Two dijet production at LHC and Tevatron in pQCD.

B.Blok (Technion, Haifa)

(based on joint work with Yu. Dokshitzer, L. Frankfurt, M. Strikman)

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Introduction

The subject of this talk: the two dijet production
at LHC and Tevatron.
The conventional jet production at colliders is two to two production

$$\frac{d\sigma^{(2\to 2)}}{d\hat{t}} \propto \frac{\alpha_s^2}{Q^4}.$$

Here the cross sections are not sensitive to nucleon structure-it comes Through D(x,Q^2)-the nucleon structure functions.

.

The most interesting property-they can be seen, i.e. there exists kinematics Where they are dominant. Indeed, they are not the leading twist process.

The 2 to 4 processes give a contribution to cross section

$$\frac{d\sigma^{(2\to 4)}}{d\hat{t}_1 d\hat{t}_2} \propto \frac{\alpha_s^4}{Q^6}$$

On the other hand 4 to 4 give the contribution

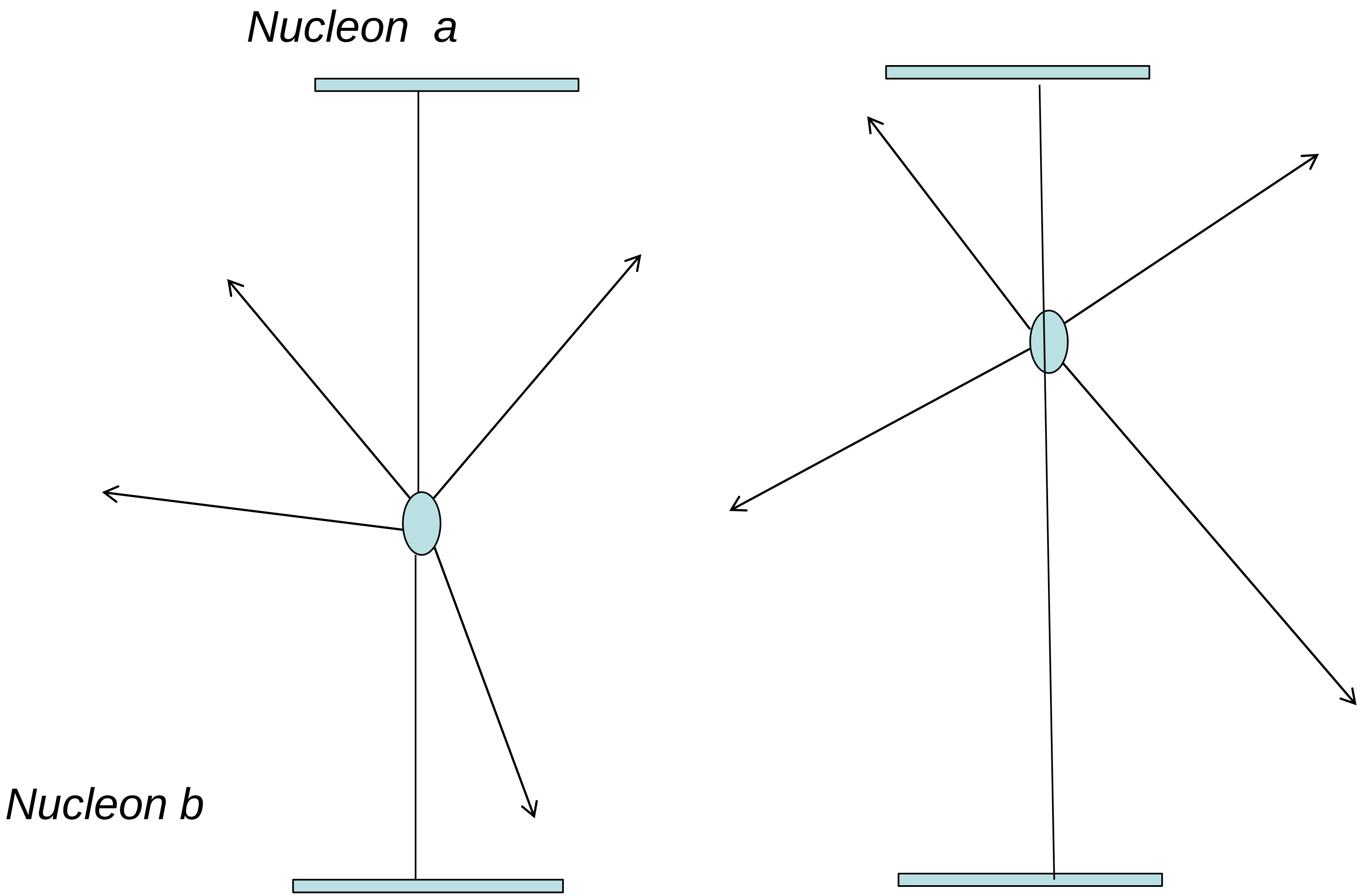
$$\frac{d\sigma^{(4\to 4)}}{d\hat{t}_1 d\hat{t}_2} \propto R^{-2} \cdot \left(\frac{\alpha_s^2}{Q^4}\right)^2 \propto \frac{\alpha_s^4}{R^2\, Q^8};$$

i.e. they can be seen as a higher twist process.

the scale R is given by

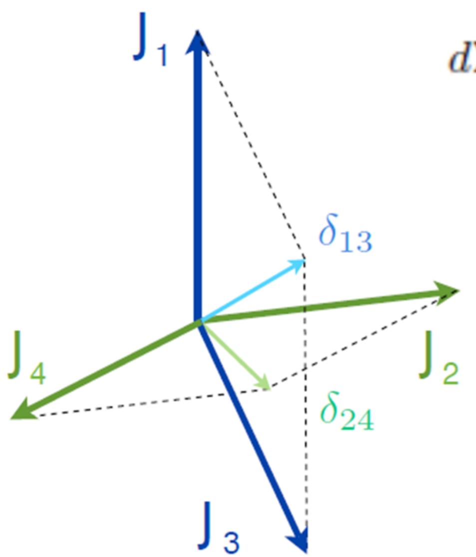
$R^2 = 1/\left\langle \Delta^2 \right\rangle$ the characteristic distance between the two partons in the hadron wave function.

$$
\begin{aligned}
d\sigma^{(4\to4)} &\propto \frac{\alpha_{\rm s}^2}{\delta_{13}^2\,\delta_{24}^2}\, d^2 j_{3\perp} d^2 j_{4\perp}\cdot d\Sigma, \\
&\qquad \delta_{13}^2 \ll Q^2, \quad \delta_{24}^2 \ll Q^2; \\
d\sigma^{(3\to4)} &\propto \frac{\alpha_{\rm s}^2}{\delta'^2\,\delta^2}\, d^2 j_{3\perp} d^2 j_{4\perp}\cdot d\Sigma, \\
&\qquad \delta'^2 \ll \delta^2 \ll Q^2,\ \delta^2 = \delta_{13}^2 \simeq \delta_{24}^2.
\end{aligned}
$$


Nucleon a
Nucleon b

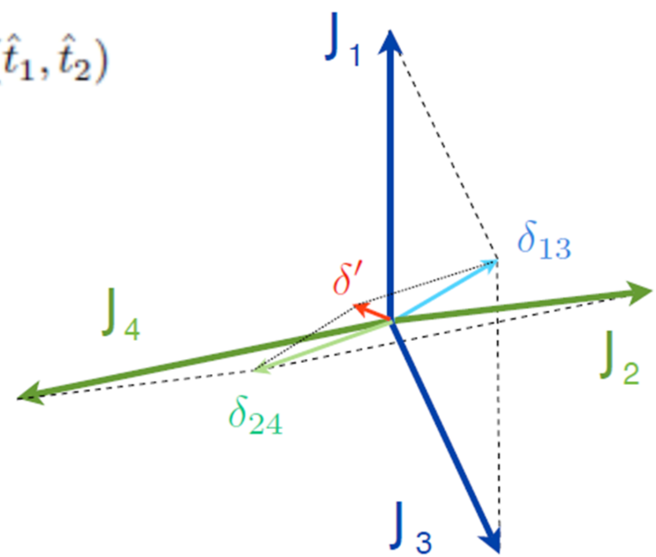
back-to-back kinematics

$\delta_{13}^2, \delta_{24}^2 \ll J_{i\perp}^2$



$d\Sigma(\hat{t}_1, \hat{t}_2)$

$\delta'^2 \ll \delta_{13}^2 \simeq \delta_{24}^2 \ll J_{i\perp}^2$



$$d\sigma^{(4\to4)} \;\propto\; \frac{\alpha_s^2}{\delta_{13}^2\,\delta_{24}^2}\, d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

$$d\sigma^{(3\to4)} \;\propto\; \frac{\alpha_s^2}{\delta'^2\,\delta^2}\, d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

The four jet cross-section in the parton model.

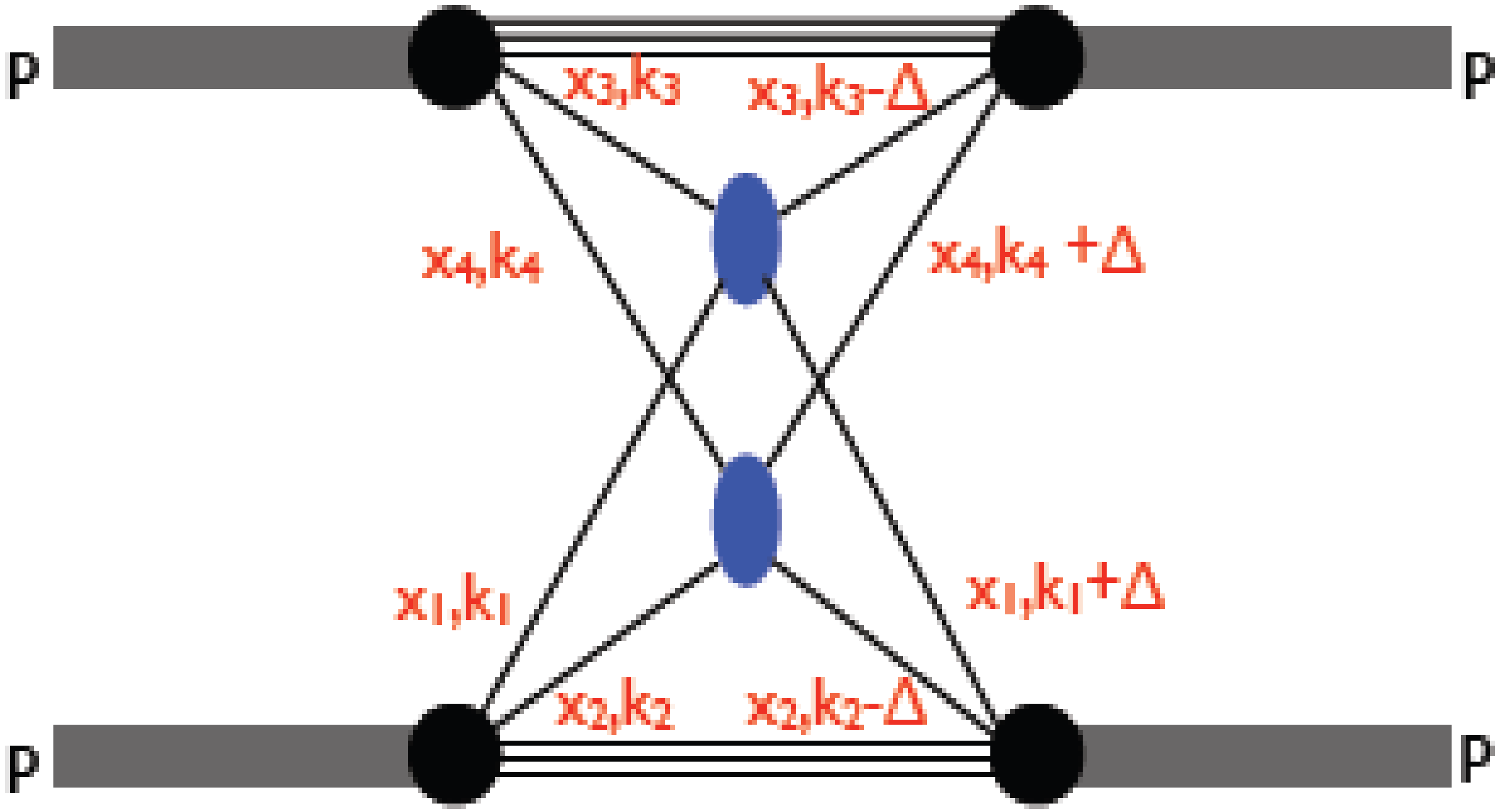
The four jet cross-section can be directly calculated in momentum space and is given by the formula:

$$\sigma_4(x_1,x_2,x_3,x_4) = \int \frac{d^2\vec{\Delta}}{(2\pi)^2} D_a(x_1,x_2,p_1^2,p_2^2,\vec{\Delta}) \times D_b(x_3,x_4,p_1^2,p_2^2,-\vec{\Delta}) \times \frac{d\sigma^{13}}{d\hat{t}_1}\frac{d\sigma^{24}}{d\hat{t}_2} d\hat{t}_1 d\hat{t}_2. \quad (2)$$

Experimentalists often denote:

$$\sigma_4 = \sigma_1\sigma_2/\pi R_{\text{int}}^2,$$

δ



p
p
p
p
x3,k3
x3,k3-Δ
x4,k4
x4,k4 +Δ
x1,k1
x1,k1+Δ
x2,k2
x2,k2-Δ

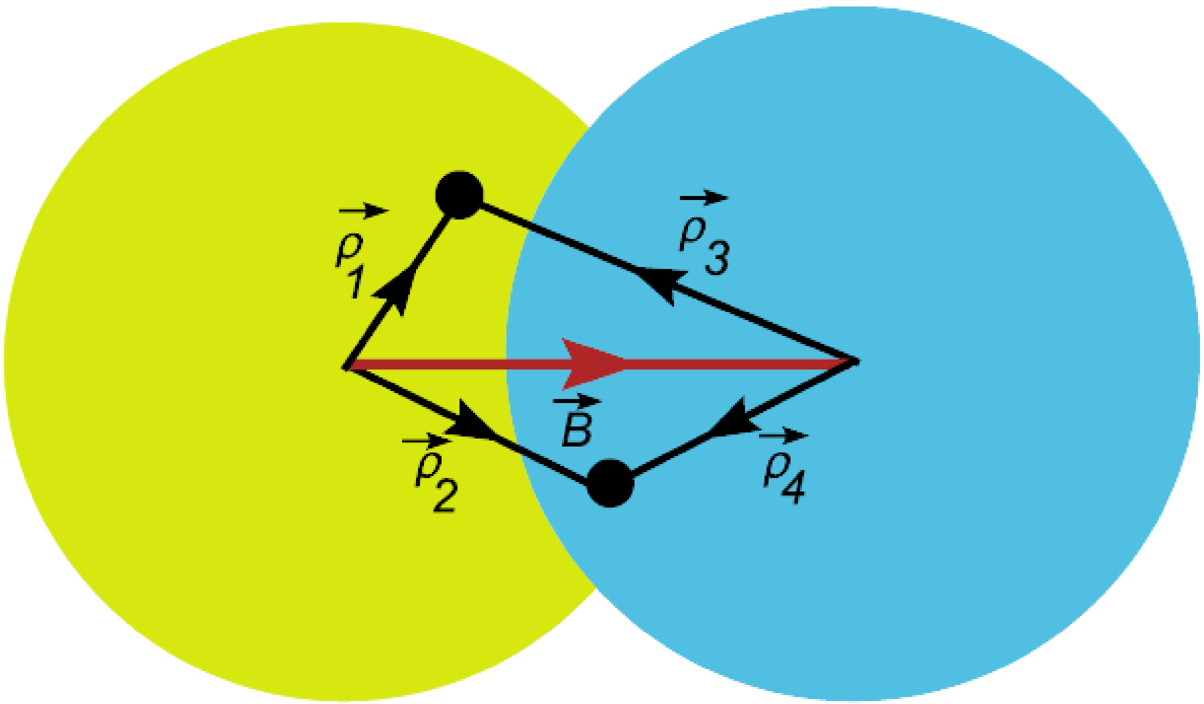


FIG. 2 (color online). Geometry of two hard collisions in impact parameter picture.

It follows from the discussion above that the area can be written explicitly in terms of these new two particle GPDs as

$$\frac{1}{\pi R_{\rm int}^2} = \int \frac{d^2\vec{\Delta}}{(2\pi)^2} \frac{D(x_1, x_2, -\vec{\Delta})D(x_3, x_4, \vec{\Delta})}{D(x_1)D(x_2)D(x_3)D(x_4)},$$

This formula is valid for inclusive dijet production. When the momentum fraction are different, the exclusive production DDT formula can be easily obtained. This formula expresses the interaction area in the model independent way as the single integral over the transverse momenta.

The new GPDs can be explicitly expressed through the light cone wave functions of the hadron as

$$
\begin{aligned}
D\ \ (x_1, x_2, p_1^2, p_2^2, \overrightarrow{\Delta}) &= \sum_{n=3}^{\infty} \int \frac{d^2 k_1}{(2\pi)^2} \frac{d^2 k_2}{(2\pi)^2} \theta(p_1^2 - k_1^2) \\
&\times\ \ \theta(p_2^2 - k_2^2) \int \prod_{i \neq 1,2} \frac{d^2 k_i}{(2\pi)^2} \int_0^1 \prod_{i \neq 1,2} dx_i \\
&\times\ \ \psi_n(x_1, \vec{k}_1, x_2, \vec{k}_2, .., \vec{k}_i, x_i..) \\
&\times\ \ \psi_n^+(x_1, \overrightarrow{k_1} + \overrightarrow{\Delta}, x_2, \overrightarrow{k_2} - \overrightarrow{\Delta}, x_3, \vec{k}_3, ...) \\
&\times\ \ (2\pi)^3 \delta(\sum_{i=1}^{i=n} x_i - 1) \delta(\sum_{i=1}^{i=n} \vec{k}_i).
\end{aligned}
$$

Here psi are the light cone wave functions of the nucleon in the initial and final states.

Note that very similar distributions arise in DIS scattering where they were denoted as quasipartonic operators
(Bukhvostov,Frolov, Lipatov and Kuraev 1985). Thus we can use their classification. There however important differences: our distributions are diagonal in longitudinal momenta and vector
Is transverse (and no integration over
Is implied)

How to use this formula? (naïve way).

The approximation of independent particles.

Suppose the multiparton wave faction factorise, i.e. we neglect possible interparton correlations and recoil effects. Then it's straightforward to see that the two particle GPDs factorise and acquire a form: •

$$D(x_1, x_2, p_1^2, p_2^2, \vec{\Delta}) = G(x_1, p_1^2, \vec{\Delta})G(x_2, p_2^2, \vec{\Delta}),$$

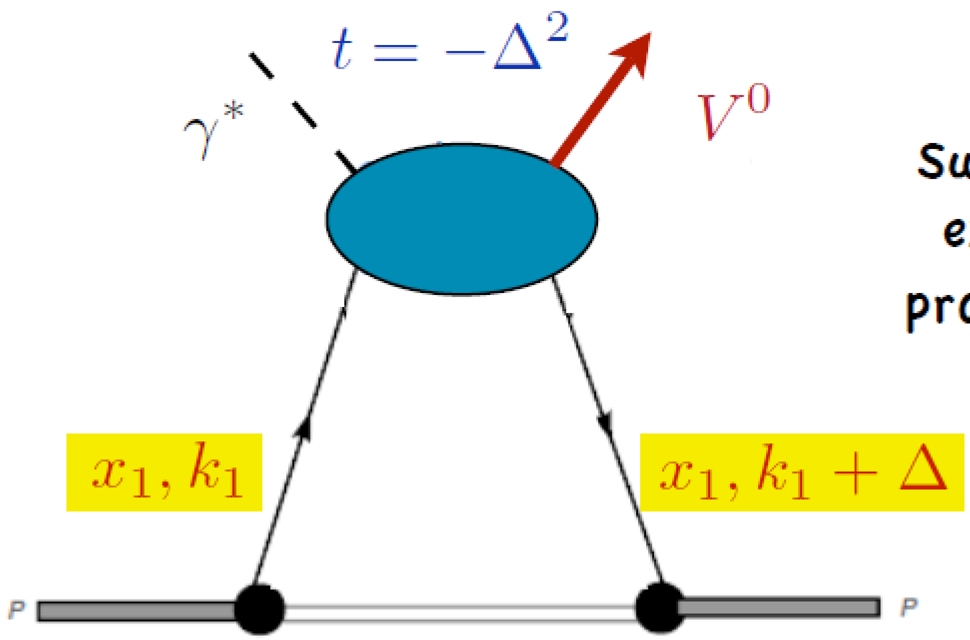
The one-particle GPD-s G are conventionally written in the dipole form:

$$G_N(x, Q^2, \vec{\Delta}) = G_N(x, Q^2)F_{2g}(\Delta)$$

G - the usual 1-parton distribution (determining DIS structure functions)

F - the two-gluon form factor of the nucleon

the dipole fit : $F_{2g}(\Delta) \simeq \dfrac{1}{(1+\Delta^2/m_g^2)^2}$ $m_g^2(x \sim 0.03, Q^2 \sim 3\mathrm{GeV}^2) \simeq 1.1\mathrm{GeV}^2$



G P D

Such an amplitude describes exclusive photo-(/electro-) production of vector mesons at HERA !

$$\frac{1}{\pi R_{\text{int}}^2} = \int \frac{d^2\Delta}{(2\pi)^2} F_{2g}^4(\Delta) = \frac{m_g^2}{28\pi}.$$

$$R_{\text{int}}^2 = 7/2r_g^2, \qquad r_g^2/4 = dF_{2g}(t)/dt_{t=0}.$$

Let us note that this result coincides with the one obtained in a geometric picture (Frankfurt,Strikman and Weiss 2003)
However the latter computation involved a complicated 6 dimensional integral that potentially could lead to large numerical uncertainties

The dependence of r^2_g on Q^2 and x is given by the approximate formula that takes into account the DGLAP evolution:

$$\langle\rho^2\rangle(x,Q^2) = \langle\rho^2\rangle(x,Q_0^2)\left(1 + A\ln\frac{Q^2}{Q_0^2}\right)^{-a},$$

$$\langle\rho^2\rangle = \frac{8}{m_g^2}\,.$$

where

$$Q_0^2 = 3\,\mathrm{GeV}^2,\quad A = 1.5,\quad a = 0.0090\ln\frac{1}{x}\,.$$

The similar analysis for quark sea leads to slightly bigger transverse area (Strikman and Weiss 2009). Recoil may be important for large x_i but also leads to smaller total cross section, i.e. to larger R_{int}

Then we see the problem: the approximation of independent particles leads to the cross section two times smaller than the experimental one (Frankfurt,Strikman and Weiss 2004), *The experimental result is 15 mb, while the use of the electromagnetic radius of the nucleon leads to this area being 60mb while we obtain in independent particle approximation 34mb*

Even more naïve way-take $\sigma_{eff} = 1/\pi R2$

(most MC generators do)

Perturbative QCD and differential cross sections

Two basic ideas (relative to conventional one dijet processes-2 to2 in our notations):

1. Double collinear enhancement in total cross sections-i.e. double pole enhancement in differential two dijet cross sections.

2. new topologies-in addition to conventional pQCD bremsstralung-parton/ladder splitting .

a) 4 to 4

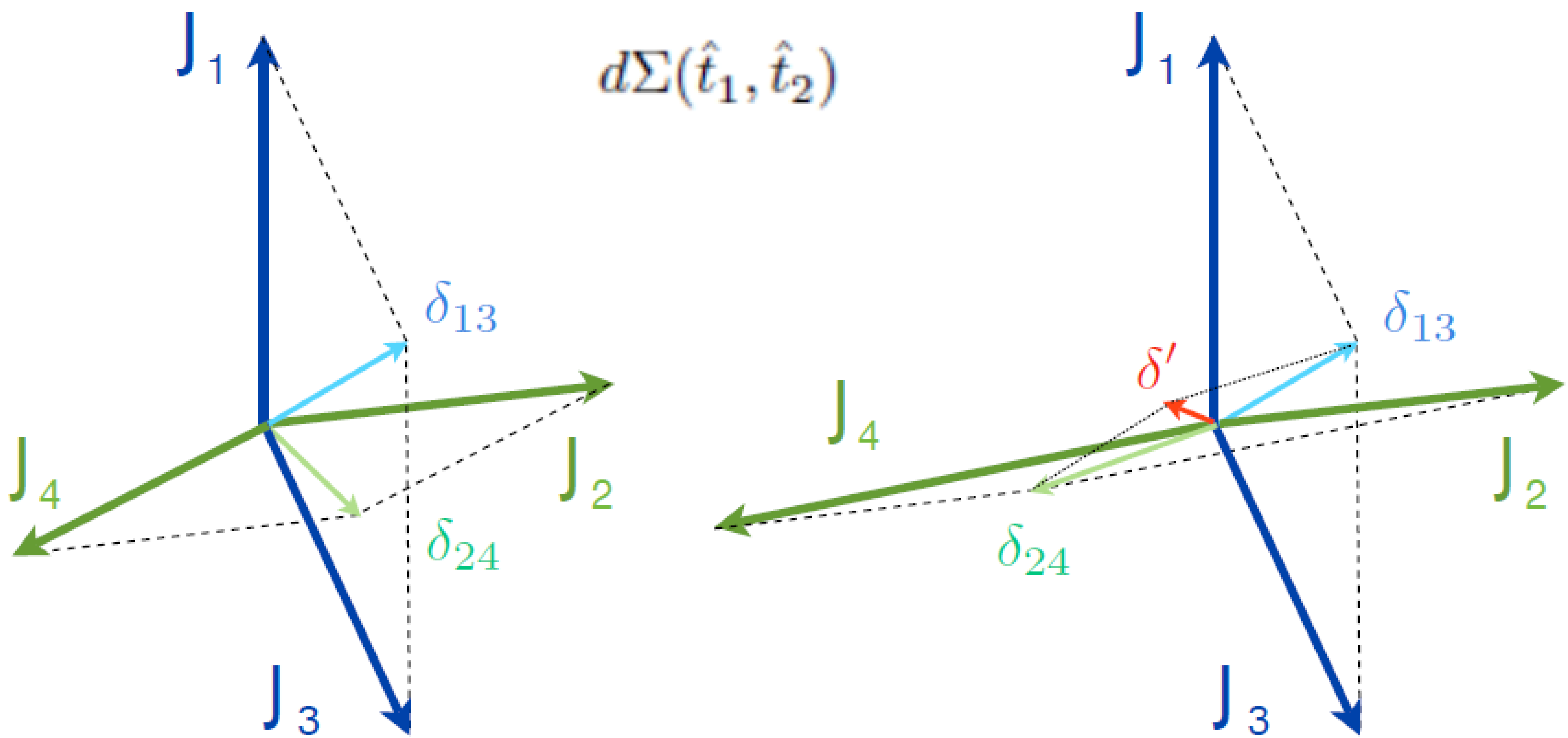
b) 3 to 4

But no 2 to 4

back-to-back kinematics

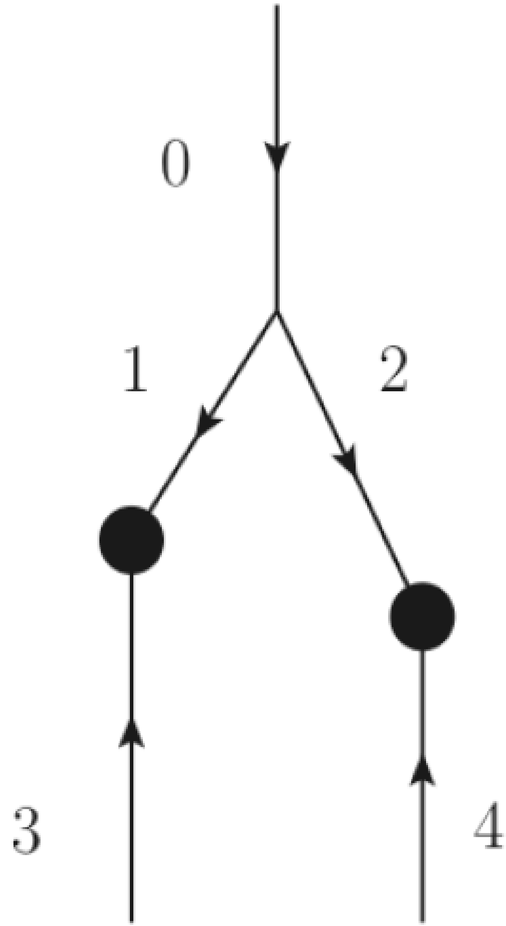
$$\delta_{13}^2, \delta_{24}^2 \ll J_{i\perp}^2$$

$$\delta'^2 \ll \delta_{13}^2 \simeq \delta_{24}^2 \ll J_{i\perp}^2$$



$$d\sigma^{(4\to 4)} \propto \frac{\alpha_s^2}{\delta_{13}^2\,\delta_{24}^2}\, d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

$$d\sigma^{(3\to 4)} \propto \frac{\alpha_s^2}{\delta'^2\,\delta^2}\, d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$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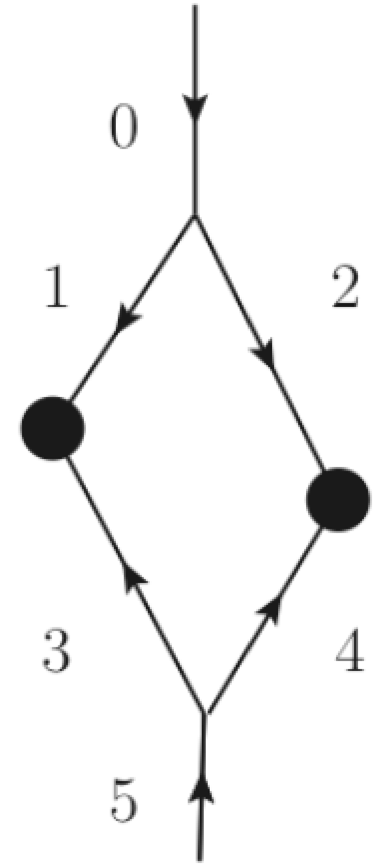
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proof

$$k_1 = (x_1 - \alpha)p_a + \beta p_b + k'_\perp,\; k_3 = (x_3 - \beta)p_b + \alpha p_a - k_\perp;$$
$$k_2 = (x_2 + \alpha)p_a - \beta p_b - k'_\perp,\; k_4 = (x_4 + \beta)p_b - \alpha p_a + k_\perp;$$
$$\vec{k}'_\perp - \vec{k}_\perp = \vec{\delta}_{12} = -\vec{\delta}_{34} \quad (\delta' \equiv 0);$$
$$k_0 \simeq (x_1 + x_2)p_a,\; k_5 \simeq (x_3 + x_4)p_b.$$

$$\frac{i}{s} \int \frac{d\beta}{2\pi i} \frac{s}{(\beta x_1 s - k_\perp^2 + i\epsilon)(\beta x_2 s + k_\perp^2 - i\epsilon)}$$
$$\times \int \frac{d\alpha}{2\pi i} \frac{s}{(\alpha x_3 s - k'^2_\perp + i\epsilon)(\alpha x_4 s + k'^2_\perp - i\epsilon)}$$
$$= \frac{i}{(x_1 + x_2)(x_3 + x_4)s} \frac{1}{k_\perp^2 \, k'^2_\perp}.$$

$$\int \frac{d^2 k_\perp}{(2\pi)^2} \frac{V}{\vec{k}_\perp^2 (\vec{k}_\perp + \vec{\delta})^2}$$

So the final result- in LLA the 2 to4 does not contribute to back to back kinematics. This can be seen directly from parton model calculation-this diagram gives only one delta function except of two for double enhancement.So 2 to 4 is out.-leads to hedgehogs.

Three to Four

There are two types of three to four contributions: The first-when there is radiation after the split, the second-when there is not. This leads to two different types of singularities: $1/\delta'^2$ $1/\delta_{13}^2$

$$\delta' = \delta_{13} + \delta_{24} \qquad \delta_{13}^2 \ll \delta_{24}^2 \simeq \delta'^2.$$

and vice versa –we call this type of singularity short split, and singularities-$1/\delta_{13}^2 1/\delta_{24}^2$

that we call a long split. The first type of pole/log terms is present only in three to four

terms, while the second is the same as in 4 to 4

terms.

$$
\pi^2 \frac{d\sigma_1^{(3\to 4)}}{d^2\delta_{13}\, d^2\delta_{24}} = \frac{d\sigma_{\rm part}}{d\hat{t}_1\, d\hat{t}_2} \cdot \frac{\partial}{\partial \delta_{13}^2} \frac{\partial}{\partial \delta_{24}^2} \Big\{ {}_{[1]}D_a^{1,2}(x_1, x_2; \delta_{13}^2, \delta_{24}^2) \cdot {}_{[2]}D_b^{3,4}(x_3, x_4; \delta_{13}^2, \delta_{24}^2)
$$

$$
\times\; S_1\left(Q^2, \delta_{13}^2\right) S_3\left(Q^2, \delta_{13}^2\right) \cdot S_2\left(Q^2, \delta_{24}^2\right) S_4\left(Q^2, \delta_{24}^2\right) \Big\}.
$$

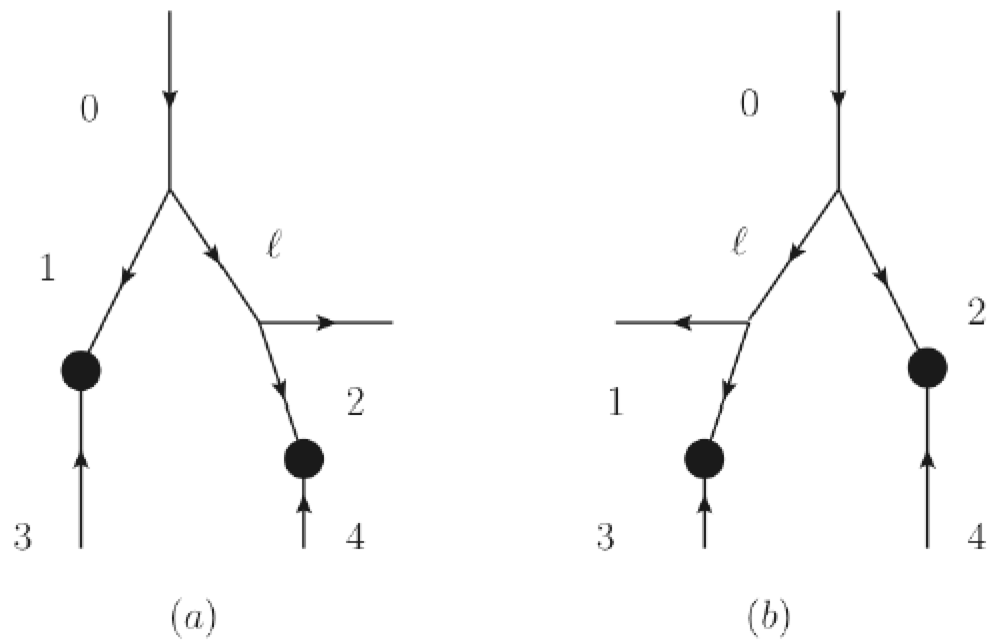

FIG. 3: Three-to-four amplitudes with extra emission from inside the splitting fork

$$\frac{d\sigma}{d^2\delta_{13}d^2\delta_{24}} \propto \frac{\alpha_s}{\delta^2}\delta(\vec{\delta}_{13}+\vec{\delta}_{24}), \quad \delta^2 \equiv \delta_{13}^2 = \delta_{24}^2.$$

$$\frac{d\sigma}{d^2\delta_{13}d^2\delta_{24}} \propto \frac{\alpha_s^2}{\delta^2\,\delta'^2}, \quad \delta'^2 \ll \delta^2 \equiv \delta_{13}^2 \simeq \delta_{24}^2.$$

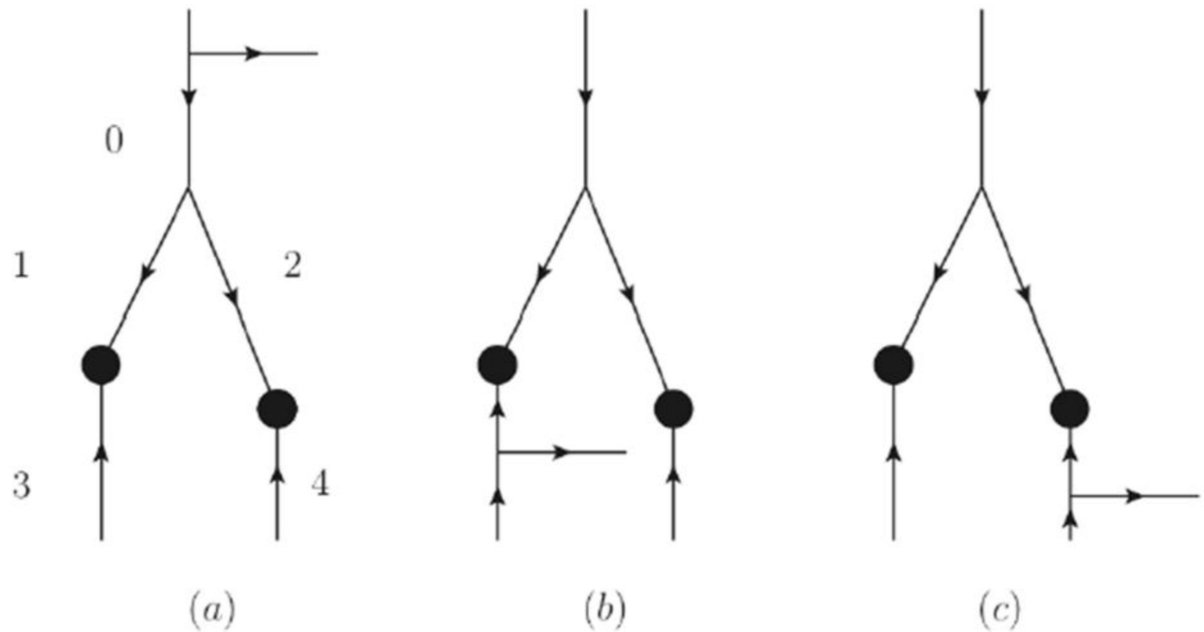

FIG. 4: $3 \to 4$ with real parton emission off the external lines

The short splits however lead to very different expression:

$$\frac{\pi^2\, d\sigma_2^{(3\to4)}}{d^2\delta_{13}\, d^2\delta_{24}} = \frac{d\sigma_{\rm part}}{d\hat{t}_1\, d\hat{t}_2}\cdot\frac{\alpha_{\rm s}(\delta^2)}{2\pi\,\delta^2}\sum_c P_c^{1,2}\left(\frac{x_1}{x_1+x_2}\right) S_1(Q^2,\delta^2)\, S_2(Q^2,\delta^2)$$

$$\times \frac{\partial}{\partial\delta'^2}\left\{S_c(\delta^2,\delta'^2)\frac{G_a^c(x_1+x_2;\delta'^2,Q_0^2)}{x_1+x_2} S_3(Q^2,\delta'^2)S_4(Q^2,\delta'^2)\times {}_{[2]}D_b^{3,4}(x_3,x_4;\delta'^2,\delta'^2)\right\}$$

- This expression contains GPD only of one of the hadrons. This contribution is not isotropic. It will depend on angle between two disbalances, with maximum, when they are opposite.

- Concequently, in the differential distributions we have 3 terms, corresponding to 4 to 4 and two diffeent types of 3 to 4 (long and short):

$$\pi^2 \frac{d\sigma^{(4\to4)}}{d^2\delta_{13}\, d^2\delta_{24}} = \frac{d\sigma_{\rm part}}{d\hat{t}_1\, d\hat{t}_2} \cdot \frac{\partial}{\partial\delta_{13}^2} \frac{\partial}{\partial\delta_{24}^2} \Big\{ {}_{[2]}D_a^{1,2}(x_1,x_2;\delta_{13}^2,\delta_{24}^2) \times {}_{[2]}D_b^{3,4}(x_3,x_4;\delta_{13}^2,\delta_{24}^2)$$

$$\times\ S_1\left(Q^2,\delta_{13}^2\right) S_3\left(Q^2,\delta_{13}^2\right) \times S_2\left(Q^2,\delta_{24}^2\right) S_4\left(Q^2,\delta_{24}^2\right) \Big\}.$$

$$\pi^2 \frac{d\sigma_1^{(3\to4)}}{d^2\delta_{13}\, d^2\delta_{24}} = \frac{d\sigma_{\rm part}}{d\hat{t}_1\, d\hat{t}_2} \cdot \frac{\partial}{\partial\delta_{13}^2} \frac{\partial}{\partial\delta_{24}^2} \Big\{ {}_{[1]}D_a^{1,2}(x_1,x_2;\delta_{13}^2,\delta_{24}^2) \cdot {}_{[2]}D_b^{3,4}(x_3,x_4;\delta_{13}^2,\delta_{24}^2)$$

$$\times\ S_1\left(Q^2,\delta_{13}^2\right) S_3\left(Q^2,\delta_{13}^2\right) \cdot S_2\left(Q^2,\delta_{24}^2\right) S_4\left(Q^2,\delta_{24}^2\right) \Big\}.$$

- here S are the corresponding Sudakov formfactors . We see that 4 to 4 and long split 3 to 4 are expressed through convolution of 2GPD of two colliding hadrons – the expressions look
- quite similar to DDT formula

Extension of DDT formula for Drell-Yan

$$\frac{d\sigma}{dq^2\, dq_\perp^2} = \frac{d\sigma_{\rm tot}}{dq^2} \times \frac{\partial}{\partial q_\perp^2} \left\{ D_a^q\left(x_1, q_\perp^2\right) D_b^{\bar{q}}\left(x_2, q_\perp^2\right) S_q^2\left(q^2, q_\perp^2\right) \right\}.$$

The total cross sections

$$\frac{d\sigma(x_1,x_2,x_3,x_4)}{d\hat{t}_1\, d\hat{t}_2} = \frac{d\sigma^{13}}{d\hat{t}_1}\,\frac{d\sigma^{24}}{d\hat{t}_2} \times \left\{ \frac{1}{S_4} + \frac{1}{S_3} \right\}.$$

$$S_4^{-1}(x_1,x_2,x_3,x_4;Q^2) = \int \frac{d^2\Delta}{(2\pi)^2} \Big\{ {}_{[2]}D_a(x_1,x_2;Q^2,Q^2;\vec{\Delta})\, {}_{[2]}D_b(x_3,x_4;Q^2,Q^2;-\vec{\Delta})$$

$$+\ {}_{[2]}D_a(x_1,x_2;Q^2,Q^2;\vec{\Delta})\, {}_{[1]}D_b(x_3,x_4;Q^2,Q^2;-\vec{\Delta})\ +\ {}_{[1]}D_a(x_1,x_2;Q^2,Q^2;\vec{\Delta})\, {}_{[2]}D_b(x_3,x_4;Q^2,Q^2;-\vec{\Delta}) \Big\},$$

and

$$S_3^{-1}(x_1,x_2,x_3,x_4;Q^2) = \sum_c \int \frac{d^2\Delta}{(2\pi)^2} P_c^{1,2}\left(\frac{x_1}{x_1+x_2}\right) \int^{Q^2} \frac{d\delta^2}{\delta^2}\,\frac{\alpha_s(\delta^2)}{2\pi} \prod_{i=1}^{4} S_i(Q^2,\delta^2)$$

$$\times\ \frac{G_a^c(x_1+x_2,\delta^2,Q_0^2)}{x_1+x_2}\, {}_{[2]}D_b^{3,4}(x_3,x_4;\delta^2,\delta^2;\vec{\Delta}) \quad + \quad (a \leftrightarrow b; 1,2 \leftrightarrow 3,4)$$

Three to Four enhancement

- The total cross section is given

$$\frac{1}{S} = \int \frac{d^2\Delta}{(2\pi)^2}\Big({}_{[2]}D_a(\Delta)\, {}_{[2]}D_b(\Delta) \; + \; {}_{[2]}D_a(\Delta)\, {}_{[1]}D_b(\Delta) \; + \; {}_{[1]}D_a(\Delta)\, {}_{[2]}D_b(\Delta) \Big),$$

- While the 4 to 4 is

$$\int \frac{d^2\Delta}{(2\pi)^2}\, F_g^2(\Delta^2) \times F_g^2(\Delta^2) = \frac{m_g^2}{7\pi}.$$

The 3 to 4 is then

$$\int \frac{d^2\Delta}{(2\pi)^2}\, {}_{[1]}D(x_1, x_2; \Delta)\, F_g^2(\Delta^2) \simeq {}_{[1]}D\Big|_{\Delta=0} \int \frac{d^2\Delta}{(2\pi)^2} F_g^2(\Delta^2),$$

$$\int \frac{d^2\Delta}{(2\pi)^2} F_g^2(\Delta^2) \; = \; \frac{m_g^2}{3\pi}.$$

- We thus see enhancement by a factor

$$\frac{7}{3} \times 2 \sim 5.$$

- This means that even small corrections to 2GPD can lead to large corrections to cross sections.

2 Parton GPD:perturbative structure.

- Two parton GPD can be also studied in pQCD. We immediately obtain that

$$D_a^{b,c}(x_1,x_2;q_1^2,q_2^2;\vec{\Delta}) = {}_{[2]}D_a^{b,c}(x_1,x_2;q_1^2,q_2^2;\vec{\Delta}) + {}_{[1]}D_a^{b,c}(x_1,x_2;q_1^2,q_2^2;\vec{\Delta}).$$

$$\begin{aligned}
{}_{[2]}D_a^{b,c}(x_1,x_2;q_1^2,q_2^2;\vec{\Delta}) &= S_b(q_1^2,Q_{\min}^2)S_c(q_2^2,Q_{\min}^2)\,{}_{[2]}D_a^{b,c}(x_1,x_2;Q_0^2,Q_0^2;\vec{\Delta}) \\
&+ \sum_{b'}\int_{Q_{\min}^2}^{q_1^2}\frac{dk^2}{k^2}\frac{\alpha_s(k^2)}{2\pi}S_b(q_1^2,k^2)\int\frac{dz}{z}P_{b'}^b(z)\,{}_{[2]}D_a^{b',c}\left(\frac{x_1}{z},x_2;k^2,q_2^2;\vec{\Delta}\right) \\
&+ \sum_{c'}\int_{Q_{\min}^2}^{q_2^2}\frac{dk^2}{k^2}\frac{\alpha_s(k^2)}{2\pi}S_c(q_2^2,k^2)\int\frac{dz}{z}P_{c'}^c(z)\,{}_{[2]}D_a^{b,c'}\left(x_1,\frac{x_2}{z};q_1^2,k^2;\vec{\Delta}\right).
\end{aligned}$$

$$Q_{\min}^2 = \max(Q_0^2,\Delta^2) \simeq Q_0^2+\Delta^2,$$

$$
{}_{[1]}D_a^{b,c}(x_1, x_2; q_1^2, q_2^2; \vec{\Delta}) = \sum_{a',b',c'} \int_{Q_{\min}^2}^{\min\,(q_1^2, q_2^2)} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int \frac{dy}{y^2}\, G_a^{a'}(y; k^2, Q_0^2)
\times \int \frac{dz}{z(1-z)}\, P_{a'}^{b'[c']}(z)\; G_{b'}^{b}\left(\frac{x_1}{zy}; q_1^2, k^2\right) G_{c'}^{c}\left(\frac{x_2}{(1-z)y}; q_2^2, k^2\right).
$$

Numerical results

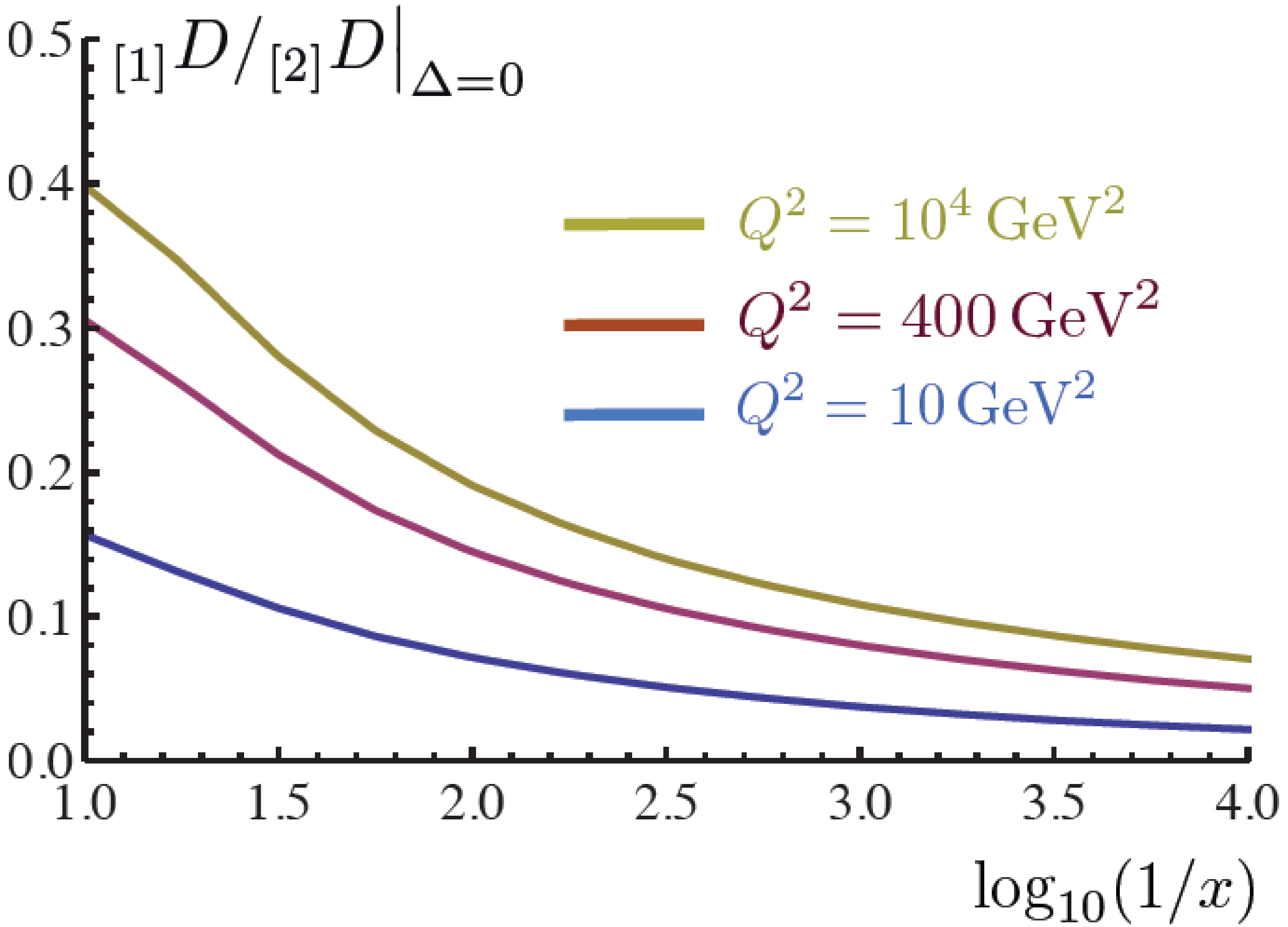

FIG. 2: The ratio of $3\to4$ to $4\to4$ PT-evolved contributions

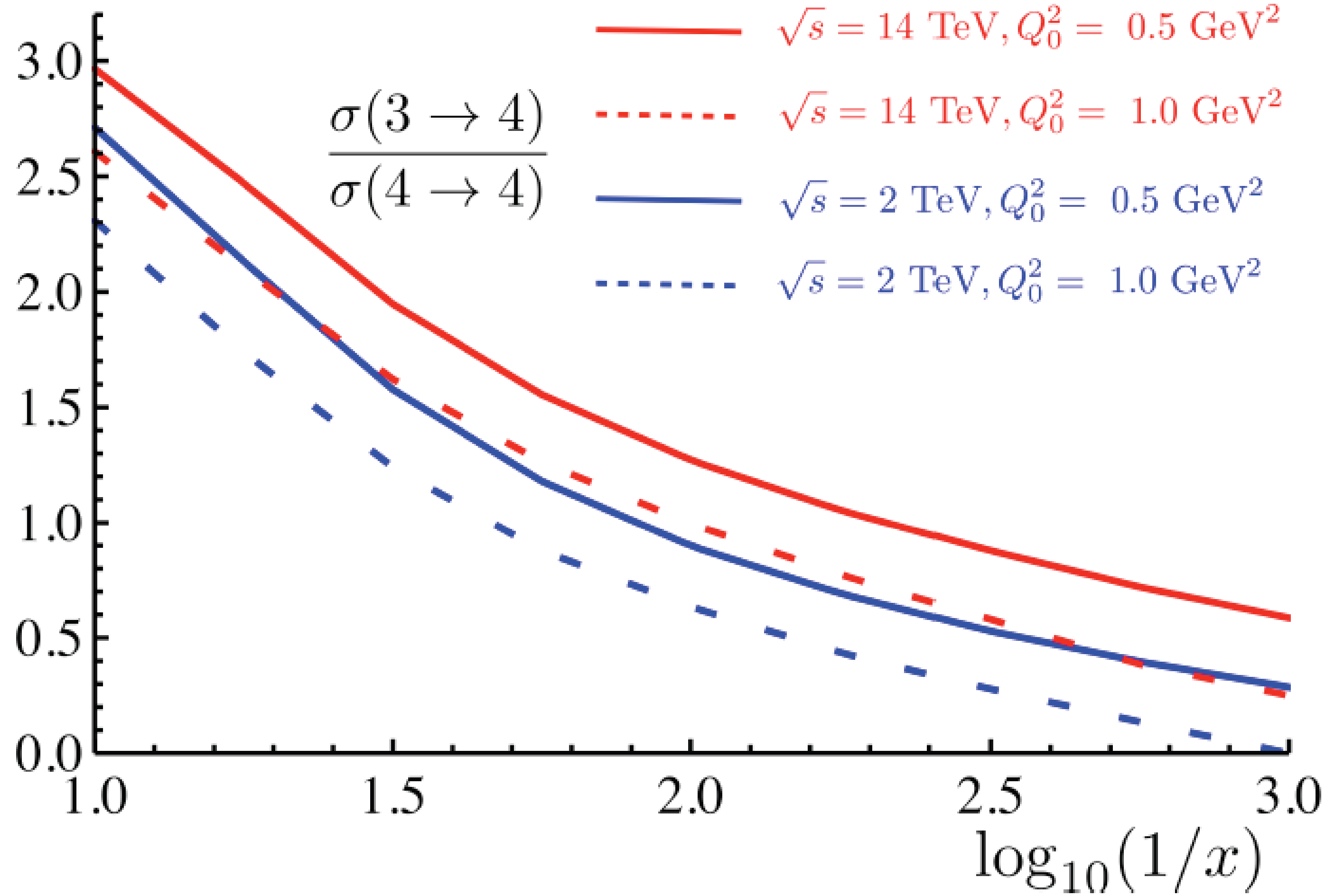

FIG. 3: The ratio of $3\to 4$ to $4\to 4$ contributions to $4g\to 4$ jets cross section ($x_i = x$) for Tevatron and LHC energies for two choices of the starting evolution scale Q_0^2; $Q^2 = x^2 s/4$.

Analytical estimate:

$$\frac{D^{bc}(x_1, x_2; 0)}{G^b(x_1)G^c(x_2)} - 1 \simeq \frac{N_c}{2(n_q C_F + n_g N_c)}.$$

For small x less 0.001- nonperturbative -QCD phenomena

dominates,leading to enhancement of order 2 –the same as for x of order 0.01

Conclusions.

1. We have now pQCD formalism that describes the multiparton .1
interactions in the LLA, and it can be a basis for further improvement.
These pQCD interactions explain observed large 4 jet cross sections .2
observed at x of order 0.01 and larger and smoothly go into small x
region(less than 0.001) dominated by soft pomerons. The enhancement
factor is of order 2 for small x.

3. The 3 to 4 corrections are new effect that distinguishes pQCD from naïve parton model extensions-new type of pQCD dynamics-ladder/parton splitting.
4. It is worthwhile to include them in MC generators. On the other hand we have seen that 2 to 4 is really part of one loop correction to 2 to 4 conventional SP mechanism and must be treated there (including in MC generators).

5. The detailed discussion of nonperturbative QCD effects (important for 5
X less than 0.001 see in arXiv:1206.5594. 6
6. Finally, we see that the QCD factorisation theorem (this work is not yet finished)
work in MPI in a much more complicated way than in conventional 2 to 2 mechanism.

6. The current formalism-can be easily extended to $n>2$ dijet production (important for minijets-low cut offs)

7. Numerical results for differential distributions-shortly.