

# Exclusive final states in diffractive excitation



LUND  
UNIVERSITY

Gösta Gustafson

Department of Theoretical Physics  
Lund University

Low x Workshop  
Cyprus, 27 - 30 June 2012

Work in coll. with C. Flensburg and L. Lönnblad

# Content

## 1. Introduction

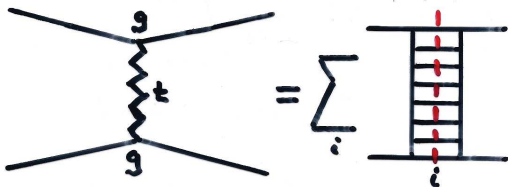
## 2. Dipole Cascades

- ▶ Mueller, LL BFKL
- ▶ Lund

## 3. Diffractive Excitation

- ▶ Good–Walker vs triple-regge
- ▶ Exclusive final states

# 1. Introduction, Reggeon theory



Elastic scattering driven by absorption.

Analogous to diffraction in optics

Single pomeron exch.:  $\sigma_{tot} \sim g^2 s^{\alpha(0)-1}$ ,  $\frac{d\sigma_{el}}{dt} \sim (g^2 s^{\alpha(t)-1})^2$

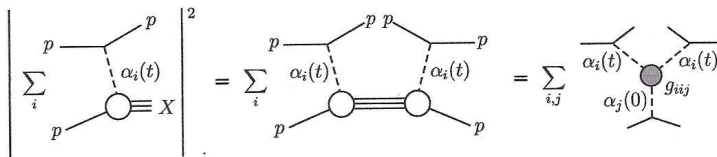
Unitarity corrections in impact parameter space:

If absorption prob. in Born approx. =  $2F$ :

Optical theorem  $\Rightarrow d\sigma_{inel}/d^2b = 1 - e^{-2F}$

$$d\sigma_{el}/d^2b = (1 - e^{-F})^2$$

# Inelastic diffraction, Mueller triple-Regge



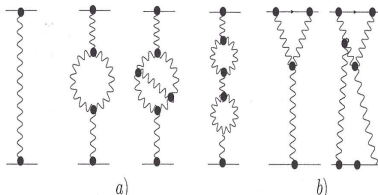
Triple pomeron coupling:  $g_{3P}$

$$\sigma \sim g_{pP}^2(t) g_{pP}(0) g_{3P} \left( \frac{s}{M_X^2} \right)^{2(\alpha(t)-1)} (M_X^2)^{(\alpha(0)-1)}$$

## Unitarity corrections

cf GLM, KMR, and others

Fit regge intercepts and couplings to exp. data



# QCD

Can the known QCD dynamics and the BFKL pomeron give more information, e.g. determine  $g_{3P}$ ?

## Problems with BFKL and low $x$ evolution

- ▶ LL BFKL not enough. Non-leading effects large
- ▶ In LL  $g_{3P}$  is singular  $\sim 1/\sqrt{-t}$
- ▶ Saturation: BK works for **large homogenous** targets
- ▶ Soft cutoff needed in parton subcollisions
- ▶ BFKL **inclusive**. For **exclusive** states: CCFM

These problems are treated in the Lund Dipole Cascade model  
DIPSY

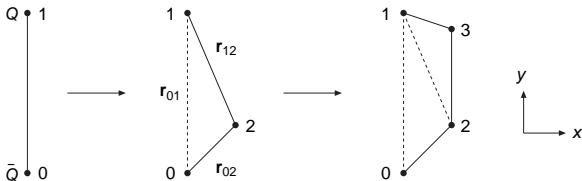
## 2. Dipole cascade models

### a. Mueller Dipol model:

LL BFKL evolution in transverse coordinate space  
Saturation effects from multiple interactions

Colour charge always accompanied by corresponding anticharge

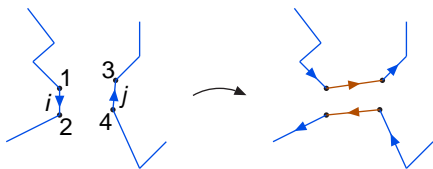
Gluon emission: dipole splits in two dipoles:



$$\text{Emission probability: } \frac{d\mathcal{P}}{dy} = \frac{\bar{\alpha}}{2\pi} d^2\mathbf{r}_2 \frac{r_{01}^2}{r_{02}^2 r_{12}^2}$$

## Dipole-dipole scattering

Single gluon exchange  $\Rightarrow$  Colour reconnection



Born amplitude:

$$f_{ij} = \frac{\alpha_s^2}{2} \ln^2 \left( \frac{r_{13} r_{24}}{r_{14} r_{23}} \right)$$

### Multiple subcollisions

BFKL stochastic process with independent subcollisions:

Sum over all dipole pairs: Born ampl.:  $F = \sum_{ij} f_{ij}$

Unitarized ampl.:  $T = 1 - e^{-\sum f_{ij}}$

## b. The Lund cascade model, DIPSY MC

Includes:

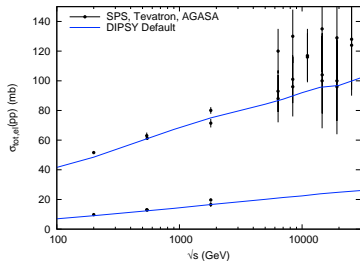
- ▶ Important non-leading effects in BFKL evol.
- ▶ Saturation from pomeron loops in the evolution  
(Not included by Mueller or in BK)
- ▶ Confinement
- ▶ MC DIPSY  
gives also fluctuations and correlations
- ▶ Applicable to collisions between electrons, protons,  
and nuclei



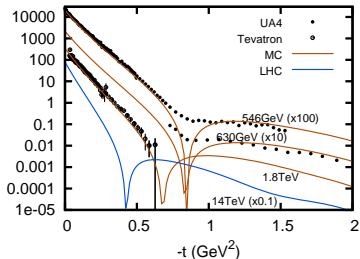
# Total and elastic cross sections

$pp$

$\sigma_{tot}$  and  $\sigma_{el}$



$d\sigma/dt$

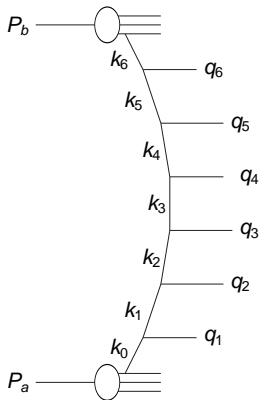


## Exclusive final states

BFKL: Inclusive

Exclusive: CCFM

In momentum space

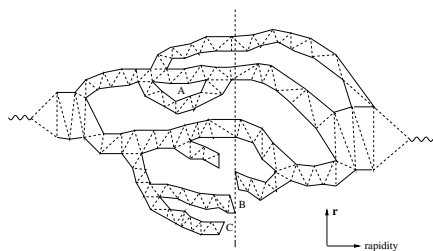


**Inclusive** cross section determined by “hard” emissions, where  $k_{\perp}$  gets a big kick, “ $k_{\perp}$ -changing emissions”

Gives initial state radiation  
 (Lund 1996, Salam 1999)

(either  $k_{\perp i} \gg k_{\perp i-1}$ ;  $q_{\perp i} \simeq k_{\perp i}$   
 or  $k_{\perp i} \ll k_{\perp i-1}$ ;  $q_{\perp i} \simeq k_{\perp i-1}$ )

## Schematic picture: BFKL is a stochastic process



Non-interacting  
branches cannot  
come on shell.

To get final states:

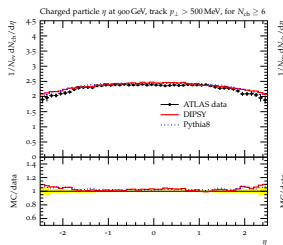
- Determine which dipoles interact
- Absorb non-interacting chains
- Determine final state radiation
- Hadronize

# Comparisons to ATLAS data

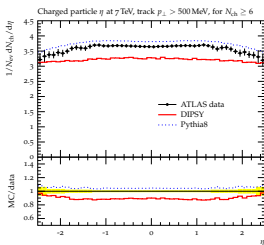
[arXiv:1103.4321]

Min bias

$\eta$  distrib. charged particles  
 0.9 TeV

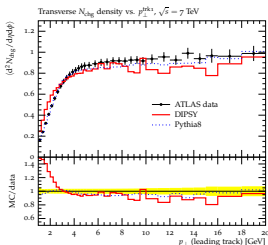


7 TeV



Underlying event

$N_{ch}$  in transv. region  
 vs  $p_{\perp}^{lead}$ , 7 TeV



Our aim to get dynamical insight, not to give precise predictions  
 At present no quarks, only gluons

### 3. Diffractive excitation

#### Alternative formalism: Good-Walker

Projectile with a substructure

The mass eigenstates,  $\Psi_k$ , can differ from the eigenstates of diffraction,  $\Phi_n$ , with eigenvalues  $T_n$

Elastic amplitude:  $\langle \Psi_{in} | T | \Psi_{in} \rangle = \langle T \rangle$

$$d\sigma_{el}/d^2b = \langle T \rangle^2$$

Ampl. for transition to state  $\Psi_k$  given by  $\langle \Psi_k | T | \Psi_{in} \rangle$

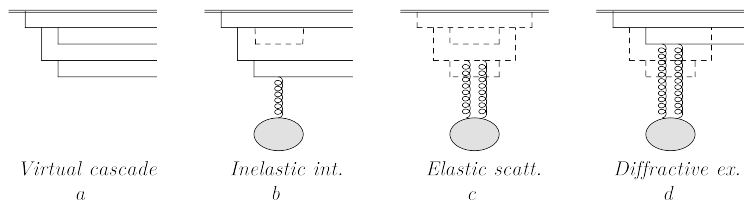
Total diffractive cross section (incl. elastic):

$$d\sigma_{diff}/d^2b = \sum_k \langle \Psi_{in} | T | \Psi_k \rangle \langle \Psi_k | T | \Psi_{in} \rangle = \langle T^2 \rangle$$

$$d\sigma_{diff\ ex}/d^2b = d\sigma_{diff} - d\sigma_{el} = \langle T^2 \rangle - \langle T \rangle^2 = V_T$$

## What are the diffractive eigenstates?

Parton cascades, which can come on shell through interaction with the target.



Continuous distrib. up to high masses (with large fluctuations)

(Also Miettinen–Pumplin (1978), Hatta *et al.* (2006))

*cf* KMR and GLM: 2 or 3 low mass states

Claim: Good–Walker and triple-pomeron are only different formulations of the same phenomenon

## a. Relation Good–Walker vs triple-pomeron

(For more details see arXiv:1206.1733)

Essential feature of the BFKL cascade:

prob. for a dipole split  $dP/dy \sim \lambda$

$\Rightarrow$  # dipoles grows  $\langle n(y) \rangle \approx e^{\lambda y}$

Large fluctuations:  $V(y) \approx e^{2\lambda y} - e^{\lambda y} = \langle n \rangle^2 (1 - e^{-\lambda y})$ ,  
 $(V \equiv \langle n^2 \rangle - \langle n \rangle^2)$

Approximate KNO scaling

2 colliding cascades, evolved  $y_1$  and  $y_2$ :

Dipole-dipole interaction prob. =  $2f \Rightarrow$

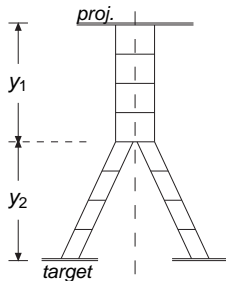
Bare pomeron:  $\sigma_{inel} \propto e^{\lambda y_1} 2f e^{\lambda y_2} = 2f e^{\lambda Y} = 2f s^\lambda$

$$\sigma_{el} \propto f^2 e^{2\lambda Y} = f^2 s^{2\lambda}$$

## Single diffr. excit.

$$M_X^2 \approx \exp(y_1)$$

$$s \approx \exp(y_1 + y_2) = \exp(Y)$$



## Triple-pomeron:

$$\frac{d\sigma_{SD}}{d \ln M^2} \approx \langle n_{proj} \rangle \lambda f^2 \langle n_{targ} \rangle^2 \approx \lambda f^2 e^{\lambda y_1} e^{2\lambda y_2} = \lambda f^2 \left(\frac{s}{M^2}\right)^{2\lambda} (M^2)^\lambda$$

Integrated cross section,  $M_X < M_{max}$ :

$$\int_{(M < M_{max})} \frac{d\sigma_{SD}}{d \ln M^2} dy_1 = f^2 e^{2\lambda Y} (1 - e^{-\lambda y_1}) = f^2 s^{2\lambda} (1 - 1/(M_{max}^2)^\lambda)$$



## Good-Walker

Gives integrated cross section for  $M_X^2 < M_{max}^2 = \exp(y_1)$

$$\begin{aligned} \sigma_{SD} &= (\text{total diffraction for projectile and elastic target}) - \sigma_{el} \\ &= \langle \langle T \rangle_{\text{targ}}^2 \rangle_{\text{proj}} - \langle \langle T \rangle_{\text{targ}} \rangle_{\text{proj}}^2 = \\ &= f^2 (2e^{2\lambda y_1} - e^{\lambda y_1})(e^{2\lambda y_2}) - f^2 (2e^{2\lambda y_1})(e^{2\lambda y_2}) = \\ &= f^2 e^{2\lambda Y} (1 - e^{-\lambda y_1}) \end{aligned}$$

Same expression as in triple-pomeron!!

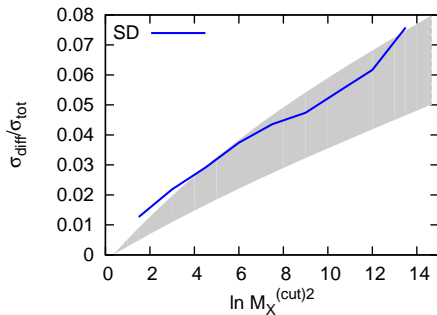
Most essential for this result is the approximate KNO scaling:

$$\frac{V(y)}{\langle n \rangle^2} = (1 - e^{-\lambda y})$$

## DIPSY: $pp$ 1.8 TeV

Single diffractive cross section for  $M_X^2 < M_{max}^2$

Shaded area: Estimate of CDF result



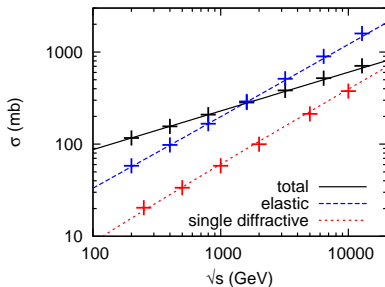
Note: Tuned only to  $\sigma_{tot}$  and  $\sigma_{el}$ . No new parameter

Saturation  $\Rightarrow$  Factorization broken between  $pp$  and DIS

# DIPSY results have the expected triple-regge form

*BARE* pomeron (Born amplitude without saturation effects)

Total, elastic and singel diffractive cross sections



Triple-Regge fit with a single pomeron pole

$$\alpha(0) = 1.21, \quad \alpha' = 0.2 \text{ GeV}^{-2}$$

$$g_{pP}(t) = (5.6 \text{ GeV}^{-1}) e^{1.9t}, \quad g_{3P}(t) \approx 1 \text{ GeV}^{-1} \text{ (dep. on def.)}$$

## b. Exclusive diffractive states

Diffraction is a quantum effect  $\Rightarrow$  interference is important  
 $\Rightarrow$  no probabilistic picture

But: positive and negative contributions to the amplitude can be generated by DIPSY, added, and squared.

### Toy model example

System with a valence particle, which can emit a single gluon

2 states: valence only  $\Psi_0 = |1, 0\rangle$

valence + gluon  $\Psi_1 = |1, 1\rangle$

Probability for emission:  $\beta^2$  prob. for no em.:  $\alpha^2 = 1 - \beta^2$

General state  $\Psi = a\Psi_0 + b\Psi_1 \equiv \begin{pmatrix} a \\ b \end{pmatrix}$

Assume an initial state  $\Psi$  which evolves to a cascade  $\Phi$  at the time of interaction with the target

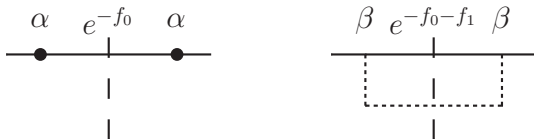
$$\Phi = U_{\text{evol}} \Psi$$

Evolution operator  $U_{\text{evol}}$  is a unitary matrix  $= \begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix}$

Eikonal interaction operator  $U_{\text{int}} = \begin{pmatrix} e^{-f_0} & 0 \\ 0 & e^{-f_0-f_1} \end{pmatrix}$

$$\Psi_{\text{out}} = S\Psi_{\text{in}} = U_{\text{evol}}^\dagger U_{\text{int}} U_{\text{evol}} \Psi_{\text{in}}$$

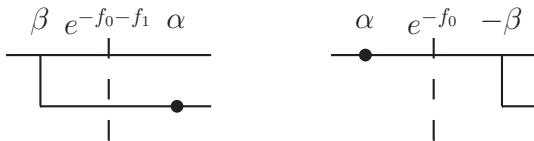
## Elastic scattering:



## Elastic amplitude

$$T_{11} = 1 - S_{11} = 1 - \alpha^2 e^{-f_0} - \beta^2 e^{-f_0-f_1} = \alpha^2(1 - e^{-f_0}) + \beta^2(1 - e^{-f_0-f_1})$$

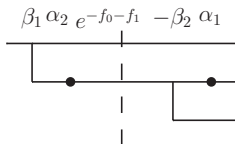
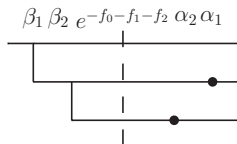
## Diffractive excitation:



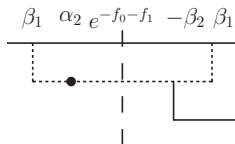
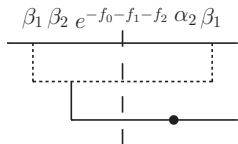
$$T_{21} = -S_{21} = -\alpha\beta e^{-f_0-f_1} - \alpha(-\beta)e^{-f_0} = \alpha\beta e^{-f_0}(1 - e^{-f_1})$$

# Cascade with 2 possible emissions

Ex.: Final state with both emissions



Final state with only the second emission



## Generalizations:

Continuous cascades

Independent gluon emissions  $\rightarrow$  dipole cascade

Include target cascade

## Calculations:

Collide many similar real cascades (emissions before and after interaction) which interfere

Collide with large no. of target cascades

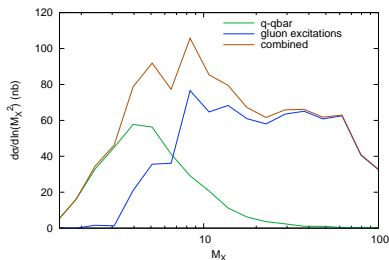
Computationally demanding, but still possible in the MC



## Preliminary results DIS

Single diffraction  $W = 120 \text{ GeV}$ ,  $Q^2 = 24 \text{ GeV}^2$

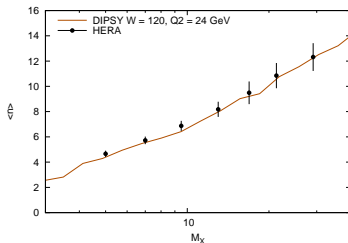
$$\frac{d\sigma}{d \ln M_X^2}$$



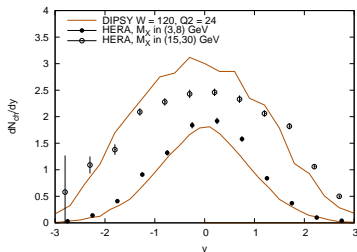
Curves show contributions from  $q\bar{q}$  state plus 0, 1, and  $\geq 2$  gluons

Peak at  $M_X^2 \approx Q^2$ . Flatter distribution for larger  $M_X$ . Cutoff for  $M_X > 50 \text{ GeV}$  from the Lorentz frame used in the calculation.

DIS:  $\langle n_{ch} \rangle$  vs  $M_X$



$dn_{ch}/d\eta$



*pp* collisions: coming soon

Future:

Double diffraction. Computationally demanding

Hard diffraction, would need a “trigger” in the MC

Might be possible to estimate the survival probability

# Conclusions

The DIPSY model is based on QCD dynamics for small  $x$  evolution plus saturation

No input structure functions:

Attempt to understand underlying dynamics, not to give optimal precision

Works well for inclusive observables

Fair description of non-diffractive final states

# Diffraction

Claim: Good–Walker and triple-pomeron describe the same dynamics

In both approaches: gap events **analogous to diffraction in optics**

Good–Walker: No extra tunable parameter

## Diffractive final states:

**Quantum effect: Interfering contributions to the amplitude**

Still possible to calculate in the dipole cascade formalism

Preliminary results for diffractive final states in DIS

*pp* coll. coming soon

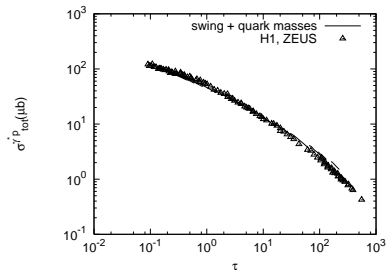
# Extra slides

# DIS, Inclusive cross sections

$\gamma^* p$  total cross section

Satisfies geometric scaling.

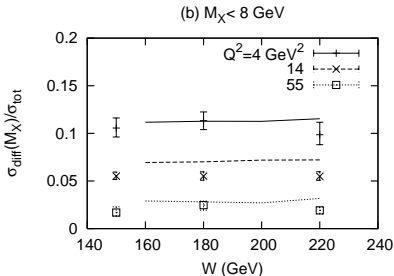
$$\tau = Q^2/Q_S^2(x), \quad Q_S^2 \propto x^{-0.3}$$



Diffractive cross section

(Data from ZEUS)

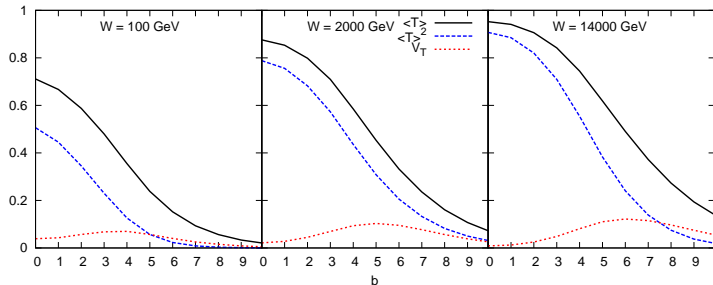
$$M_X < 8 \text{ GeV}, \quad Q^2 = 4, 14, 55 \text{ GeV}^2$$



## Impact parameter profile

Central collisions:  $\langle T \rangle$  large  $\Rightarrow$  Fluctuations small

Peripheral collisions:  $\langle T \rangle$  small  $\Rightarrow$  Fluctuations small

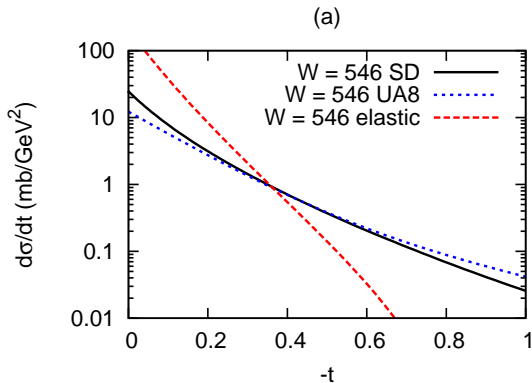


Largest fluctuations when  $\langle F \rangle \sim 1$  and  $\langle T \rangle \sim 0.5$

Circular ring expanding to larger radius at higher energy

# $t$ -dependence

## Single diffractive and elastic cross sections



Agrees with fit to UA8 data



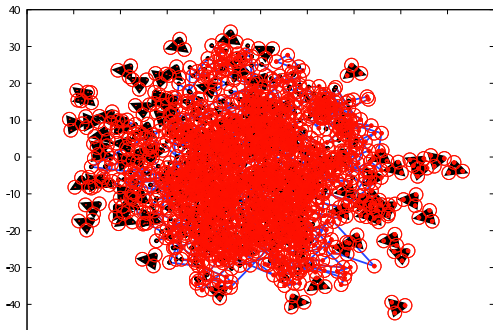
# Nucleus collisions

Gives full partonic picture:

Energy & momentum density  $\sim$  initial conditions in hydro

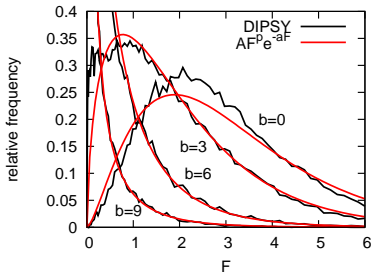
Fluctuations  $\Rightarrow$  e.g.  $v_3$

Ex.: *Pb – Pb* 200 GeV/N

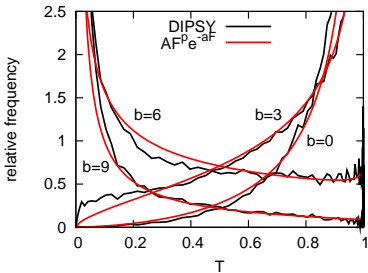


# Prob. distrib. for $pp$ amplitudes

Born ampl.  $F$   $W = 2 \text{ TeV}$



Unitarized ampl.  $T = 1 - e^{-F}$



Born approximation: large fluctuations

$\langle F \rangle$  is large  $\Rightarrow$  Unitarity effects important

$\sim$  enhanced diagrams in triple-regge formalism

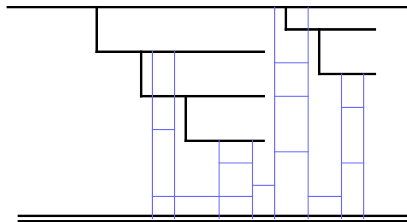
Fluctuations strongly reduced for central collisions

## Exclusive diffractive excitation

- ▶ A single real final state cascade is collided with a large number of virtual cascades.
- ▶ Collide several similar real final states, to calculate fluctuations.
  - ▶ Takes time, but possible to calculate at amplitude level.
- ▶ No extra parameters! Exclusive diffractive excitation predicted from inclusive and non-diffractive minimum bias.

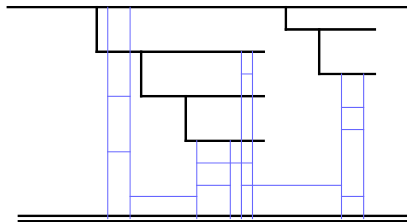
## Exclusive diffractive excitation

- ▶ A single real final state cascade is collided with a large number of virtual cascades.
- ▶ Collide several similar real final states, to calculate fluctuations.
  - ▶ Takes time, but possible to calculate at amplitude level.
- ▶ **No extra parameters!** Exclusive diffractive excitation predicted from inclusive and non-diffractive minimum bias.



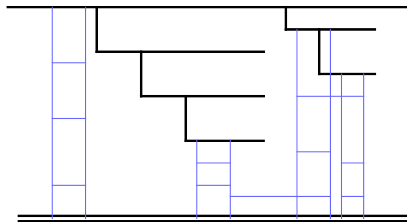
## Exclusive diffractive excitation

- ▶ A single real final state cascade is collided with a large number of virtual cascades.
- ▶ Collide several similar real final states, to calculate fluctuations.
  - ▶ Takes time, but possible to calculate at amplitude level.
- ▶ **No extra parameters!** Exclusive diffractive excitation predicted from inclusive and non-diffractive minimum bias.



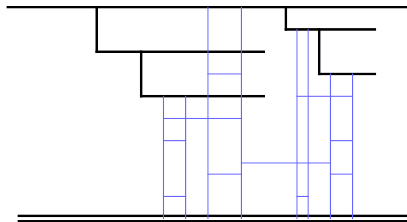
## Exclusive diffractive excitation

- ▶ A single real final state cascade is collided with a large number of virtual cascades.
- ▶ Collide several similar real final states, to calculate fluctuations.
  - ▶ Takes time, but possible to calculate at amplitude level.
- ▶ No extra parameters! Exclusive diffractive excitation predicted from inclusive and non-diffractive minimum bias.



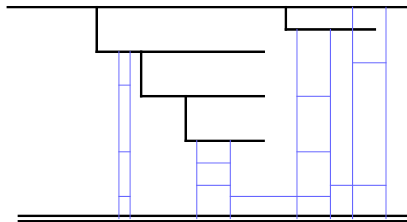
## Exclusive diffractive excitation

- ▶ A single real final state cascade is collided with a large number of virtual cascades.
- ▶ Collide several similar real final states, to calculate fluctuations.
  - ▶ Takes time, but possible to calculate at amplitude level.
- ▶ **No extra parameters!** Exclusive diffractive excitation predicted from inclusive and non-diffractive minimum bias.



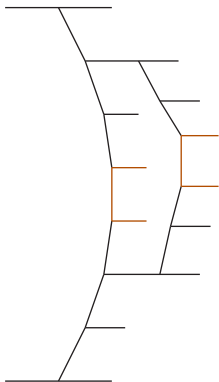
## Exclusive diffractive excitation

- ▶ A single real final state cascade is collided with a large number of virtual cascades.
- ▶ Collide several similar real final states, to calculate fluctuations.
  - ▶ Takes time, but possible to calculate at amplitude level.
- ▶ No extra parameters! Exclusive diffractive excitation predicted from inclusive and non-diffractive minimum bias.





Assume: High energy collisions driven by parton-parton subcollisions (à la PYTHIA)



Low  $x$ : BFKL evolution

High  $p_{\perp}$  also within evolution

Multiple int.  $\Rightarrow$  saturation

Pomeron loops

MC  $\rightarrow$  Fluctuations

Diffractive excit., Correlations

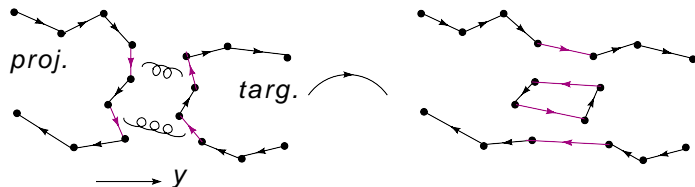
$e$ ,  $p$ , and  $A$  collisions

## The Lund model includes also nonleading effects in the evolution

- ▶ Energy conservation ( $\sim$  non-sing. terms in  $P(z)$ )  
small dipole — high  $p_{\perp} \sim 1/r$   
Cascade ordered in  $p_{+}$   
 $\Rightarrow$  small dipoles suppressed for small  $\delta y$
- ▶ “Energy scale terms”  $\sim$  “consistency constraint”  
 $\Rightarrow$  Cascade ordered in  $p_{-}$   
A single chain is left-right symmetric
- ▶ Running  $\alpha_S$

# Saturation

Multiple interactions  $\Rightarrow$  colour loops



Multiple interaction in one frame

$\Rightarrow$  colour loop within evolution in another frame

## Colour swing

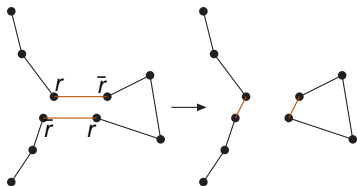
Gluon emission  $\sim \bar{\alpha} = \frac{N_C}{\pi} \alpha_s$

Gluon scattering  $\sim \alpha_s^2$ . Color suppressed

$\Rightarrow$  Loop formation color suppressed. Related to identical colors.

Two dipoles with same colour form a quadrupole

May be better described by recoupled smaller dipoles



Weight favouring small dipoles  $\Rightarrow$  near frame indep. result