Quantitative, model independent analysis of Geometrical Scaling in DIS

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What is geometrical scaling?

$$\sigma_{\gamma^* p}(x, Q^2) \quad \sim \quad \frac{F_2(x, Q^2)}{Q^2} = \mathcal{F}\left(Q^2/Q^2_{\mathrm{sat}}(x)\right)$$

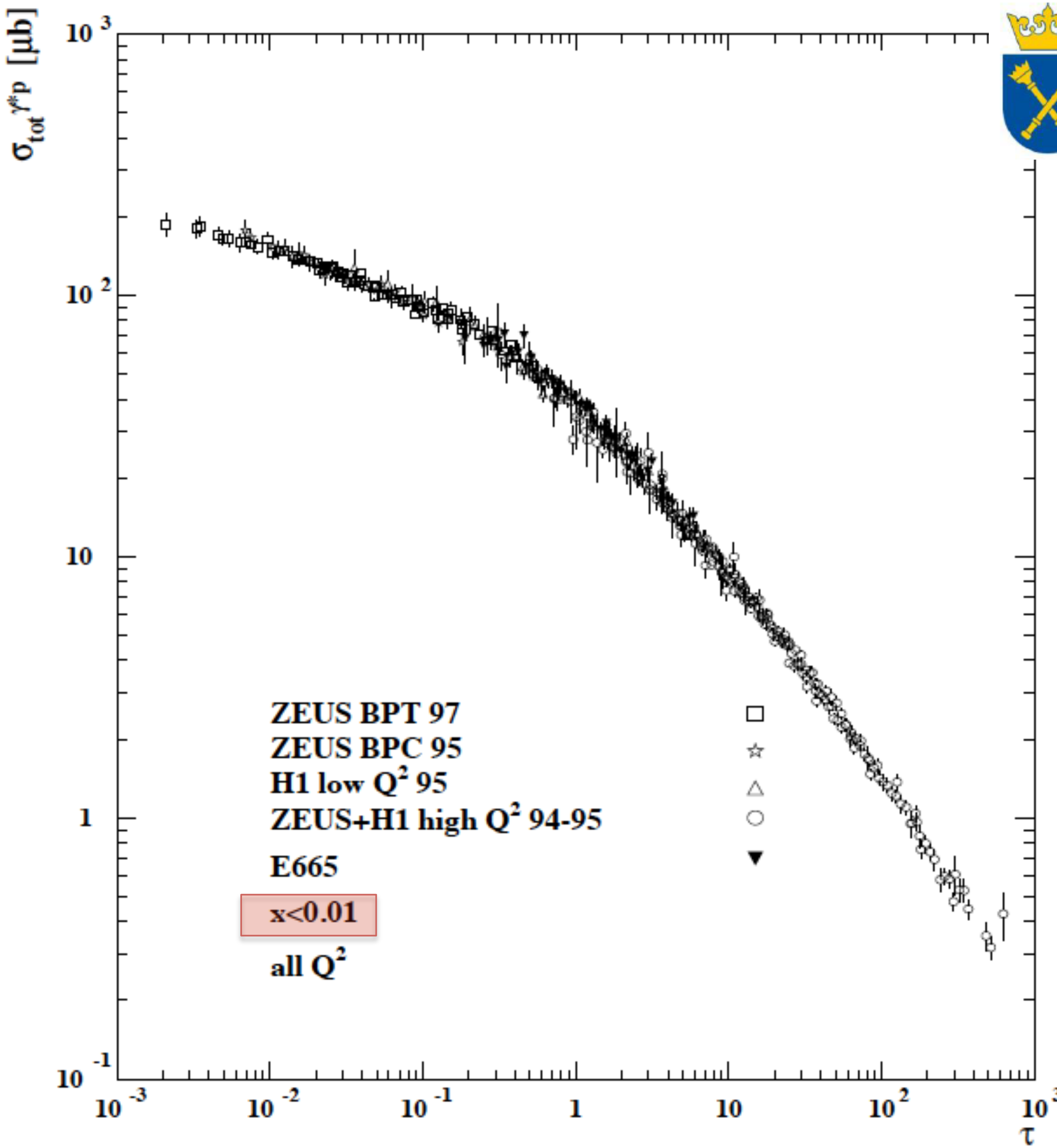
should hold for small x (large W) and any Q^2

$$Q^2_{\mathrm{sat}}(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{-\lambda}$$

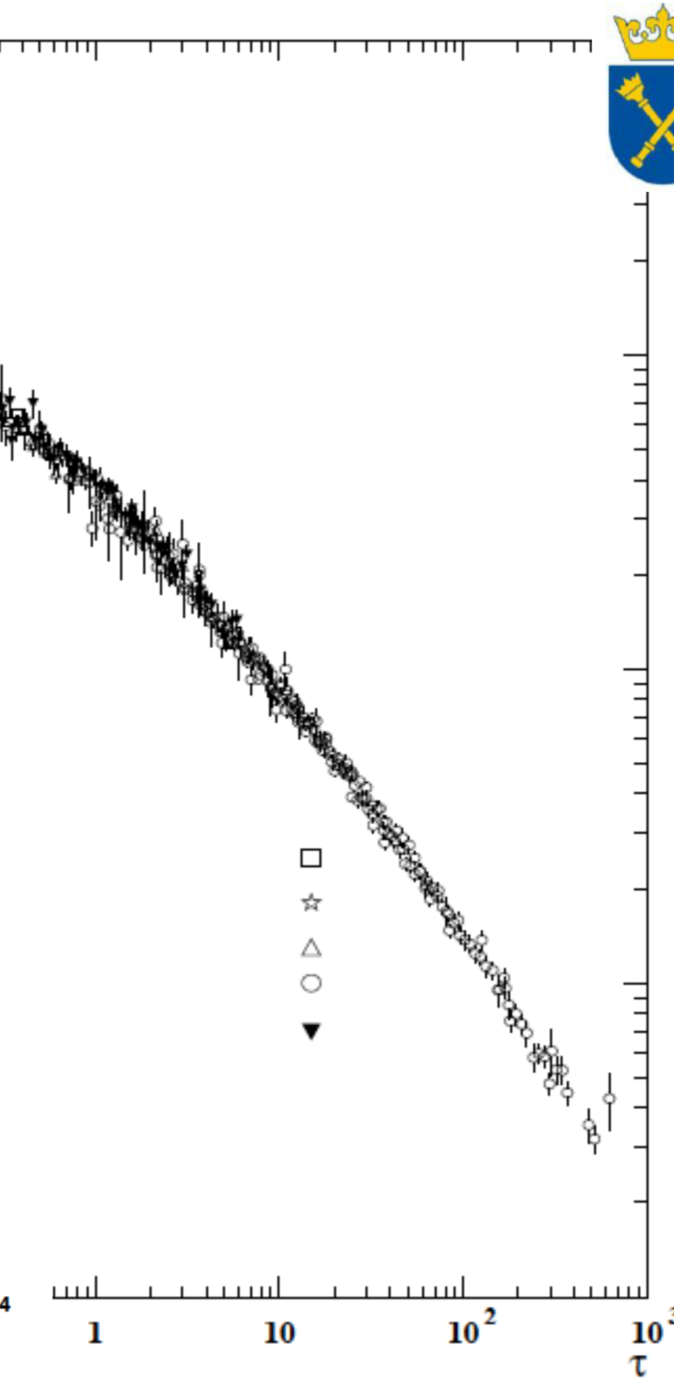
2000

Geometric scaling for the total $\gamma^* p$ cross section in the low x region

A. M. Staśto[(a,b)], K. Golec-Biernat[(b,c)] and J. Kwieciński[(b)]



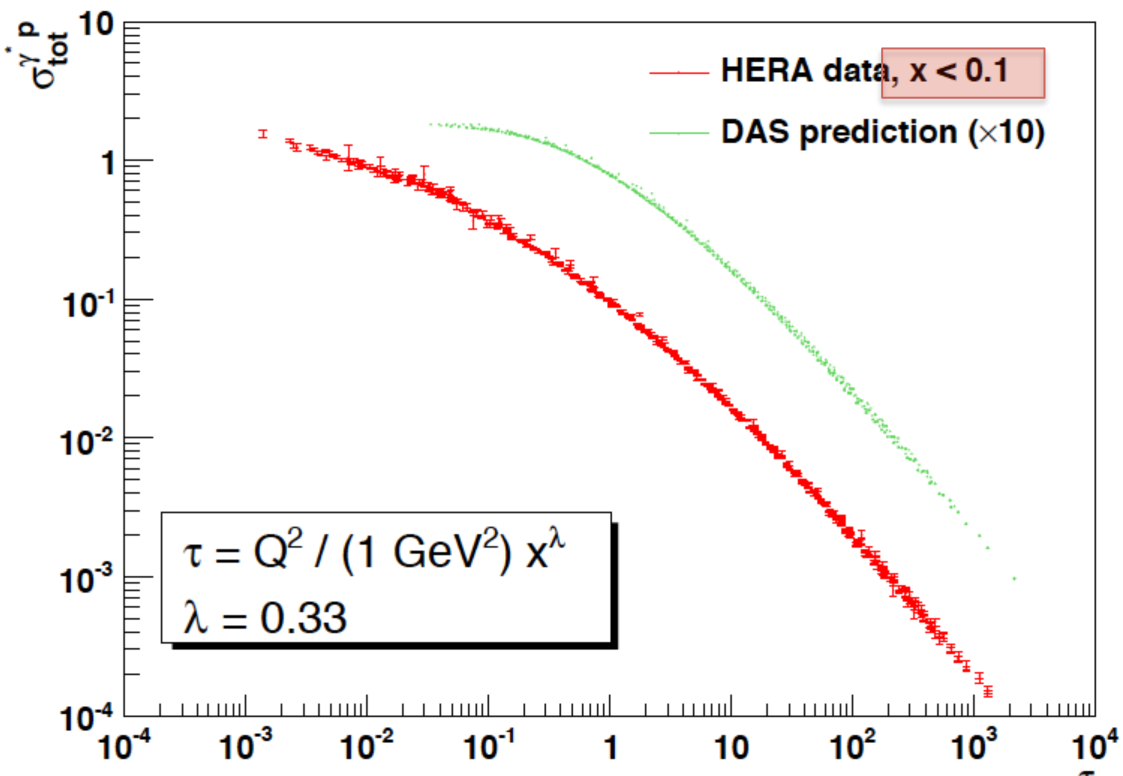
2000



HERA data and DGLAP evolution: theory and phenomenology

Fabrizio Caola, Stefano Forte and Juan Rojo,

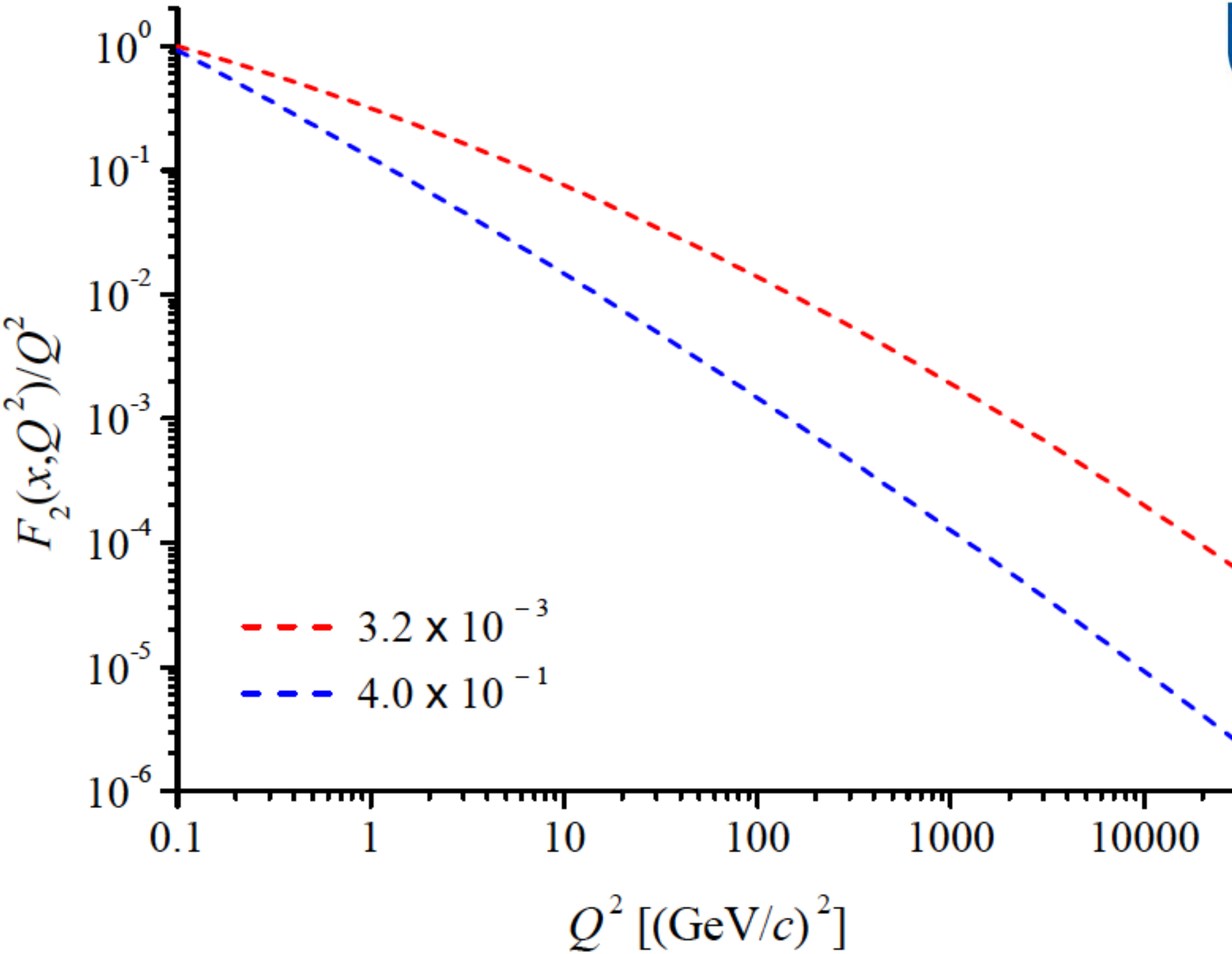
2010



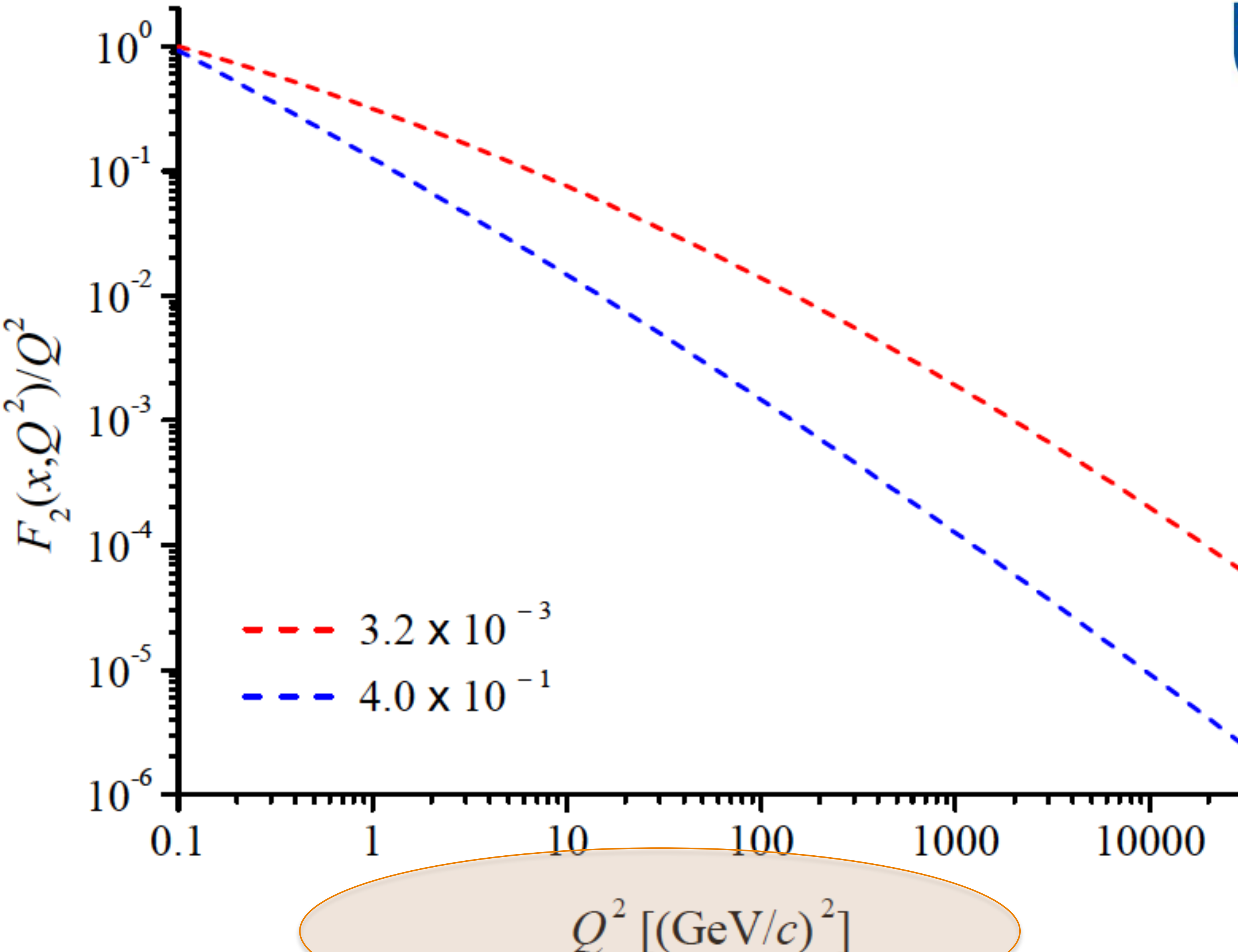
Can one establish in a model independent way:

- what is kinematical range where GS is working?
- what is the best value of exponent lambda?
- is lambda constant?

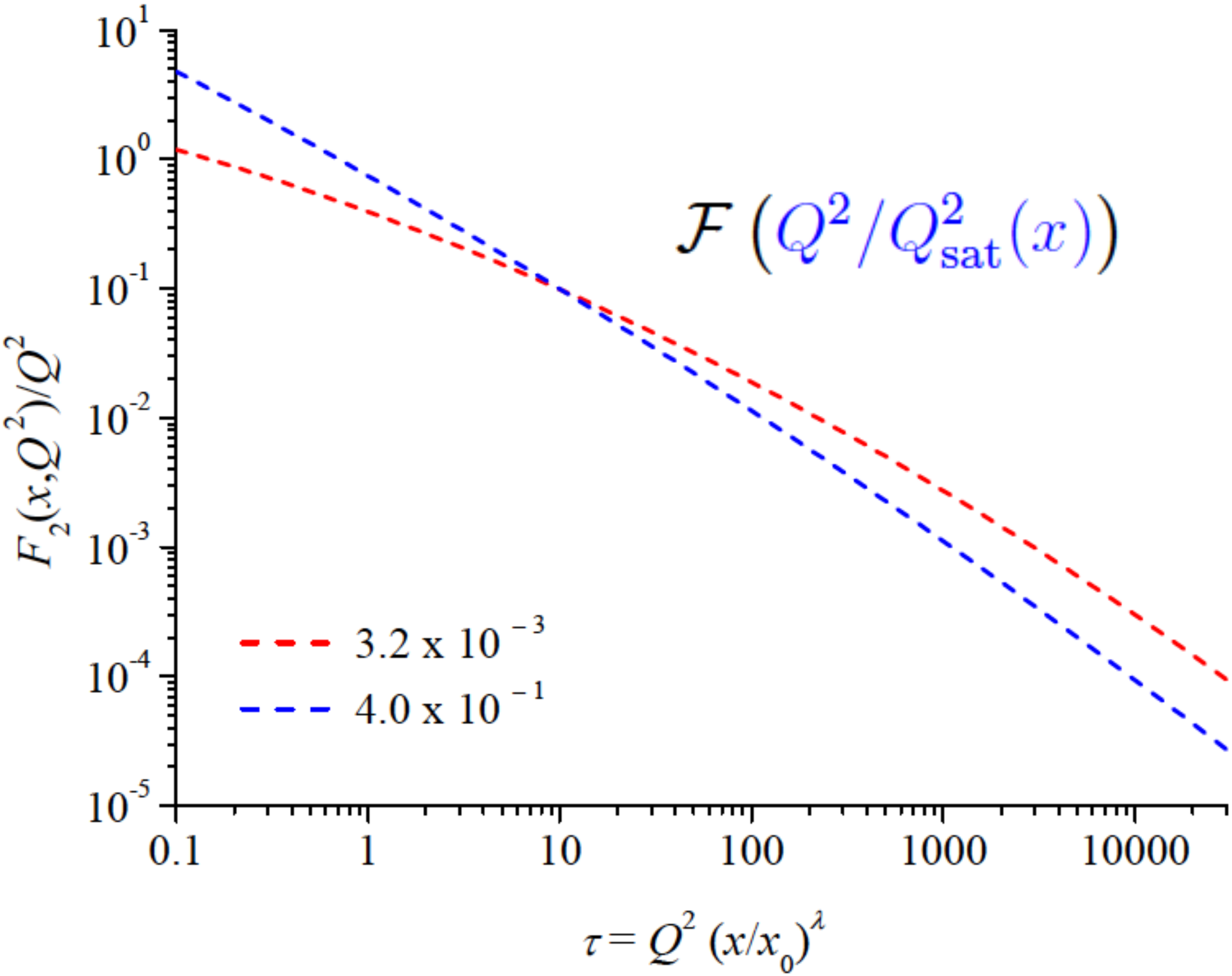
quantitaive criterion for GS

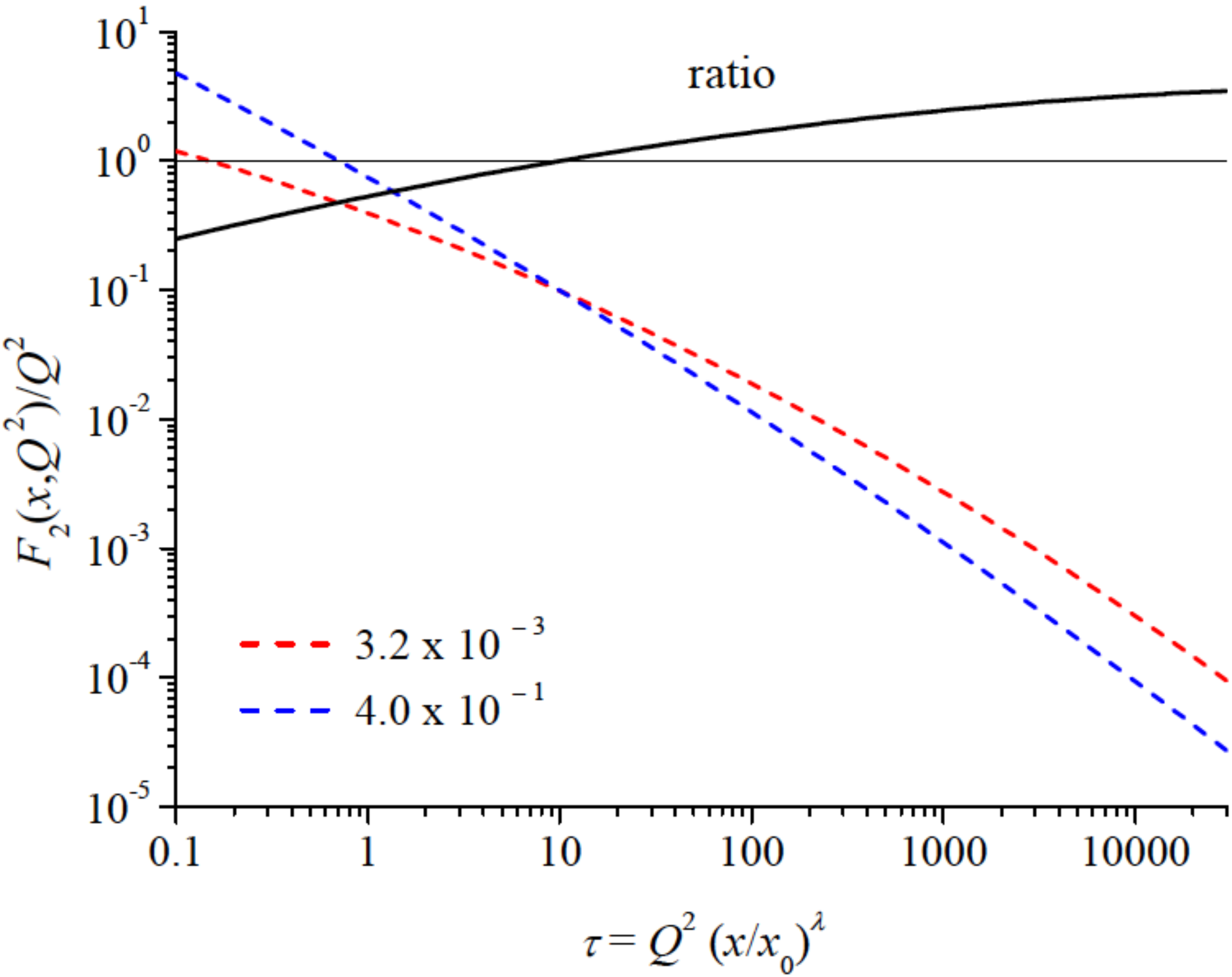


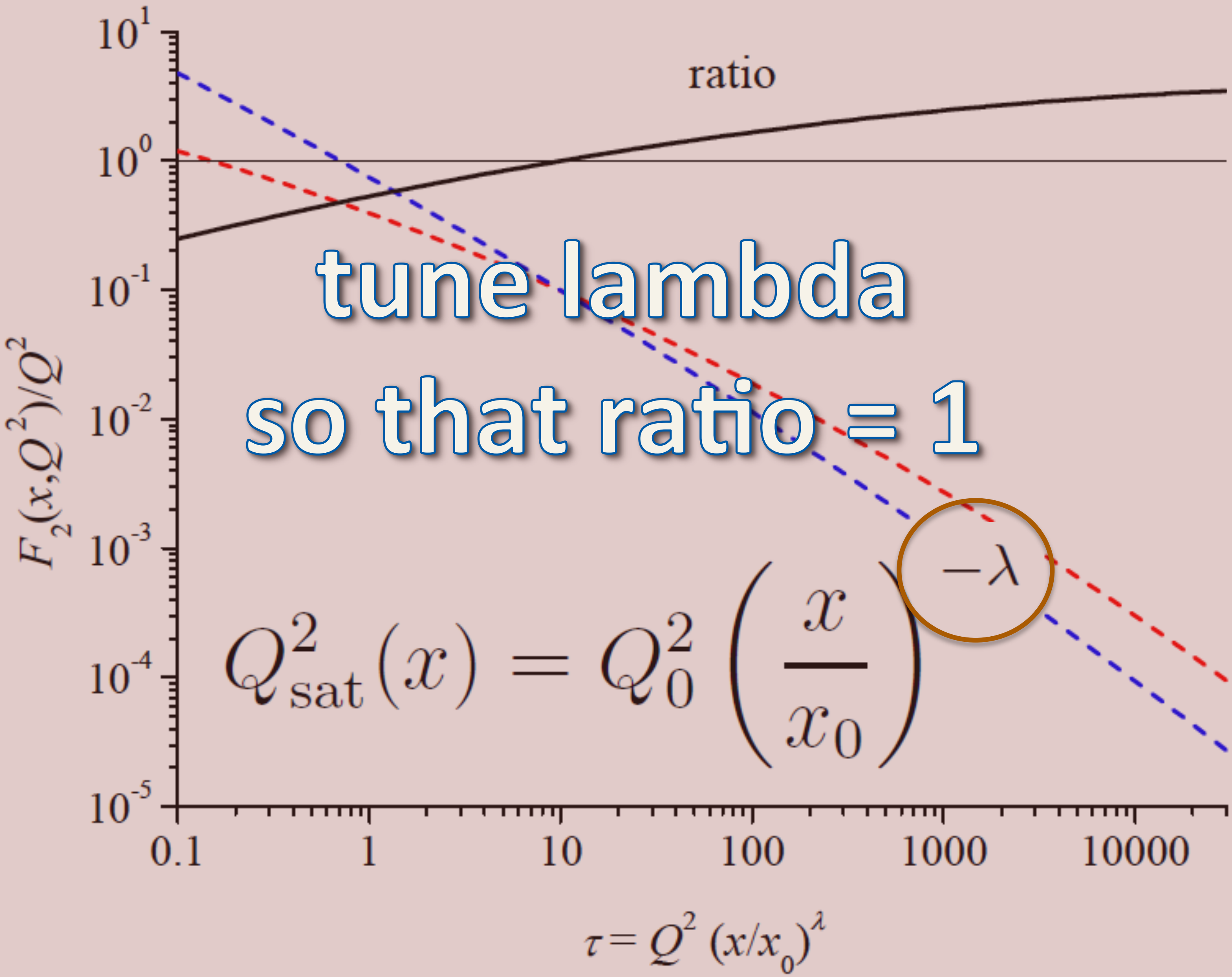
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$F_2(x,Q^2)/Q^2$
3.2×10^{-3}
4.0×10^{-1}
0.1
1
10
100
1000
10000
Q^2 [(GeV/c)^2]



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$F_2(x,Q^2)/Q^2$
3.2×10^{-3}
4.0×10^{-1}
0.1
1
10
100
1000
10000
Q^2 [(GeV/c)^2]

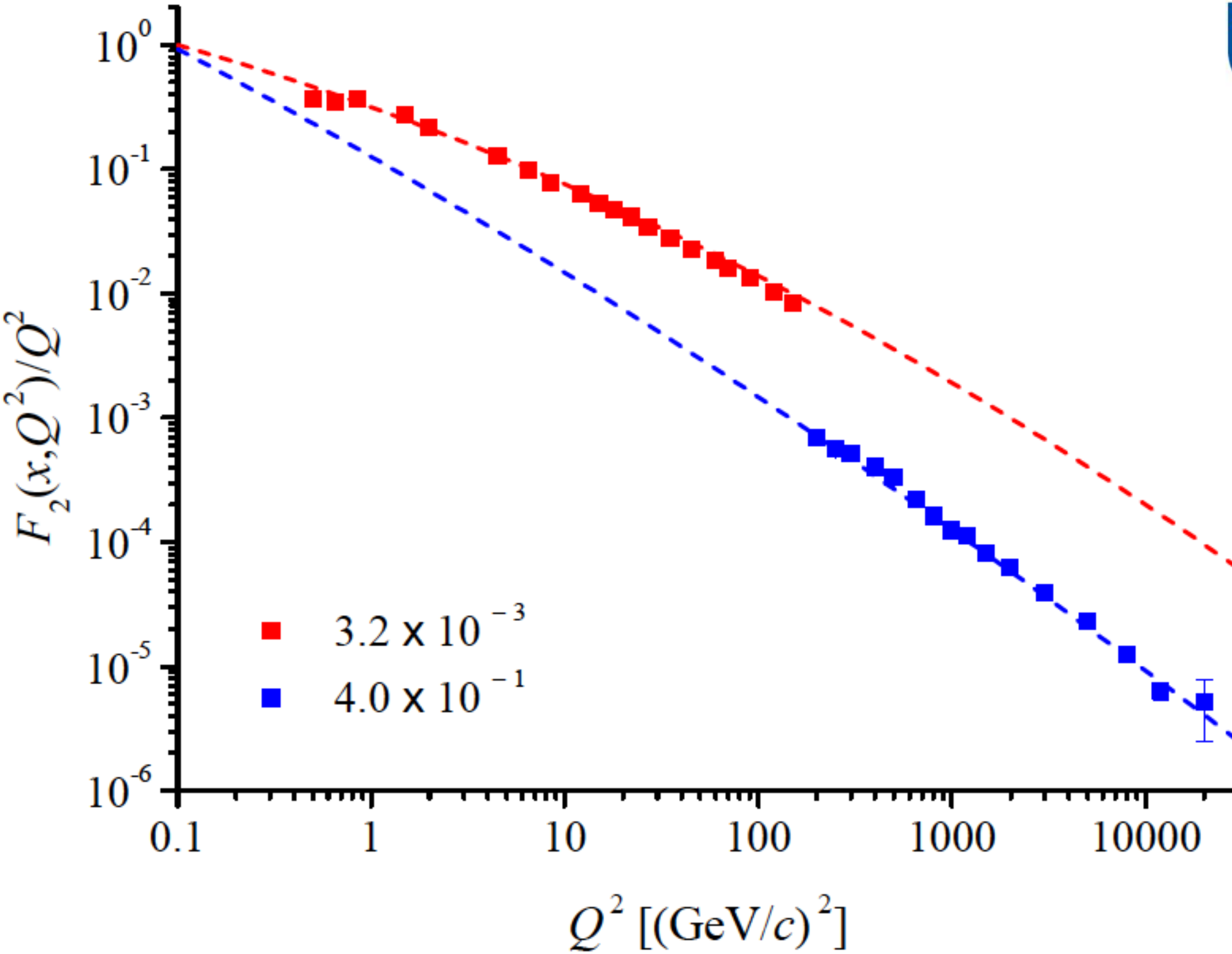

$\mathcal{F}\left(Q^2/Q^2_{\text{sat}}(x)\right)$
$F_2(x,Q^2)/Q^2$
$\tau = Q^2\,(x/x_0)^\lambda$
3.2×10^{-3}
4.0×10^{-1}
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.1
1
10
100
1000
10000


ratio
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$F_2(x,Q^2)/Q^2$
3.2 x 10 $^{-3}$
4.0 x 10 $^{-1}$
0.1
1
10
100
1000
10000
$\tau = Q^2\,(x/x_0)^\lambda$

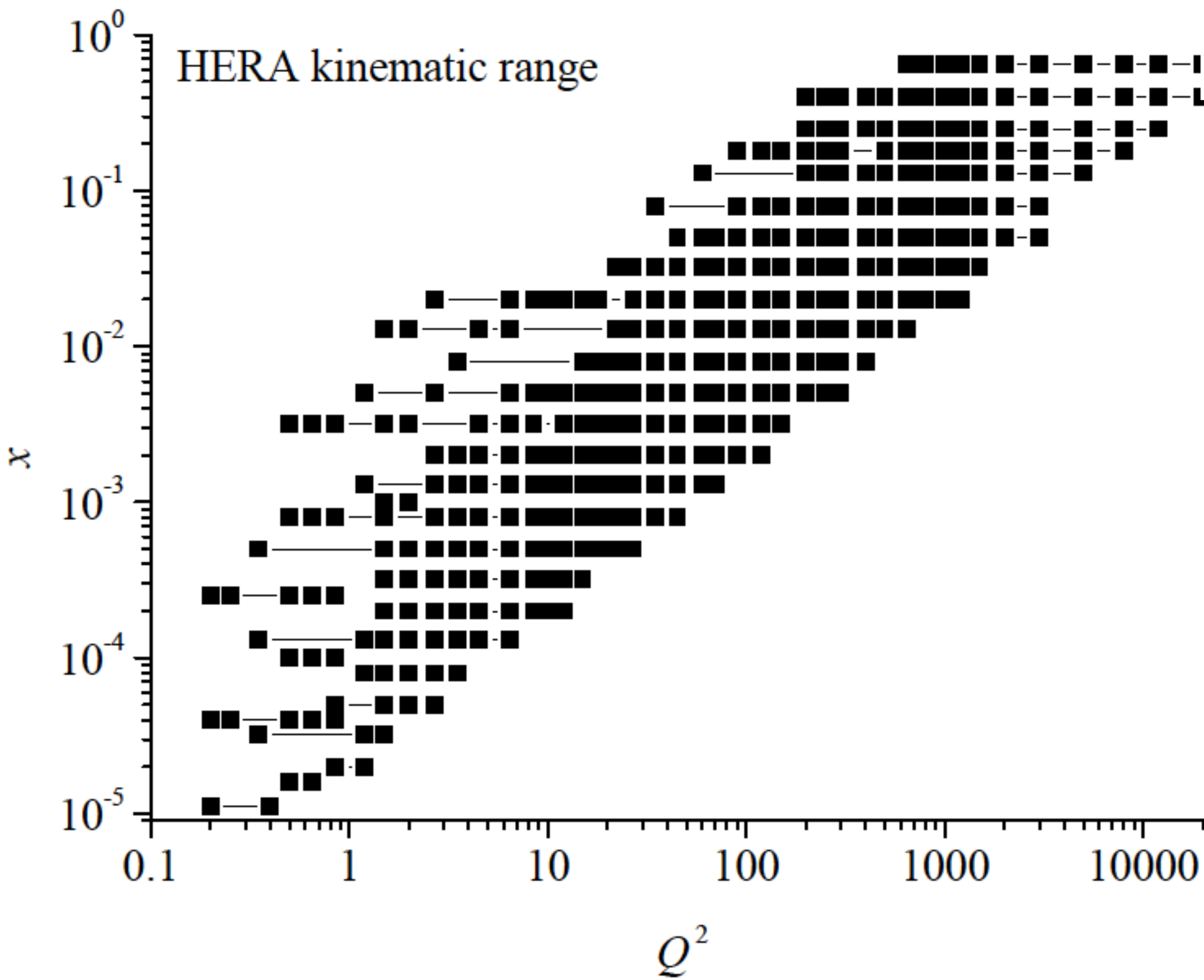


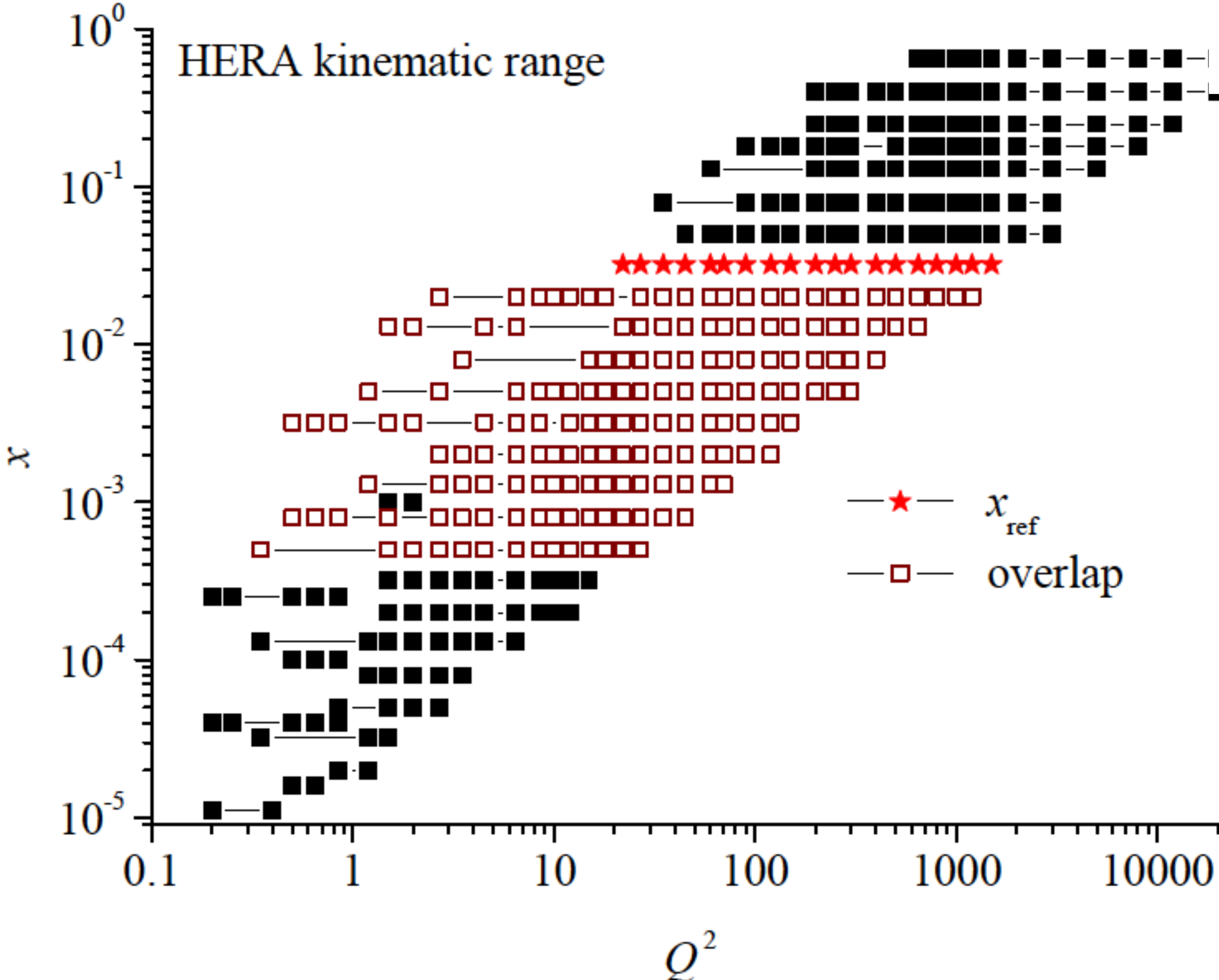
tune lambda so that ratio = 1

$$Q^2_{\rm sat}(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{-\lambda}$$

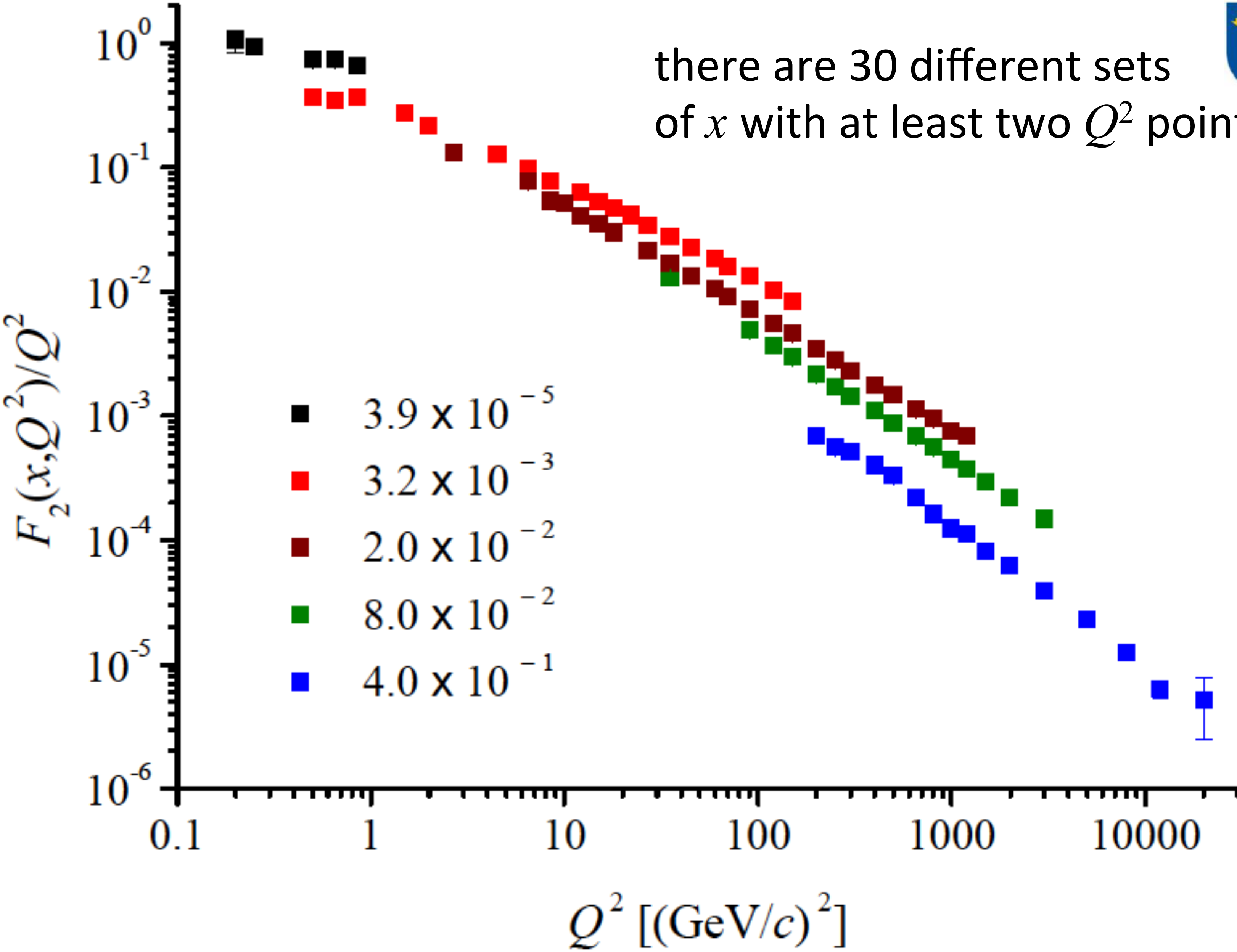


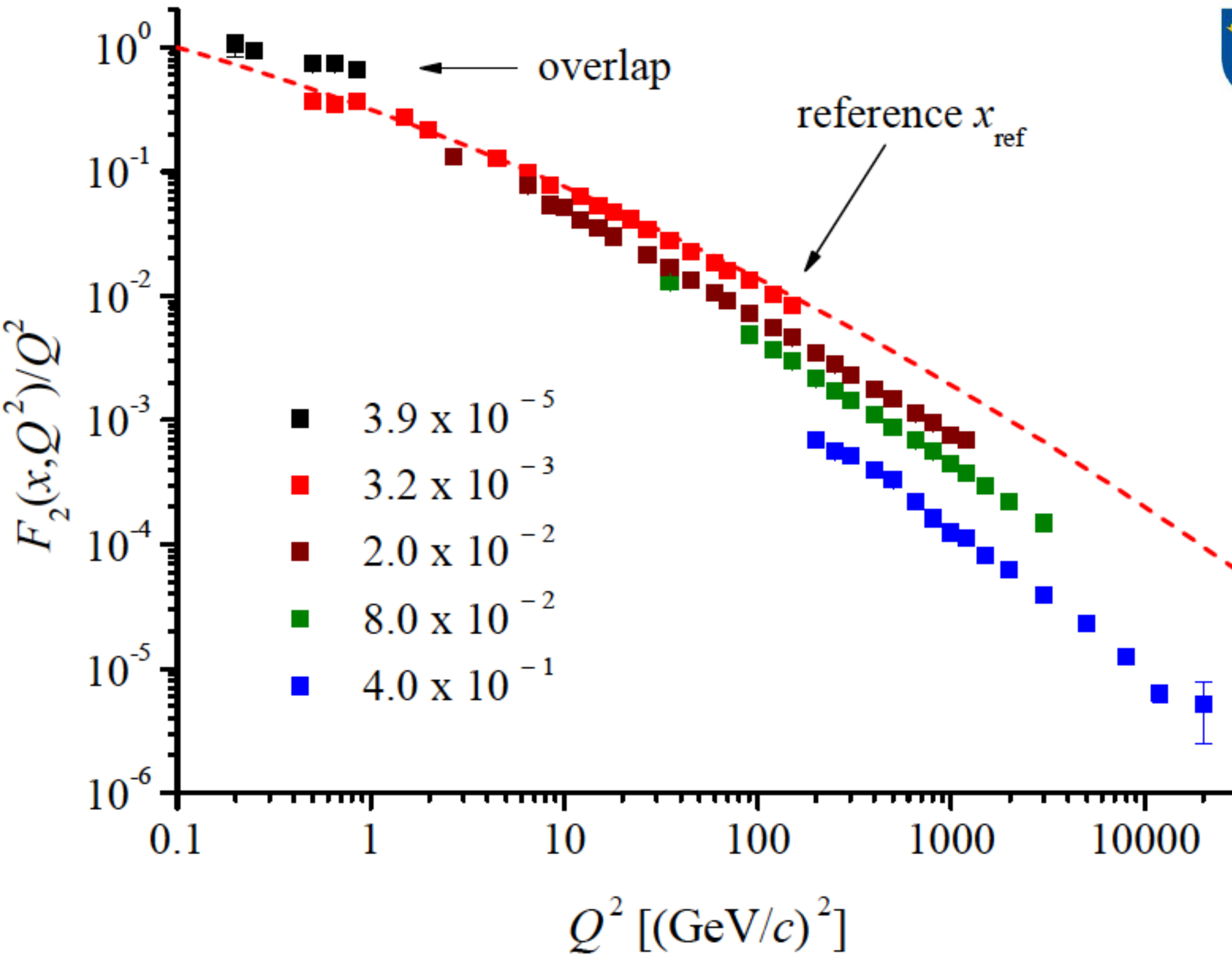
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$F_2(x,Q^2)/Q^2$
3.2×10^{-3}
4.0×10^{-1}
0.1
1
10
100
1000
10000
Q^2 [(GeV/c)^2]


HERA kinematic range
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
x
0.1
1
10
100
1000
10000
Q^2


HERA kinematic range
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
x
x_{ref}
overlap
0.1
1
10
100
1000
10000
Q^2

there are 30 different sets of x with at least two Q^2 points

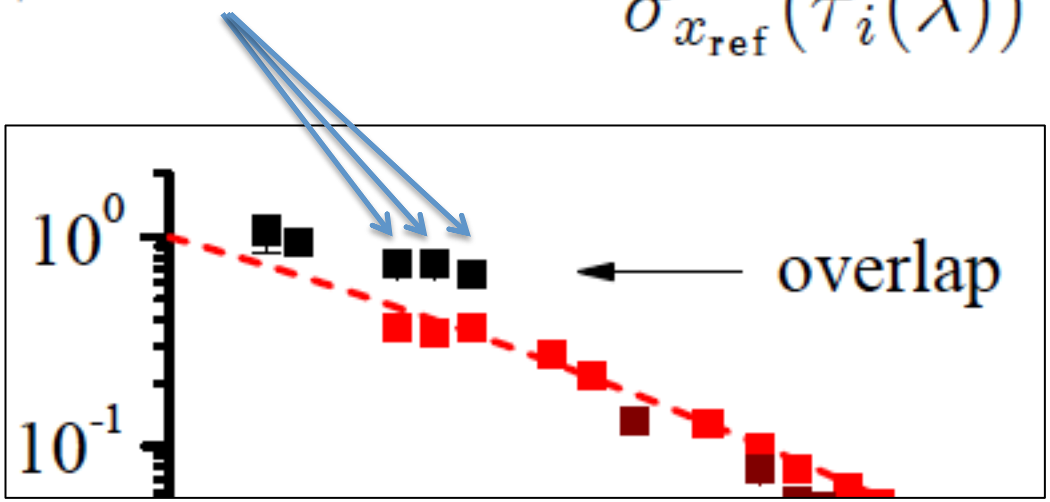




overlap
reference x_{ref}
3.9×10^{-5}
3.2×10^{-3}
2.0×10^{-2}
8.0×10^{-2}
4.0×10^{-1}
$F_2(x,Q^2)/Q^2$
Q^2 [(GeV/c)^2]
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0.1
1
10
100
1000
10000

$$\sigma_x(\tau_i(\lambda)) = \frac{F_2}{Q^2}(\tau_i(\lambda))$$

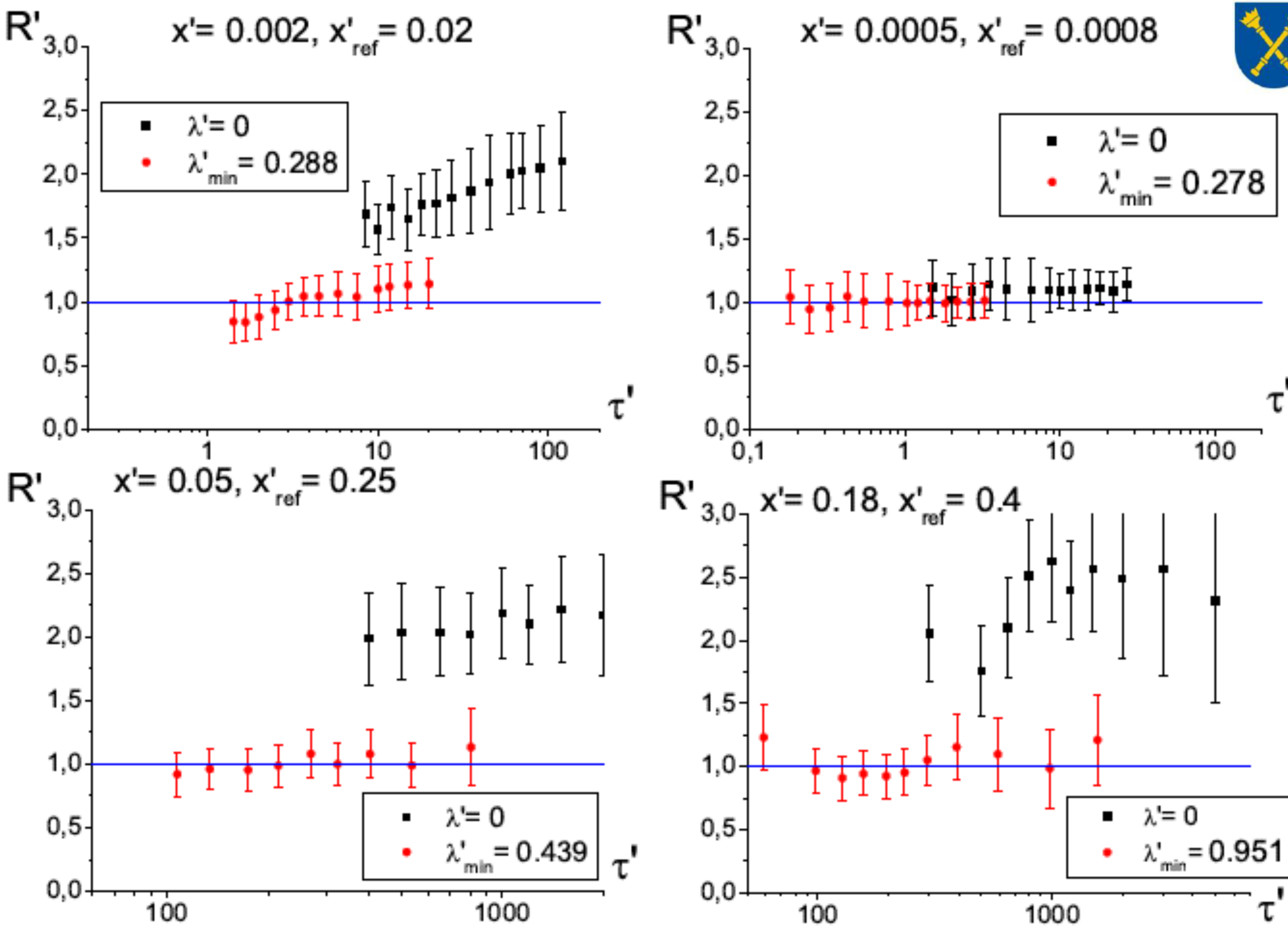
$$R_{x/x_{\mathrm{ref}}}(\tau_i(\lambda)) = \frac{\sigma_x(\tau_i(\lambda))}{\sigma_{x_{\mathrm{ref}}}(\tau_i(\lambda))}$$



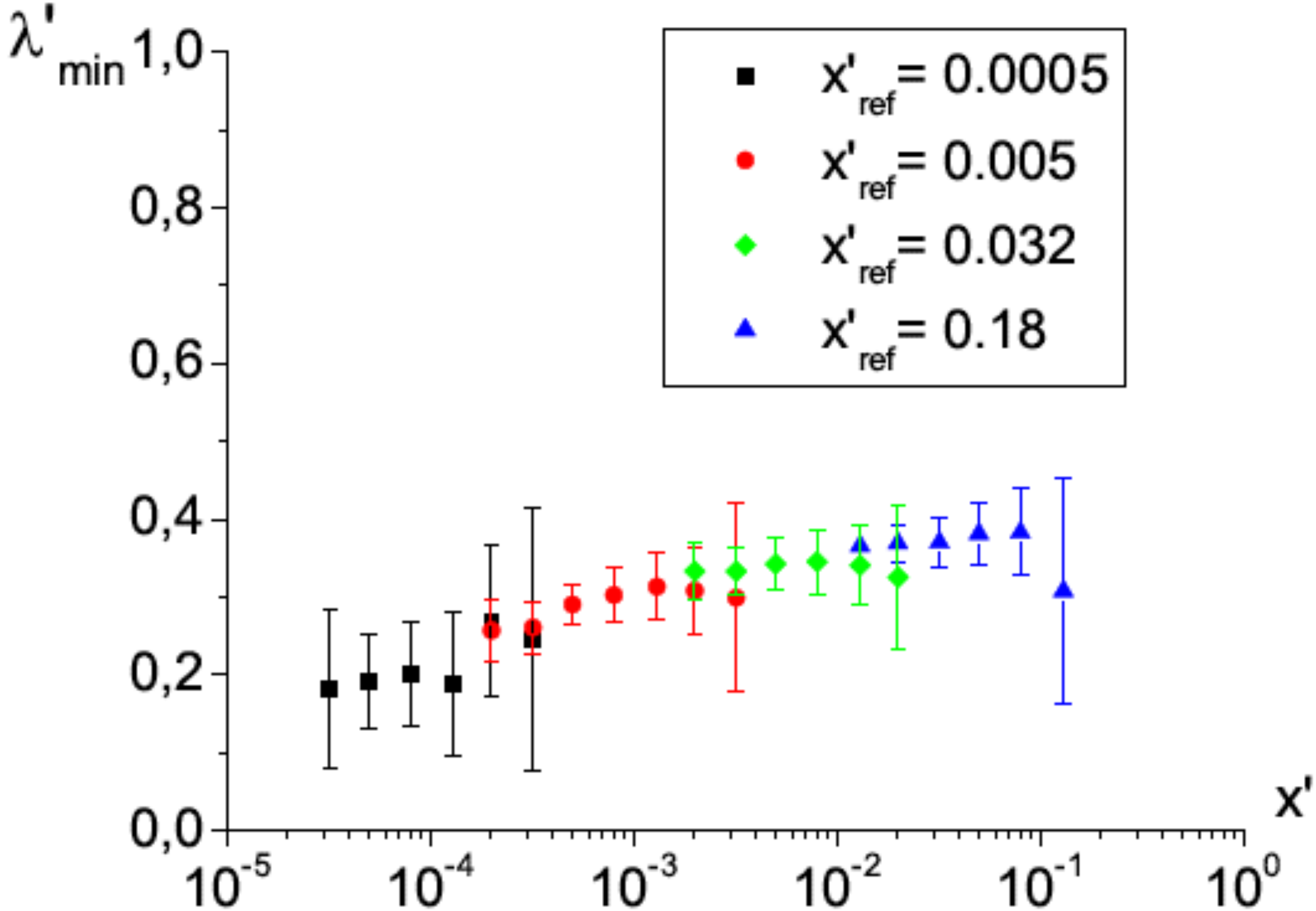
$$\sigma_x(\tau_i(\lambda)) = \frac{F_2}{Q^2}(\tau_i(\lambda))$$

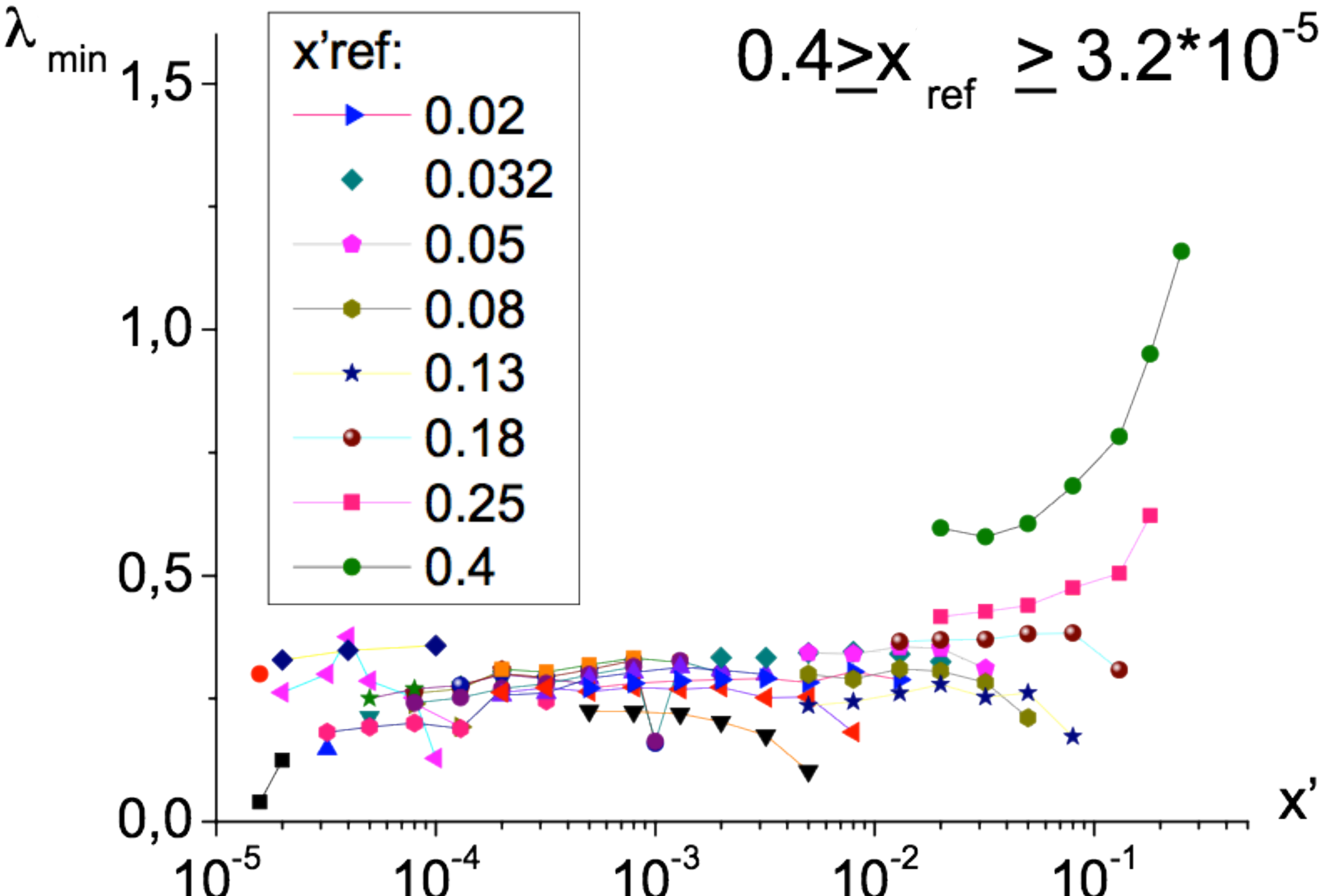
$$R_{x/x_{\mathrm{ref}}}(\tau_i(\lambda)) = \frac{\sigma_x(\tau_i(\lambda))}{\sigma_{x_{\mathrm{ref}}}(\tau_i(\lambda))}$$

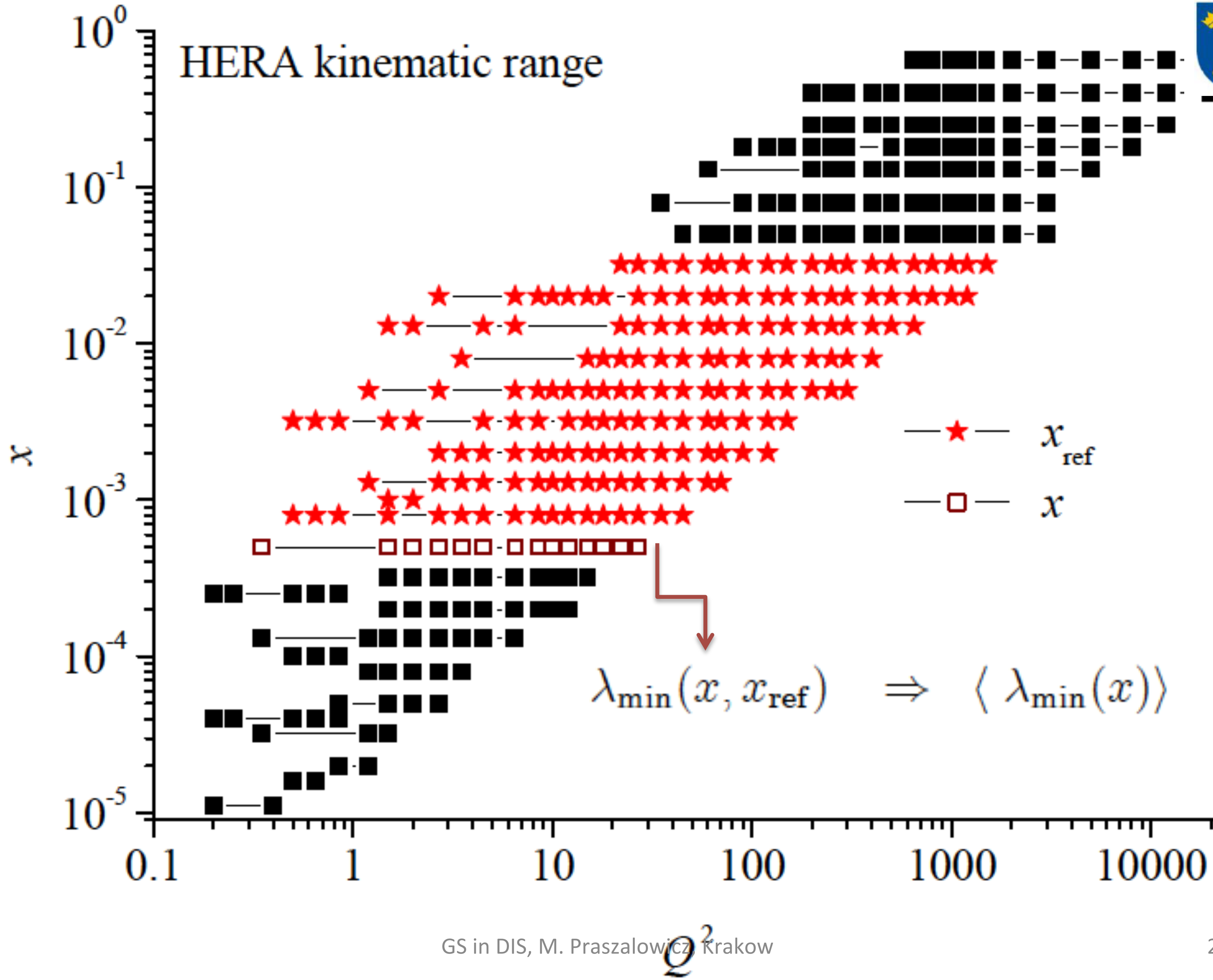
$$\chi^2(x, x_{\mathrm{ref}}; \lambda) = \sum_{\mathrm{overlap}} \frac{(R_{x/x_{\mathrm{ref}}}(\tau_i(\lambda)) - 1)^2}{\Delta R^2_{x/x_{\mathrm{ref}}}(\tau_i(\lambda))} \Rightarrow \lambda_{\mathrm{min}}(x, x_{\mathrm{ref}})$$

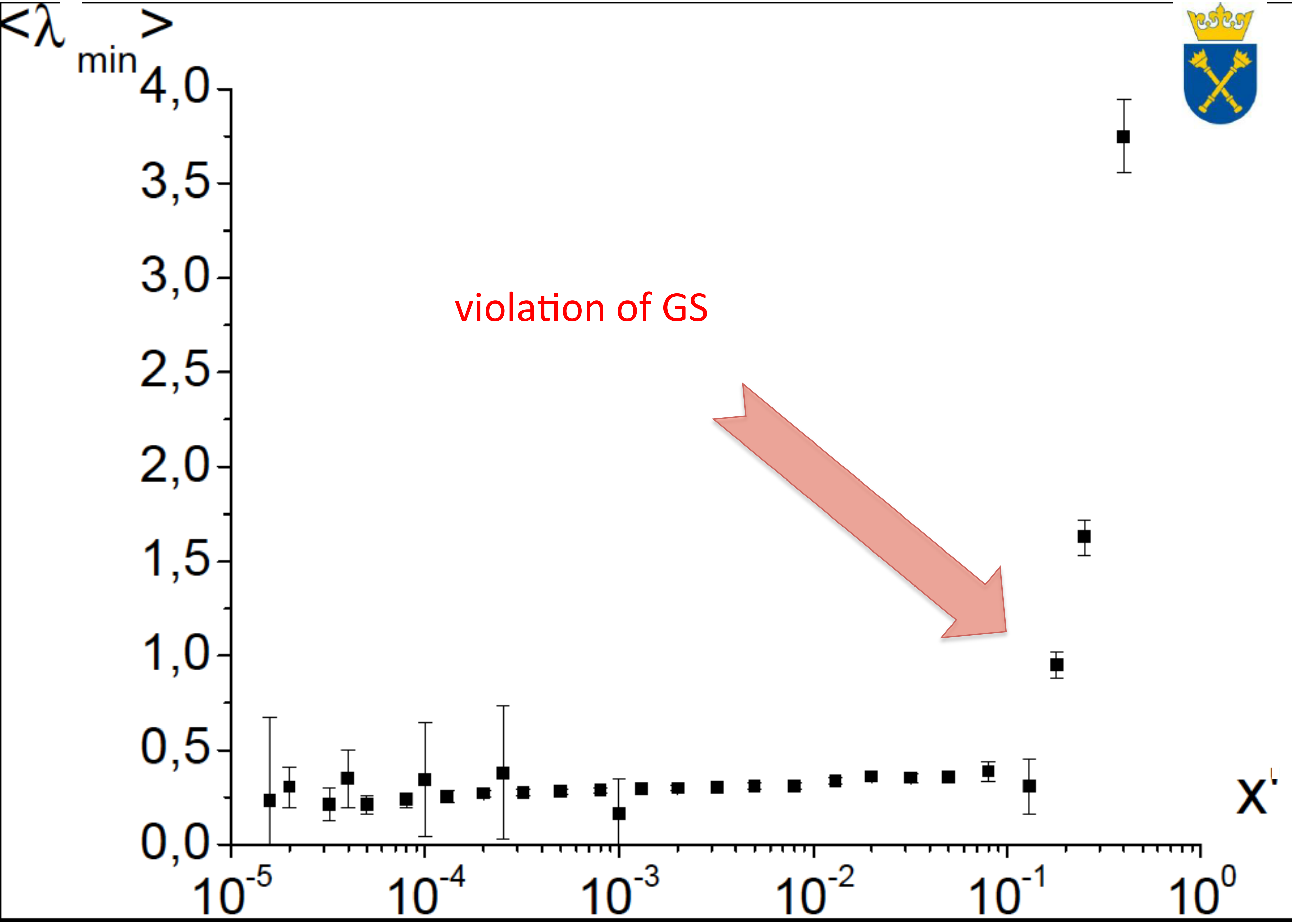

R'
x'= 0.002, x'ref= 0.02
λ'= 0
λ'min= 0.288
τ'
x'= 0.0005, x'ref= 0.0008
λ'= 0
λ'min= 0.278
x'= 0.05, x'ref= 0.25
λ'= 0
λ'min= 0.439
x'= 0.18, x'ref= 0.4
λ'= 0
λ'min= 0.951

an example for a few selcted x_{ref}

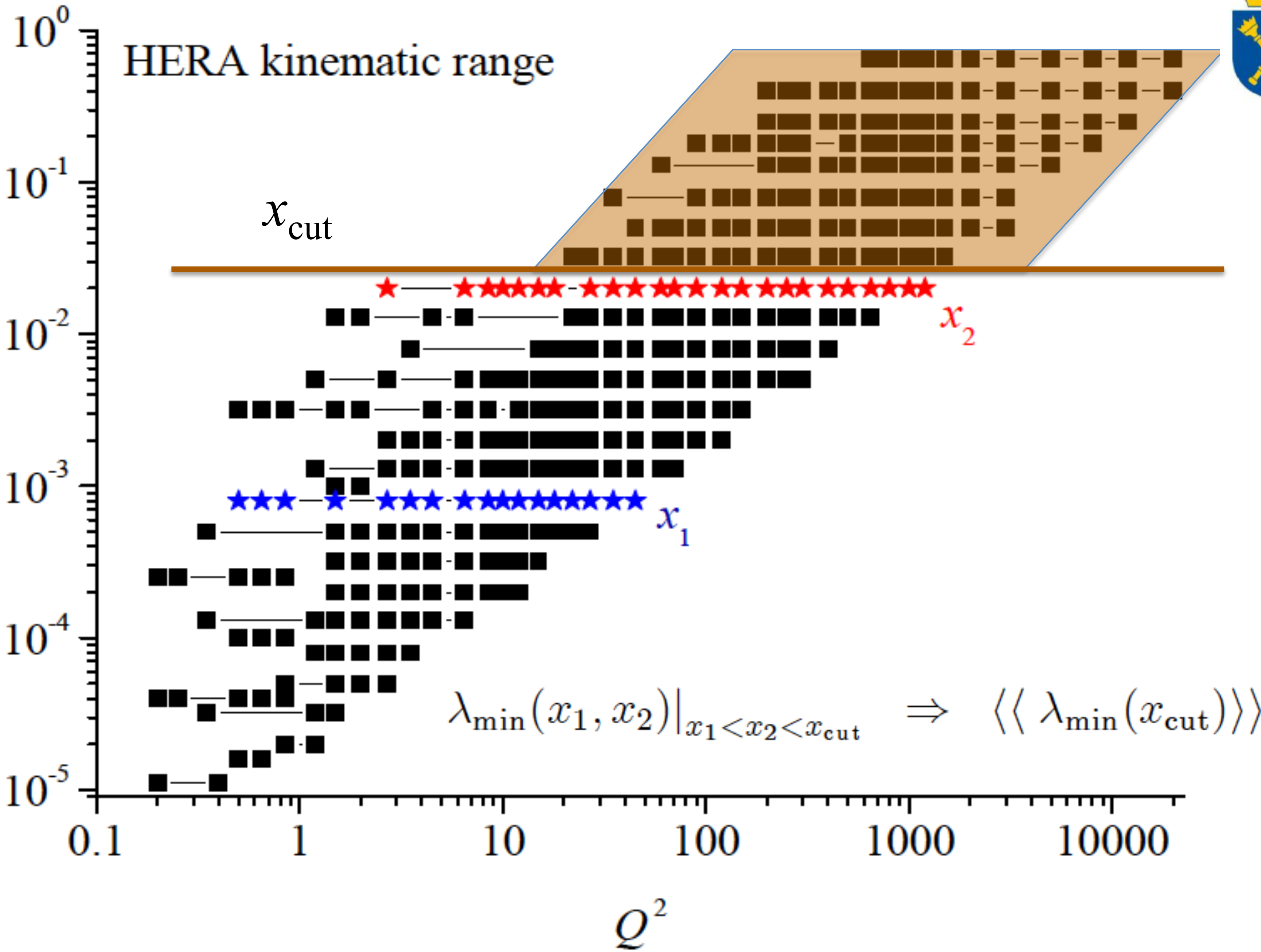


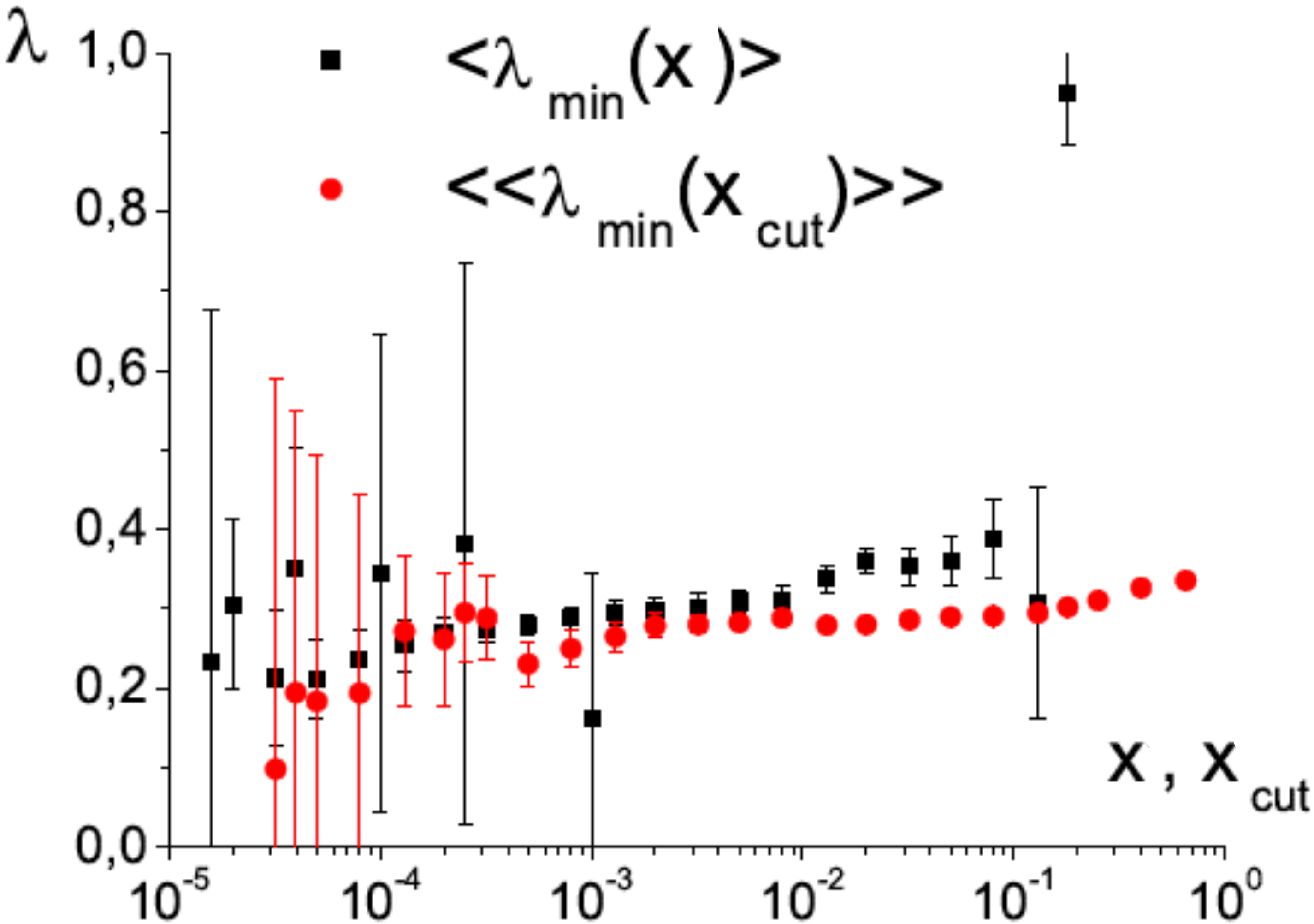

λ min
1,5
1,0
0,5
0,0
x'ref:
0.02
0.032
0.05
0.08
0.13
0.18
0.25
0.4
0.4≥x ref ≥ 3.2*10-5
x'
10-5
10-4
10-3
10-2
10-1


HERA kinematic range
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
x
x_{ref}
x
$\lambda_{\min}(x, x_{\text{ref}}) \Rightarrow \langle \lambda_{\min}(x) \rangle$
0.1
1
10
100
1000
10000
Q^2



<λ min>
4,0
3,5
3,0
2,5
2,0
1,5
1,0
0,5
0,0
violation of GS
10-5
10-4
10-3
10-2
10-1
100
x'


HERA kinematic range
x_{cut}
x_2
x_1
$\lambda_{\min}(x_1, x_2)|_{x_1<x_2<x_{cut}} \Rightarrow \langle\langle \lambda_{\min}(x_{cut}) \rangle\rangle$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.1
1
10
100
1000
10000
Q^2


λ
1,0
0,8
0,6
0,4
0,2
0,0
<λ min(x)>
<<λ min(x cut)>>
x , x cut
10-5
10-4
10-3
10-2
10-1
100

Energy binning

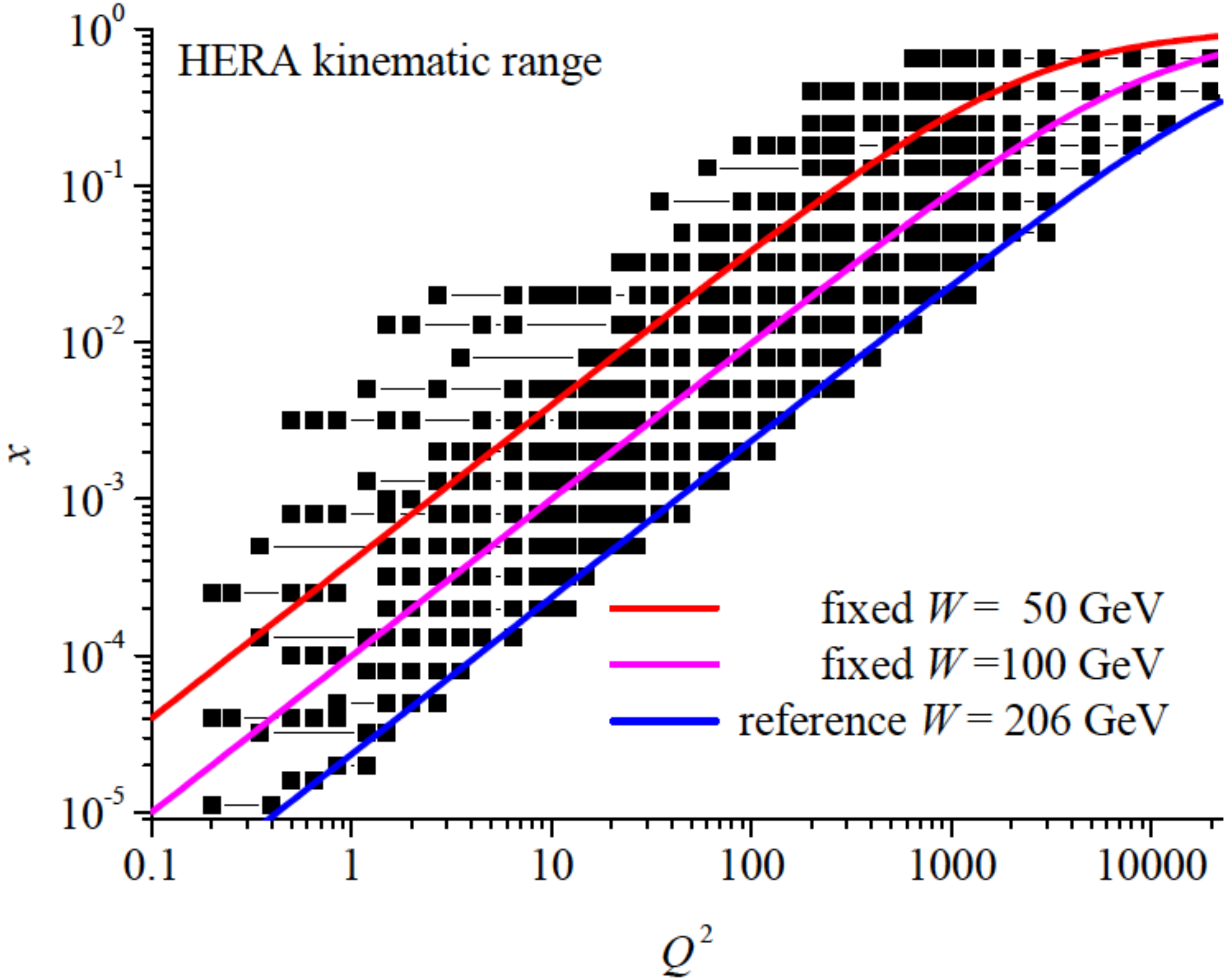
$$(x, Q^2) \rightarrow (W, Q^2)$$

Advantages:

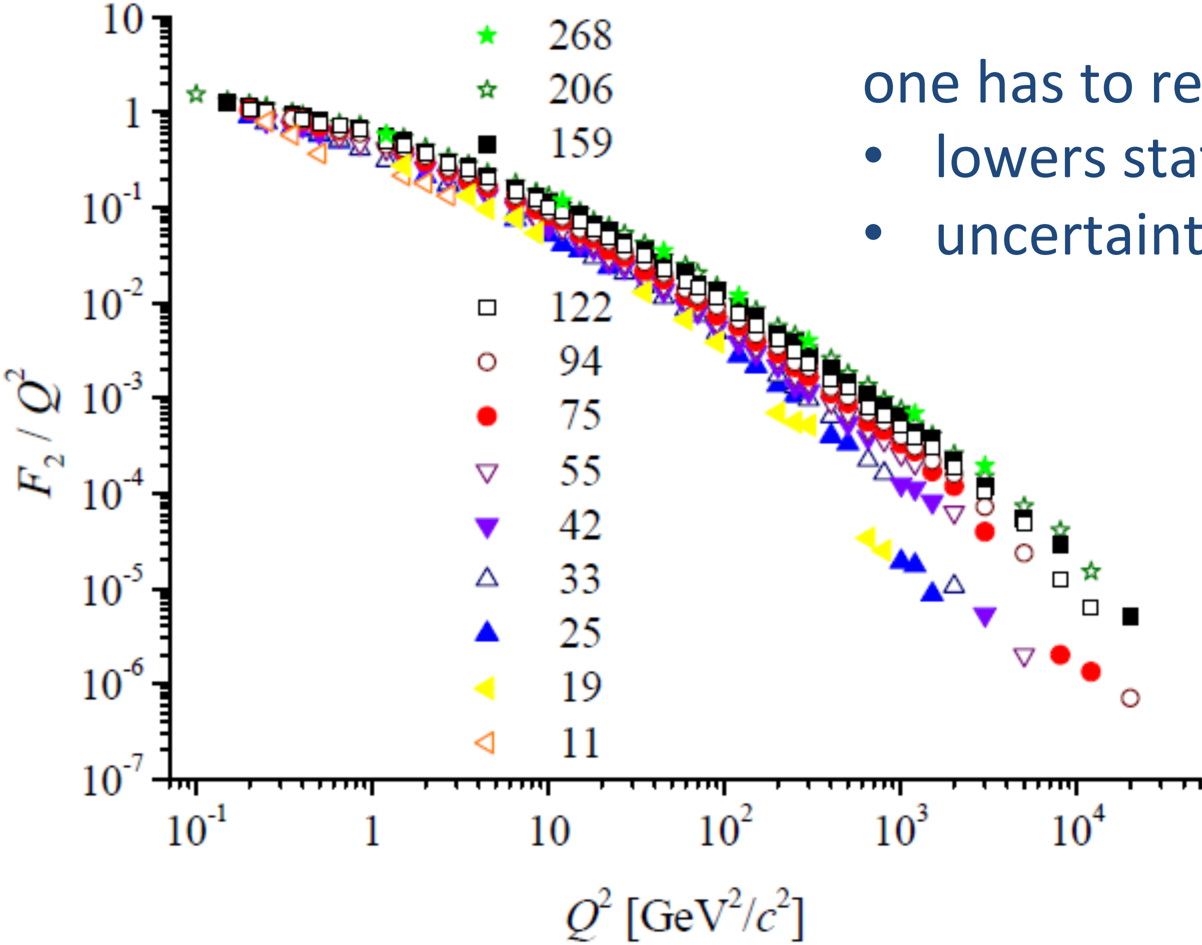
- direct comparison with hadron-hadron collisions
- no problems with overlap

Disanvantages:

- one has to rebin data (arbitrariness)
- lower statistics
- uncontrolable errors

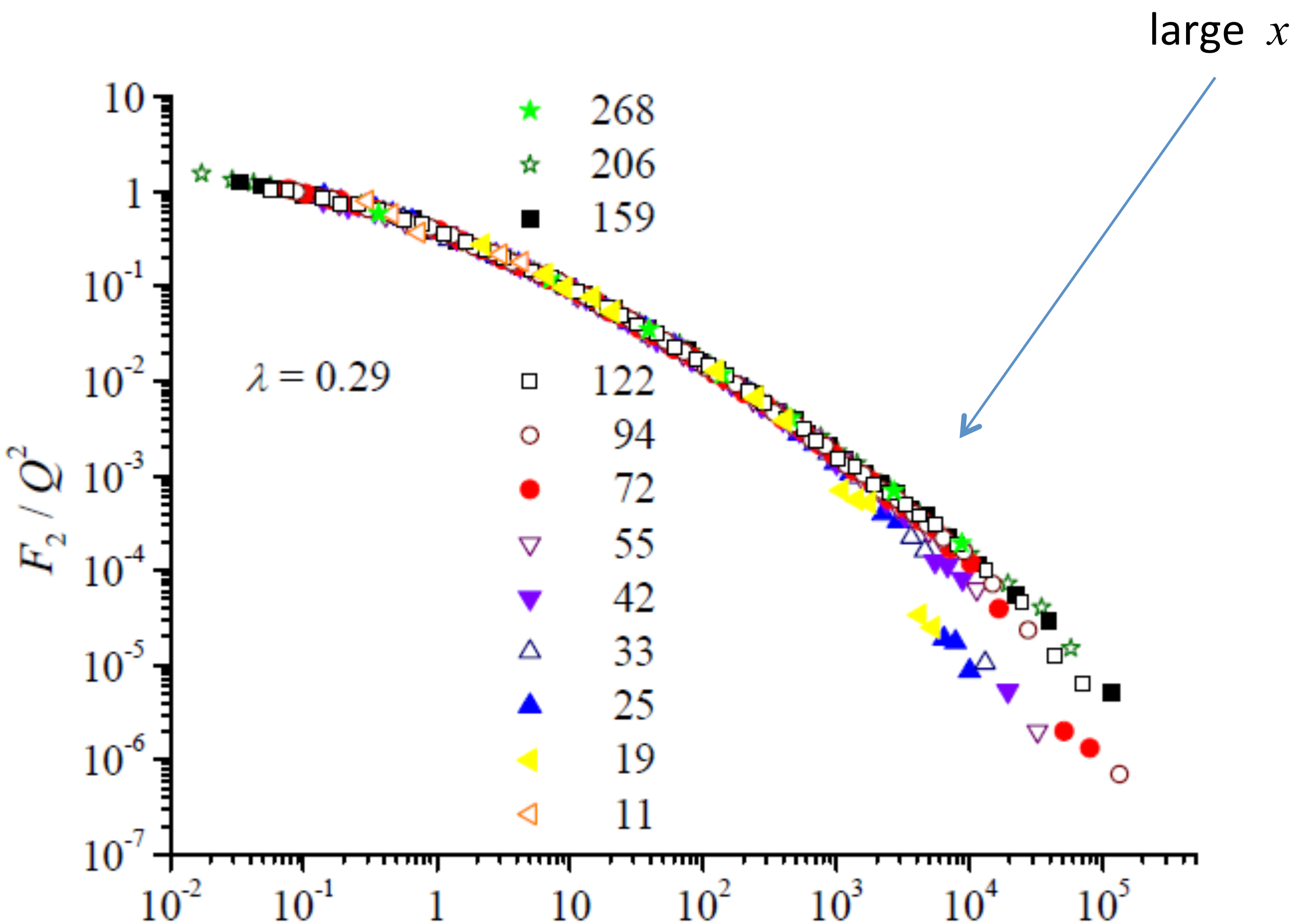

HERA kinematic range
fixed W = 50 GeV
fixed W =100 GeV
reference W = 206 GeV
x
Q²
0.1
1
10
100
1000
10000

Combined HERA data 2009 for e⁺ in W bins

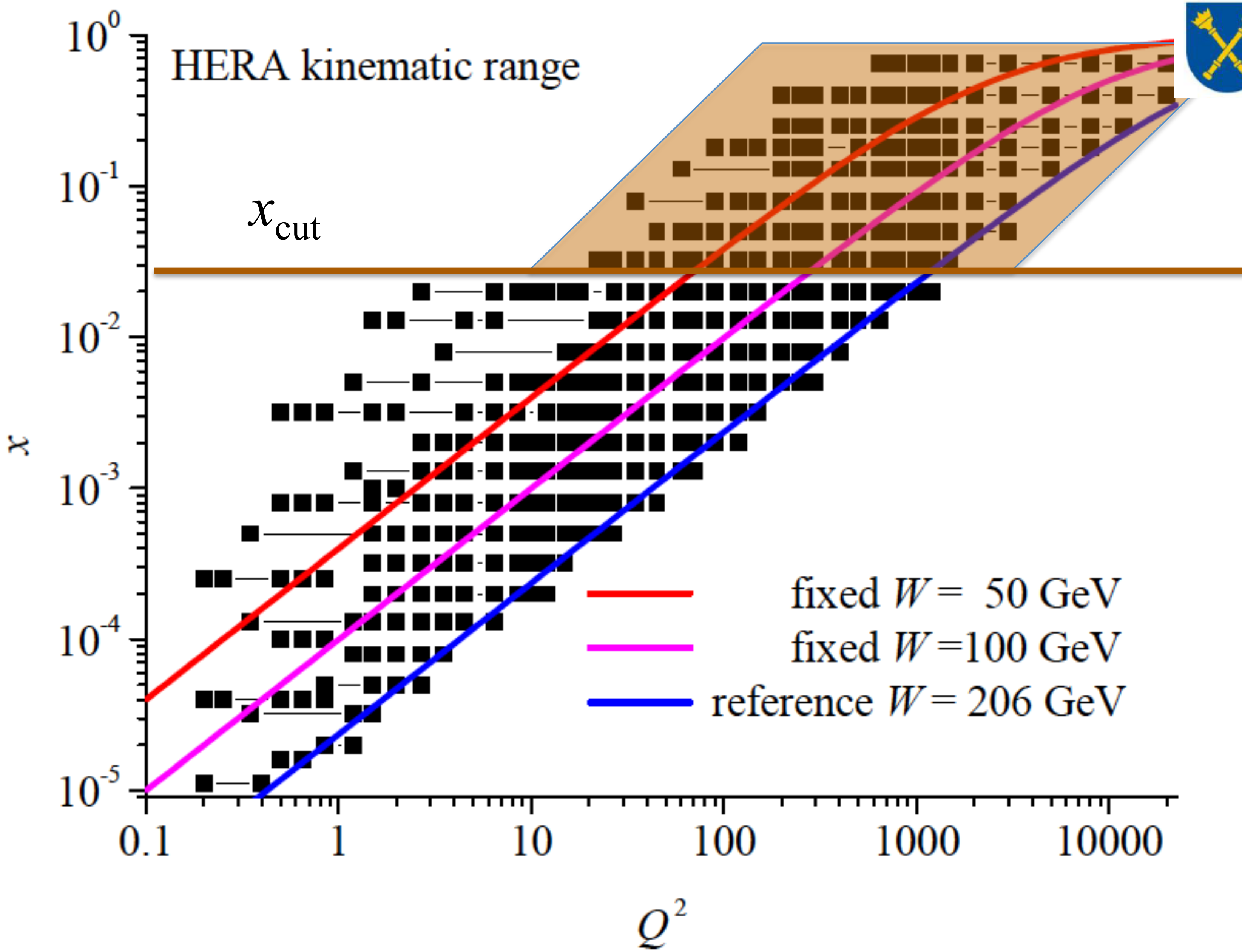


one has to rebin the data:

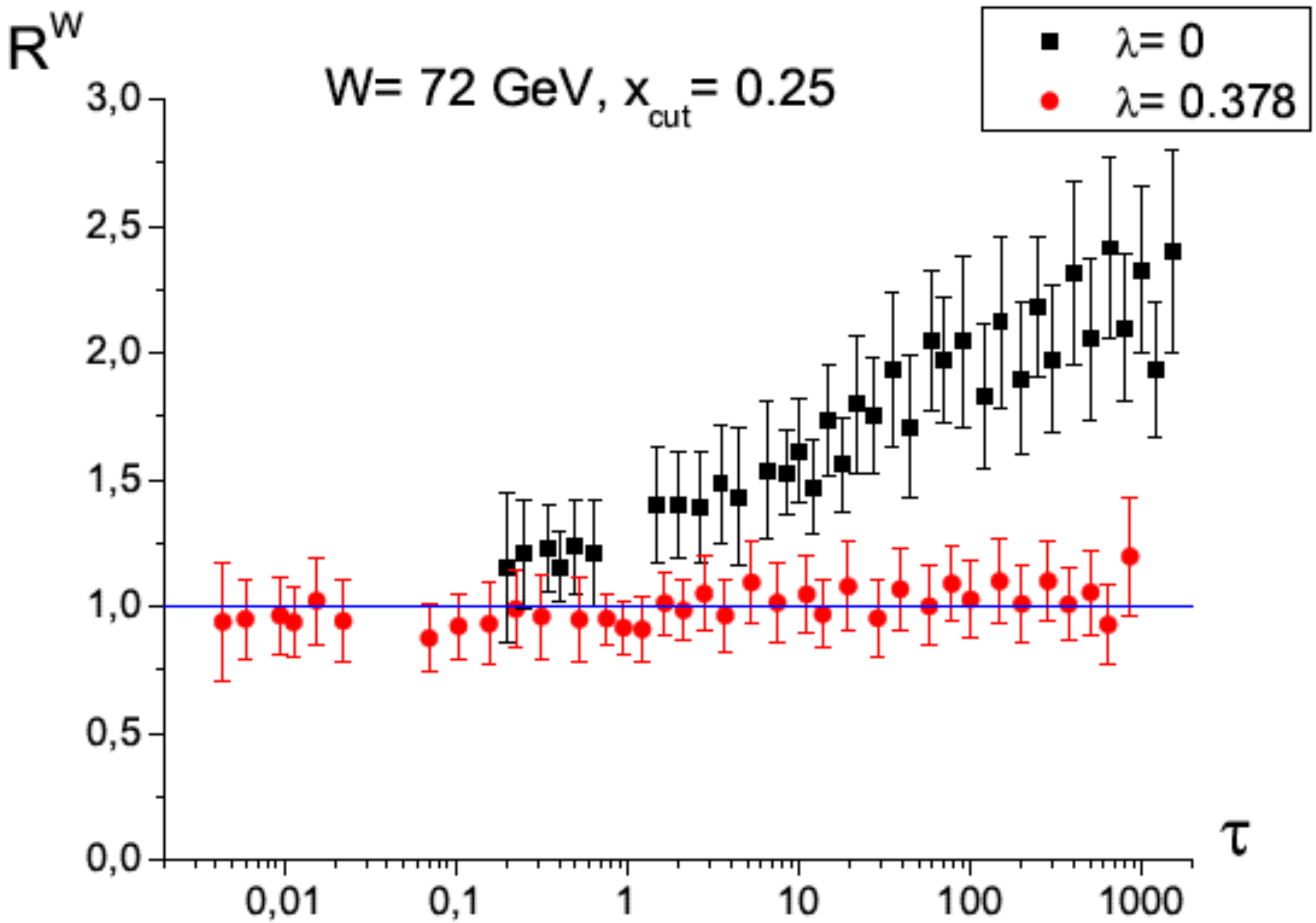
- lowers statisctics
- uncertainties



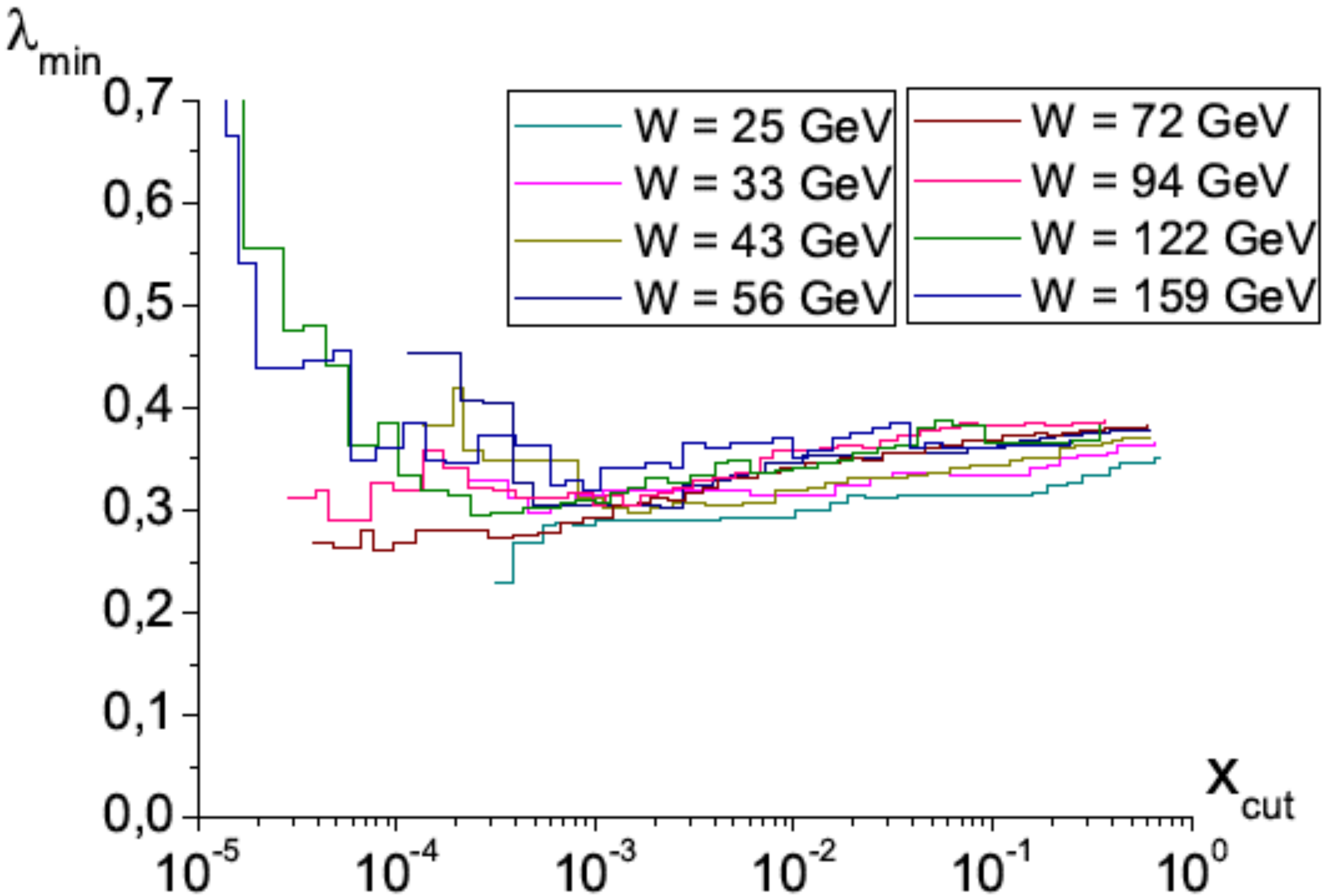
$$\tau = \frac{Q^2}{Q^2_{\rm sat}(x)} \quad Q^2_{\rm sat}(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{-\lambda} \quad \lambda = 0.29$$



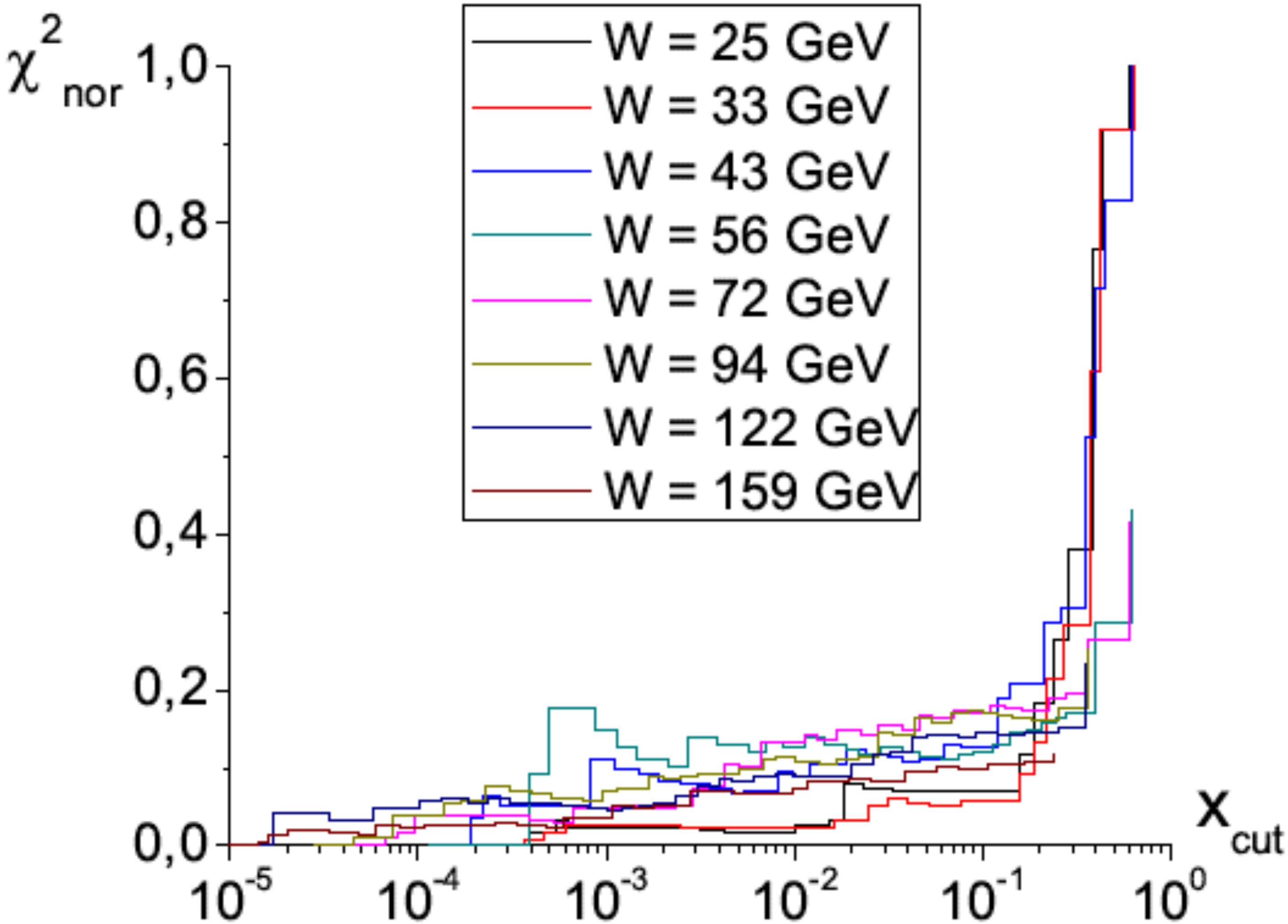
HERA kinematic range
x_{cut}
fixed W = 50 GeV
fixed W =100 GeV
reference W = 206 GeV
x
Q^2
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.1
1
10
100
1000
10000

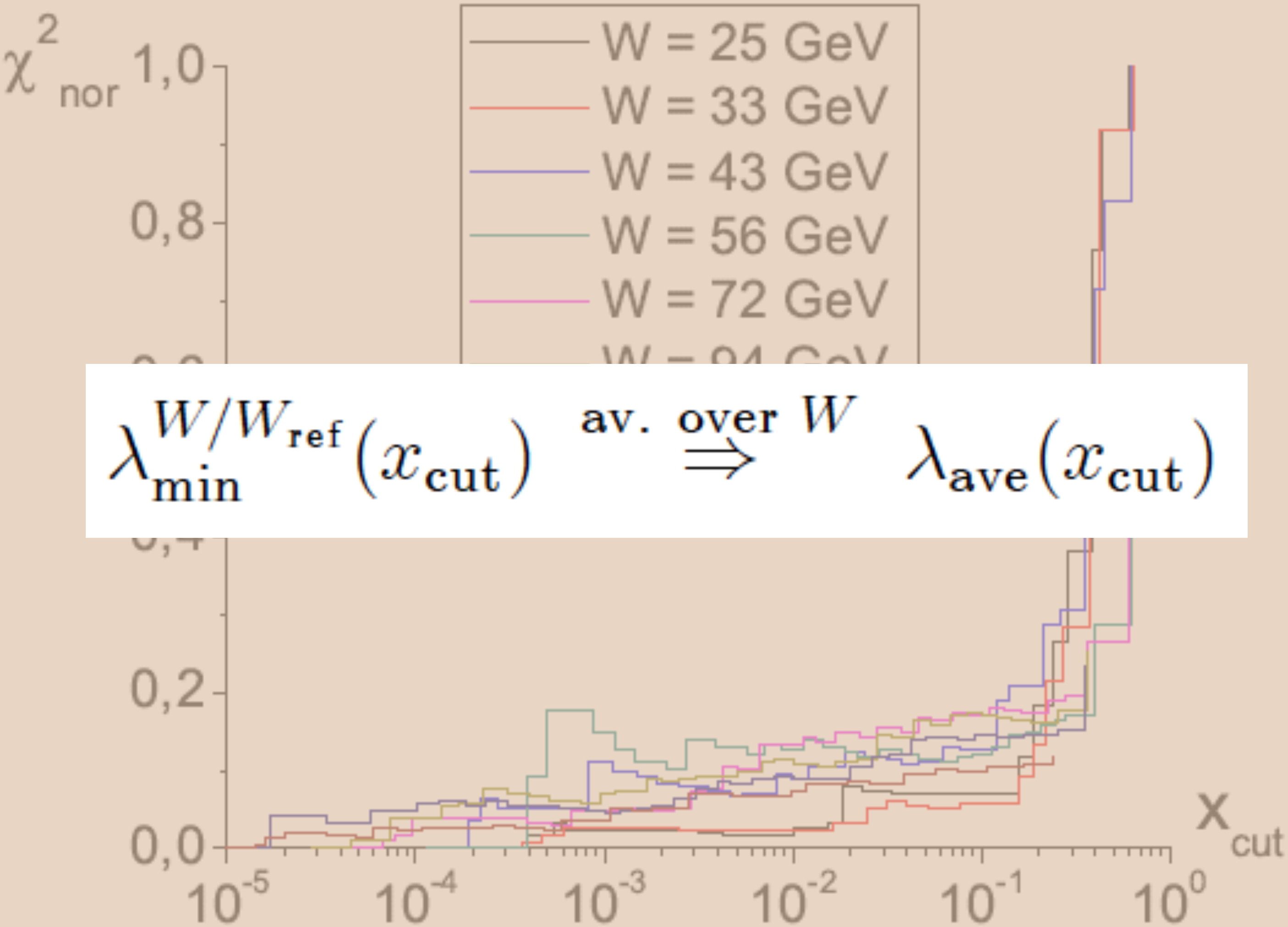


R^W
W= 72 GeV, x_{cut}= 0.25
λ= 0
λ= 0.378
3,0
2,5
2,0
1,5
1,0
0,5
0,0
0,01
0,1
1
10
100
1000
τ

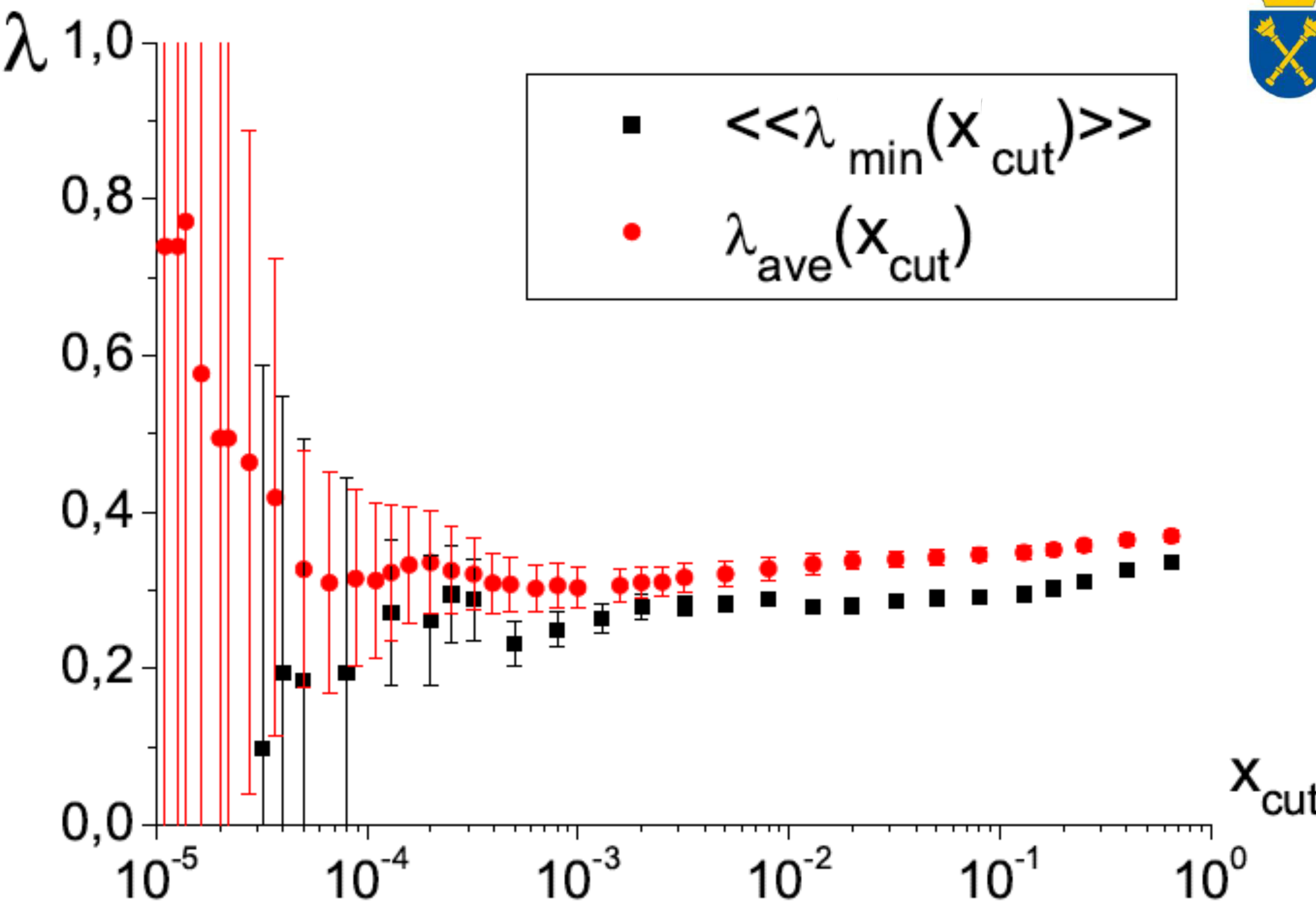


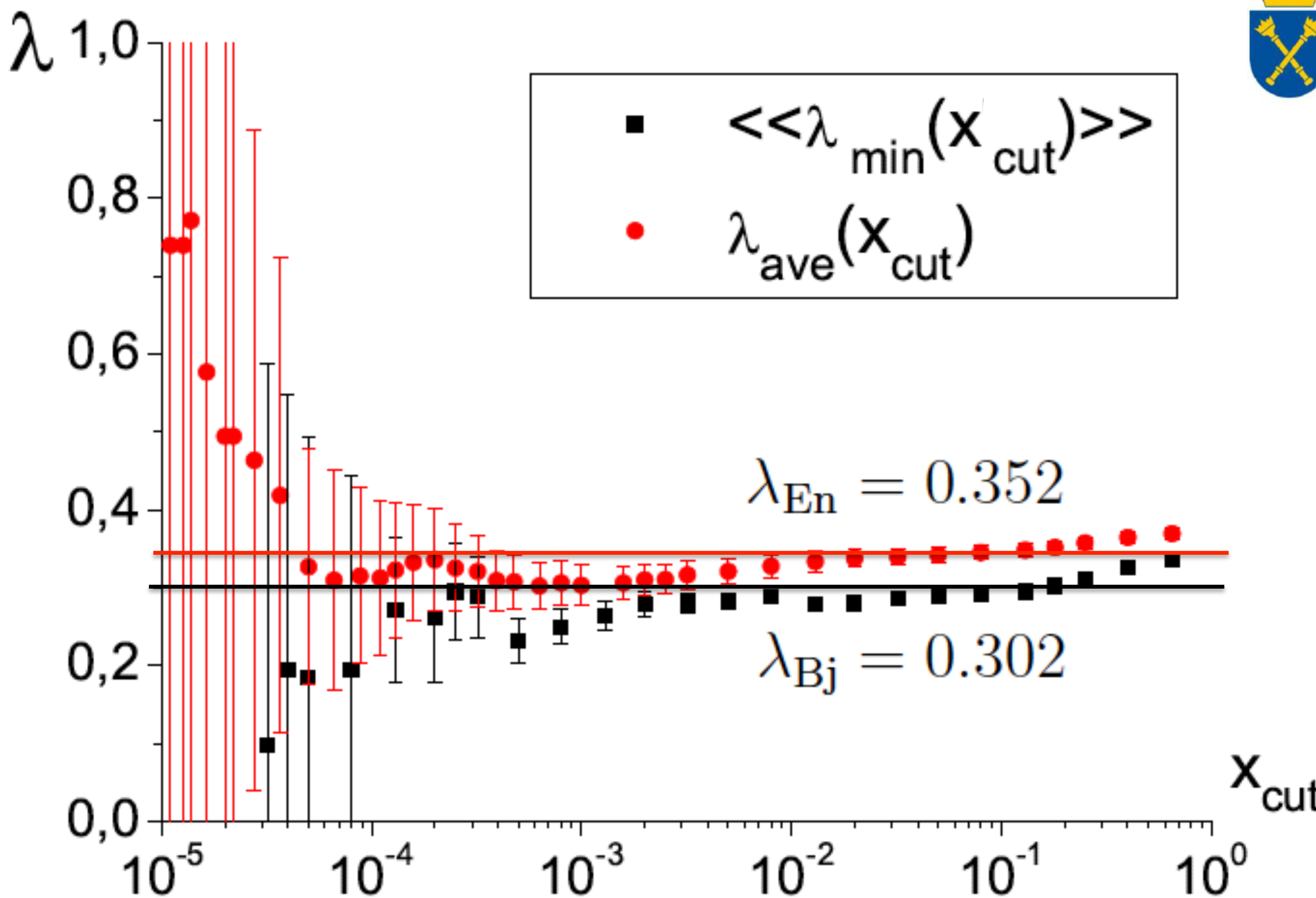
λmin
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
W = 25 GeV
W = 33 GeV
W = 43 GeV
W = 56 GeV
W = 72 GeV
W = 94 GeV
W = 122 GeV
W = 159 GeV
xcut
10-5
10-4
10-3
10-2
10-1
100


W = 25 GeV
W = 33 GeV
W = 43 GeV
W = 56 GeV
W = 72 GeV
W = 94 GeV
W = 122 GeV
W = 159 GeV
χ^2_{nor}
1,0
0,8
0,6
0,4
0,2
0,0
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
x_{cut}



$$\lambda_{\mathrm{min}}^{W/W_{\mathrm{ref}}}(x_{\mathrm{cut}}) \quad \overset{\text{av. over } W}{\Rightarrow} \quad \lambda_{\mathrm{ave}}(x_{\mathrm{cut}})$$

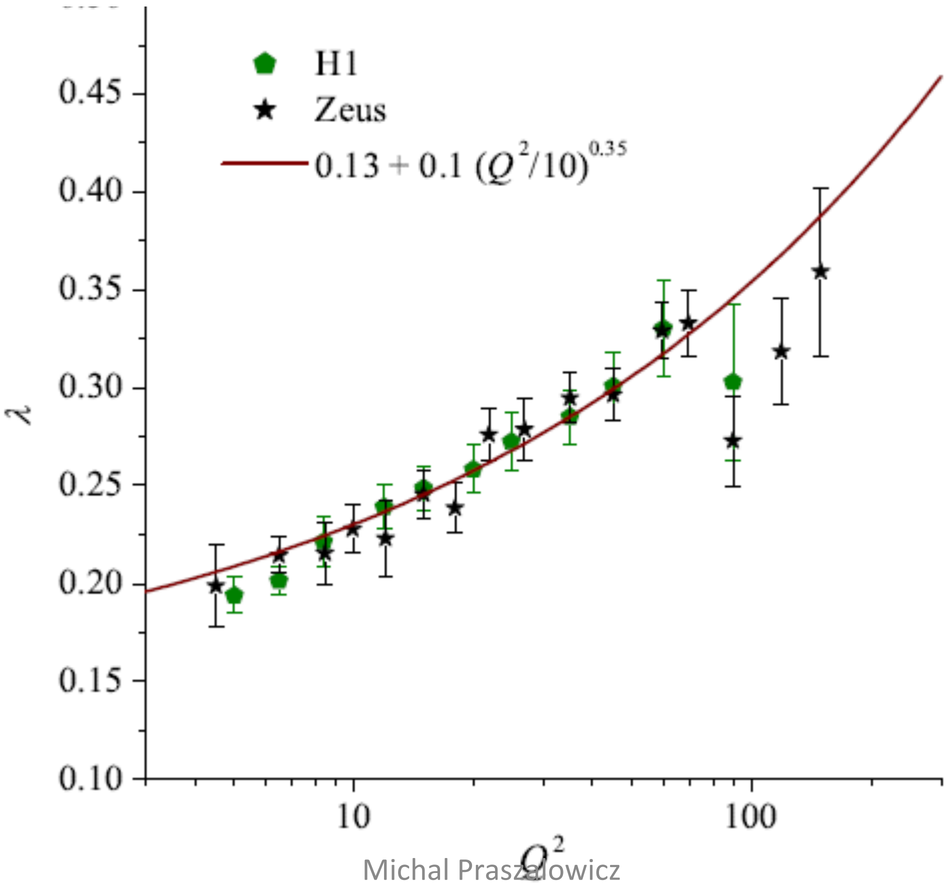

λ
1,0
0,8
0,6
0,4
0,2
0,0
<<λ min(x cut)>>
λave(xcut)
xcut
10-5
10-4
10-3
10-2
10-1
100



λ
1,0
0,8
0,6
0,4
0,2
0,0
<<λ min(x cut)>>
λ ave(x cut)
$\lambda_{\text{En}} = 0.352$
$\lambda_{\text{Bj}} = 0.302$
x cut
10^-5
10^-4
10^-3
10^-2
10^-1
10^0

Geometrical Scaling with $\lambda(Q^2)$

$$F_2(x,Q^2) \sim \sigma_0 Q^2_{\rm sat} \sim \frac{1}{x^{\lambda(Q^2)}}$$

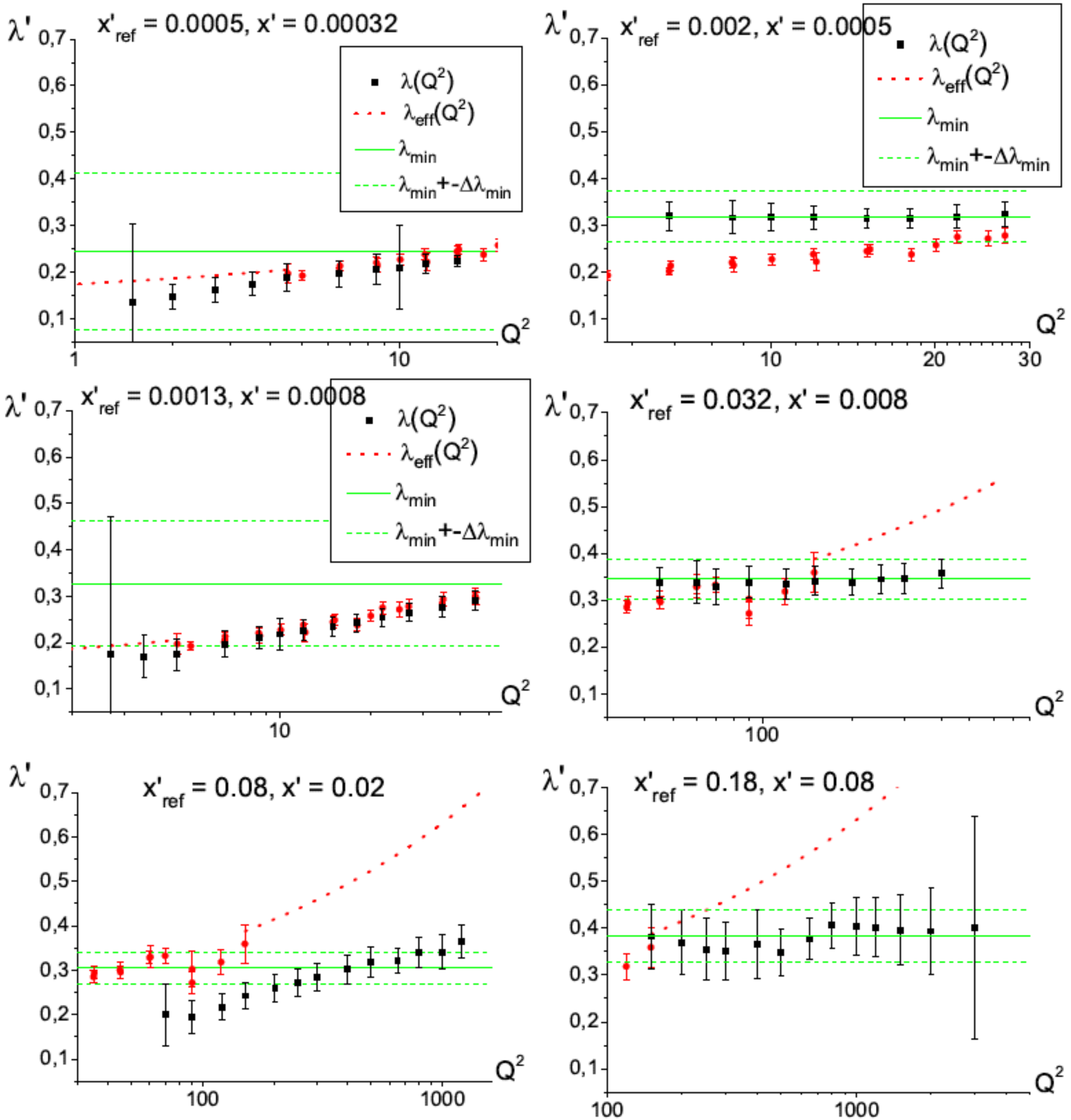


H. Kowalski, L.N. Lipatov, D.A. Ross,
G. Watt, Eur.Phys.J.C70:983-998,2010.

Momentum dependence of *lambda*

Same method, but with *lambda*(Q^2)
(only for x binning)

- very flat chi^2
- possibility to fall into a false minimum
- some dependence on the initial *lambda*
- no firm conclusion



x'ref = 0.0005, x' = 0.00032
x'ref = 0.002, x' = 0.0005
x'ref = 0.0013, x' = 0.0008
x'ref = 0.032, x' = 0.008
x'ref = 0.08, x' = 0.02
x'ref = 0.18, x' = 0.08
λ(Q²)
λeff(Q²)
λmin
λmin+-Δλmin
λ'
Q²

Summary

- We have proposed a simple procedure to extablish whether GS is present, only by manipulating data
- Some „*termometers*” are not sensitive to GSV
- Geometrical Scaling in DIS (for combined HERA data) is observed up to high values of Bjorken x

$$x \sim 0.1$$

- both x binnig and W binning have been considered
- exponent λ of saturation scale is 0.3
- no definitive conlusion on $\lambda(Q^2)$
- no handle on the absolute value of the saturation scale