

# $\pi$ on structure function at small $x$

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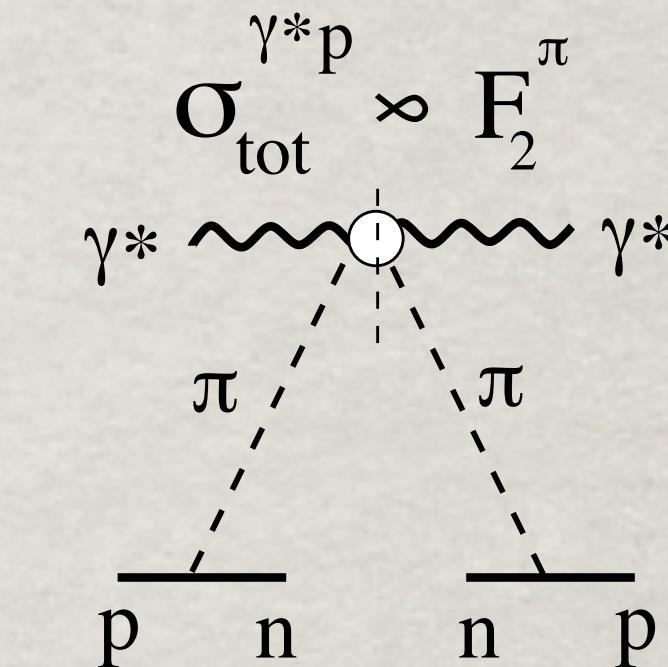
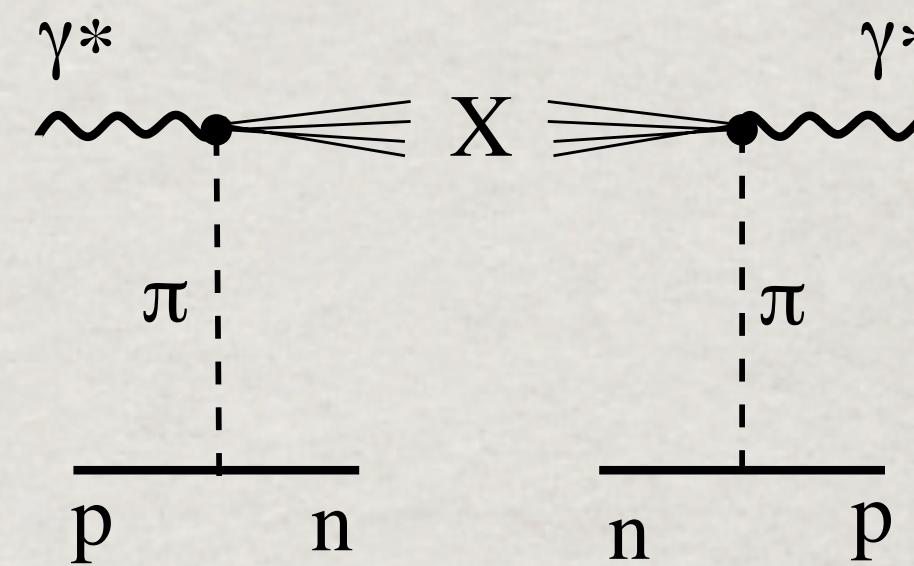
# Píon pole

$$\gamma^* + p \rightarrow X + n$$

$$z = \frac{p_n^+}{p_p^+} \rightarrow 1$$

$$M_X^2 = (1 - z)s$$

$$\sum_X$$



The source of information about  $F_2^\pi(x_\pi, Q^2)$  at small  $x_\pi = \frac{x}{1-z}$

The amplitude includes both non-flip and spin-flip terms

$$A_{p \rightarrow n}^B(\tilde{q}, z) = \bar{\xi}_n \left[ \sigma_3 q_L + \frac{1}{\sqrt{z}} \tilde{\sigma} \cdot \tilde{q}_T \right] \xi_p \phi^B(q_T, z)$$

$$q_L = \frac{1-z}{\sqrt{z}} m_N$$

$$\phi^B(q_T, z) = \frac{\alpha'_\pi}{8} G_{\pi^+ pn}(t) \eta_\pi(t) (1-z)^{-\alpha_\pi(t)} A_{\gamma^* \pi \rightarrow X}(M_X^2)$$

$$\sum_X |A_{\gamma^* \pi^+ \rightarrow X}(M_X^2)|^2 = \frac{4\pi^2 \alpha_{\text{em}}}{x_\pi} F_2^\pi(x_\pi, Q^2)$$

# Educated guess

$$R_{\pi/p}(x, Q^2) = F_2^\pi(x, Q^2)/F_2^p(x, Q^2) = ?$$

The small  $q$ - $\bar{q}$  dipole,  $\gamma^* \rightarrow \bar{q}q$ , is a good counter of valence quarks, so one could (naively) expect  $R_{\pi/p}(x, Q^2) = 2/3$

However, the proton has a considerable pion component:  $p \rightarrow N\pi$

It can be evaluated relying on the observed deviation from the Gottfried sum rule:

$$R_{\pi/p} = \frac{2}{3 + 2\langle n_\pi \rangle}$$

DY E866:  $\langle n_\pi \rangle = 0.36$

NMC:  $\langle n_\pi \rangle = 0.44$

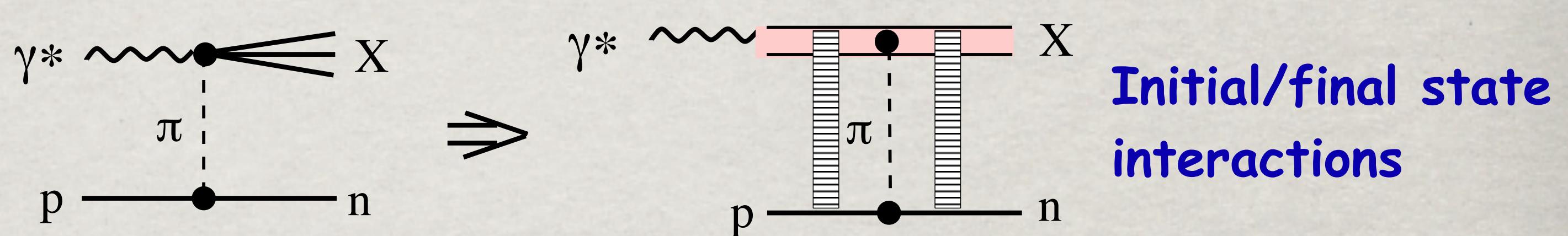
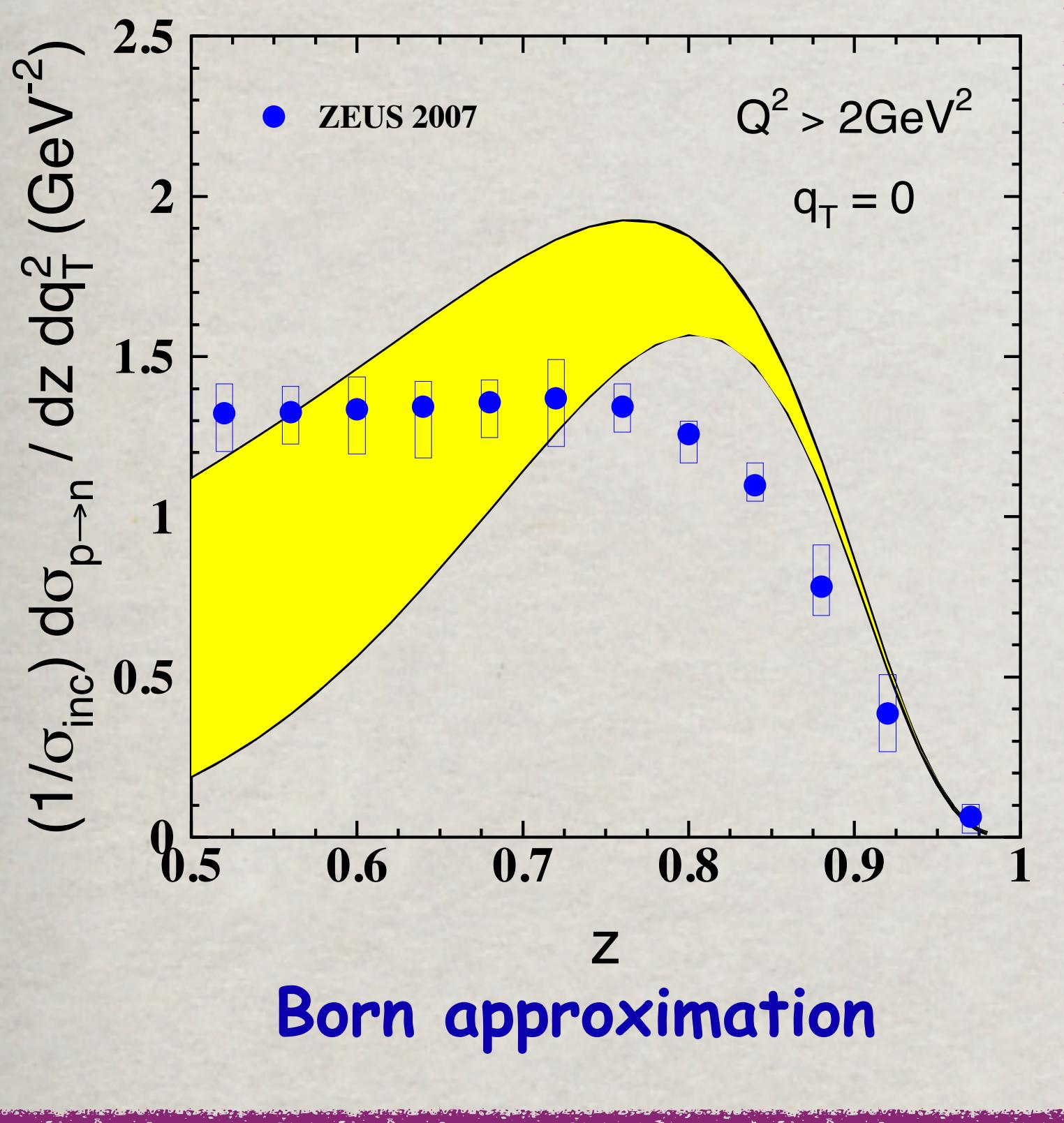
HERMES  $\langle n_\pi \rangle = 0.48$

Adding the poorly known contribution of the iso-scalar mesons, our guess is

$$R_{\pi/p} = \frac{1}{2}$$

This is our trial value for further calculations

# Absorptive corrections



**The Born amplitude in impact parameters:**

$$f_{p \rightarrow n}^B(\tilde{b}, z) = \bar{\xi}_n \left[ \sigma_3 q_L \theta_0^B(b, z) - i \frac{\tilde{\sigma} \cdot \tilde{b}}{b \sqrt{z}} \theta_s^B(b, z) \right] \xi_p$$

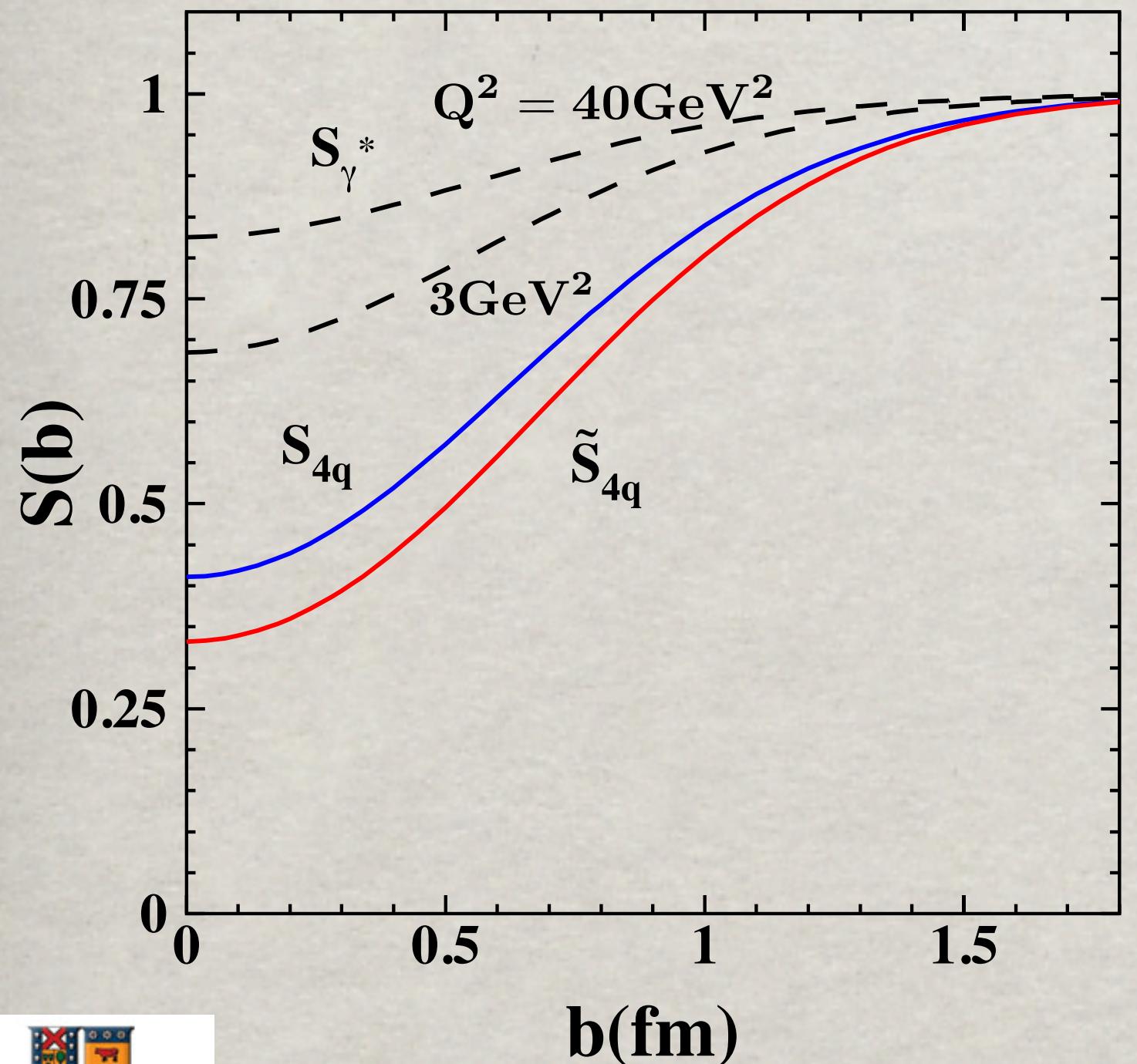
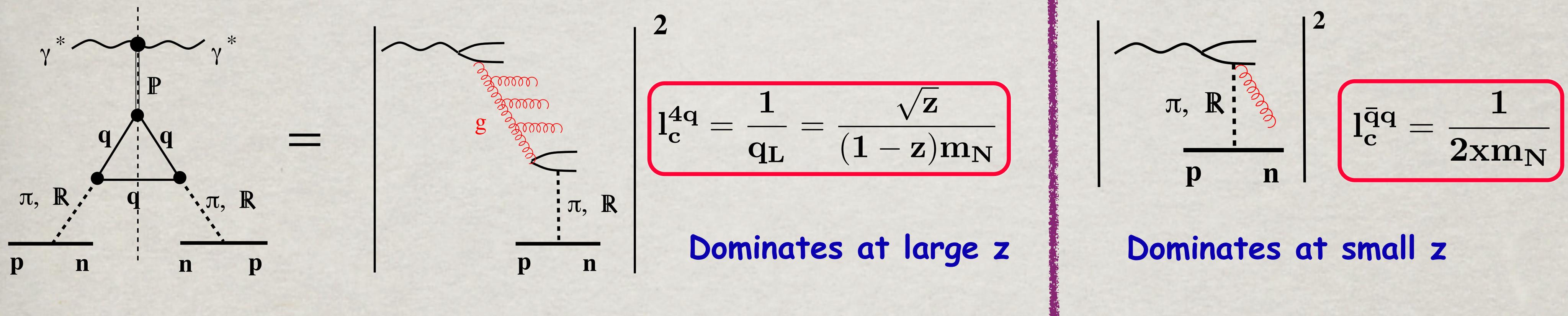
$$\theta_0^B(b, z) = \int d^2 q_T e^{i \tilde{b} \tilde{q}_T} \phi^B(q_T, z)$$

$$\theta_s^B(b, z) = \frac{1}{b} \int d^2 q_T e^{i \tilde{b} \tilde{q}_T} (\tilde{b} \cdot \tilde{q}) \phi^B(q_T, z)$$

$$\theta_{0,s}^B(b, z) \Rightarrow \theta_{0,s}^B(b, z) S_{abs}(b, z) \longrightarrow S_{abs}(b) = 1 - \text{Im } f_{el}(b)$$

**And back to momentum representation**

# Absorptive corrections

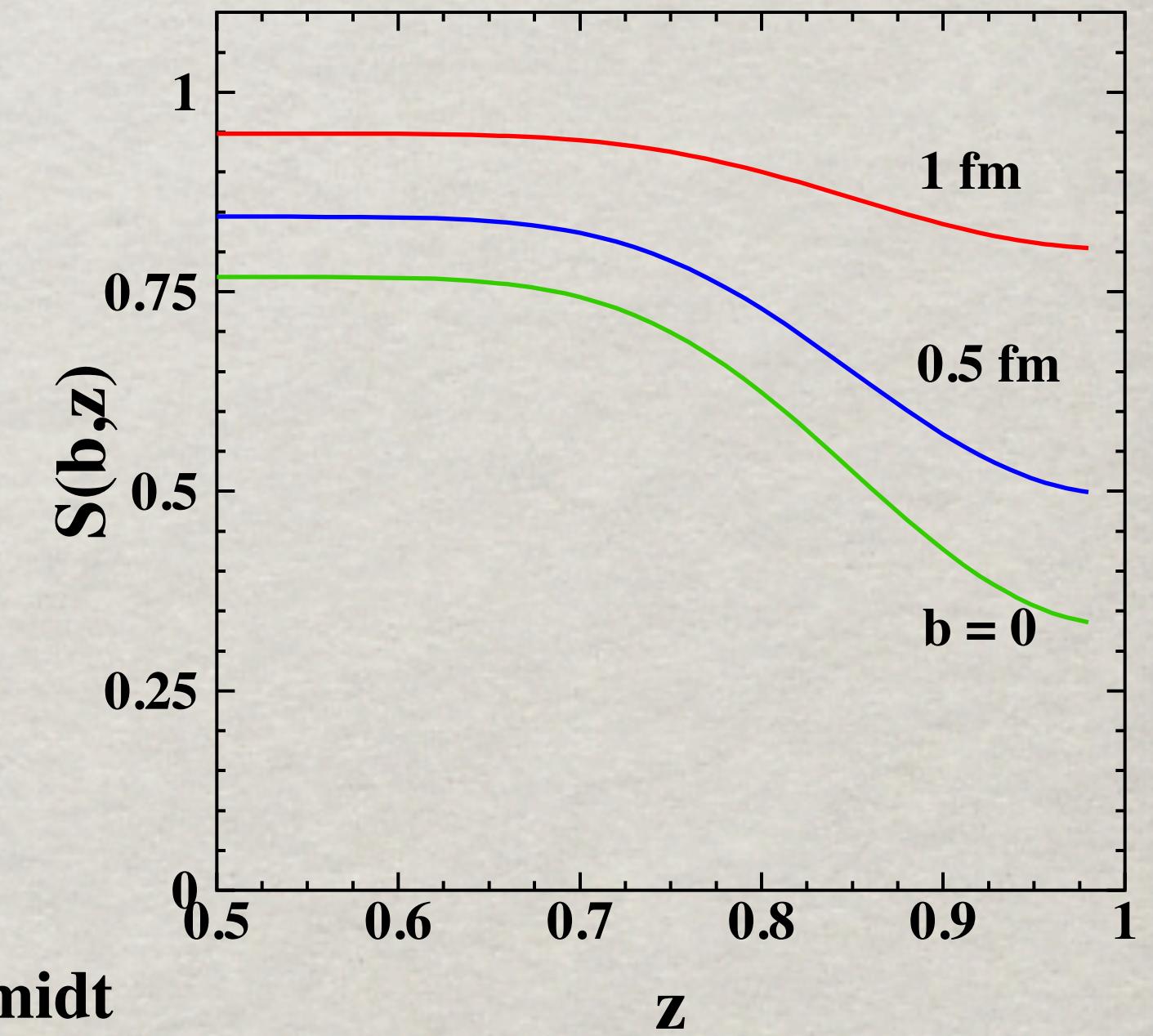


$$S(b) = S_{4q}(b) F_N(q_L) + S_{\gamma^*}(b) [1 - F_N(q_L)]$$

$$F_N(q_L) = (1 + q_L^2 L^2)^{-1}$$

$$L = 1 \text{ fm}$$

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# Other Reggeons

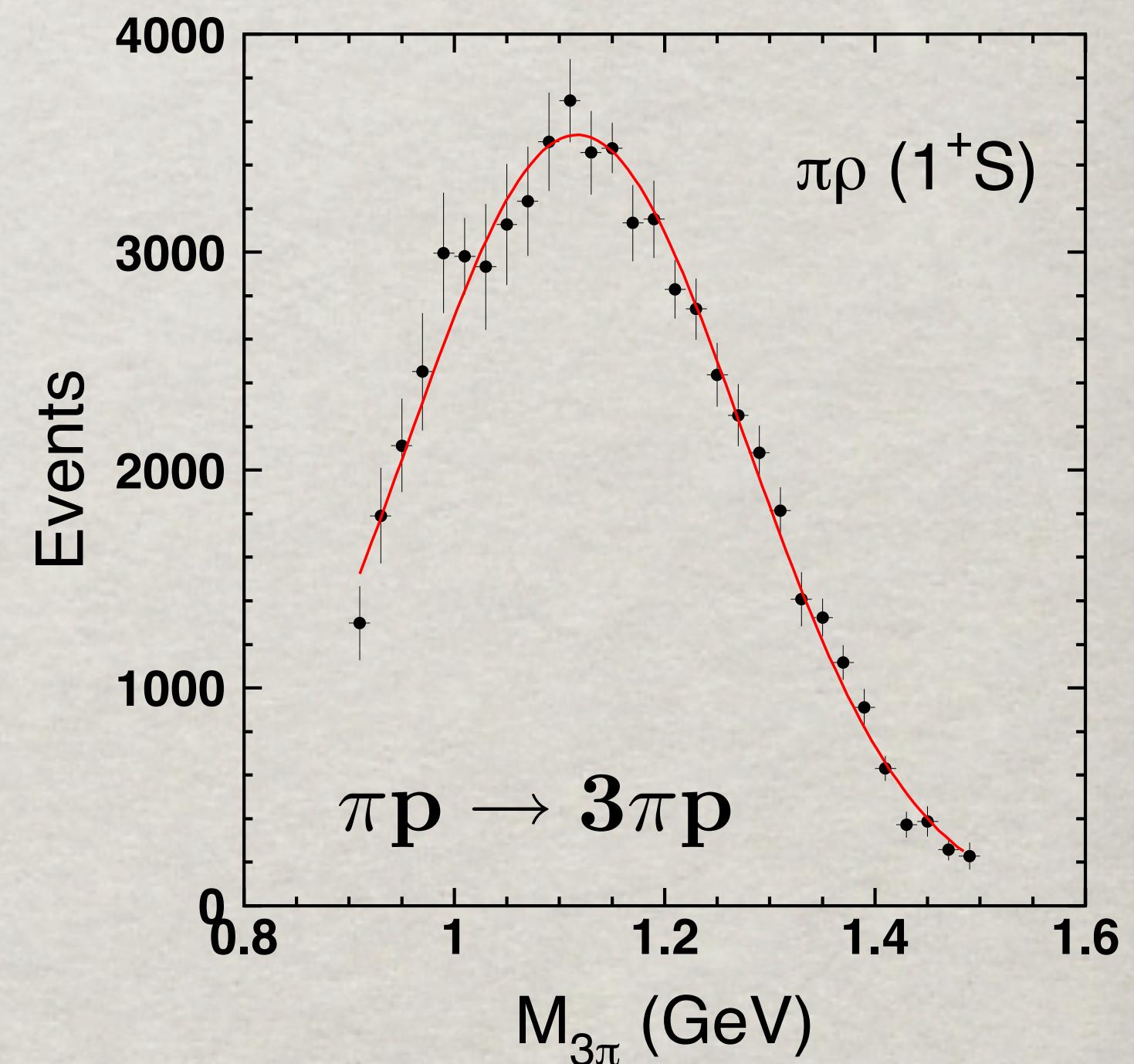
The natural parity Reggeons  $\rho$  and  $a_2$  are well fixed by the Regge phenomenology. They are spin-flip and important for neutron production at  $z \rightarrow 1$ , because have higher intercepts.

$a_1$  is a very weak pole in the dispersion relation, no  $a_1$ -dominance in the axial current. The  $\pi\rho$  cut can be treated as an effective  $\tilde{a}_1$ -pole.

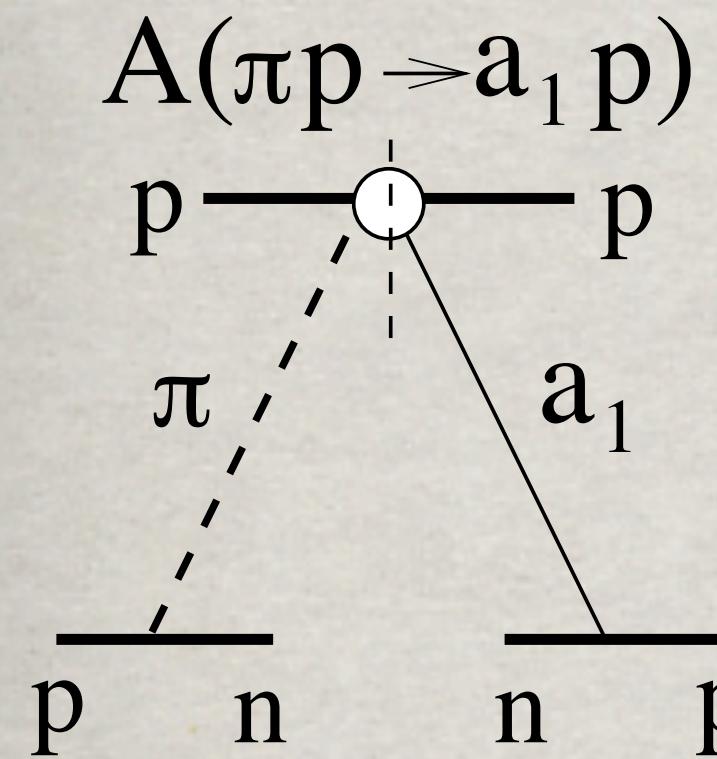
PCAC and the 2d Weinberg sum rule lead to

$$\frac{g_{aN\bar{N}}}{g_{\pi N\bar{N}}} = \frac{m_a^2 f_\pi}{2 m_N f_\rho} \approx 0.5$$

$$f_a = f_\rho = \frac{\sqrt{2} m_\rho^2}{\gamma_\rho}$$

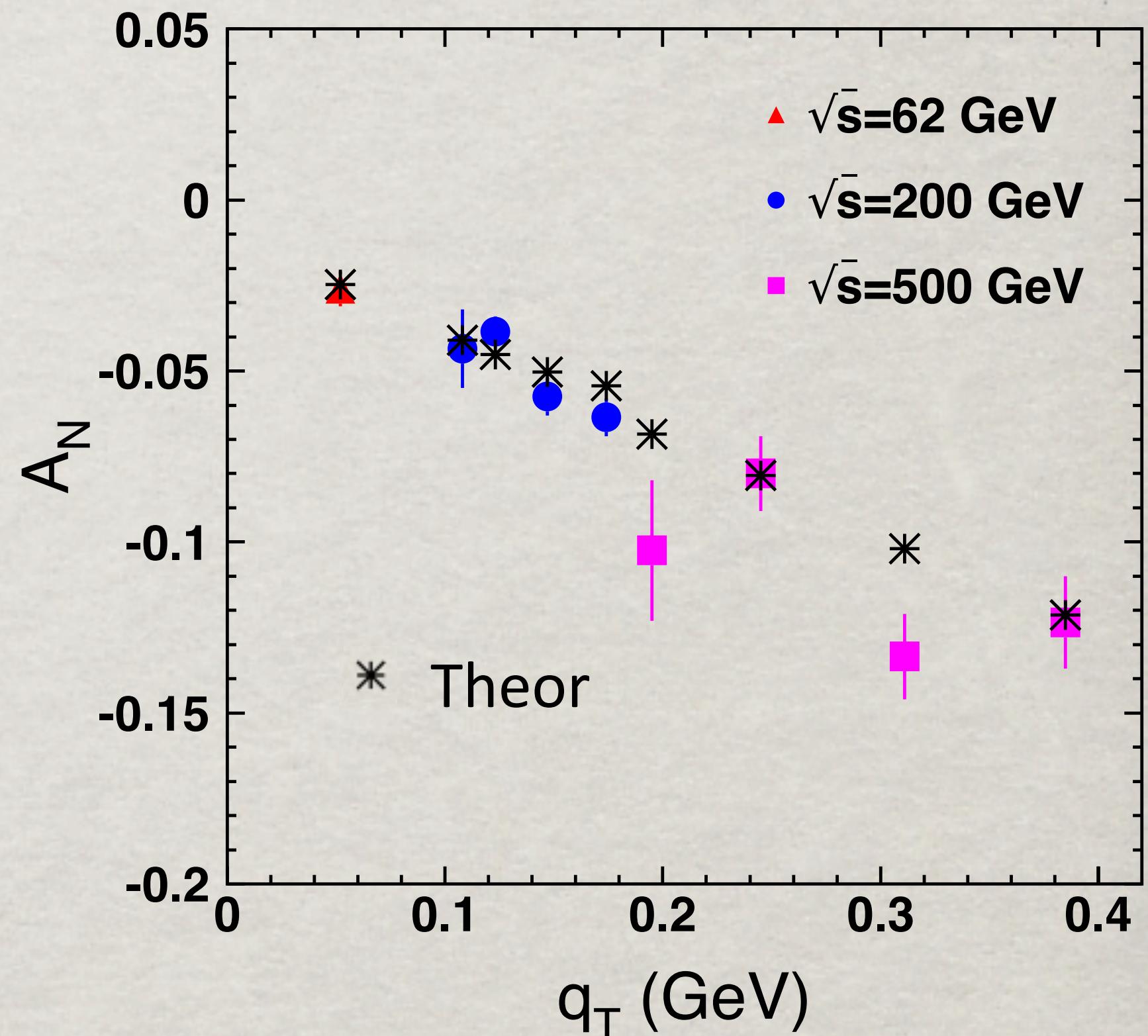
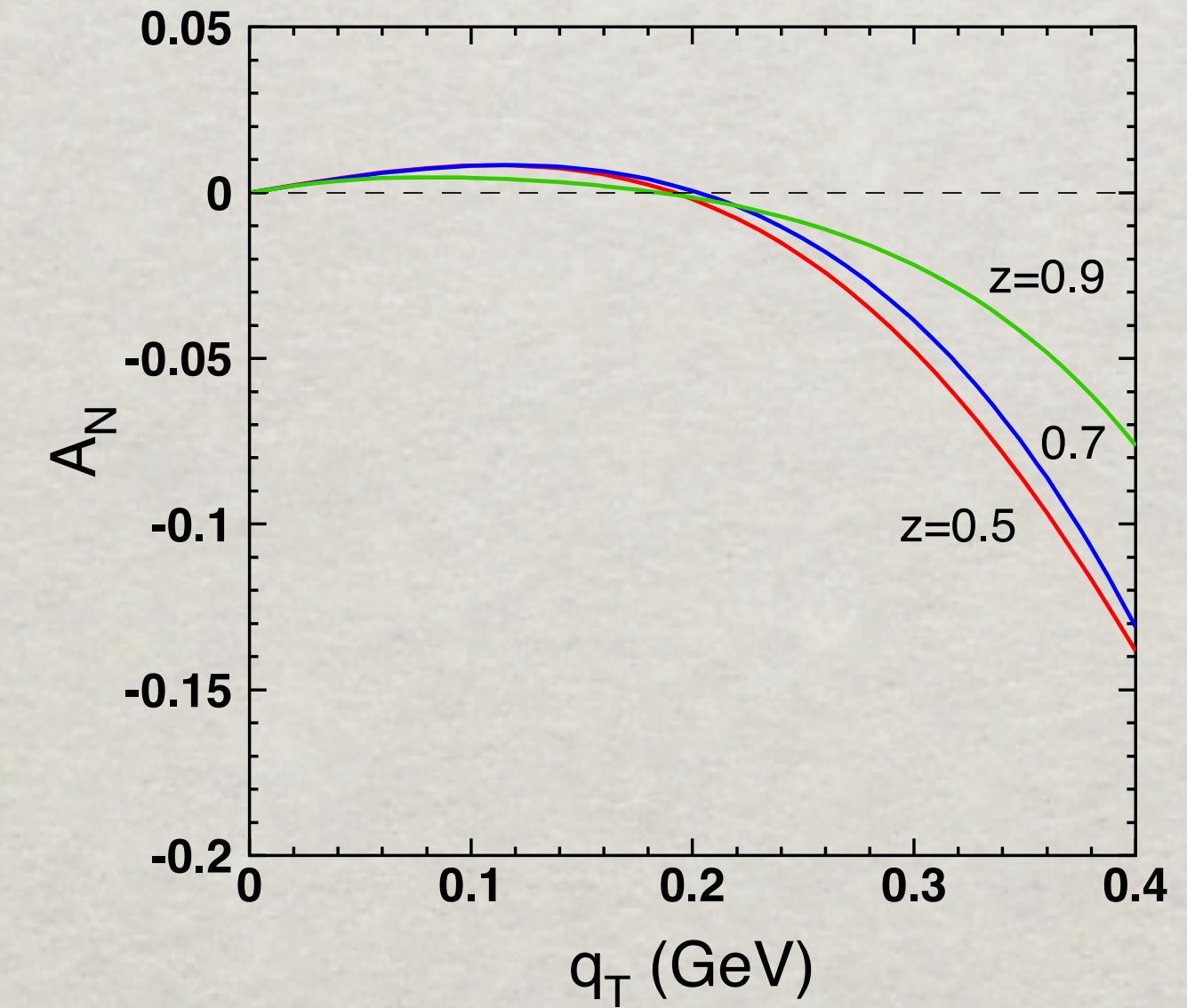


# Other Reggeons



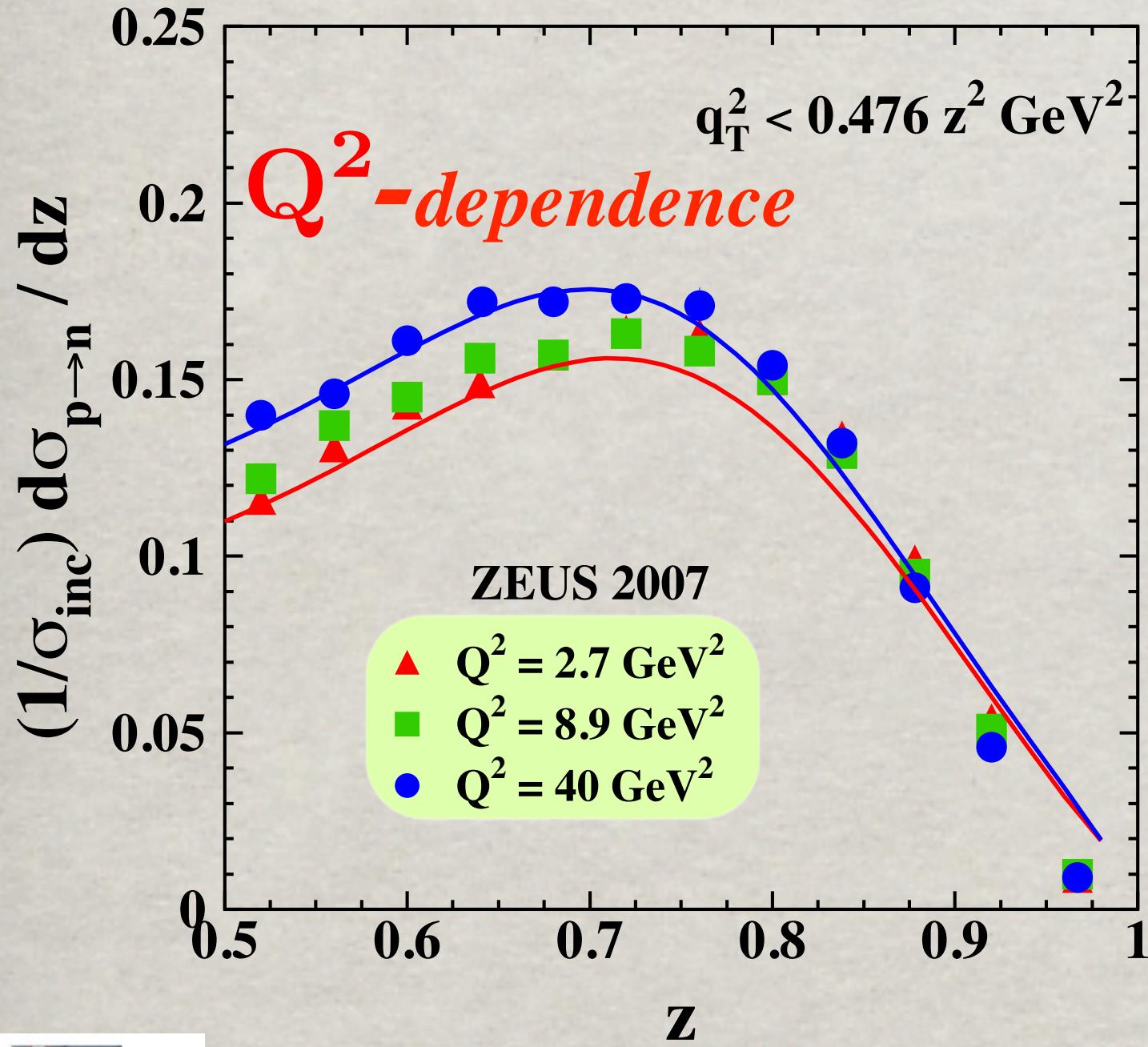
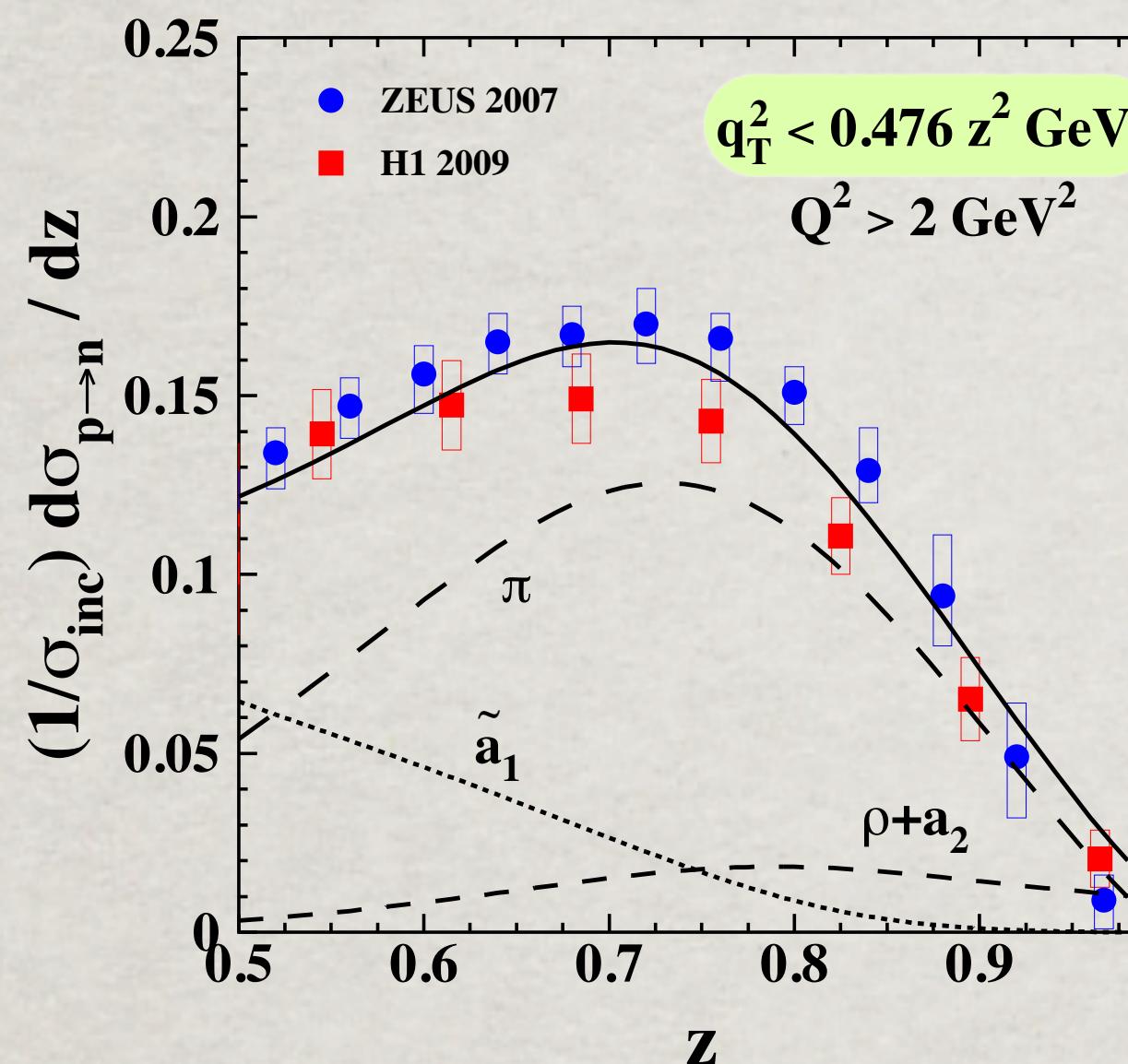
With such a coupling  $\pi - \tilde{a}_1$  interference well explains PHENIX data on azimuthal asymmetry of neutrons.

Neither the pion pole, nor adding the absorptive corrections can explain the observed asymmetry.

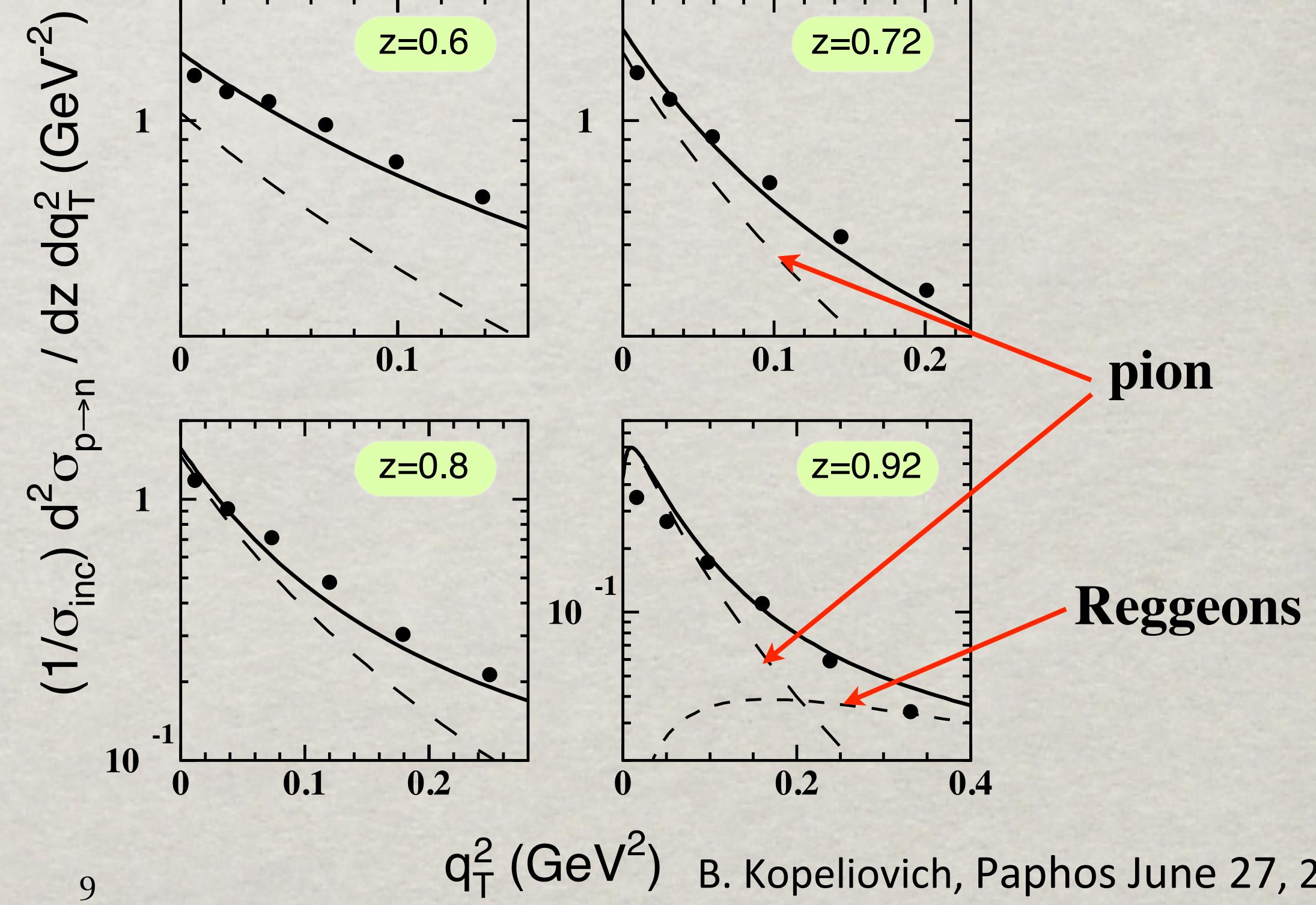


# Results

*z-dependence*



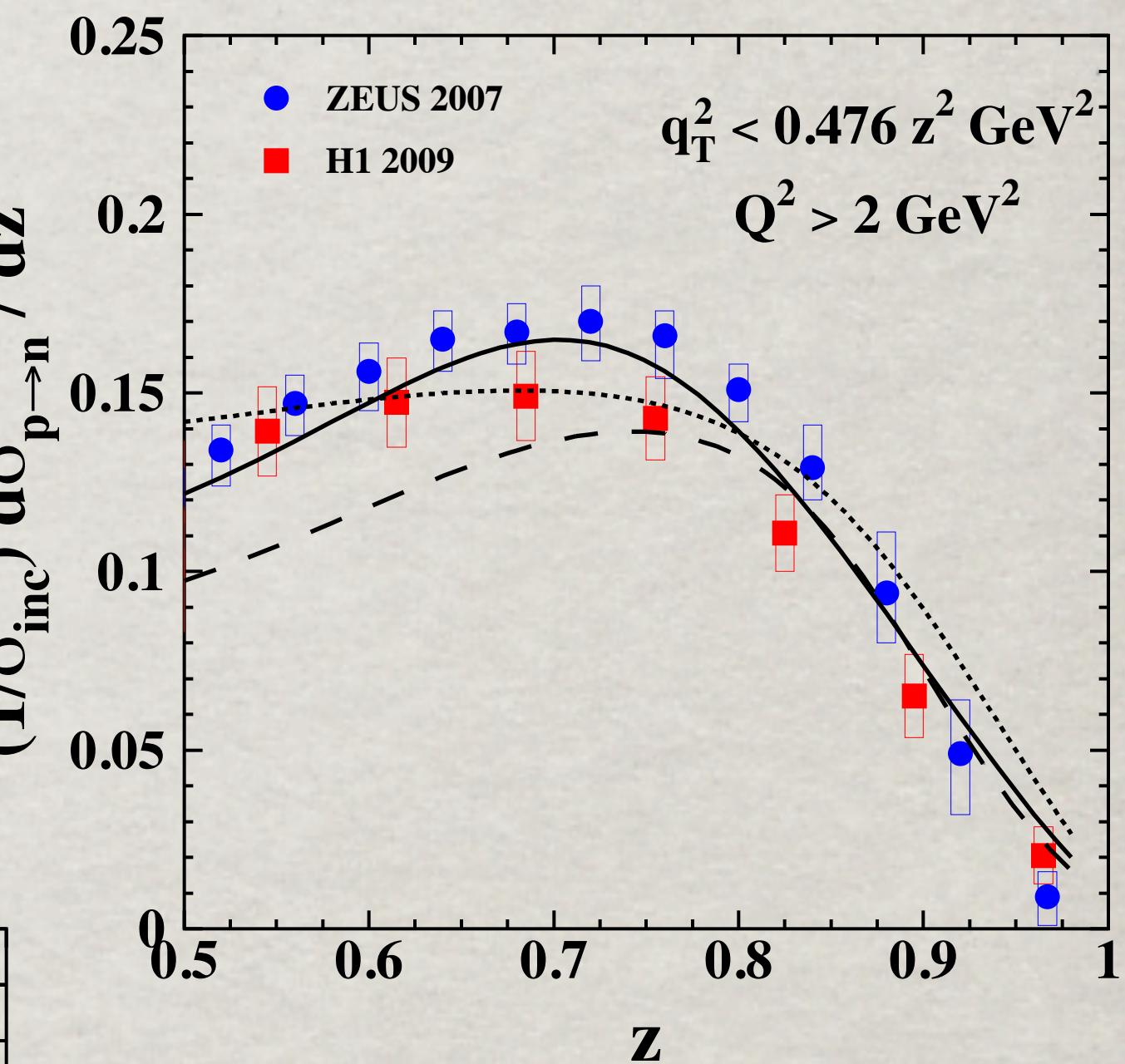
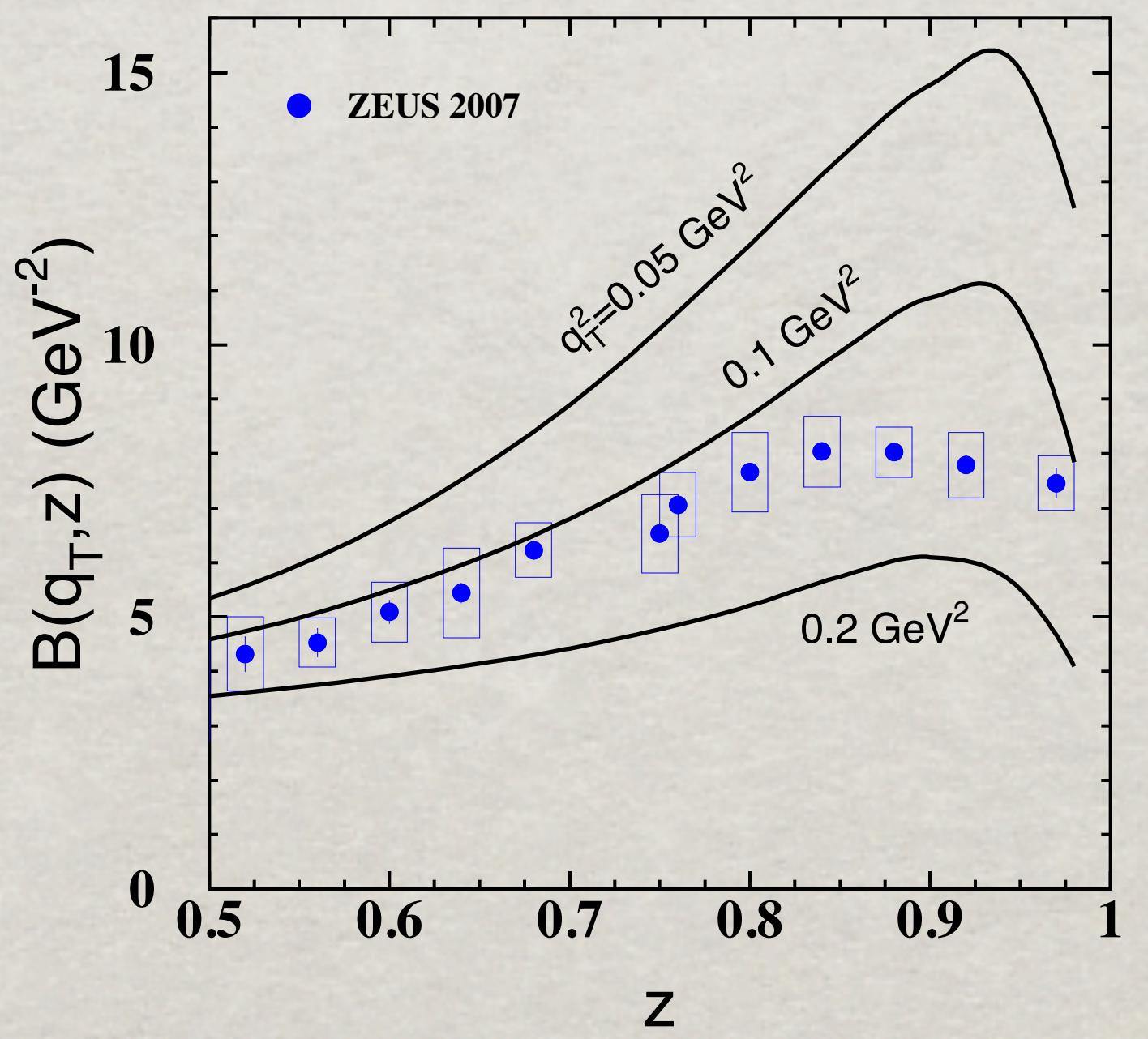
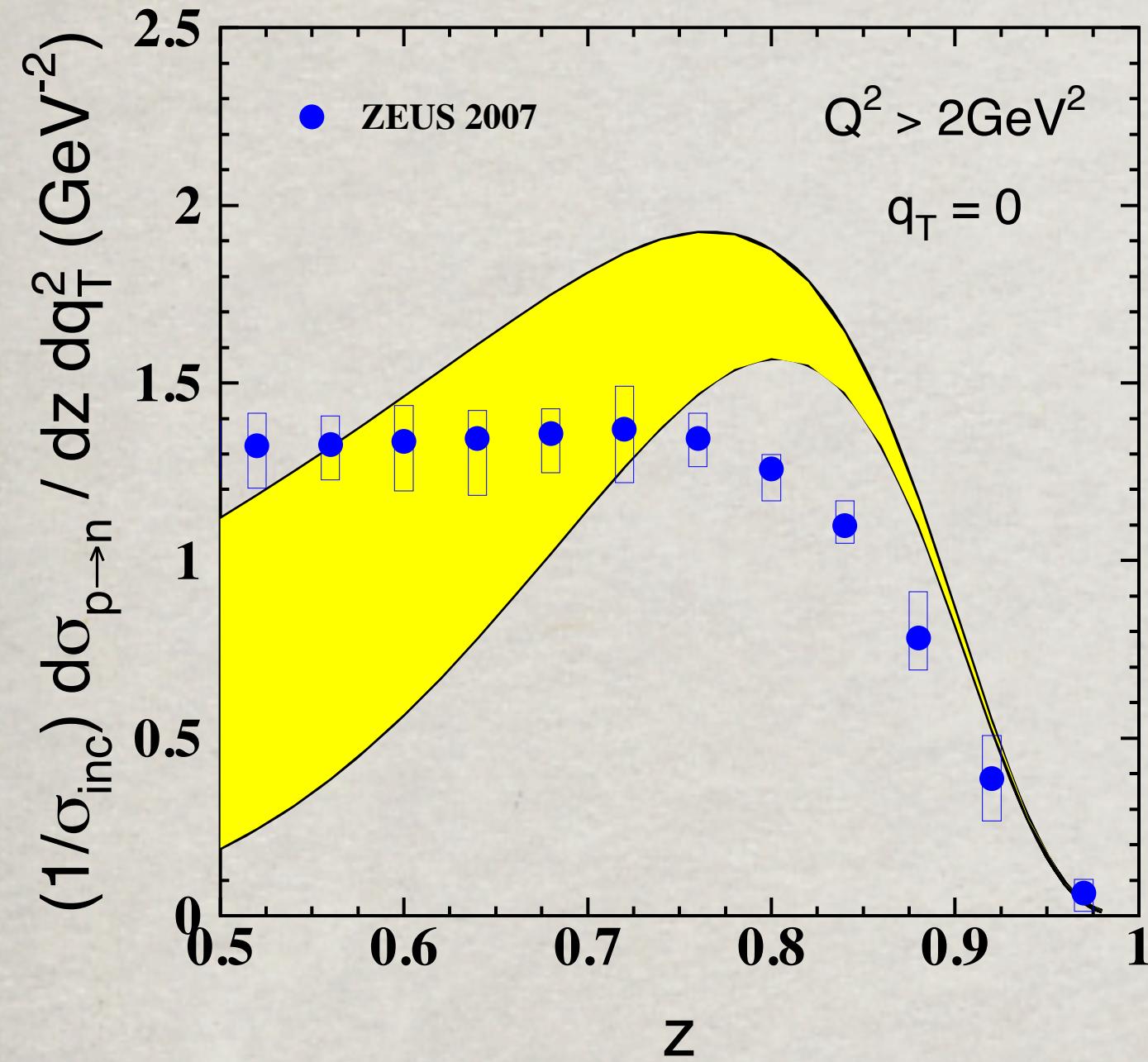
*q<sub>T</sub>-dependence*



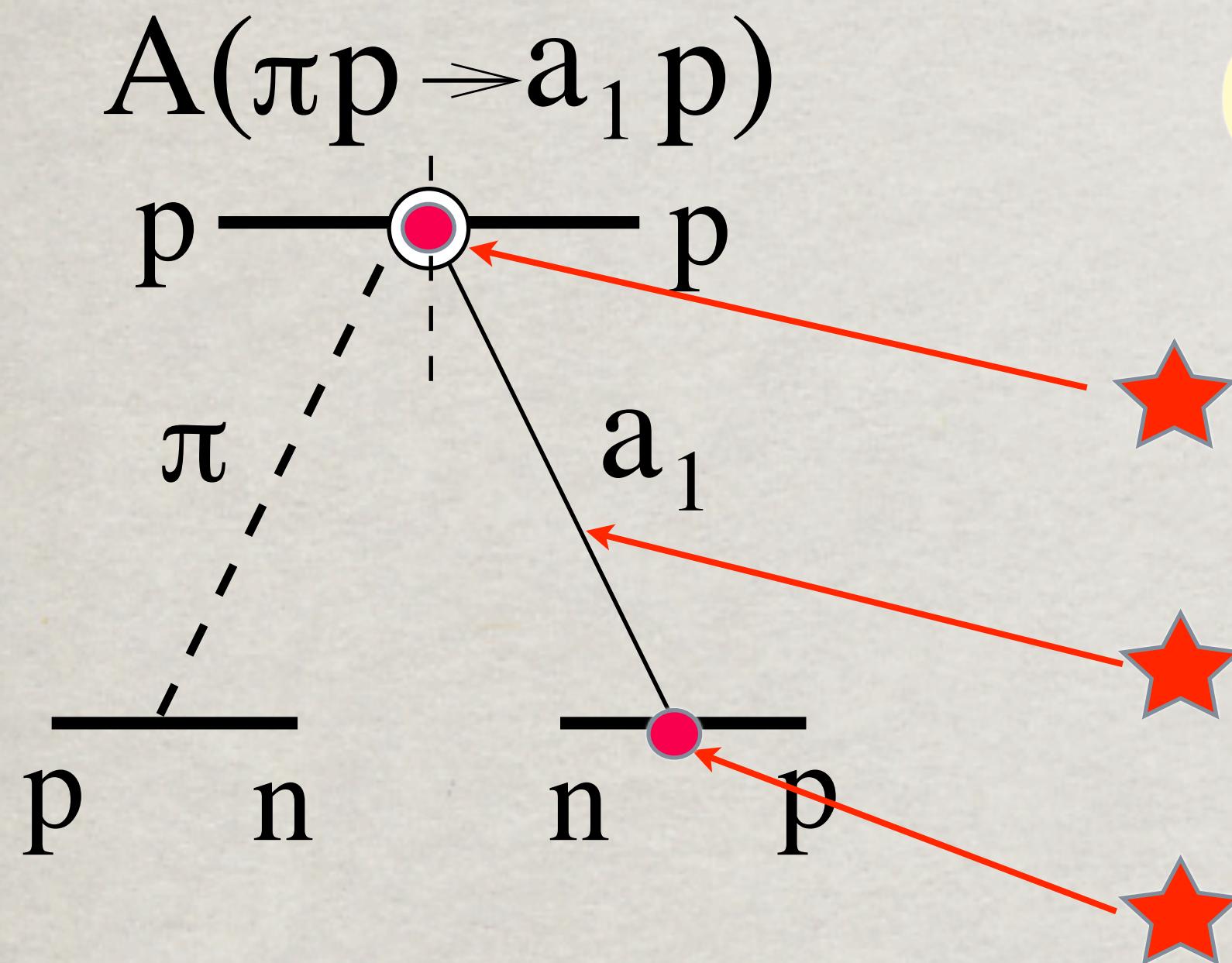
# Summary

- Starting the bottom-up description with a plausible assumption  $R_{\pi/p} = F_2^\pi(x, Q^2)/F_2^p(x, Q^2) = 1/2$  we reached a good agreement with DIS data, basing on model dependent calculations of the absorptive corrections and iso-vector Reggeons.
- The top-down strategy, determination of the pion structure function starting from DIS data on leading neutron production, is supposed to be least model dependent. However, the necessity of correcting for absorption brings a considerable theoretical uncertainty. Even with a plausible assumption that  $F_2^\pi(x, Q^2) \propto F_2^p(x, Q^2)$  the coefficient  $R_{\pi/p}$  can be extracted from data with a theoretic uncertainty of 20-30%.

# Backups



# Backups



Three unknowns:

★  $A(\pi p \rightarrow a_1 p) = \sqrt{d\sigma(\pi p \rightarrow a_1 p)/dq_T^2|_{q_T=0}}$

Regge trajectory  $\alpha_{a_1}(t)$

$a_1$ -nucleon coupling  $g_{a_1 np}$

$$A_N^{(\pi-a_1)}(q_T, z) = q_T \frac{4m_N q_L}{|t|^{3/2}} (1-z)^{\alpha_\pi(t) - \alpha_{a_1}(t)} \frac{\text{Im } \eta_\pi^*(t) \eta_{a_1}(t)}{|\eta_\pi(t)|^2}$$

$$\times \left( \frac{d\sigma_{\pi p \rightarrow a_1 p}(M_X^2)/dt|_{t=0}}{d\sigma_{\pi p \rightarrow \pi p}(M_X^2)/dt|_{t=0}} \right)^{1/2} \frac{g_{a_1^+ pn}}{g_{\pi^+ pn}}$$

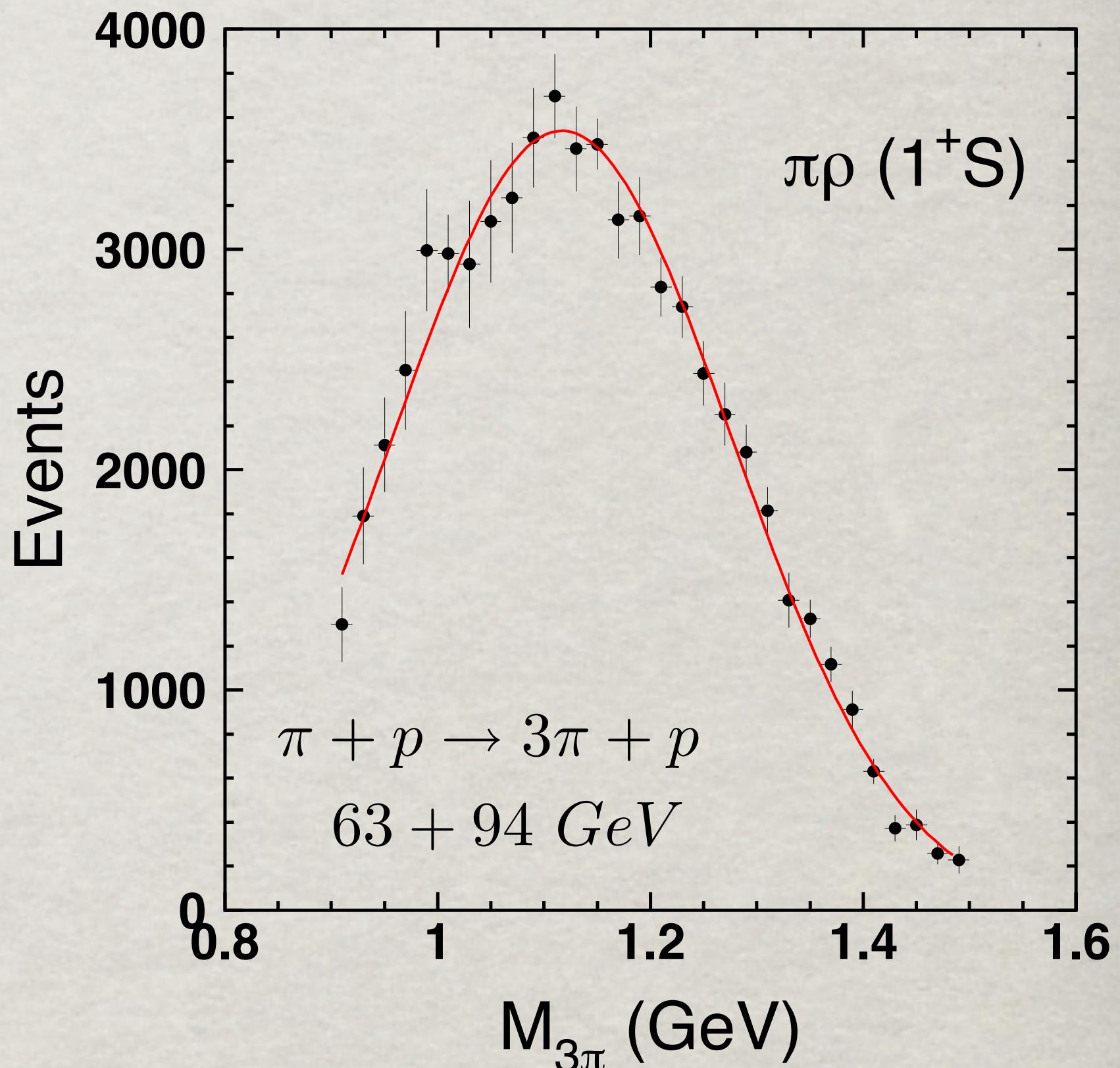
# Backups

The  $a_1$  is a very weak pole: no axial-vector dominance for the axial current.

Nevertheless, the invariant mass distribution of diffractively produced  $\pi-\rho$  in  $1^+S$  state forms a peak, dominated by the Deck mechanism, with a similar position and width as  $a_1$ . This singularity in the dispersion relation can be treated as an effective pole "a" with mass  $m_a = 1.1 \text{ GeV}$ .

The cross section of  $\pi + p \rightarrow (\pi\rho)_{1^+S} + p$  was measured up to 94 GeV.

$$\frac{d\sigma_{\pi p \rightarrow ap}(E_{\text{lab}} = 94 \text{ GeV})}{dq_T^2} \Big|_{q_T=0} = 0.8 \pm 0.08 \frac{\text{mb}}{\text{GeV}^2}$$



Extrapolated to the RHIC energy range correcting for absorption.

# Backups

PCAC miraculously relates the pion-nucleon coupling with the axial constant

$G_A$  represents the contribution to the dispersion relation of all axial-vector states heavier than pion. Assuming dominance of the  $1^+S$  a-peak, we get

The dispersion integrals for vector and axial currents are related by the 2d Weinberg sum rule

Thus,

$$\frac{g_{aNN}}{g_{\pi NN}} = \frac{m_a^2 f_\pi}{2 m_N f_\rho} \approx 0.5$$

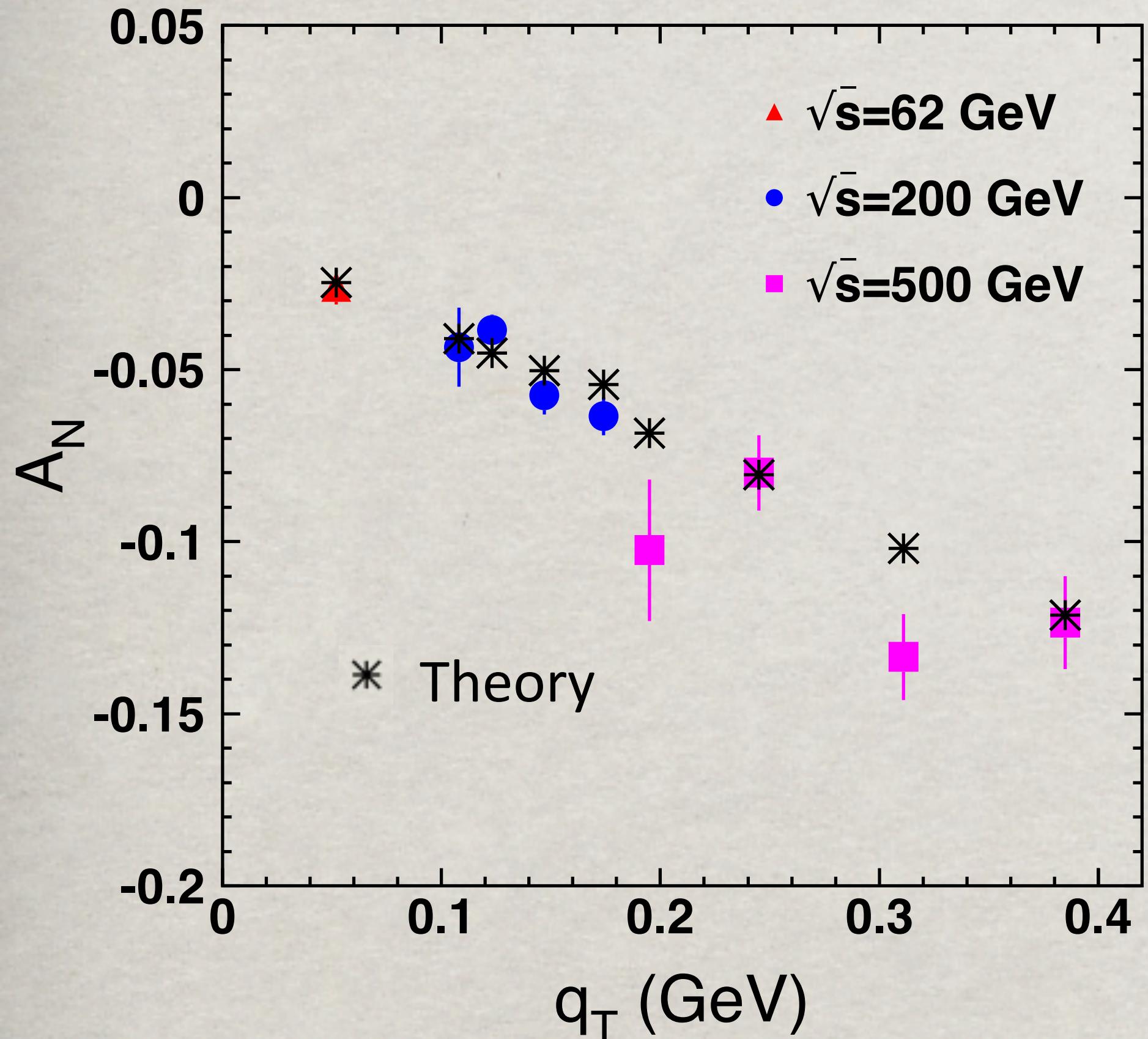
$$g_{\pi NN} = \frac{\sqrt{2} m_N G_A}{f_\pi}$$

Goldberger-Treiman relation

$$G_A = \frac{\sqrt{2} f_a g_{aNN}}{m_a^2}$$

$$f_a = f_\rho = \frac{\sqrt{2} m_\rho^2}{\gamma_\rho}$$

# Backups



The data agree well with independence of energy

$$A_N^{(\pi-a)}(q_T, z) = q_T \frac{4m_N q_L}{|t|^{3/2}} (1-z)^{\alpha_\pi(t)-\alpha_a(t)} \times \frac{\text{Im } \eta_\pi^*(t) \eta_a(t)}{|\eta_\pi(t)|^2} \left( \frac{d\sigma_{\pi p \rightarrow ap}(M_X^2)/dt|_{t=0}}{d\sigma_{\pi p \rightarrow \pi p}(M_X^2)/dt|_{t=0}} \right)^{1/2} \frac{g_{apn}}{g_{\pi pn}}$$

Theoretical uncertainty is not large, about 30%