STRING-GAUGE DUAL (ADS/CFT) DESCRIPTION OF DEEP-INELASTIC SCATTERING

Chung-ITan, Brown University Low-x Workshop, Paphos, Cyprus June 27 - July 1, 2012

Richard Brower, Marko Djurić, Ina Sarcević and Chung-I Tan: *String-Gauge Dual Description of DIS and Small-x*, 10.1007/JHEP 11(2010)051, arXiv:1007.2259

Brower, Polchinski, Strassler, Tan (BPST) The Pomeron and Gauge/String Duality (2006)

HERA vs LHeC region: dots are HI-ZEUS small-x data points



Effective Pomeron Intercept from HERA data:

$$F_2 \simeq C(Q^2) \ x^{-\epsilon_{eff}}$$



Executive Summary:

Gauge/String Duality (AdS/CFT) \longrightarrow 2-GLUONS \simeq GRAVITON

Status Report:

Establishing "Pomeron" in QCD non-perturbatively,

Unification of Soft and Hard Physics in High Energy Collision

 New phenomenology based on "Large Pomeron intercept", e.g., DIS at small-x: (DGLAP vs Pomeron); Central Diffractive Higgs Production.

Outline

- QCD High Energy Scattering with AdS/CFT
- Deep Inelastic Scattering at Small-x: (universality)
- Higher Order Effects: Saturation, Confinement, etc.
- Summary and Outlook

I. Gauge-String Duality: AdS/CFT

Weak Coupling:

Gluons and Quarks: Gauge Invariant Operators: $\begin{aligned} A^{ab}_{\mu}(x), \psi^{a}_{f}(x) \\ \bar{\psi}(x)\psi(x), \ \bar{\psi}(x)D_{\mu}\psi(x) \\ S(x) &= TrF^{2}_{\mu\nu}(x), \ O(x) = TrF^{3}(x) \\ T_{\mu\nu}(x) &= TrF_{\mu\lambda}(x)F_{\lambda\nu}(x), \ etc. \end{aligned}$

$$\mathcal{L}(x) = -TrF^2 + \bar{\psi}\mathcal{D}\psi + \cdots$$

Strong Coupling:

Metric tensor: $G_{mn}(x)$ Anti-symmetric tensor (Kalb-Ramond fields): Dilaton, Axion, etc. Other differential forms:

 $G_{mn}(x) = g_{mn}^{(0)}(x) + h_{mn}(x)$ elds): $b_{mn}(x)$ $\phi(x), a(x), etc.$ $C_{mn}...(x)$

 $\mathcal{L}(x) = \mathcal{L}(G(x), b(x), C(x), \cdots)$

$$\mathcal{N} = 4$$
 SYM Scattering at High Energy

$$\langle e^{\int d^4 x \phi_i(x) \mathcal{O}_i(x)} \rangle_{CFT} = \mathcal{Z}_{string} \left[\phi_i(x, z) |_{z \sim 0} \to \phi_i(x) \right]$$

Bulk Degrees of Freedom from type-IIB Supergravity on AdS₅:

- metric tensor: G_{MN}
- Kalb-Ramond 2 Forms: B_{MN} , C_{MN}
- Dilaton and zero form: ϕ and C_0

$$\lambda = g^2 N_c \to \infty$$

Supergravity limit

- Strong coupling
- Conformal
- Pomeron as Graviton in AdS

WHAT IS THE BARE POMERON ? LEADING I/N TERM CYLINDER EXCHANGE

WEAK: TWO GLUON <=> STRONG: ADS GRAVITON





$$J_{cut} = 1 + 1 - 1 = 1$$

$$S = \frac{1}{2\kappa^2} \int d^4x dz \sqrt{-g(z)} \left(-\mathcal{R} + \frac{12}{R^2} + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi \right)$$

F.E. Low. Phys. Rev. D 12 (1975), p. 163. S. Nussinov. Phys. Rev. Lett. 34 (1975), p. 1286.

AdS Witten Diagram: Adv. Theor. Math. Physics 2 (1998)253



One Graviton Exchange at High Energy

- Draw all "Witten-Feynman" Diagrams in AdS₅,
- High Energy Dominated by Spin-2 Exchanges



$$T^{(1)}(p_1, p_2, p_3, p_4) = g_s^2 \int \frac{dz}{z^5} \int \frac{dz'}{z'^5} \tilde{\Phi}_{\Delta}(p_1^2, z) \tilde{\Phi}_{\Delta}(p_3^2, z) \mathcal{T}^{(1)}(p_i, z, z') \tilde{\Phi}_{\Delta}(p_2^2, z') \tilde{\Phi}_{\Delta}(p_4^2, z')$$

$$\mathcal{T}^{(1)}(p_i, z, z') = (z^2 z'^2 s)^2 G_{++, --}(q, z, z') = (z z' s)^2 G_{\Delta=4}^{(5)}(q, z, z')$$

Additional Steps for QCD:



Brower, Polchinski, Strassler, and Tan: "The Pomeron and Gauge/String Duality," hep-th/063115

Diffusion in AdS: $(\lambda < \infty)$



At finite λ , due to Diffusion in AdS

Graviton (Pomeron) becomes j-plane singularity at

$$j_0: 2 \to 2 - 2/\sqrt{\lambda}$$

 Conformal: No scale and it is a branch cut, not a Regge trajectory





Confinement Deformation: Glueball Spectrum $(\lambda = \infty)$





Four-Dimensional Mass:

 $E^2 = (p_1^2 + p_2^2 + p_3^2) + M^2$

5-Dim Massless Mode:

 $0 = E^2 - (p_1^2 + p_2^2 + p_3^2 + p_r^2)$



BASIC BUILDING BLOCK

- Elastic Vertex:
- Pomeron/Graviton Propagator:

$$\mathcal{K}(s,b,z,z') = -\left(\frac{(zz')^2}{R^4}\right) \int \frac{dj}{2\pi i} \left(\frac{1+e^{-i\pi j}}{\sin \pi j}\right) \,\widehat{s}^j \,G_j(z,x^\perp,z',x'^\perp;j)$$

conformal:
$$G_j(z, x^{\perp}, z', x'^{\perp}) = \frac{1}{4\pi z z'} \frac{e^{(2-\Delta(j))\xi}}{\sinh \xi}$$
, $\Delta(j) = 2 + \sqrt{2} \lambda^{1/4} \sqrt{(j-j_0)}$ confinement: $G_j(z, x^{\perp}, z', x'^{\perp}; j)$ \longrightarrow discrete sum

• Universality and Holographic:

By choosing wave functions, Φ , can treat

DIS, Higgs Production, Proton-Proton, etc., on equal footing.



Comparison of strong vs weak coupling kernel at t=0

Strong Coupling: $\mathcal{K}(r,r',s) = rac{s^{j_0}}{\sqrt{4\pi \mathcal{D} \ln s}} e^{-(\ln r - \ln r')^2/4\mathcal{D} \ln s}$ Diffusion in "warped co-ordinate" $j_0 = 2 - \frac{2}{\sqrt{q^2 N}} + O(1/q^2 N)$ $\mathcal{D} = \frac{1}{2\sqrt{q^2 N}} + O(1/q^2 N)$. Weak Coupling: $K(s, k_{\perp}, k'_{\perp}) \approx \frac{s^{\alpha(0)-1}}{\sqrt{\pi \ln s}} e^{-\left[(\ln k'_{\perp} - \ln k_{\perp})^2/4\mathcal{D} \ln s\right]}$ $\mathcal{D} = \frac{14\zeta(3)}{g^2 N/4\pi^2}.$ $j_0 = 1 + \ln(2)g^2 N/\pi^2$

Unified Hard (conformal) and Soft (confining) Pomeron

At finite λ , due to Confinement in AdS, at t > 0aymptotical linear Regge trajectories



AdS/CFT ===>

In gauge theories with string-theoretical dual descriptions, the <u>Pomeron</u> emerges <u>unambiguously</u>.

Pomeron can be associated with a Reggeized Graviton.

Both the IR (soft) Pomeron and the UV (BFKL) Pomeron are dealt in a unified single step.

R. Brower, J. Polchinski, M. Strassler, and C-I Tan, "The Pomeron and Gauge/String Duality", (hep-th/0603115.)

- Perturbative QCD
- Short-Distance
- $\alpha_{BFKL}(0) \sim 1.4$
- Increasing Virtuality
- No Shrinkage of elastic peak
- Fixed-cut in t
- Diffusion in Virtuality

- Non-Perturbative
- Long-distance: Confinement
- $\alpha_P(0) \sim 1.08$
- Fixed trans. Momenta
- Shrinkage of Elastic Peak: <|t|> $\sim 1/\log s$
- $\alpha'(0) \sim 0.3 \text{ Gev}^{-2}$
- Diffusion in impact space

Note: Slide from 1990

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Unified treatment in terms of diffusion

-- in Impact Space and in Virtuality

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"Heterotic Pomeron"--Levin-CIT (hep-ph/9302308)

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Unified treatment in terms of diffusion in AdS with confinement deformation

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Unified treatment in terms of diffusion in AdS with confinement deformation

"Heterotic Pomeron"--Levin-CIT (hep-ph/9302308) Brower, Polchinski, Strassler, CIT (hep-th/0603115)

II. Deep Inelastic Scattering (DIS) at small-x

Deep Inelastic Scattering (DIS)



$$F_2(x, Q2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \left[\sigma_T(\gamma^* p) +_L (\gamma^* p) \right]$$
$$x \equiv \frac{Q^2}{s}$$

$$Small \ x: \ \frac{Q^2}{s} \to 0$$
 $Optical \ Theorem$

$$\sigma_{total}(s, Q^2) = (1/s) \text{Im } A(s, t = 0; Q^2)$$

HERA vs LHeC region: dots are HI-ZEUS small-x data points



Effective Pomeron Intercept from HERA data:

$$F_2 \simeq C(Q^2) \ x^{-\epsilon_{eff}}$$



Questions on HERA DIS small-x data:

- ▶ Why $\alpha_{eff} = 1 + \epsilon_{eff}(Q^2)$?
- Confinement? (Perturbative vs. Non-perturbative?)
- Saturation? (evolution vs. non-linear evolution?)

ADS BUILDING BLOCKS BLOCKS For 2-to-2 $A(s,t) = \Phi_{13} * \mathcal{K}_P * \Phi_{24}$ $A(s,t) = g_0^2 \int d^3 \mathbf{b} d^3 \mathbf{b}' \ e^{i\mathbf{q}_{\perp} \cdot (\mathbf{x} - \mathbf{x}')} \ \Phi_{13}(z) \ \mathcal{K}(s, \mathbf{x} - \mathbf{x}', z, z') \ \Phi_{24}(z')$ $d^3 \mathbf{b} \equiv dz d^2 x_\perp \sqrt{-g(z)}$ where $g(z) = \det[g_{nm}] = -e^{5A(z)}$ For 2-to-3 $A(s, s_1, s_2, t_1, t_2) = \Phi_{13} * \mathcal{K}_P * V * \mathcal{K}_P * \Phi_{24} ,$

ELASTIC VS DIS ADS BUILDING BLOCKS

$$A(s, x_{\perp} - x'_{\perp}) = g_0^2 \int d^3 \mathbf{b} d^3 \mathbf{b}' \Phi_{12}(z) G(s, x_{\perp} - x'_{\perp}, z, z') \Phi_{34}(z')$$

$$\sigma_T(s) = \frac{1}{s} ImA(s,0)$$

for $F_2(x,Q)$

$$\Phi_{13}(z) \to \Phi_{\gamma^*\gamma^*}(z,Q) = \frac{1}{z} [Qz)^4 (K_0^2(Qz) + K_1^2(Qz)]$$

 $d^3 \mathbf{b} \equiv dz d^2 x_\perp \sqrt{-g(z)}$ where $g(z) = \det[g_{nm}] = -e^{5A(z)}$

DIS in String Theory continued

$$F_{2}(x,Q^{2}) \text{ from AdS/CFT}$$

$$F_{2} = c \frac{Q}{Q'} \frac{(Q_{0}^{2} \frac{Q}{Q'} \frac{1}{x})^{1-\rho}}{\sqrt{\log(Q_{0}^{2} \frac{Q}{Q'} \frac{1}{x})}} \exp(-\frac{\log^{2}(\frac{Q}{Q'})}{\rho \log(Q_{0}^{2} \frac{Q}{Q'} \frac{1}{x})}$$

- This is the expression we will use later for comparing to data. Let's make a few comments about this function.
- c is a dimensionless normalization constant. I have grouped here all the constants the multiply F₂, including the coupling constant that comes from χ, and only appears as product together with normalization.
- ► At any Q² fixed, we see that at small x the term (¹/_x)^(1-ρ) dominates. This leads to a violation of the Froissart bound.



red – Conformal kernel, green – Hard-Wall kernel

MOMENTS AND ANOMALOUS DIMENSION $M_n(Q^2) = \int_0^1 dx \; x^{n-2} F_2(x,Q^2) \to Q^{-\gamma_n}$



Simultaneous compatible large Q^2 and small x evolutions! Energy-Momentum Conservation MOMENTS AND ANOMALOUS DIMENSION $M_n(Q^2) = \int_0^1 dx \; x^{n-2} F_2(x,Q^2) \to Q^{-\gamma_n}$



Simultaneous compatible large Q^2 and small x evolutions! Energy-Momentum Conservation built-in automatically. $\mathcal{N} = 4$ SYM Leading Twist $\Delta(J)$ vs J: Anomalous Dimensions $\lambda = 0$ DGLAP (DIS moments) 3 $Tr[F_{+\mu}D_{+}^{j-2}F_{+}^{\mu}]$ 2.5 $\lambda \gg 1$ (0,2) $T_{\mu\nu} \gamma = 0$ 1.5 $\lambda \ll 1$ -1-0.5 $\lambda = 0$, BFKL -1 1 3 4 5 Δ $\lambda = g^2 N = 0$ $j = j_0 @ min \Delta$

II-b. Deep Inelastic Scattering (DIS) at small-x:

Confinement !!!

QCD: EMERGENCE OF 5-DIM: ADS

"Fifth" co-ordinate is size z / z' of proj/target



2-d Longitudinal $p^{\pm} = p^0 \pm p^3$ 2-d Transverse space: x'_{\perp} - $x_{\perp} = b_{\perp}$

1-d Resolution:

 $\begin{array}{l} p^{\pm} = p^0 \pm p^3 \simeq exp[\ \pm \ log(s/\varLambda_{\it qcd})] \\ x'_{\perp} \mbox{-} \ x_{\perp} = b_{\perp} \\ z = 1/Q \ (or \ z' = 1/Q') \end{array}$

Geometry of Transverse AdS-3



DIS in String Theory The Hard-wall Model continued

We will take over the structure function formula we had before, and just replace the Pomeron exchange kernel with the new version.

 $F_2(x,Q^2)$ from hard-wall AdS/CFT

$$F_{2} = c \frac{Q}{Q'} \frac{(Q_{0}^{2} \frac{Q}{Q'} \frac{1}{x})^{1-\rho}}{\sqrt{\log(Q_{0}^{2} \frac{Q}{Q'} \frac{1}{x})}} (\exp(-\frac{\log^{2}(\frac{Q}{Q'})}{\rho \log(Q_{0}^{2} \frac{Q}{Q'} \frac{1}{x})}) + \mathcal{F} \exp(-\frac{\log(\frac{Q_{0}^{2}}{QQ'})^{2}}{\rho \log(Q_{0}^{2} \frac{Q}{Q'} \frac{1}{x})}))$$

we see that this part is the same as before, while this part is new. The function \mathcal{F} is given by

$$\mathcal{F}(x, Q, Q')$$
 $= 1 - 4\sqrt{\pi \log(Q_0^2 \frac{Q}{Q'} \frac{1}{x})} e^{\eta^2} erfc(\eta)$

where

$$\eta = \frac{\log(\frac{x}{Q^2} (Q_0^2 \frac{Q}{Q'} \frac{1}{x})^{1-\rho})}{\sqrt{\rho \log(Q_0^2 \frac{Q}{Q'} \frac{1}{x})}}$$





red – Conformal kernel, green – Hard-Wall kernel

Confining Pomeron: $\chi^2 = 1.07$



 \leftarrow

III. Beyond Pomeron

Sum over all Pomeron graph (string perturbative, 1/N²)
 Eikonal summation in AdS₃

Constraints from Conformal Invariance, Unitarity, Analyticity, Confinement, Universality, etc.

@Froissart Bound?

"non-perturbative" (e.g., blackhole production)

Higher Orders Witten Diagrams:



• Eikonal Sum: derived both via Cheng-Wu or by Shock-wave method

$$A_{2\to 2}(s,t) \simeq -2is \int d^2b \ e^{-ib^{\perp}q_{\perp}} \int dz dz' P_{13}(z) P_{24}(z') \left[e^{i\chi(s,b^{\perp},z,z')} - 1 \right]$$

$$P_{13}(z) = (z/R)^2 \sqrt{g(z)} \Phi_1(z) \Phi_3(z) \qquad P_{24}(z) = (z'/R)^2 \sqrt{g(z')} \Phi_2(z') \Phi_4(z')$$

transverse AdS₃ space !!

$$\chi(s, x^{\perp} - x'^{\perp}, z, z') = \frac{g_0^2 R^4}{2(zz')^2 s} \mathcal{K}(s, x^{\perp} - x'^{\perp}, z, z')$$

• <u>Saturation:</u>

$$\chi(s, x^{\perp} - x'^{\perp}, z, z') = O(1)$$

• Universality:

e.g., Choose Φ_1 and Φ_3 for DIS.



 \leftarrow



 $\widehat{\mathbf{n}}$

 $F_2(x,Q^2) \sim (1/x)^{\epsilon_{effecti}}$

 $\hat{\mathbf{n}}$



 \leftarrow

Standard expectation (from Itakura's RIKEN lectures)





Scattering in Conformal Limit:

Use the condition:
$$\chi(s, x^{\perp} - {x'}^{\perp}, z, z') = O(1)$$

Elastic Ring:

Ν

$$b_{\text{diff}} \sim \sqrt{zz'} \ (zz's/N^2)^{1/6}$$

$$\sigma_{total} \sim s^{1/3}$$

. . . .

Inner Absorptive Disc:

$$b_{\text{black}} \sim \sqrt{zz'} \quad \frac{(zz's)^{(j_0-1)/2}}{\lambda^{1/4}N} \qquad b_{\text{black}} \sim \sqrt{zz'} \left(\frac{(zz's)^{j_0-1}}{\lambda^{1/4}N}\right)^{1/\sqrt{2}\sqrt{\lambda(j_0-1)}}$$

Inner Core: "black hole" production ?

Saturation of Froissart Bound

- The Confinement deformation gives an exponential cutoff for b
 b_{max} ~c log (s/s₀),
- Coefficient c ~ I/m₀, m₀ being the <u>mass of lightest tensor</u> <u>glueball.</u>
- Froissart is respected and saturated.

 $\Delta b \sim \log(s/s_0)$

Disk picture



bmax determined by confinement.

IV. Summary and Outlook

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ELASTIC, DIS, DOUBLE HIGGS: ADS BUILDING BLOCKS

$$A(s,t) = g_0^2 \int d^3 \mathbf{b} d^3 \mathbf{b}' \ e^{i\mathbf{q}_{\perp} \cdot (\mathbf{x} - \mathbf{x}')} \ \Phi_{13}(z) \ \mathcal{K}(s, \mathbf{x} - \mathbf{x}', z, z') \ \Phi_{24}(z')$$

 $\sigma_{T}(s) = \frac{1}{s} ImA(s,0)$ $d^{3}\mathbf{b} \equiv dzd^{2}x_{\perp}\sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$ for $F_{2}(x,Q)$ $\Phi_{13}(z) \to \Phi_{\gamma^{*}\gamma^{*}}(z,Q) = \frac{1}{z} [Qz)^{4} (K_{0}^{2}(Qz) + K_{1}^{2}(Qz)]$

For Double Diffractive Higgs

 $A(s_1, s_2, s, t_1, t_2) = \Phi_{13} * \mathcal{K}_1 * \mathcal{V}_H * \mathcal{K}_2 * \Phi_{24}$

