Double parton distributions and their evolution

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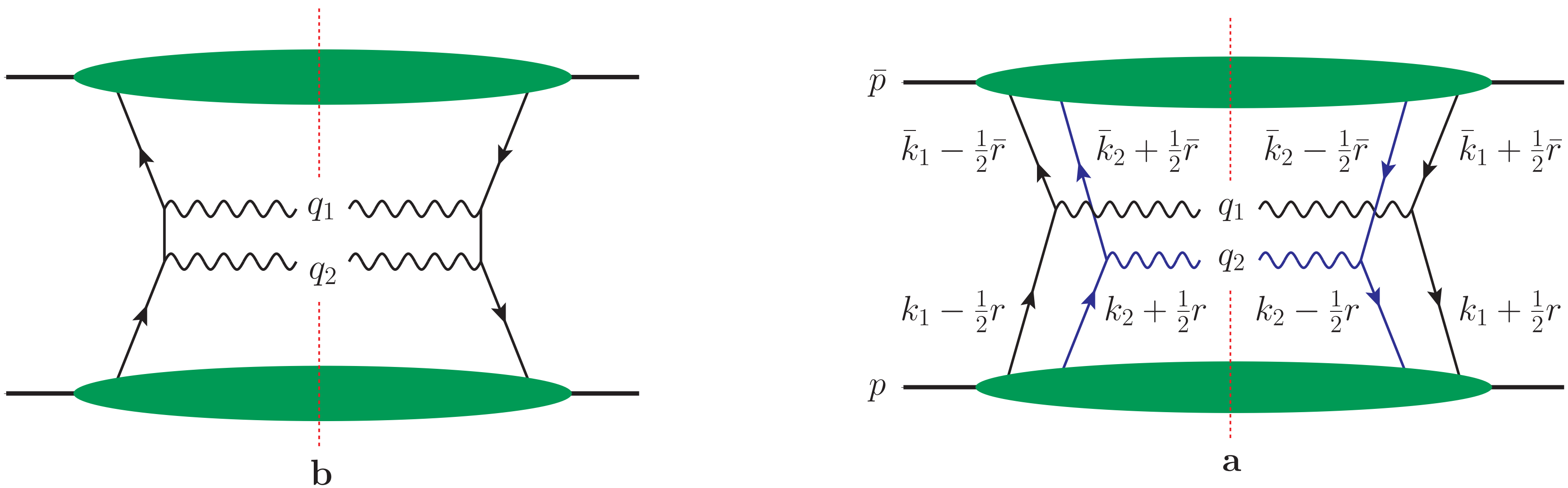
Motivation

- Growing interest in multiparton scattering in hadronic collisions
- Two parton scattering: Snigirev, Gaunt, Stirling, Ceccopieri, Ryskin, Bartels, Diehl, A. Schaefer, Dokshitzer, Frankfurt, Strikman, Blok, Kulesza, Szczurek,.....
- Double parton distributions in the cross section, e.g.,

$$\sigma_{pp\to WWX} = D(x_1, x_2) \otimes \hat{\sigma}_{qq\to W} \otimes \hat{\sigma}_{qq\to W} \otimes D(x_1, x_2)$$

Single PDF versus double PDF

- Two vector bosons from two and four partons scattering.



- One and two hard scatterings with single and double PDF:

$$D_f(x,\mu)\,, \qquad\qquad D_{f_1 f_2}(x_1,x_2,\mu_1,\mu_2)$$

- Obvious condition: $0 < x_1 + x_2 \leq 1$.

- Ansatz $D_{f_1 f_2}(x_1,x_2,\mu_1,\mu_2) = D_{f_1}(x_1,\mu_1) \cdot D_{f_2}(x_2,\mu_2)$ is not general.

Evolution equations for single PDF

- Markov process with time $t = \ln \mu$ and $f = q_i, \bar{q}_i, g$

$$D_f(x, t + \delta t) = \sum_{f'} \int_x^1 du \underbrace{\delta t \, P^r_{f f'}(x, u) \, D_{f'}(u, t)}_{real\ emission} + \underbrace{\big(1 - \delta t \, P^v_f(x)\big) \, D_f(x, t)}_{virtual\ emission}$$

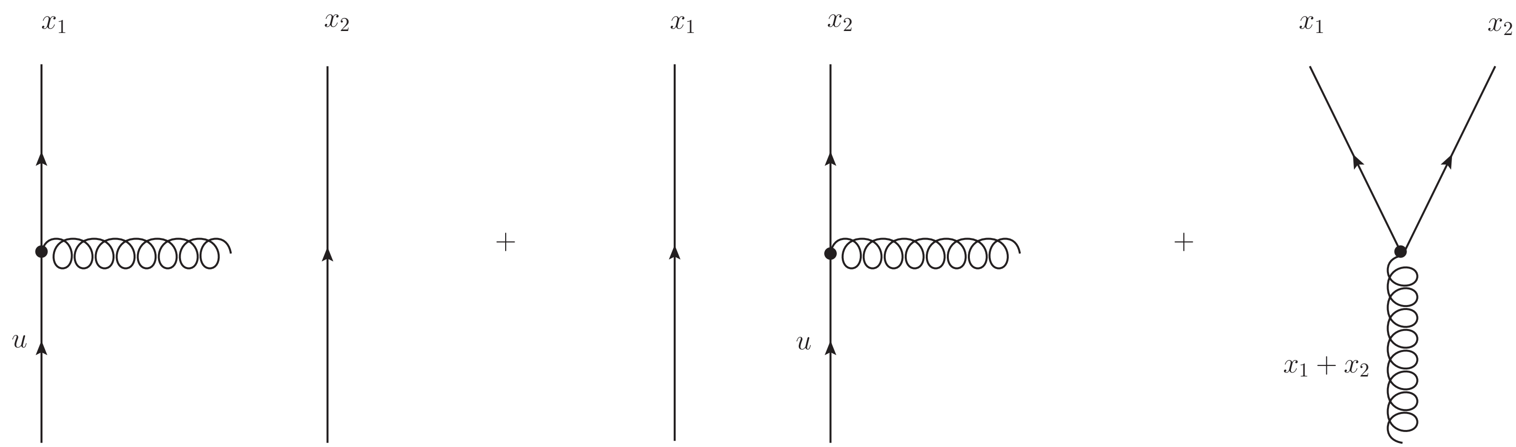
- In the limit $\delta t \to 0$, a set of linear differential equations (DGLAP)

$$\partial_t \, D_f(x, t) = \sum_{f'} \int_x^1 du \, P^r_{f f'}(x, u) \, D_{f'}(u, t) - P^v_f(x) \, D_f(x, t)$$

- Virtual $P^v_f(x)$ from momentum sum rule: $\int_0^1 dx \sum_f x D_f(x, t) = 1$.

Evolution equations for dPDF (LLA)

- Pictorially: $D_{f_1 f_2}(x_1, x_2, \mu, \mu)\,, \quad t = \ln\mu$



- Analytically:

$$
\begin{aligned}
\partial_t D_{f_1 f_2}(x_1, x_2, t) &= \sum_{f'} \int_0^{1-x_2} du\, P_{f_1 f'}(x_1, u)\, D_{f' f_2}(u, x_2, t) \\
&+ \sum_{f'} \int_0^{1-x_1} du\, P_{f_2 f'}(x_2, u)\, D_{f_1 f'}(x_1, u, t) \\
&+ \sum_{f'} P_{f' \to f_1 f_2}\left(\frac{x_1}{x_1 + x_2}\right) \underbrace{D_{f'}(x_1 + x_2, t)}_{sPDF}
\end{aligned}
$$

Evolution equations for dPDF

- For different scales: $t_1 = \ln \mu_1 \; < \; t_2 = \ln \mu_2$

$$D_{f_1 f_2}(x_1, x_2, t_0, t_0) \; \to \; D_{f_1 f_2}(x_1, x_2, t_1, t_1) \; \to \; D_{f_1 f_2}(x_1 = fixed, x_2, t_1, t_2)$$

- sPDF evolution for x_2 from $t_1 \to t_2$ with fixed x_1:

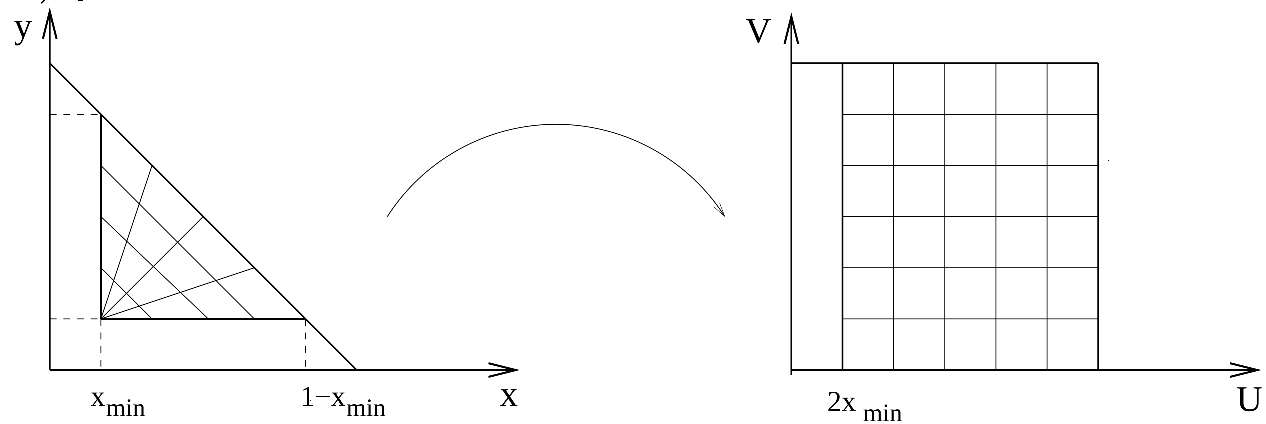
$$\partial_{t_2} D_{f_1 f_2}(x_1, x_2, t_1, t_2) = \sum_{f'} \int_0^{1-x_1} du \, P_{f_2 f'}(x_2, u) \, D_{f_1 f_2}(x_1, u, t_1, t_2)$$

- No parton exchange symmetry:

$$D_{f_1 f_2}(x_1, x_2, t_1, t_2) \neq D_{f_2 f_1}(x_2, x_1, t_1, t_2)$$

Finding solutions

- Chebyshev polynomial approximation method on a grid of points in the (x_1, x_2) plane.

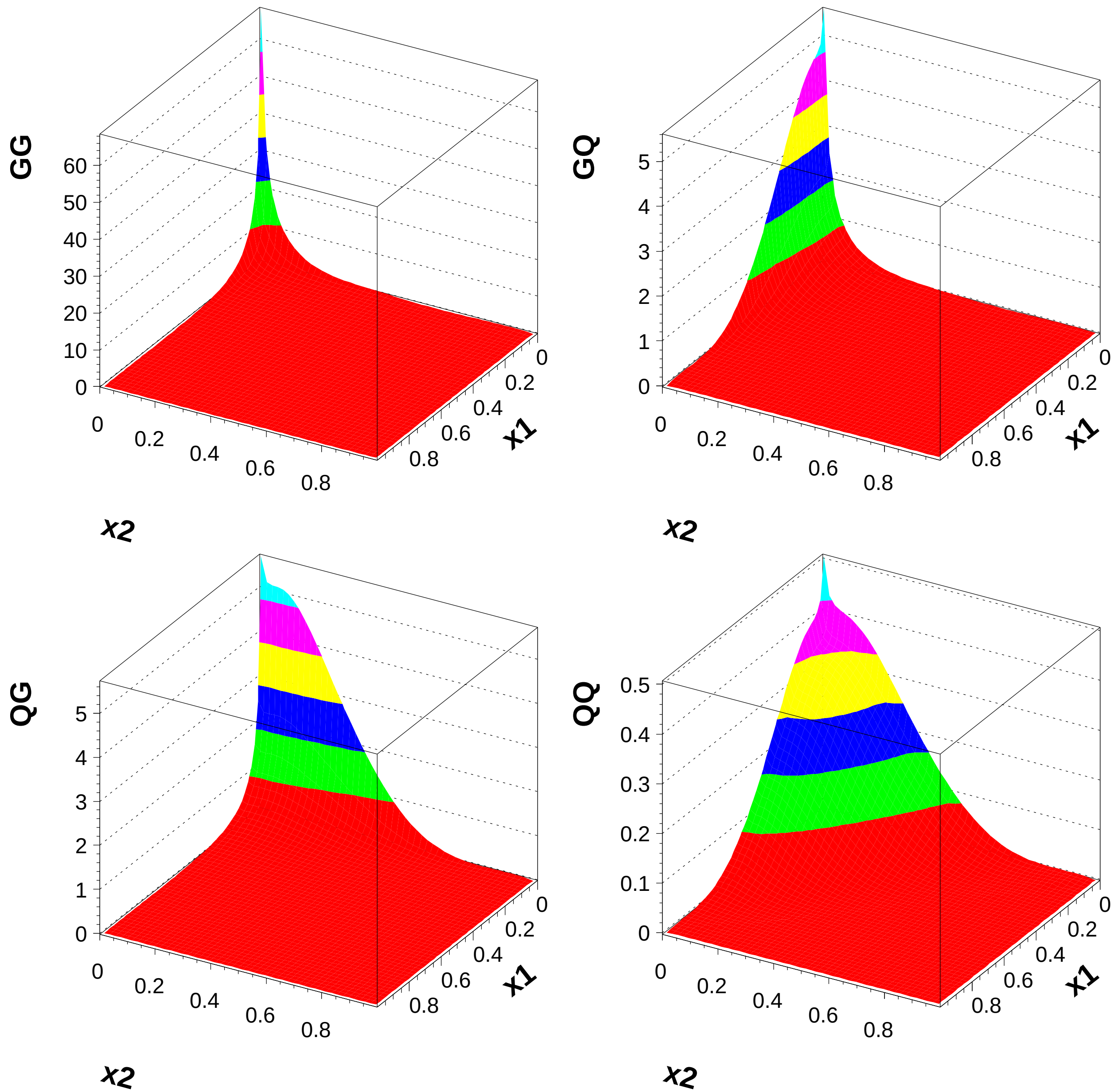


- Set of first order differential equations for dPDF at grid points

$$\frac{d}{dt} D_{nm}(t) = \sum_{k,l} A_{nm|kl}\, D_{kl}(t) + \sum_{k} B_{nmk}\, D_k(t)$$

- Problems: stability, agreement with carefully chosen tests,...

An example



Momentum sum rule

- Conditional probability with known second parton:

$$\sum_{f_1} \int_0^{1-x_2} dx_1 \, x_1 \, \frac{D_{f_1 f_2}(x_1, x_2)}{D_{f_2}(x_2)} = (1 - x_2)$$

- Relation conserved by evolution equations once imposed at initial t_0

$$\sum_{f_1} \int_0^{1-x_2} dx_1 \, x_1 \, D_{f_1 f_2}(x_1, x_2, t) = (1 - x_2) D_{f_2}(x_2, t)$$

- Sum rule for dPDF after using sPDF momentum sum rule:

$$\sum_{f_1, f_2} \int_0^1 dx_2 \int_0^{1-x_2} dx_1 \, \frac{x_1 x_2}{1 - x_2} D_{f_1 f_2}(x_1, x_2, t) = 1$$

How to fulfill momentum sum rule?

- Initial conditions (Korotkin, Snigirev, Gaunt, Stirling):

$$D_{f_1 f_2}(x_1, x_2) = D_{f_1}(x_1) D_{f_2}(x_2) \frac{(1 - x_1 - x_2)^n}{(1 - x_1)^{n_1} (1 - x_2)^{n_2}}$$

- Our initial conditions (no parton exchange symmetry):

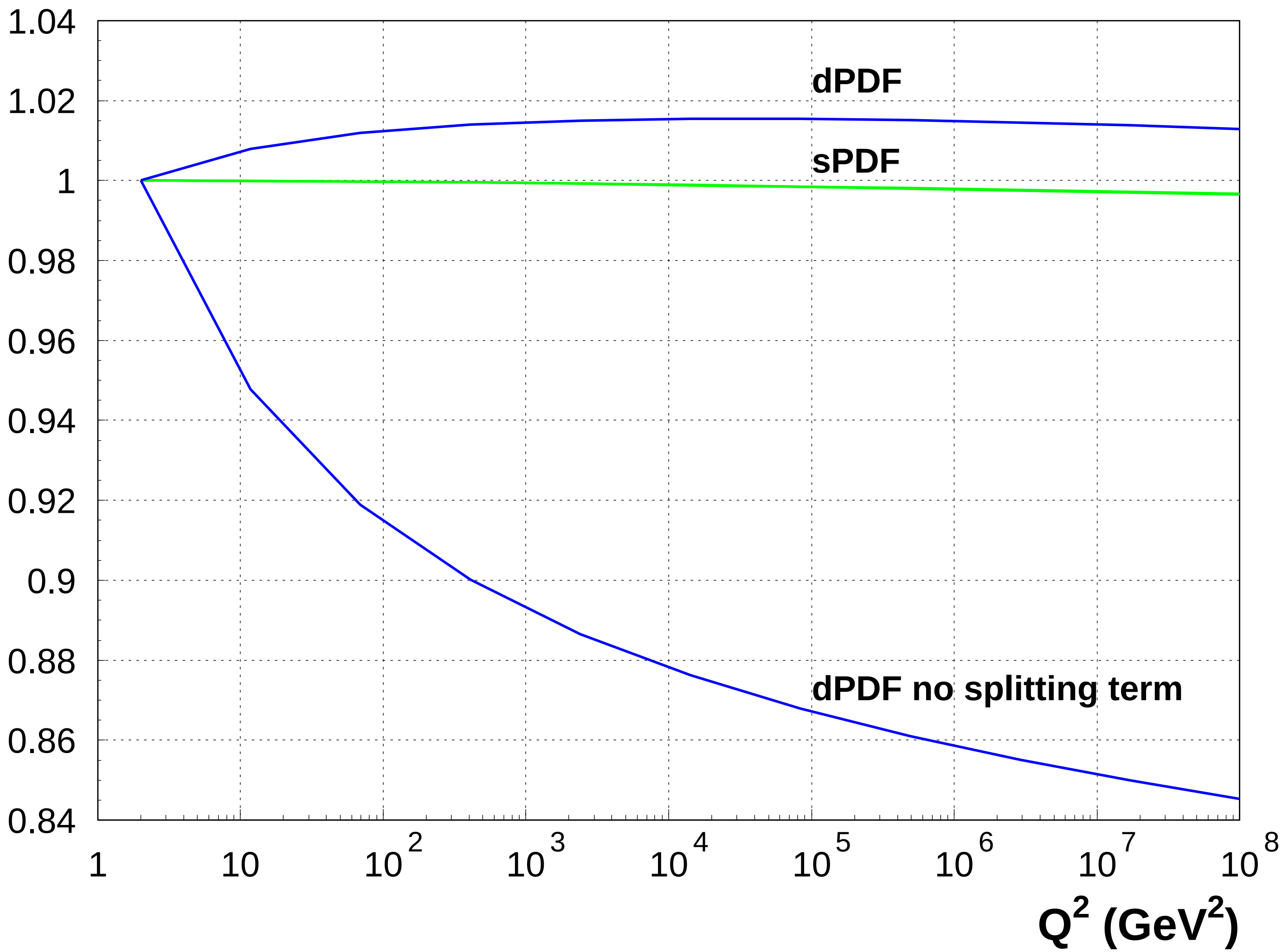
$$D_{f_1 f_2}(x_1, x_2) = \frac{1}{1 - x_2} D_{f_1}\left(\frac{x_1}{1 - x_2}\right) D_{f_2}(x_2)$$

- Satisfies also valence number quark rule

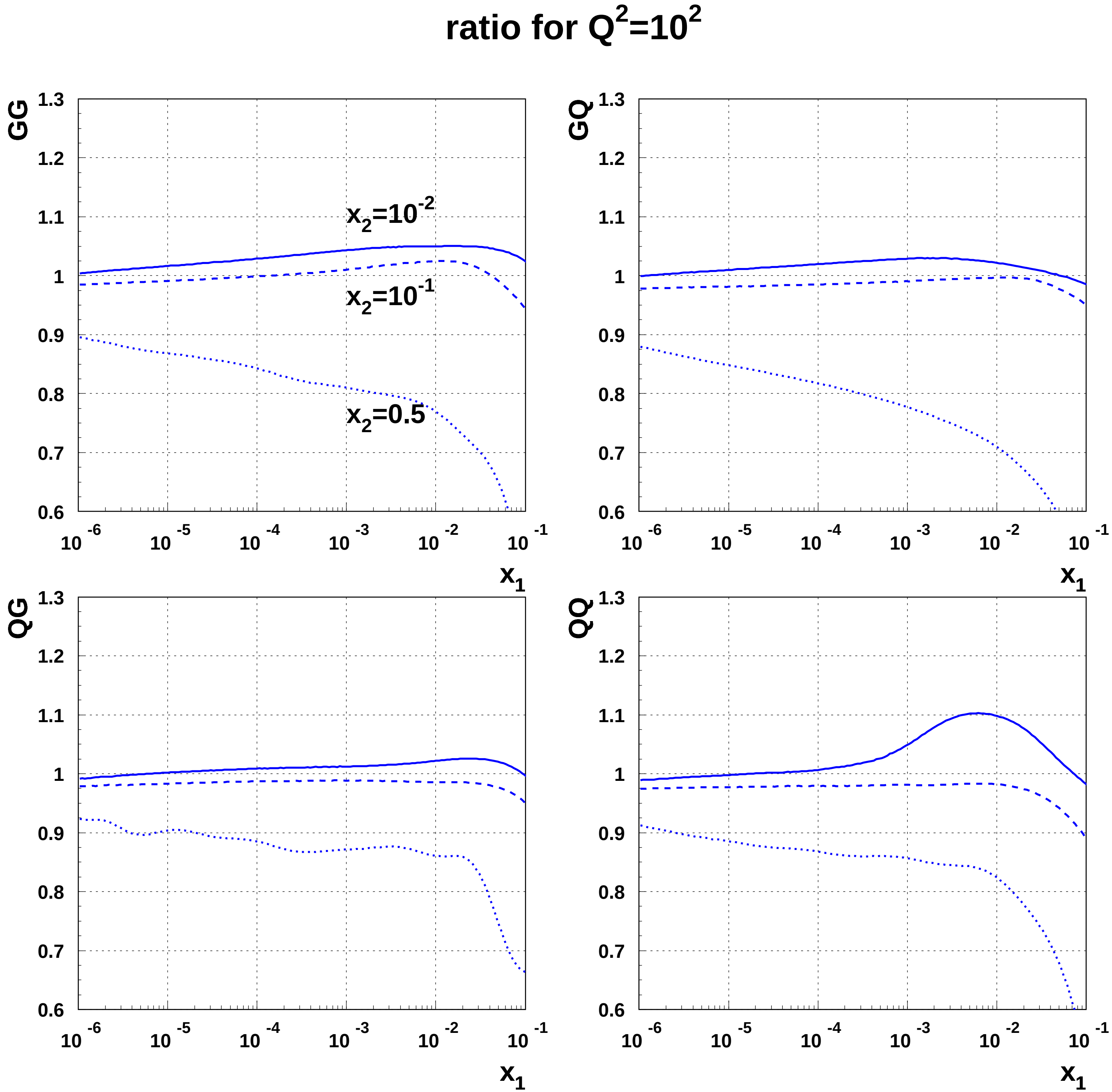
$$\int_0^{1-x_2} dx_1 \left\{ D_{f_1 f_2}(x_1, x_2) - D_{\bar{f}_1 f_2}(x_1, x_2) \right\} = N_{f_1} D_{f_2}(x_2)$$

with $N_u = 2$ and $N_d = 1$ for the proton.

Momentum sum rule in practice



$D_{f_1 f_2}(x_1, x_2)$ versus $D_{f_1}(x_1) D_{f_2}(x_2)$



Summary

- Sum rules play important role in dPDF evolution LLA.
- For small x_1, x_2, a naive product of sPDF not so bad, but ...
- ...remember about higher order corrections, parton recombination effects, transverse momentum dependend dPDF,...
- We are still at the beginnig of the understanding of multiparton scattering.