \simeq 0.2$ and $\alpha'_{I\!P} \simeq 0.$

In conventional Regge models, $\Delta_{I\!P}$ controls the amplitudes energy dependences and $\alpha'_{I\!P}$ the shrinkage of the forward t-cones. In updated $I\!P$ models these features are controlled by s and t unitarity screenings.

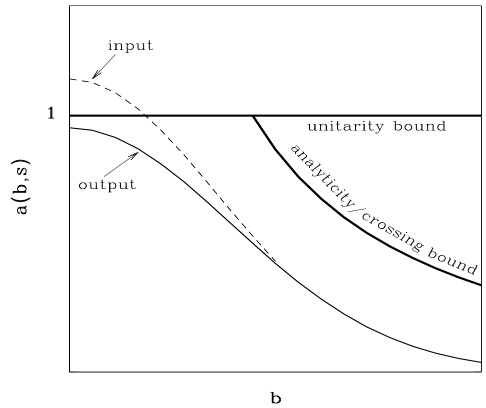
Following are the elements of the updated $I\!P$ models architecture relevant to the goal of this presentation:

- GLM has 3 input $I\!P$ exchange b-space elastic amplitudes of GW 2 channel interaction matrix eigen states: $A_{1,1}(s,b)$, $A_{2,2}(s,b)$ and $A_{1,2}(s,b) = A_{2,1}(s,b)$. KMR is a 3 channel model.

- Eikonal screening secures that the scattering amplitudes are bounded by the black s-channel unitarity bound. $A_{i,k}(s,b) = i\{1 - e^{-\Omega_{i,k}(s,b)/2}\} \leq 1$. $\Omega_{i,k}$ is real, equal to the imaginary part of the input (i,k) $I\!P$ Born amplitude.

- t-channel unitarity is coupled to multi $I\!P$ t-channel interactions leading to non GW "high mass" diffraction.

- S^2, the survival probability factor, has an eikonal and a multi $I\!P$ components. It is a suppression factor induced by the screening of non GW diffraction.



FROISSART-MARTIN BOUND

Given $\Omega(s,b)$, the non screened total, elastic and inelastic cross sections are:

$$\sigma_{tot} = 2\int d^2b\left(1 - e^{-\Omega(s,b)/2}\right), \quad \sigma_{el} = \int d^2b\left(1 - e^{-\Omega(s,b)/2}\right)^2, \quad \sigma_{inel} = \int d^2b\left(1 - e^{-\Omega(s,b)}\right).$$

The figure above shows the effect of s-channel screening, securing that the screened elastic amplitude is bounded by unity. The figure illustrates, also, the bound implied by analyticity/crossing on the expanding b-amplitude.

Saturating s-channel unitarity and analyticity/crossing bounds, we get the Froissart-Martin bound, $\sigma_{tot} \leq C ln^2(s/s_0)$**.** $s_0 = 1GeV^2$, $C = \pi/2m_\pi^2 \simeq 30mb.$

C is too large to be of use. At W=100 TeV the bound is $\simeq 1.6 \cdot 10^4 mb$.

The Froissart-Martin $ln^2 s$ **behavior relates to the bound, NOT to the total cross section which can grow more rapidly than** $ln^2 s$ **as long as it is below the unitarity bound.**

In t-space, σ_{tot} **is proportional to a single point,** $d\sigma_{el}/dt(t=0)$**.**

σ_{tot} **in b-space is obtained from a** b^2 **integration of** $2(1-e^{-\frac{1}{2}\Omega(s,b)})$.

Consequently, saturation in b-space is a differential feature attained first at b=0 and then expands slowly.

In a non GW single dimension representation, $\sigma_{el} \leq \frac{1}{2}\sigma_{tot}$ **and** $\sigma_{inel} \geq \frac{1}{2}\sigma_{tot}$.

At saturation, regardless at what energy it is attained, $\sigma_{el} = \sigma_{inel} = \frac{1}{2}\sigma_{tot}$.

Two intriguing questions remain opened for investigation:

1) Are the bounds, just presented, significantly different in a GW model?

2) Is black disc saturation attainable below the Planck scale?

UNITARITY BOUNDS IN A MULTI CHANNEL GW MODEL

The elastic, SD and DD amplitudes in GLM GW model are:

$$a_{el}(s,b) = i\{\alpha^4 A_{1,1} + 2\alpha^2\beta^2 A_{1,2} + \beta^4 A_{2,2}\},$$

$$a_{sd}(s,b) = i\alpha\beta\{-\alpha^2 A_{1,1} + (\alpha^2 - \beta^2) A_{1,2} + \beta^2 A_{2,2}\},$$

$$a_{dd}(s,b) = i\alpha^2\beta^2\{A_{1,1} - 2A_{1,2} + A_{2,2}\}.$$

GW mixing parameters $\alpha^2 + \beta^2 = 1$. **GW** $A_{i,k}$ **amplitudes are bounded by unity.**

$a_{el}(s,b)$ **reaches this bound, at a given (s,b), when and only when,**

$A_{1,1}(s,b) = A_{1,2}(s,b) = A_{2,2}(s,b) = 1$**, independent of the GW mixing parameter.**

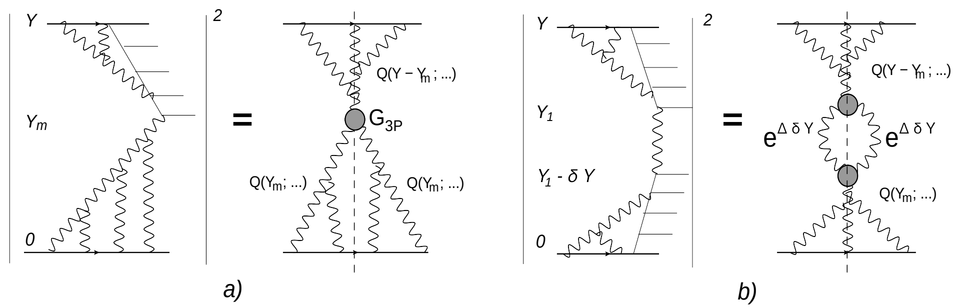
Single channel models neglect the GW mechanism. In GW multi channel models, we distinguish between GW and non GW diffraction.

In such models, we obtain the Pumplin bound: $(\sigma_{el} + \sigma^{GW}_{diff}) \leq \frac{1}{2}\sigma_{tot}$**.**

Consequently, $\sigma_{el} \leq \frac{1}{2}\sigma_{tot} - \sigma^{GW}_{diff}$, $\sigma_{inel} \geq \frac{1}{2}\sigma_{tot} + \sigma^{GW}_{diff}$, $\sigma^{GW}_{diff} = \sigma^{GW}_{sd} + \sigma^{GW}_{dd}$**.**

When, $a_{el}(s,b) = 1$, $a_{sd}(s,b) = a_{dd}(s,b) = 0$**.**

Accordingly, at saturation $\sigma^{GW}_{diff} = 0$ **and** $\sigma_{el} = \sigma_{inel} = \frac{1}{2}\sigma_{tot}$**.**



HIGH MASS DIFFRACTION

Mueller(1971) applied 3 body unitarity to equate the cross section of $a + b \to M^2 + b$ to the triple Regge diagram $a + b + \bar{b} \to a + b + \bar{b}$.

The signature of this presentation is a triple vertex with a leading $3I\!\!P$ term.

The $3I\!\!P$ approximation is valid when $\frac{m_p^2}{M^2} << 1$ and $\frac{M^2}{s} << 1$.

The leading energy/mass dependences are $\frac{d\sigma^{3I\!\!P}}{dt\,dM^2} \propto s^{2\Delta_{I\!\!P}}\left(\frac{1}{M^2}\right)^{1+\Delta_{I\!\!P}}$.

Mueller's $3I\!\!P$ approximations for the non GW "high mass" SD and DD are the lowest order of multi $I\!\!P$ interactions. This feature is compatible with t-channel unitarity. The figure shows the SD and DD GLM diagrams. Multi $I\!\!P$ vertexes are reduced to chains of $3I\!\!P$ vertexes. Other $I\!\!P$ models use different procedures.

The experimental signature of a $I\!P$ exchanged reaction is a large rapidity gap (LRG), devoid of hadrons in the $\eta - \phi$ Lego plot.

S^2, the LRG survival probability, is a unitarity induced suppression factor of non GW diffraction, soft or hard: $S^2 = \sigma_{diff}(screened)/\sigma_{diff}(non\, screened)$.

Denote the gap survival factor initiated by s-channel eikonalization S^2_{eik}, and the one initiated by t-channel multi $I\!P$ interactions S^2_{enh}. S^2 is obtained from a convolution of S^2_{eik} and S^2_{enh}, approximated by $S^2 \simeq S^2_{eik} \cdot S^2_{enh}$.

Assume no screenings, $\Delta_{I\!P}$ can be obtained from either the s dependence of $\sigma_{tot}, \sigma_{el}$, σ_{sd} and σ_{dd}, or from the diffractive non GW "high mass" distribution.

s and t screenings initiate a decrease in the effective output value of $\Delta_{I\!P}$ with growing s. It is denoted $\Delta^{eff}_{I\!P}$.

The M^2 dependence of S^2 in non GW diffraction is much more moderate than S^2 dependence on s. Hence, the fitted value of $\Delta^{eff}_{I\!P}$ from an "high mass" distribution is close to its input value.

The analysis of soft diffraction, is hindered by the lack of uniform experimental and theoretical definitions of its signatures and bounds.

Commonly, the low mass bound of non GW "high mass" diffraction is 2-4.5 GeV and its high bound is 0.05s. It is not always consistent with a LRG signature.

Following Kaidalov, it is common to bound the GW diffraction from above by the lower bound of the non GW diffraction. The resulting "low mass" cross section is sensitive to the arbitrary choice of its upper bound.

In GLM, GW and non GW diffraction have the same upper bound at 0.05s. Consequently, GLM diffraction is predominantly GW. Kaidalov diffraction is predominantly non GW.

The complexity of $I\!\!P$ diffractive diagrams leads to different summing procedures. GLM and KMR multi-$I\!\!P$ calculation validity is $\simeq$100TeV.

Thus, I have omitted the "high mass" diffraction from the forthcoming analysis.

IS SATURATION ATTAINABLE?

As the available data base is well below saturation, if it exists, I shall try to offer a reasonable guess of its attainability based on an investigation of relevant features in the outputs of GLM and KMR, which are $I\!P$ models, and BH, which is single channeled based on a logarithmic parametrization.

As it stands, model predictions at exceedingly high energies are based on the analysis of ISR-LHC7 data.

Note, though, that models which have compatible outputs at ISR-LHC7, may have diverging predictions at higher energies.

	$7\,TeV$			$14\,TeV$			$57\,TeV$		$100\,TeV$			$1.2 \cdot 10^{16}$ TeV	
	GLM	KMR	BH	GLM	KMR	BH	GLM	BH	GLM	KMR	BH	GLM	BH
σ_{tot}	94.2	97.4	95.4	104.0	107.5	107.3	125.0	134.8	134.0	138.8	147.1	393	2067
σ_{inel}	71.3	73.6	69.0	77.9	80.3	76.3	92.2	92.9	98.5	100.7	100.0	279	1131
$\frac{\sigma_{inel}}{\sigma_{tot}}$	0.76	0.76	0.72	0.75	0.75	0.71	0.74	0.70	0.74	0.73	0.68	0.71	0.55

A) Total and Inelastic Cross Sections:

In the Table above, I am comparing σ_{tot} and σ_{inel} outputs of GLM, KMR and BH in the energy range of 7 TeV up to Planck scale.

As can be easily seen, the 3 models have compatible outputs up to 100 TeV.

At Planck scale, the results of GLM and BH are different by factors of 3.5-5.

The critical observation is that $\frac{\sigma_{inel}}{\sigma_{tot}} > 0.5$, up to the Planck scale.

i.e. IN THE MODELS CONSIDERED, SATURATION HAS NOT BEEN ATTAINED.

TeV	$1.8 \to 7.0$	$7.0 \to 14.0$	$14.0 \to 57.0$	$57.0 \to 100.0$	$14.0 \to 100.0$	$100.0 \to 1.22 \cdot 10^{16}$
$\Delta_{eff}(GLM)$	0.081	0.071	0.065	0.062	0.064	0.017
$\Delta_{eff}(KMR)$	0.076	0.071			0.065	
$\Delta_{eff}(BH)$	0.088	0.085	0.081	0.078	0.080	0.041

B) σ_{tot} Dependence on Energy:

When compared with input $\Delta_{I\!P}$, $\Delta_{I\!P}^{eff}$ serves as a simple measure of the rate of cross section growth. The screenings of $\sigma_{tot}, \sigma_{el}, \sigma_{sd}$ and σ_{dd} are not identical. Hence, their $\Delta_{I\!P}^{eff}$ values are different.

The cleanest determination of $\Delta_{I\!P}^{eff}$ is from the energy dependence of σ_{tot}.

All other options require also a determination of $\alpha'_{I\!P}$.

The table above compares $\Delta_{I\!P}^{eff}$ values obtained by GLM, KMR and BH.

The continuous reduction of $\Delta_{I\!P}^{eff}$ in all 3 models is a consequence of eikonal screening disregarding the different inputs.

THE VALUES OF $\Delta_{I\!P}^{eff}$ ARE INCOMPATIBLE WITH A $ln^2\,s$ DEPENDENCE UP TO THE PLANCK SCALE.

	$7\,TeV$		$14\,TeV$		$57\,TeV$	$100\,TeV$		$1.2\cdot 10^{16}$ TeV
	GLM	KMR	GLM	KMR	GLM	GLM	KMR	GLM
σ_{tot}	94.2	97.4	104.0	107.5	125.0	134.0	138.8	393
σ_{el}	22.9	23.8	26.1	27.2	32.8	35.5	38.1	114
σ_{sd}^{GW}	10.5	7.3	11.2	8.1	12.8	13.4	10.4	35.4
σ_{dd}^{GW}	6.0	0.9	6.6	1.1	7.7	8.1	1.6	0.7
$\frac{\sigma_{el}+\sigma_{dif}^{GW}}{\sigma_{tot}}$	0.42	0.33	0.42	0.34	0.43	0.43	0.36	0.38

C) GW Diffractive Cross Sections at Exceedingly High Energies:

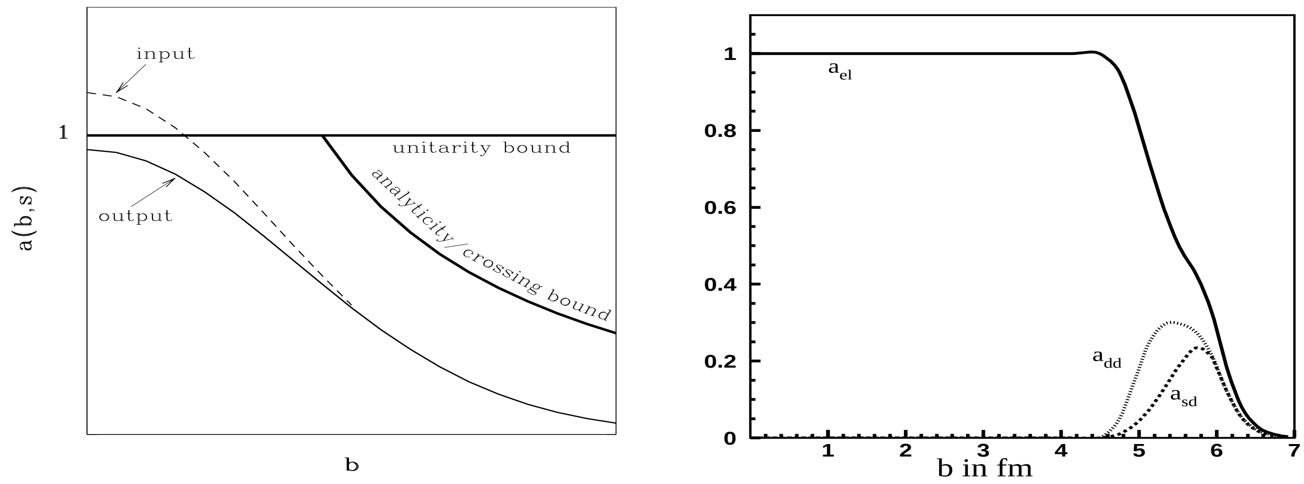
Dynamic GW features in a $I\!\!P$ model translate into 2 essential relations:

1) The Pumplin bound: $(\sigma_{el} + \sigma_{diff}^{GW}) \leq \frac{1}{2}\sigma_{tot}$.

2) When, $a_{el}(s,b) = 1, \quad a_{sd}(s,b) = a_{dd}(s,b) = 0$.

As seen, GLM GW σ_{sd} and σ_{dd} are larger than KMR. KMR σ_{tot} and σ_{el} outputs are larger than GLM.

Both features are inbuilt depending on leverages given by the diffractive mass bounds and number of eikonal channels. In our context, we observe that $\frac{\sigma_{el}+\sigma_{dif}^{GW}}{\sigma_{tot}} < 0.5$ up to the Planck scale. i.e. NON SATURATION IS SUPPORTED!



I wish to point out a pathology induced by the weakness of the analiticity/crossing bound on the expansion of the elastic b-space amplitude. This is shown in the figure on the left.

The output elastic b-space amplitude, has a Gaussian like high b tail, which is smaller than 1. This tail is aside or below the Froissart-Martin R_{el} bound in b-space. This enables high b diffraction to survive.

Note that, the analyticity/crossing R_{el} bound is not effective even at the Planck scale. This is shown in the right hand figure.